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Joint State and Fault Estimation of Complex Networks under Measurement Saturations and Stochastic Nonlinearities

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***Abstract*—In this paper, the joint state and fault estimation problem is investigated for a class of discrete-time complex networks with measurement saturations and stochastic nonlinearities. The difference between the actual measurement and the saturated measurement is regarded as an unknown input and the system is thus re-organized as a singular system. An appropriate estimator is designed for each node which aims to estimate the system states and the loss of the actuator effectiveness simultaneously. In the presence of measurement saturations and stochastic nonlinearities, upper bounds of the error covariances of the fault estimates are recursively obtained and then minimized. Sufficient conditions are proposed to guarantee the existence, unbiasedness, and boundeness of the developed estimator. Our developed estimator design algorithm is distributed because it depends only on the local information and the information from the neighboring nodes, thereby avoiding the usage of a center estimator. Finally, simulation results are presented to show the performance of the proposed strategy in simultaneously estimating the states and faults.**

***Index Terms*—Fault estimation; unbiased estimation; complex network; measurement saturation; stochastic nonlinearity.**

I. Introduction

The past few decades have witnessed ever-increasing demands on system safety and reliability owing to the growing complexity of modern processes. The fault diagnosis (FD) has proven to be an attractive yet promising research direction in engineering practice. So far, a model-based FD framework has been established that consists of two steps: 1) a residual signal is employed to detect the possible faults in the underlying system, and 2) the residual generator is then designed to guarantee that the desired residual is robust against disturbances and sensitive to the faults via parity space method [52], H_{-}/H_{∞} observer [10] and/or some other techniques. Further research has been carried out to determine the location and the amplitude of the faults, that is, isolate and estimate the faults by properly exploiting the system inputs/outputs. A great number of results have been reported in the literature on the general fault diagnosis, isolation and estimation problems, see e.g. [18], [24], [28], [34], [50], [51].

In real-world systems, due mainly to physical and technical constraints, sensors cannot generate signals whose amplitudes are unlimited, and the resulting saturation phenomena are typically described by nonlinearities which, in turn, pose extra challenges to the analysis/synthesis problems of the overall systems. Up to now, the control/estimation/FD problems subject to saturations have received much research attention [1]–[3], [38]. The FD problem has been investigated in electric power systems with current transformers [16], [33], where the three-phase currents at the sending end and receiving end may be saturated. Furthermore, some appealing algorithms have been developed in [19] to simultaneously estimate the system states and reconstruct the additive faults. In the deterministic case, the sector-bounded conditions have been widely employed to guarantee that the saturation-induced nonlinearities are bounded by some linear terms, and then the linear matrix inequality technique can be adopted to solve certain optimization problems. In the stochastic case, the saturation levels have been employed to calculate the upper bounds of the error covariances and the estimator parameters have been selected to minimize these bounds in the minimum variance sense.

It is noted that, when the sensor saturation levels are significantly out of the normal range, it appears inappropriate to directly use these levels to determine the bounds of the estimation error covariances as this would inevitably introduce severe conservatism and subsequently degrade the FD performances. Intuitively, if the difference between the actual value and the saturated value can be estimated and then compensated in the FD unit, the accuracy of the FD results can be much improved. Unfortunately, this is a rather challenging task since the dynamics of such a difference is dependent on the unavailable system states, and therefore cannot be accurately obtained.

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Also, calculating the fault estimation error covariances is a non-trivial task when taking into account the differences between the actual and the saturated measurements, and this is further complicated by the need of characterizing the FD unit in the minimum variance framework. In view of these identified challenges, we are motivated to investigate the FD problem for systems with saturation phenomena by developing a novel yet efficient estimation strategy.

Complex networks have been an attractive research topic because their successful applications in a variety of practical systems [17]. In the analysis of complex networks, both the dynamics of individual nodes and the coupling configuration between different nodes should be simultaneously taken into account. Until now, much research attention has been devoted to the synchronization [6], [42] and state estimation [12], [36] problems of complex networks. The FD problem for complex networks has mostly been investigated in the robust framework [21], [32], [46], [49] where the worst-case performance of the disturbance attenuation has been considered. It is noted that many of the above-mentioned results have used the information from *all* the nodes and, in this case, a center system would be required to collect information from all the nodes. This is certainly inconvenient for distributed execution (as preferred in the setting of complex networks) of the developed algorithms. In fact, it is much desirable in practice that each node in a complex network can realize FD with only *local* information and the information from its neighboring nodes because, in doing so, the transmission burden can be greatly reduced and the real-time FD performance can be improved as well.

The distributed and decentralized FD problem has stirred some research attention. In [4], [5], the distributed fault detection problem has been studied for a class of complex networks by resorting to the concept of Plug-and-Play [37], [41], where Schur stability of the estimation and detection strategy has been discussed in details as well. In the presence of bounded disturbances and unmodeled dynamics, [23], [35] have analyzed the FD and fault-tolerant control problems in distributed systems, where the conditions guaranteeing the detection and isolation of faults have been quantitatively established. The fault estimation and fault-tolerant control issues have been considered in distributed multi-agent systems in [26], [27] via sliding-mode and hierarchical-structure approaches, respectively. Note that the saturation phenomenon has not been considered in the distributed estimation methodologies proposed in [30], [31]. Furthermore, almost all the existing results concerning fault estimation problems have been obtained based on the assumption that certain knowledge about the fault dynamics is known *a priori*, but such an assumption is not always realistic since the positions/amplitudes of actual faults are usually unavailable. Hence, the main motivation of this work is to estimate the faults (without known dynamics) for complex networks with measurement saturations.

Apart from the measurement saturation, another frequently encountered phenomenon in the complex networks is the nonlinearities. To date, the analysis/synthesis problems of nonlinear systems with distributed structures have been extensively studied in [8], [14], [25], [47], where the addressed nonlinear functions have mostly been handled under some linear constraints such as sector-bounded conditions and Lipschitz conditions. Nonetheless, the FD-related results have been quite scarce for nonlinear complex networks. Furthermore, in practice, certain nonlinearities may occur in a random manner because of imperfect data transmissions, variations of the working points, changes of the operation environment, etc [40]. Systems with stochastic nonlinearities have recently stirred some research interests, see [13] and the references therein. The FD problem has been addressed for practical centralized systems with stochastic nonlinearities such as rotating machinery [15] and buck converter [45]. Unfortunately, it remains challenging as how to address the distributed FD problem for complex networks with both measurement saturations and stochastic nonlinearities (for which the substantial challenges may result from the modeling complexity and the algorithm feasibility), and this provides us with another motivation for shortening such a gap.

Based on the above discussion, the aim of this article is to investigate the distributed FD problem for a class of discrete complex networks with measurement saturations and stochastic nonlinearities. Some components of outputs are subject to saturation phenomenon at each node. The difference between the actual measurement and the saturated measurement is estimated and then used to jointly estimate the system states and the fault at each node. An upper bound of the estimation error covariance is obtained and subsequently minimized by appropriately designing the estimator parameters via solving recursive matrix equations. The unbiasedness and existence conditions are explicitly presented for the developed estimator. It is noticeable that the developed algorithm can be implemented in a truly *distributed* way since only local information and information from the neighboring nodes are utilized. Finally, some simulation examples are provided to demonstrate the validity of the proposed strategy.

The main contributions of the paper are highlighted as follows: 1) a novel idea of estimating and then compensating the difference induced by the measurement saturation is proposed for the first time, which proves to help achieve accurate FD results; 2) a new estimator is established that jointly estimates the system states and faults by minimizing an upper bound of the estimation error covariance for each node; and 3) the proposed algorithm is distributed since only the information from the local node itself and the neighboring nodes is adopted.

Notations. The notation used in the paper is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The variables A^T and A^{-1} denote the transpose and inverse of matrix A, respectively. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). I is the identity matrix with compatible dimension. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\mathrm{tr}\{A\}$ stands for the trace of a square matrix A. $\mathrm{sign}(\cdot)$ denotes the signum function. When a variable has more than one subscript, the first one denotes the node identifier, and the last one corresponds to the time step.

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II. Problem Formulation

Consider the following class of discrete-time complex networks with N nodes:

$$\begin{cases} x_{i,s+1} = A_{i,s}x_{i,s} + g_{i,s}(x_{i,s},\alpha_{i,s}) + B_{i,s}\Upsilon_{i,s}u_{i,s} \\ \qquad\qquad + \sum_{j=1}^{N} a_{ij,s}\Gamma x_{j,s} + w_{i,s}, \\ y_{i,1,s} = \hat{C}_{i,1,s}x_{i,1,s} + \hat{C}_{i,2,s}x_{i,2,s} + v_{i,1,s}, \\ y_{i,2,s} = \sigma^{[i]}\left(C_{i,2,s}x_{i,2,s} + v_{i,2,s}\right), \\ u_{i,s} = Q^{[P]}_{i,s}y_{i,s} + Q^{[I]}_{i,s}\sum_{l=1}^{M} y_{i,s-l}, \end{cases} \tag{1}$$

where the subscript s denotes the time step, $x_{i,s} \in \mathbb{R}^n$, $u_{i,s} \in \mathbb{R}^l$ and $y_{i,s} \in \mathbb{R}^m$ are the system state, control input and measurement output of the ith node, respectively. $x_{i,s}$ and $y_{i,s}$ are partitioned as $x_{i,s} = [x^T_{i,1,s}, x^T_{i,2,s}]^T$, $y_{i,s} = [y^T_{i,1,s}, y^T_{i,2,s}]^T$, where $y_{i,1,s} \in \mathbb{R}^{m_1}$ is those measurement components which are free of saturations, and $y_{i,2,s} \in \mathbb{R}^{m_2}$ corresponds to elements subject to saturations. $w_{i,s} \in \mathbb{R}^n$ and $v_{i,s} = [v^T_{i,1,s}, v^T_{i,2,s}]^T \in \mathbb{R}^m$ are the process noise and the measurement noise of the ith node, respectively, which are mutually uncorrelated zero-mean sequences with $\mathbb{E}\{w_{i,s}w^T_{i,s}\} = W_{i,s}$ and $\mathbb{E}\{v_{i,s}v^T_{i,s}\} = V_{i,s}$. The matrices $A_{i,s}$, $B_{i,s}$, $\hat{C}_{i,1,s}$, $\hat{C}_{i,2,s}$, $C_{i,2,s}$, $Q^{[P]}_{i,s}$ and $Q^{[I]}_{i,s}$ are known with appropriate dimensions. The superscripts [P] and [I] mean that $Q^{[P]}_{i,s}$ and $Q^{[I]}_{i,s}$ are the proportional and integral control gains, respectively. $\Gamma = \mathrm{diag}\{r_1,\dots,r_n\}$ is the inner-coupling matrix. $\Upsilon_{i,s} = \mathrm{diag}\{\gamma_{i,1,s},\dots,\gamma_{i,l,s}\}$ represents the possible loss of control effectiveness. $\mathcal{A}_s = [a_{ij,s}]_{N\times N}$ is the coupling configuration matrix of the given complex network with $a_{ij,s} \geq 0$ for $i \neq j$. The diffusive coupling condition $a_{ii,s} = -\sum_{j=1,j\neq i}^{N} a_{ij,s}$ holds for every node [44].

The variable $\alpha_{i,s}$ is random and zero-mean. Furthermore, $g_{i,s}(x_{i,s},\alpha_{i,s})$ satisfies the following conditions:

$$\mathbb{E}\{g_{i,s}(x_{i,s},\alpha_{i,s})|x_{i,s}\} = 0, \tag{2}$$

$$\mathbb{E}\{g_{i,s}(x_{i,s},\alpha_{i,s})g^T_{j,h}(x_{j,h},\alpha_{j,h})|x_{i,s},x_{j,h}\} = 0, \quad \text{if} \quad i\neq j \ \text{ or } \ s\neq h \tag{3}$$

$$\mathbb{E}\{g_{i,s}(x_{i,s},\alpha_{i,s})g^T_{i,s}(x_{i,s},\alpha_{i,s})|x_{i,s}\} = \Theta_{i,s}x^T_{i,s}\Psi_{i,s}x_{i,s}, \tag{4}$$

where $\Theta_{i,s}$ and $\Psi_{i,s}$ are known positive semidefinite matrices with appropriate dimensions. Moreover, $\alpha_{i,s}$ is independent of $w_{i,s}$ or $v_{i,s}$.

For a vector $\rho = [\rho_1,\dots,\rho_{m_2}]^T$, the saturation function of the ith node $\sigma^{[i]} : \mathbb{R}^{m_2} \to \mathbb{R}^{m_2}$ is defined as:

$$\sigma^{[i]}(\rho) = \left[\sigma^{[i]}_1(\rho_1),\dots,\sigma^{[i]}_{m_2}(\rho_{m_2})\right]^T, \tag{5}$$

where $\sigma^{[i]}_j(\rho_j) = \mathrm{sign}(\rho_j)\min\left(b^{[i]}_j, |\rho_j|\right)$ and $b^{[i]}_j \geq 0$ means the saturation level for all $j = 1,\dots,m_2$.

Setting

$$u_{i,s} \triangleq [u_{i,1,s},\dots,u_{i,l,s}]^T,$$
$$U_{i,s} \triangleq \mathrm{diag}\{u_{i,1,s},\dots,u_{i,l,s}\},$$
$$\gamma_{i,s} \triangleq [\gamma_{i,1,s},\dots,\gamma_{i,l,s}]^T,$$

the first equation in (1) can be organized as:

$$x_{i,s+1} = A_{i,s}x_{i,s} + g_{i,s}(x_{i,s},\alpha_{i,s}) + B_{i,s}U_{i,s}\gamma_{i,s} + \sum_{j=1}^{N} a_{ij,s}\Gamma x_{j,s} + w_{i,s}. \tag{6}$$

Defining the measurement error (induced by the saturation phenomenon) as

$$d_{i,s} \triangleq \sigma^{[i]}\left(C_{i,2,s}x_{i,2,s} + v_{i,2,s}\right) - C_{i,2,s}x_{i,2,s} - v_{i,2,s},$$

the third equation in (1) can be written as

$$y_{i,2,s} = C_{i,2,s}x_{i,2,s} + d_{i,s} + v_{i,2,s}. \tag{7}$$

Constructing an augmented state $\bar{x}_{i,s} \triangleq [x^T_{i,s}, d^T_{i,s}]^T$, system (6) and (7) can be rewritten in the following *singular* form:

$$\begin{cases} E\bar{x}_{i,s+1} = \bar{A}_{i,s}\bar{x}_{i,s} + g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s}) + B_{i,s}U_{i,s}\gamma_{i,s} \\ \qquad\qquad + \sum_{j=1}^{N} a_{ij,s}\bar{\Gamma}\bar{x}_{j,s} + w_{i,s}, \\ y_{i,1,s} = \bar{C}_{i,1,s}\bar{x}_{i,s} + v_{i,1,s}, \\ y_{i,2,s} = \bar{C}_{i,2,s}\bar{x}_{i,s} + v_{i,2,s}, \end{cases} \tag{8}$$

where

$$\bar{A}_{i,s} = [A_{i,s}, 0], \quad \bar{\Gamma} = [\Gamma, 0], \quad E = [I, 0],$$
$$\bar{C}_{i,1,s} = \left[\hat{C}_{i,1,s}, \hat{C}_{i,2,s}, 0\right], \quad \bar{C}_{i,2,s} = [0, C_{i,2,s}, I].$$

For system (8), the following state and fault estimator is to be established:

$$\tilde{z}_{i,s+1} = M_{i,s}\hat{x}_{i,s} + \sum_{j=1}^{N} H_{ij,s}\hat{x}_{j,s}, \tag{9}$$

$$\tilde{x}_{i,s+1} = \tilde{z}_{i,s+1} + K_{i,s+1}y_{i,2,s+1}, \tag{10}$$

$$\hat{\gamma}_{i,s} = R_{i,s+1}\left(y_{i,s+1} - \bar{C}_{i,s+1}\tilde{x}_{i,s+1}\right), \tag{11}$$

$$\hat{z}_{i,s+1} = M_{i,s}\hat{z}_{i,s} + J_{i,s}y_{i,2,s} + S_{i,s}\hat{\gamma}_{i,s} + \sum_{j=1}^{N} H_{ij,s}\hat{z}_{j,s} + \sum_{j=1}^{N} L_{ij,s}y_{j,2,s}, \tag{12}$$

$$\hat{x}_{i,s+1} = \hat{z}_{i,s+1} + K_{i,s+1}y_{i,2,s+1}, \tag{13}$$

where $\bar{C}_{i,s} = \left[\bar{C}^T_{i,1,s}, \bar{C}^T_{i,2,s}\right]^T$. $\hat{\gamma}_{i,s} \in \mathbb{R}^l$ and $\hat{x}_{i,s} \in \mathbb{R}^{n+m_2}$ are the estimates of $\gamma_{i,s}$ and $\bar{x}_{i,s}$, respectively. $\tilde{z}_{i,s+1} \in \mathbb{R}^{n+m_2}$ and $\hat{z}_{i,s+1} \in \mathbb{R}^{n+m_2}$ are the estimator states, and $\tilde{x}_{i,s+1} \in \mathbb{R}^{n+m_2}$ is an interim variable to estimate $\gamma_{i,s}$. $M_{i,s}$, $H_{ij,s}$, $K_{i,s+1}$, $R_{i,s+1}$, $J_{i,s}$, $S_{i,s}$ and $L_{ij,s}$ are the parameters to be designed.

Denote $\tilde{e}_{i,s} \triangleq \bar{x}_{i,s} - \tilde{x}_{i,s}$, $e_{i,s} \triangleq \bar{x}_{i,s} - \hat{x}_{i,s}$, $e^{[\gamma]}_{i,s} \triangleq \gamma_{i,s} - \hat{\gamma}_{i,s}$. Our goal is to design an estimator in the form of (9)-(13) for system (8) which is capable of obtaining the unbiased estimates of $\gamma_{i,s}$ and $\bar{x}_{i,s}$ in the presence of the measurement saturations and the interconnections between different nodes. Furthermore, the estimator parameters at every node will be determined with aim to minimize an upper bound of the fault estimation error covariance at each time step. The scheme of each node in the complex network and the distributed estimator is presented in Fig. 1.

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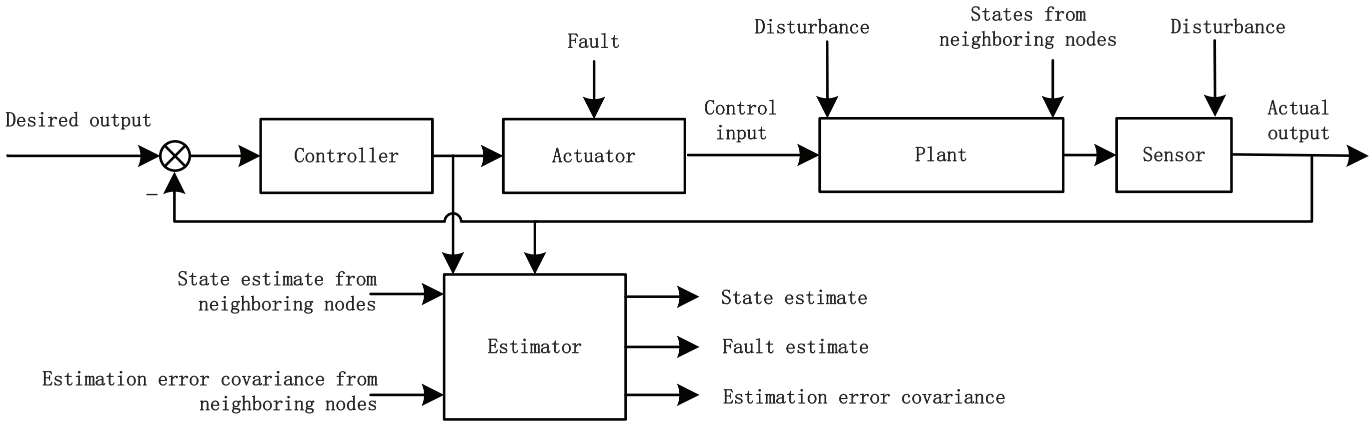

Fig. 1. Scheme of each node and distributed estimator

Remark 1: In this paper, the considered complex network and the estimator are both *time-varying*, which improves the applicability of the method. The diffusive coupling condition stems from the relative state (the difference between system states of different nodes) in the information exchange [44]. Additive plant/actuator faults can also be readily coped with in the proposed framework via replacing $B_{i,s}U_{i,s}$. State-dependent multiplicative noises can be seen as special cases of the considered stochastic nonlinearities. In the proposed estimator (9)-(13), the information on the dynamics of neither $\gamma_{i,s}$ nor $d_{i,s}$ is required, which facilitates the effective handling of the fault (without *priori* knowledge) and the saturation. It will be illustrated later that a group of properly selected parameters can guarantee the unbiasedness of the estimation results of $\gamma_{i,s}$ and $\bar{x}_{i,s}$ under several existence conditions. By introducing the variables $\tilde{x}_{i,s}$ and $\tilde{z}_{i,s}$, the right-hand side of (11) is a linear combination of the fault, stochastic nonlinearity, external noises and estimation error in the previous time step. It will be shown that the coefficient of the fault $\gamma_{i,s}$ can be a unit matrix under certain conditions and an unbiased estimation result of the fault can be obtained correspondingly. Moreover, since it is overly complicated to calculate the accurate estimation error covariance in the presence of the saturations and the stochastic nonlinearities, an upper bound of the fault estimation error covariance will be derived and then minimized via determining the estimator parameters.

Remark 2: In case that the saturated measurement $y_{i,2,s}$ is dependent on all the elements of the system state, the measurement equations in (1) can be obtained after a coordinate transformation. As long as the matrix $C_{i,2,s}$ is of full row rank, there always exists an invertible matrix $S_{i,2,s}$ such that

$$C_{i,2,s}S_{i,2,s} = \left[0, \tilde{C}_{i,2,s}\right], \tag{14}$$

where $\tilde{C}_{i,2,s} \in \mathbb{R}^{m_2 \times m_2}$ is invertible. Considering the coordinate transformation $x^{[t]}_{i,s} = S^{-1}_{i,2,s}x_{i,s}$, the second measurement equation becomes

$$\begin{aligned} y_{i,2,s} =& \sigma^{[i]}\left(C_{i,2,s}S_{i,2,s}x^{[t]}_{i,s} + v_{i,2,s}\right) \\ =& \sigma^{[i]}\left(\left[0, \tilde{C}_{i,2,s}\right]x^{[t]}_{i,s} + v_{i,2,s}\right) \\ =& \sigma^{[i]}\left(\tilde{C}_{i,2,s}x^{[t]}_{i,2,s} + v_{i,2,s}\right), \end{aligned} \tag{15}$$

where $x^{[t]}_{i,s} = \left[\left(x^{[t]}_{i,1,s}\right)^T, \left(x^{[t]}_{i,2,s}\right)^T\right]^T$. Thus, it is evident that the measurement equations can still be written in the form of (1) after the coordinate transformation even if the saturated measurement is related to every element of the original system state.

III. Estimator Design

In this section, the desired state and fault estimator is to be parameterized, and the existence conditions of the estimator will be established as well. Firstly, two matrices $X_{i,s} \in \mathbb{R}^{(n+m_2)\times n}$ and $K_{i,s} \in \mathbb{R}^{(n+m_2)\times m_2}$ are provided to satisfy

$$[X_{i,s}, K_{i,s}]\begin{bmatrix} E \\ \bar{C}_{i,2,s} \end{bmatrix} = I. \tag{16}$$

It follows from the definitions of matrices E and $\bar{C}_{i,2,s}$ that

$$\begin{aligned} \begin{bmatrix} E \\ \bar{C}_{i,2,s} \end{bmatrix}^{-1} &= \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & C_{i,2,s} & I \end{bmatrix}^{-1} \\ &= \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & -C_{i,2,s} & I \end{bmatrix}. \end{aligned} \tag{17}$$

Therefore, the invertibility of the matrix $\begin{bmatrix} E \\ \bar{C}_{i,2,s} \end{bmatrix}$ can be guaranteed. Furthermore, it is obvious that

$$X_{i,s} = \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & -C_{i,2,s} \end{bmatrix}, \quad K_{i,s} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}. \tag{18}$$

The matrices $X_{i,s}$ and $K_{i,s}$ will play important roles in the following design procedure. It can be seen that $K_{i,s}$ is in fact constant, so its subscripts will be omitted in the following calculations. Now, the unbiasedness of the estimator is to be discussed.

Theorem 1: If the condition

$$R_{i,s+1}\bar{C}_{i,s+1}X_{i,s+1}B_{i,s}U_{i,s} = I \tag{19}$$

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holds for every time step, then with an unbiased initial condition $\mathbb{E}\{e_{i,0}\} = 0$ for all the nodes, the state and fault estimation results are both unbiased with $X_{i,s+1}$ and K given by (18), and the other parameters provided as follows:

$$H_{ij,s} = a_{ij,s}X_{i,s+1}\bar{\Gamma}, \tag{20}$$
$$M_{i,s} = X_{i,s+1}\bar{A}_{i,s}, \tag{21}$$
$$S_{i,s} = X_{i,s+1}B_{i,s}U_{i,s}, \tag{22}$$
$$J_{i,s} = M_{i,s}K, \tag{23}$$
$$L_{ij,s} = H_{ij,s}K. \tag{24}$$

Proof: See Appendix A. ■

Remark 3: Based on Theorem 1, with the unbiased initial condition, the unbiased fault and state estimation can be achieved under condition (19). In this unified framework, the original system states, the multiplicative fault and the output error (brought in by the measurement saturation) are all estimated simultaneously. Compared to some existing results where the saturation levels have been used to develop a robust estimator [19], [38], the method proposed in this article directly estimates the saturation-induced error and then compensates it in the state and fault estimation. The consideration of the distributed structure in complex networks constitutes another contribution of our method with respect to [19]. It is noted that, when (19) holds, $R_{i,s+1}$ cannot be determined uniquely and, to deal with this issue, $R_{i,s+1}$ is to be selected in the minimum variance sense.

Set $P_{i,s} \triangleq \mathbb{E}\{e_{i,s}e_{i,s}^T\}$, $P_{i,s}^{[\gamma]} \triangleq \mathbb{E}\left\{e_{i,s}^{[\gamma]}\left(e_{i,s}^{[\gamma]}\right)^T\right\}$. In the following lemma, the state and fault estimation error covariances are provided.

Lemma 1: If (19) holds, then $P_{i,s}^{[\gamma]}$ obeys the following equation:

$$P_{i,s}^{[\gamma]} = R_{i,s+1}Q_{i,s+1}^{[\gamma]}R_{i,s+1}^T, \tag{25}$$

where

$$\begin{aligned} Q_{i,s+1}^{[\gamma]} =& \bar{C}_{i,s+1}X_{i,s+1}R_{i,s+1}^{[\gamma]}X_{i,s+1}^T\bar{C}_{i,s+1}^T + \big(\bar{C}_{i,s+1} \\ &\times KF - I\big)V_{i,s+1}\left(\bar{C}_{i,s+1}KF - I\right)^T, \end{aligned} \tag{26}$$

$$\begin{aligned} R_{i,s+1}^{[\gamma]} =& \left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)P_{i,s}\left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)^T + \big(\bar{A}_{i,s} \\ &+ a_{ii,s}\bar{\Gamma}\big)\sum_{j=1,j\neq i}^{N} a_{ij,s}\mathbb{E}\left\{e_{i,s}e_{j,s}^T\right\}\bar{\Gamma}^T \\ &+ \sum_{j=1,j\neq i}^{N} a_{ij,s}\bar{\Gamma}\mathbb{E}\left\{e_{j,s}e_{i,s}^T\right\}\left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)^T \\ &+ \sum_{j=1,j\neq i}^{N}\sum_{h=1,h\neq i}^{N} a_{ij,s}a_{ih,s}\bar{\Gamma}\mathbb{E}\Big\{e_{j,s}e_{h,s}^T\Big\}\bar{\Gamma}^T \\ &+ \Theta_{i,s}\mathbb{E}\left\{\bar{x}_{i,s}^TE^T\Psi_{i,s}E\bar{x}_{i,s}\right\} + W_{i,s}. \end{aligned} \tag{27}$$

Proof: See Appendix B. ■

We can see that it is quite complicated to directly compute $P_{i,s}^{[\gamma]}$ based on Lemma 1 since there are many cross-covariances between estimation errors of different nodes. Naturally, it is difficult to characterize the estimator in the sense of minimizing the accurate estimation error covariance. To facilitate the estimator design, we will calculate an upper bound of the estimation error covariance and then locally minimize it.

The following lemma is to be used in the subsequent procedures.

Lemma 2: [19] For any two vectors $x, y \in \mathbb{R}^n$, the inequality

$$xy^T + yx^T \leq \varepsilon xx^T + \varepsilon^{-1}yy^T \tag{28}$$

holds where $\varepsilon > 0$ is a constant scalar.

Lemma 2 can be proved easily with the renowned inequality of arithmetic and geometric means. In this paper, this lemma is introduced to deal with the cross-covariances between estimation errors of different nodes.

Theorem 2: Let ε_1 and ε_2 be positive scalars. Assume that $\mathbb{E}\{e_{i,\kappa}\} = 0$ and $P_{i,\kappa} \leq \bar{P}_{i,\kappa}$ hold for every node $i = 1, \ldots, n$ and every time step $\kappa \leq s$. Define the following variable:

$$\bar{P}_{i,s}^{[\gamma]} = R_{i,s+1}\bar{Q}_{i,s+1}^{[\gamma]}R_{i,s+1}^T, \tag{29}$$

where

$$\begin{aligned} \bar{Q}_{i,s+1}^{[\gamma]} =& \bar{C}_{i,s+1}X_{i,s+1}\bar{R}_{i,s+1}^{[\gamma]}X_{i,s+1}^T\bar{C}_{i,s+1}^T + \big(\bar{C}_{i,s+1} \\ &\times KF - I\big)V_{i,s+1}\left(\bar{C}_{i,s+1}KF - I\right)^T, \end{aligned} \tag{30}$$

$$\begin{aligned} \bar{R}_{i,s+1}^{[\gamma]} =& \left(1 + \varepsilon_1\sum_{j=1,j\neq i}^{N} a_{ij,s}\right)\left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)\bar{P}_{i,s} \\ &\times\left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)^T + \left(\varepsilon_1^{-1} + \sum_{j=1,j\neq i}^{N} a_{ij,s}\right) \\ &\times\sum_{j=1,j\neq i}^{N} a_{ij,s}\bar{\Gamma}\bar{P}_{j,s}\bar{\Gamma}^T + \left(1 + \varepsilon_2^{-1}\right)\Theta_{i,s}\mathrm{tr}\{E \\ &\times \hat{x}_{i,s}\hat{x}_{i,s}^TE^T\Psi_{i,s}\} + (1 + \varepsilon_2)\Theta_{i,s}\mathrm{tr}\{E\bar{P}_{i,s}E^T \\ &\times \Psi_{i,s}\} + W_{i,s}. \end{aligned} \tag{31}$$

Then, we have $P_{i,s}^{[\gamma]} \leq \bar{P}_{i,s}^{[\gamma]}$. Moreover, if the following condition is satisfied:

$$\mathrm{rank}\left(\left[\Phi_{i,s+1}^T, \hat{F}^T\right]\right) = \mathrm{rank}\left(\Phi_{i,s+1}^T\right), \tag{32}$$

where

$$\Phi_{i,s+1} = \begin{bmatrix} \Delta_{i,s+1} & \bar{Q}_{i,s+1}^{[\gamma]} \\ 0 & -\Delta_{i,s+1}^T \end{bmatrix}, \tag{33}$$
$$\Delta_{i,s+1} = \bar{C}_{i,s+1}X_{i,s+1}B_{i,s}U_{i,s}, \tag{34}$$
$$\hat{F} = [I, 0], \tag{35}$$

then with

$$R_{i,s+1} = \tilde{F}_{i,s+1}[I, 0]^T, \tag{36}$$
$$\tilde{F}_{i,s+1} = \hat{F}\Phi_{i,s+1}^{\dagger} + \Xi\left(I - \Phi_{i,s+1}\Phi_{i,s+1}^{\dagger}\right), \tag{37}$$

where Ξ is an arbitrary matrix with appropriate dimension, the gain $R_{i,s+1}$ in (36) can minimize $\mathrm{tr}\left\{\bar{P}_{i,s+1}^{[\gamma]}\right\}$ at each time step.

Proof: See Appendix C. ■

Remark 4: Based on Theorem 1, it can be seen that the unbiased fault estimator can be established with the upper bounds of the distributed state estimation error. In the subsequent

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steps, the upper bound will be obtained with the parameters designed in Theorems 1 and 2. $S_{i,s}$ can be updated at each time step when the control input is known. Under condition (32), the parameter in (36) can always guarantee that assumption (19) holds.

Let us examine the rank of $\Phi_{i,s+1}$ in detail. According to the definitions of $\bar{C}_{i,s}$, $X_{i,s}$ and K, $\bar{Q}^{[\gamma]}_{i,s+1}$ can be written as

$$\bar{Q}^{[\gamma]}_{i,s+1} = \begin{bmatrix} \hat{C}_{i,1,s+1} & \hat{C}_{i,2,s+1} \\ 0 & 0 \end{bmatrix} \bar{R}^{[\gamma]}_{i,s+1} \begin{bmatrix} \hat{C}_{i,1,s} & \hat{C}_{i,2,s} \\ 0 & 0 \end{bmatrix}^T + \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} V_{i,s+1} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}. \tag{38}$$

Since $\bar{R}^{[\gamma]}_{i,s+1}$ and $V_{i,s+1}$ are both positive definite matrices according to their definitions, we can write $\bar{Q}^{[\gamma]}_{i,s+1}$ as

$$\bar{Q}^{[\gamma]}_{i,s+1} = \begin{bmatrix} \Pi_{i,s+1} & 0 \\ 0 & 0 \end{bmatrix}, \tag{39}$$

where $\Pi_{i,s+1}$ is a positive definite matrix.

The matrix $\Delta_{i,s+1}$ can be re-organized as

$$\Delta_{i,s+1} = \begin{bmatrix} \hat{C}_{i,1,s+1} & \hat{C}_{i,2,s+1} \\ 0 & 0 \end{bmatrix} B_{i,s} U_{i,s} = \begin{bmatrix} \breve{\Delta}_{i,s+1} \\ 0 \end{bmatrix}, \tag{40}$$

where

$$\breve{\Delta}_{i,s+1} = \left[\hat{C}_{i,1,s+1}, \hat{C}_{i,2,s+1}\right] B_{i,s} U_{i,s}. \tag{41}$$

It follows that

$$\Phi_{i,s+1} = \begin{bmatrix} \breve{\Delta}_{i,s+1} & \Pi_{i,s+1} & 0 \\ 0 & 0 & 0 \\ 0 & -\breve{\Delta}^T_{i,s+1} & 0 \end{bmatrix}, \tag{42}$$

and

$$\left[\Phi^T_{i,s+1}, \hat{F}^T\right] = \begin{bmatrix} \breve{\Delta}^T_{i,s+1} & 0 & 0 & I \\ \Pi_{i,s+1} & 0 & -\breve{\Delta}_{i,s+1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{43}$$

Noticing that $\Pi_{i,s+1}$ is a positive definite matrix of full rank, we have

$$\text{rank}\left(\left[\Phi^T_{i,s+1}, \hat{F}^T\right]\right) = m_1 + l, \tag{44}$$

where m_1 and l are the dimensions of $\Pi_{i,s+1}$ and I, respectively.

According to (42), it can be seen that

$$\text{rank}\left(\Phi_{i,s+1}\right) = \text{rank}\left(\begin{bmatrix} \breve{\Delta}_{i,s+1} & \Pi_{i,s+1} \\ 0 & -\breve{\Delta}^T_{i,s+1} \end{bmatrix}\right). \tag{45}$$

Based on (44) and (45), we can conclude that if

$$\text{rank}\left(\begin{bmatrix} \breve{\Delta}_{i,s+1} & \Pi_{i,s+1} \\ 0 & -\breve{\Delta}^T_{i,s+1} \end{bmatrix}\right) = m_1 + l, \tag{46}$$

then (32) is satisfied and the existence of the desired $R_{i,s+1}$ can be ensured.

Remark 5: In (68), Lemma 2 has been applied with $\varepsilon = 1$ to reduce the conservatism in the calculation of the upper bound. A detailed explanation is given as follows. Select an arbitrary ε, and then (68) becomes

$$\begin{aligned} &\sum_{j=1,j\neq i}^{N} \sum_{h=1,h\neq i}^{N} a_{ij,s} a_{ih,s} \bar{\Gamma} \mathbb{E}\left\{e_{j,s} e^T_{h,s}\right\} \bar{\Gamma}^T \\ =&\frac{1}{2} \sum_{j=1,j\neq i}^{N} \sum_{h=1,h\neq i}^{N} a_{ij,s} a_{ih,s} \bar{\Gamma} \mathbb{E}\left\{e_{j,s} e^T_{h,s} + e_{h,s} e^T_{j,s}\right\} \bar{\Gamma}^T \\ \leq&\frac{1}{2} \sum_{j=1,j\neq i}^{N} \sum_{h=1,h\neq i}^{N} a_{ij,s} a_{ih,s} \bar{\Gamma} \mathbb{E}\left\{\varepsilon e_{j,s} e^T_{j,s} + \frac{1}{\varepsilon} e_{h,s} e^T_{h,s}\right\} \bar{\Gamma}^T \\ =&\frac{1}{2}\left(\varepsilon + \frac{1}{\varepsilon}\right) \sum_{j=1,j\neq i}^{N} a_{ij,s} \sum_{j=1,j\neq i}^{N} a_{ij,s} \bar{\Gamma} \mathbb{E}\left\{e_{j,s} e^T_{j,s}\right\} \bar{\Gamma}^T. \end{aligned}$$

It is obvious that when $\varepsilon = 1$, the coefficient $\frac{1}{2}\left(\varepsilon + \frac{1}{\varepsilon}\right)$ is minimum and its value is 1. A tight upper bound of the equation naturally leads to less conservatism.

Based on Theorem 2, $R_{i,s+1}$ can be obtained with a group of $\bar{P}_{i,s} \geq P_{i,s}$, and these upper bounds of estimation errors are provided in the following theorem.

Theorem 3: Assume that the conditions (19) and (32) hold for every node and every time step, and the estimator is characterized based on Theorems 1 and 2. Consider the following matrix:

$$\begin{aligned} \bar{P}_{i,s+1} =& \left(I - S_{i,s} R_{i,s+1} \bar{C}_{i,s+1}\right) X_{i,s+1} \bar{R}^{[\gamma]}_{i,s+1} X^T_{i,s+1} \big(I \\ &- S_{i,s} R_{i,s+1} \bar{C}_{i,s+1}\big)^T + \big[S_{i,s} R_{i,s+1} \big(\bar{C}_{i,s+1} \\ &\times KF - I\big) - KF\big] V_{i,s+1} \big[S_{i,s} R_{i,s+1} \\ &\times \left(\bar{C}_{i,s+1} KF - I\right) - KF\big]. \end{aligned} \tag{47}$$

Then, $\bar{P}_{i,s}$ is an upper bound of $P_{i,s}$.

Proof: See Appendix D. ■

The overall algorithm is summarized as follows to show the determination of the parameters at each time step.

Algorithm:	
Step 1.	Determine the initial value $\bar{P}_{i,0} \geq P_{i,0}$.
Step 2.	Calculate the $\bar{P}^{[\gamma]}_{i,s}$, $\bar{Q}^{[\gamma]}_{i,s+1}$ and $\bar{R}^{[\gamma]}_{i,s+1}$ based on (29)-(31).
Step 3.	If (32) does not hold, the solution does not exist and stop. Otherwise, compute $R_{i,s+1}$ with (36).
Step 4.	Choose $M_{i,s}$, $H_{ij,s}$, K, $J_{i,s}$, $S_{i,s}$ and $L_{ij,s}$ according to (18) and (20)-(24).
Step 5.	Update $\bar{P}_{i,s+1}$ with (47).
Step 6.	Set $s = s + 1$ and go to *Step 2*.

Remark 6: Theorem 3 has provided a way to calculate an upper bound of the state estimation error covariance. So far, the state and fault estimation problem has been solved in the paper for a class of complex networks subject to sensor saturations and stochastic nonlinearities. The structure of the estimator has been properly selected such that the fault can be decoupled from the state estimation error under condition (19). The system has been written in a singular form, where the saturation error has been integrated into the system state. In

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this way, the covariances of the state/fault estimation errors can be obtained even in the presence of unknown faults and saturation errors. The applicability and feasibility of the established estimator have been greatly enhanced since a center node is not required. The upper bound of the estimation error covariance has been calculated in consideration of the interconnections between different nodes, and the estimator has been parameterized allowing for the sparse structure of the complex network.

Remark 7: In the parameters of the estimator (9)-(13), $M_{i,s}$, $H_{ij,s}$, $K_{i,s}$, $J_{i,s}$, $S_{i,s}$ and $L_{ij,s}$ have been obtained based on (18) and (20)-(24), such that the unbiased fault/state estimates can be achieved. By resorting to (36), $R_{i,s+1}$ has been computed to minimize an upper bound of the fault estimation error under condition (32).

Now let us analyze the stability of the proposed estimation strategy. When $\lim_{s\to\infty} x_{i,s} \to \infty$, it is obvious that $\bar{P}_{i,s}$ is divergent due to the term $g_{i,s}(x_{i,s}, \alpha_{i,s})$, whose second-order moment is a quadratic function of $x_{i,s}$. Next, the dynamics of the estimation error will be discussed in the absence of the stochastic nonlinearity $g_{i,s}(x_{i,s}, \alpha_{i,s})$. Denote the following variables:

$$\begin{aligned}
\mathcal{D}_{1,s} \triangleq& \mathrm{diag}\left\{\left(I - S_{1,s}R_{1,s+1}\bar{C}_{1,s+1}\right)X_{1,s+1}, \ldots,\right.\\
&\left.\left(I - S_{N,s}R_{N,s+1}\bar{C}_{N,s+1}\right)X_{N,s+1}\right\},\\
\mathcal{D}_{2,s} \triangleq& \mathrm{diag}\left\{S_{1,s}R_{1,s+1}\left(\bar{C}_{1,s+1}KF - I\right) - KF, \ldots,\right.\\
&\left.S_{N,s}R_{N,s+1}\left(\bar{C}_{N,s+1}KF - I\right) - KF\right\},\\
\tilde{A}_s \triangleq& \mathrm{diag}\left\{\bar{A}_{1,s} + a_{11,s}\bar{\Gamma}, \ldots, \bar{A}_{N,s} + a_{NN,s}\bar{\Gamma}\right\},\\
\tilde{W}_s \triangleq& \mathrm{diag}\left\{W_{1,s}, \ldots, W_{N,s}\right\}, \ \ \tilde{\Gamma}_1 \triangleq \mathrm{diag}\{\underbrace{\bar{\Gamma}, \ldots, \bar{\Gamma}}_{N}\},\\
\tilde{V}_s \triangleq& \mathrm{diag}\left\{V_{1,s}, \ldots, V_{N,s}\right\}, \ \ \tilde{\Gamma}_2 \triangleq [\underbrace{\bar{\Gamma}, \ldots, \bar{\Gamma}}_{N}],\\
\mathcal{A}_{1,s} \triangleq& \mathrm{diag}\left\{1 + \varepsilon\sum_{j=2}^{N} a_{1j,s}, \ldots, 1 + \varepsilon\sum_{j=1}^{N-1} a_{Nj,s}\right\} \otimes I,\\
\mathcal{A}_{2,s} \triangleq& \mathrm{diag}\left\{\varepsilon^{-1} + \sum_{j=2}^{N} a_{1j,s}, \ldots, \varepsilon^{-1} + \sum_{j=1}^{N-1} a_{Nj,s}\right\} \otimes I,\\
\mathcal{A}_{3,s} \triangleq& [\tilde{a}_{ij,s}]_{N\times N} \otimes I, \ \ \text{where} \ \ \tilde{a}_{ij,s} = \begin{cases} a_{ij,s}, & \text{if} \ \ i \neq j,\\ 0, & \text{if} \ \ i = j. \end{cases}
\end{aligned} \tag{48}$$

With the variables defined above, the following assumption is proposed.

Assumption 1: There are positive real numbers $\bar{w}_i$, $\bar{v}_i$, $\bar{b}_i$, $\bar{u}_i$, $\bar{c}_i$ and $\underline{\tau}_i$, such that the following bounds on various matrices are fulfilled for every $1 \leq i \leq N$ and $s \geq 0$:

$$\begin{aligned}
&\|W_{i,s}\| \leq \bar{w}_i, \ \ \|V_{i,s}\| \leq \bar{v}_i, \ \ \|B_{i,s}\| \leq \bar{b}_i,\\
&\|U_{i,s}\| \leq \bar{u}_i, \ \ \|C_{i,s}\| \leq \bar{c}_i,\\
&\bar{C}_{i,s+1}X_{i,s+1}B_{i,s}U_{i,s}U_{i,s}^T B_{i,s}^T X_{i,s+1}^T \bar{C}_{i,s+1}^T \geq \underline{\tau}_i I,
\end{aligned} \tag{49}$$

and the following inequality holds:

$$\|\mathcal{A}_{1,s}\| \left\|\mathcal{D}_{1,s}\tilde{A}_s\right\|^2 + \left\|\mathcal{A}_{2,s}\mathcal{A}_{3,s}\tilde{\Gamma}_1\right\| \left\|\tilde{\Gamma}_2\right\| < 1. \tag{50}$$

Theorem 4: Consider the complex network (8) (without the stochastic nonlinearity $g_{i,s}(x_{i,s}, \alpha_{i,s})$) with the distributed estimator (9)-(13), whose parameters are determined by resorting to Theorems 2 and 3. If the initial estimation error is bounded and condition (32) holds, then under Assumption 1, the state and fault estimation errors are bounded in mean square, i.e.,

$$\sup_{s\in\mathbb{N}} \sum_{i=1}^{N} \mathbb{E}\left\{e_{i,s}^T e_{i,s}\right\} < \infty, \tag{51}$$

and

$$\sup_{s\in\mathbb{N}} \sum_{i=1}^{N} \mathbb{E}\left\{\left(e_{i,s}^{[\gamma]}\right)^T e_{i,s}^{[\gamma]}\right\} < \infty. \tag{52}$$

Proof: See Appendix E. ■

Remark 8: The main differences between the methods developed in this paper and those in [30], [31] mainly lie in two aspects. 1) The dynamics of the faults has not been required in our current investigation, while faults in [30], [31] have been modeled in a polynomial form (with respect to time step s), which might not be the case in engineering practice. In this paper, to deal with the unavailable fault dynamics, we have provided (19) to guarantee the unbiased fault/state estimations, and then presented the sufficient condition (32) that ensures (19) to hold. 2) The measurement saturation phenomenon has been introduced in this paper, which has brought in extra nonlinearities/errors. To tackle such an extra complexity, we have constructed a singular system (8) accounting for the saturation error, and the corresponding structure of the estimator (9)-(13) has been different from that of the Kalman-like filter in [30], [31].

IV. Illustrations

Let us consider a time-varying complex network with four nodes. When $i \neq j$, $a_{ij,s} = 0.1$ or 0. A group of independent Bernoulli distributed sequences is employed to characterize the dynamic topology of the system. Define $\varrho_{ij} = \mathrm{Prob}(a_{ij,s} = 0.1)(i \neq j)$ with

$$\begin{bmatrix} \varrho_{12} & \varrho_{13} & \varrho_{14}\\ \varrho_{21} & \varrho_{23} & \varrho_{24}\\ \varrho_{31} & \varrho_{32} & \varrho_{34}\\ \varrho_{41} & \varrho_{42} & \varrho_{43} \end{bmatrix} = \begin{bmatrix} 0.25 & 0.35 & 0.45\\ 0.35 & 0.45 & 0.55\\ 0.45 & 0.55 & 0.65\\ 0.55 & 0.65 & 0.75 \end{bmatrix}.$$

The simulation example is inspired by the three-tank system presented in [29], where each node represents a three-tank system. Three-tank system is a typical nonlinear system widely used in control and filtering disciplines. Because of the output signal range of the plate capacitor and the vibration induced by the water inlet, the actual three-tank system is subject to both measurement saturations and stochastic nonlinearities. Set the steady liquid levels to be $h_\star = [0.6813, 0.3321, 0.5534]^T$m and the sampling period $T_s = 1$s. Then, the system model of the system can be obtained as follows along the similar line in [48]:

$$A_{i,s} = \begin{bmatrix} 0.9908 & 0 & 0.0091\\ 0 & 0.9856 & 0.0072\\ 0.0091 & 0.0072 & 0.9836 \end{bmatrix}$$

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$$
\begin{aligned}
&+\sin(s/10)\times\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -0.0001\\ 0 & 0 & 0.0001\end{bmatrix},\\
&B_{i,s}=\begin{bmatrix}64.6627\\ 0.0007\\ 0.2978\end{bmatrix}+\sin(s/10)\times\begin{bmatrix}0\\ 0.0001\\ -0.0001\end{bmatrix},\\
&\hat{C}_{i,1,s}=1,\ \ \hat{C}_{i,2,s}=\begin{bmatrix}0 & 0\end{bmatrix},\ \ C_{i,2,s}=\begin{bmatrix}1 & 0\end{bmatrix},\\
&Q_{i,s}^{[P]}=10^{-3}\times\begin{bmatrix}-31.9 & 9.6\end{bmatrix},\\
&Q_{i,s}^{[I]}=10^{-3}\times\begin{bmatrix}-1.5 & 2.1\end{bmatrix},\ \ \Gamma=0.1I.
\end{aligned}
$$

Based on the values of $\hat{C}_{i,1,s}$, $\hat{C}_{i,2,s}$ and $C_{i,2,s}$, it can be seen that the measurement of Tank 2 is subject to saturation phenomenon where the saturation level is set to be 0.02.

The disturbances $w_{i,s}$ and $v_{i,s}$ are 2×10^{-5} times of unit Gaussian white noises for $i=1,\dots,4$. The parameters ε_1 and ε_2 is determined to be 1.2 and 1.5, respectively. Every element of the initial system state is uniformly distributed over $[-0.02-0.01i, 0.08-0.01i]$ for the ith node $(i=1,\dots,4)$. The stochastic nonlinear function is selected as:

$$
g_{i,s}(x_{i,s},\alpha_{i,s})=\begin{bmatrix}0.4\\ 0.3\\ 0.1\end{bmatrix}\sum_{r=1}^{3}\text{sign}\left(x_{i,s}^{(r)}\right)x_{i,s}^{(r)}\alpha_{i,s}^{(r)},
$$

where $\alpha_{i,s}=\left[\alpha_{i,s}^{(1)},\alpha_{i,s}^{(2)},\alpha_{i,s}^{(3)}\right]^T$ is 5×10^{-3} times of unit Gaussian white noise for $i=1,\dots,4$. It can be readily verified that such a nonlinear function satisfies (2)-(4) with

$$
\Theta_{i,s}=\begin{bmatrix}0.16 & 0.12 & 0.04\\ 0.12 & 0.09 & 0.03\\ 0.04 & 0.03 & 0.01\end{bmatrix},\ \ \Psi_{i,s}=2.5\times10^{-5}I.
$$

The following multiplicative loss of actuator effectiveness is considered for Node 4:

$$
\Upsilon_{4,s}=\begin{cases}1, & \text{if}\quad s\le 40,\\ 1-(s-20)/40, & \text{otherwise.}\end{cases}
$$

A multiplicative ramp fault, which is common in practice due to ubiquitous component degradations, is considered in the simulation. It is noted that other types of faults can be handled with the proposed strategy as well since there is no limitation on the dynamics of the fault in our estimator.

Firstly, the measurement outputs subject to the saturation phenomenon at all the nodes are depicted in Fig. 2. It can be seen that the outputs in Nodes 1 and 4 are saturated at the first 22 and 13 time steps, respectively, when the system states are distant from the equilibrium points in the early stage. By resorting to Theorems 1-3, the estimates of the states and the actuator faults can be obtained at every node. The Euclidean norms of state estimation errors are illustrated in Fig. 3. The state estimation is satisfying in the presence of the saturations and the multiplicative faults. The actual fault at Node 4 and the estimate are both presented in Fig. 4. It is clear that the fault can be estimated well after some unsteady transient steps. Therefore, the proposed method can estimate the state and fault well simultaneously. The condition (32) can be verified in this simulation example by making sure that $u_{i,s}\neq 0$ at each time instant.

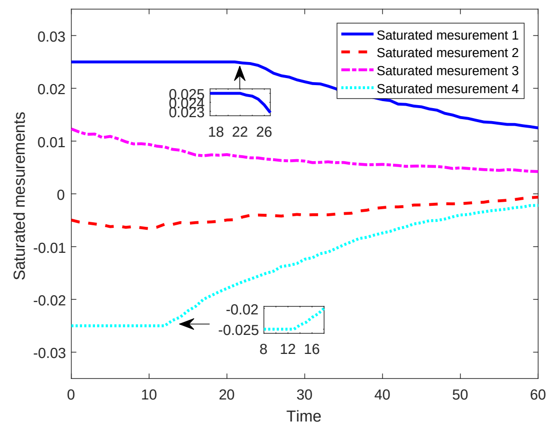

Fig. 2. Saturated measurements

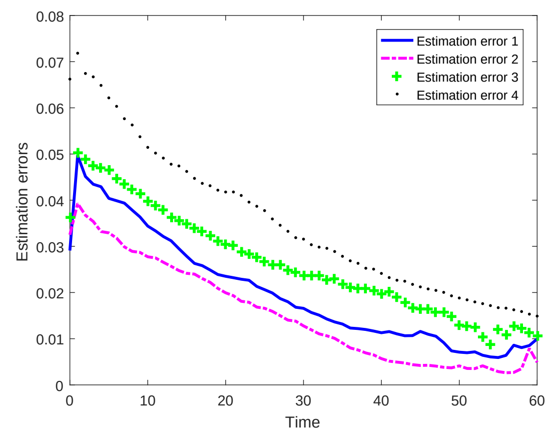

Fig. 3. State estimation errors

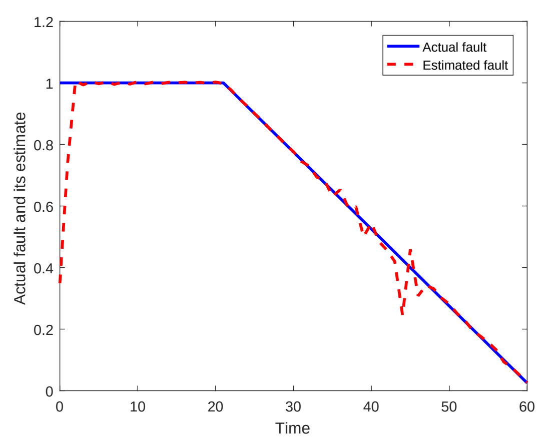

Fig. 4. Actual fault and its estimate

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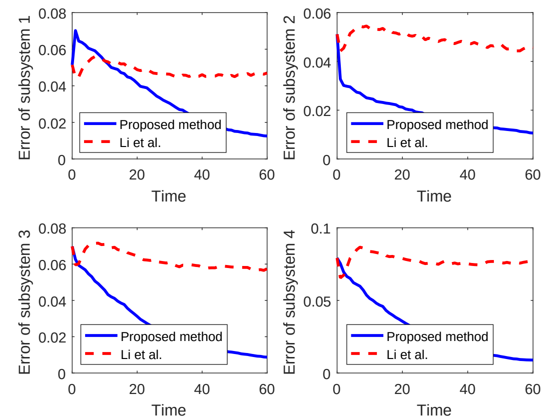

Fig. 5. State estimation error comparisons

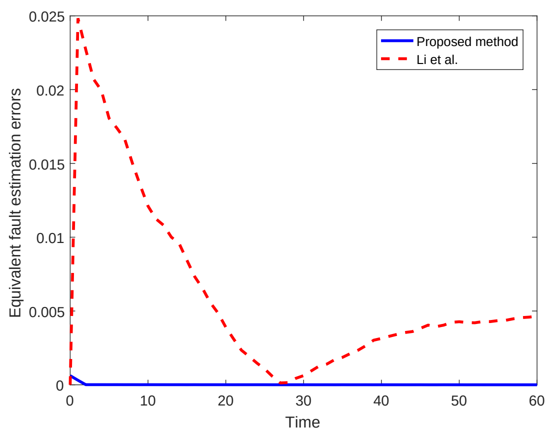

Fig. 6. Equivalent additive fault estimation errors

To further illustrate the estimation performance of our approach, the estimation result of the method proposed in [25] that can cope with distributed systems is compared with that of the developed estimator. The state estimation errors of all the nodes are presented in Fig. 5, and it is transparent that our method can achieve smaller errors at almost every time step. Moreover, the equivalent additive fault (i.e., $u_{i,s} - \Upsilon_{i,s} u_{i,s}$ in our problem formulation) estimation errors are compared in Fig. 6 because the method in [25] is inapplicable to deal with the multiplicative parameters. Our estimator can realize more accurate fault estimation because the saturation effects are compensated in the framework, and such a strategy leads to less conservatism.

To show the applicability of the provided method, the unplugging and plug-in time of each node is illustrated in Table I. Due to the space limitation, the unplugging and plug-in time only in the first 20 time steps are presented. It can be seen that each node is plugged-in and unplugged at least once, and our method can still achieve the satisfying estimation results. Therefore, it can be asserted that the developed method is suitable for complex networks over dynamic topology.

To illustrate the performance of the established estimation

TABLE I
PLUG-IN AND UNPLUGGING OF EACH NODE IN 0-20 TIME STEPS

Node	Unplugging time	Plug-in time
1	4	6
	10	11
	12	13
	18	19
2	15	17
	19	20
3	18	19
4	9	10

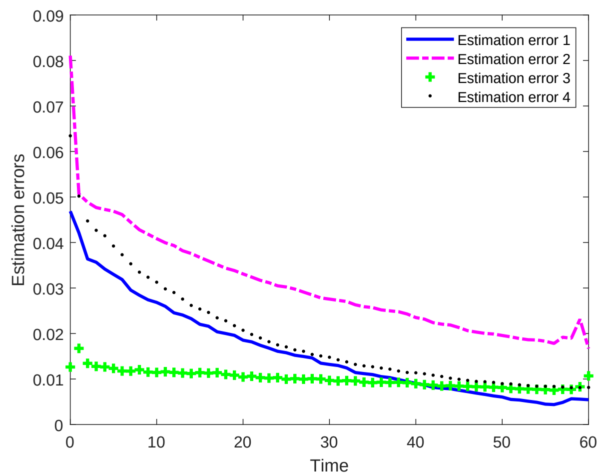

Fig. 7. State estimation errors in the unstable case

strategy in an unstable system, we reset $Q_{i,s}^{[P]}$ as

$$Q_{i,s}^{[P]} = 10^{-2} \times \begin{bmatrix} -4.35 & 1.30 \end{bmatrix}.$$

The state and fault estimation results are depicted in Fig. 7 and Fig. 8, respectively. It is clear that the state/fault can be tracked well in an unstable complex network.

V. Conclusion

The joint state and fault estimation problem has been studied in the paper for a class of complex networks with measurement saturations and stochastic nonlinearities. The difference between the actual measurement and the saturated measurement has been formulated as an unknown input. An augmented state composed of the original system state and the unknown input has been constructed and the system has been written in a singular structure. To cater for the singular form, an estimator in the proper structure has been put forward at each node with only locally available information. The parameters have been obtained ensuring the unbiasedness of the estimation results and the minimization of the upper bounds of the fault estimation error covariances. Sufficient conditions have been established which can guarantee the existence, unbiasedness, and boundedness of the desired estimator. Some simulation examples have been demonstrated to show the effectiveness of the proposed algorithm. Further research topics would be

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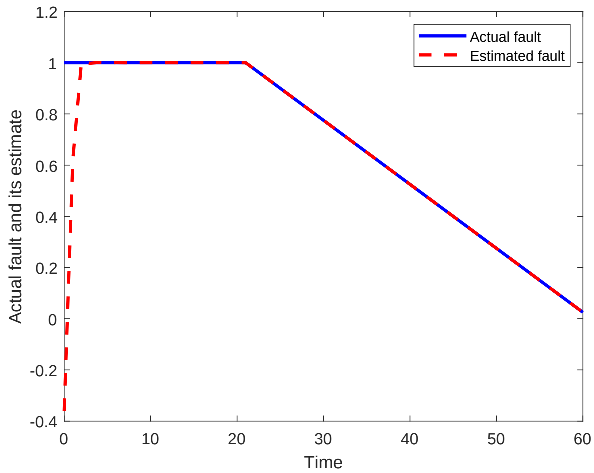

Fig. 8. Actual fault and its estimate in the unstable case

the extension of the main results of this paper to more complex systems with more network-induced phenomena [7], [9], [11], [20], [22], [39], [43], [53].

Appendix A
Proof of Theorem 1

The theorem can be proved by induction. Considering the initial condition $\mathbb{E}\{e_{i,0}\}=0$, we can assume $\mathbb{E}\{e_{i,\kappa}\}=0$ for all the nodes and every $\kappa \le s$, and it remains to show that $\mathbb{E}\left\{e_{i,s}^{[\gamma]}\right\}=0$ and $\mathbb{E}\{e_{i,s+1}\}=0$. The unbiasedness of the fault estimation is to be dealt with first.

From (10), it follows that

$$\begin{aligned}\tilde{e}_{i,s+1}=&\bar{x}_{i,s+1}-\tilde{x}_{i,s+1}\\ =&\left(I-K\bar{C}_{i,2,s+1}\right)\bar{x}_{i,s+1}-\tilde{z}_{i,s+1}-Kv_{i,2,s+1}.\end{aligned} \tag{53}$$

Considering (8), (9) and (16), we have

$$\begin{aligned}\tilde{e}_{i,s+1}=&X_{i,s+1}\Bigg[\bar{A}_{i,s}\bar{x}_{i,s}+g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s})+B_{i,s}U_{i,s}\gamma_{i,s}\\ &+\sum_{j=1}^{N}a_{ij,s}\bar{\Gamma}\bar{x}_{j,s}+w_{i,s}\Bigg]-\sum_{j=1}^{N}H_{ij,s}\hat{x}_{j,s}-M_{i,s}\hat{x}_{i,s}\\ &-Kv_{i,2,s+1}.\end{aligned} \tag{54}$$

Substituting (20) and (21) into (54) yields

$$\begin{aligned}\tilde{e}_{i,s+1}=&M_{i,s}e_{i,s}+X_{i,s+1}g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s})+X_{i,s+1}B_{i,s}U_{i,s}\\ &\times\gamma_{i,s}+\sum_{j=1}^{N}H_{ij,s}e_{j,s}+X_{i,s+1}w_{i,s}-Kv_{i,2,s+1}.\end{aligned} \tag{55}$$

Now, let us calculate the fault estimation error. Based on (11) and (55), we have

$$\begin{aligned}e_{i,s}^{[\gamma]}=&\gamma_{i,s}-R_{i,s+1}\left(y_{i,s+1}-\bar{C}_{i,s+1}\tilde{x}_{i,s+1}\right)\\ =&\gamma_{i,s}-R_{i,s+1}\left(\bar{C}_{i,s+1}\tilde{e}_{i,s+1}+v_{i,s+1}\right)\\ =&\left(I-R_{i,s+1}\bar{C}_{i,s+1}X_{i,s+1}B_{i,s}U_{i,s}\right)\gamma_{i,s}-R_{i,s+1}\\ &\times\bar{C}_{i,s+1}\eta_{i,s+1}+R_{i,s+1}\left(\bar{C}_{i,s+1}KF-I\right)v_{i,s+1},\end{aligned} \tag{56}$$

where

$$\begin{aligned}\eta_{i,s+1}=&M_{i,s}e_{i,s}+X_{i,s+1}g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s})\\ &+\sum_{j=1}^{N}H_{ij,s}e_{j,s}+X_{i,s+1}w_{i,s},\end{aligned} \tag{57}$$

$$F=[0,I]. \tag{58}$$

According to the assumptions that $\mathbb{E}\{e_{i,\kappa}\}=0$ for every $\kappa\le s$ and that the noises are zero-mean, it can be seen that $\mathbb{E}\left\{e_{i,s}^{[\gamma]}\right\}=0$ if (19) holds. Therefore, the fault estimate is unbiased and we have

$$\begin{aligned}e_{i,s}^{[\gamma]}=&-R_{i,s+1}\bar{C}_{i,s+1}\eta_{i,s+1}+R_{i,s+1}(\bar{C}_{i,s+1}KF\\ &-I)v_{i,s+1}.\end{aligned} \tag{59}$$

Next, the unbiasedness of the state estimate is to be proved. According to (13), we have

$$\begin{aligned}e_{i,s+1}=&\bar{x}_{i,s+1}-\hat{x}_{i,s+1}\\ =&\left(I-K\bar{C}_{i,2,s+1}\right)\bar{x}_{i,s+1}-\hat{z}_{i,s+1}-Kv_{i,2,s+1}.\end{aligned} \tag{60}$$

From (8), (12) and (16), it follows that

$$\begin{aligned}e_{i,s+1}=&X_{i,s+1}\Bigg(\bar{A}_{i,s}\bar{x}_{i,s}+g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s})+B_{i,s}U_{i,s}\gamma_{i,s}\\ &+\sum_{j=1}^{N}a_{ij,s}\bar{\Gamma}\bar{x}_{j,s}+w_{i,s}\Bigg)-M_{i,s}\hat{z}_{i,s}-J_{i,s}y_{i,2,s}\\ &-S_{i,s}\hat{\gamma}_{i,s}-\sum_{j=1}^{N}H_{ij,s}\hat{z}_{j,s}-\sum_{j=1}^{N}L_{ij,s}y_{j,2,s}\\ &-Kv_{i,2,s+1}.\end{aligned} \tag{61}$$

According to (20)-(22), we have

$$\begin{aligned}e_{i,s+1}=&M_{i,s}\bar{x}_{i,s}+X_{i,s+1}g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s})+S_{i,s}e_{i,s}^{[\gamma]}\\ &+\sum_{j=1}^{N}H_{ij,s}\bar{x}_{j,s}+X_{i,s+1}w_{i,s}-M_{i,s}\hat{z}_{i,s}\\ &-J_{i,s}y_{i,2,s}-\sum_{j=1}^{N}H_{ij,s}\hat{z}_{j,s}-\sum_{j=1}^{N}L_{ij,s}y_{j,2,s}\\ &-Kv_{i,2,s+1}.\end{aligned} \tag{62}$$

Based on (13), (59) and the definition of $e_{i,s}$, we have

$$\begin{aligned}e_{i,s+1}=&M_{i,s}\bar{x}_{i,s}+X_{i,s+1}g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s})-S_{i,s}R_{i,s+1}\\ &\times\bar{C}_{i,s+1}\eta_{i,s+1}+S_{i,s}R_{i,s+1}(\bar{C}_{i,s+1}KF-I)v_{i,s+1}\\ &+\sum_{j=1}^{N}H_{ij,s}\bar{x}_{j,s}+X_{i,s+1}w_{i,s}-M_{i,s}\big(\bar{x}_{i,s}-e_{i,s}\\ &-Ky_{i,2,s}\big)-J_{i,s}y_{i,2,s}-\sum_{j=1}^{N}H_{ij,s}\big(\bar{x}_{j,s}-e_{j,s}\\ &-K_{j,s}y_{j,2,s}\big)-\sum_{j=1}^{N}L_{ij,s}y_{j,2,s}-Kv_{i,2,s+1}.\end{aligned} \tag{63}$$

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It follows from (23) and (24) that

$$\begin{aligned} e_{i,s+1} =& M_{i,s}e_{i,s} + X_{i,s+1}g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s}) - S_{i,s}R_{i,s+1} \\ &\times \bar{C}_{i,s+1}\eta_{i,s+1} + S_{i,s}R_{i,s+1}(\bar{C}_{i,s+1}KF - I)v_{i,s+1} \\ &+ \sum_{j=1}^{N} H_{ij,s}e_{j,s} + X_{i,s+1}w_{i,s} - KFv_{i,s+1}. \end{aligned} \quad (64)$$

Again, since it has been assumed that $\mathbb{E}\{e_{i,\kappa}\}=0$ for every $\kappa \le s$, it can be concluded that $\mathbb{E}\{e_{i,s+1}\}=0$. The proof is complete.

Appendix B
Proof of Lemma 1

Considering (20) and (21), $\eta_{i,s+1}$ in (57) can be written as

$$\eta_{i,s+1} = X_{i,s+1}\hat{\eta}_{i,s+1}, \quad (65)$$

where

$$\hat{\eta}_{i,s+1} = \bar{A}_{i,s}e_{i,s} + g_{i,s}(E\bar{x}_{i,s},\alpha_{i,s}) + \sum_{j=1}^{N} a_{ij,s}\bar{\Gamma}e_{j,s} + w_{i,s}. \quad (66)$$

It is obvious that $R^{[\gamma]}_{i,s+1} = \mathbb{E}\left\{\hat{\eta}_{i,s+1}\hat{\eta}^T_{i,s+1}\right\}$. Then, (25) is easily accessible from (59), where the detailed proof is omitted here.

Appendix C
Proof of Theorem 2

First, let us show that $R^{[\gamma]}_{i,s+1} \le \bar{R}^{[\gamma]}_{i,s+1}$. Based on Lemma 2, we have

$$\begin{aligned} &\left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)\sum_{j=1,j\neq i}^{N} a_{ij,s}\mathbb{E}\left\{e_{i,s}e^T_{j,s}\right\}\bar{\Gamma}^T \\ &+ \sum_{j=1,j\neq i}^{N} a_{ij,s}\bar{\Gamma}\mathbb{E}\left\{e_{j,s}e^T_{i,s}\right\}\left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)^T \\ \le& \sum_{j=1,j\neq i}^{N} a_{ij,s}\Big[\varepsilon_1\left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)\mathbb{E}\left\{e_{i,s}e^T_{i,s}\right\}\big(\bar{A}_{i,s} \\ &+ a_{ii,s}\bar{\Gamma}\big)^T + \varepsilon_1^{-1}\bar{\Gamma}\mathbb{E}\left\{e_{j,s}e^T_{j,s}\right\}\bar{\Gamma}^T\Big], \end{aligned} \quad (67)$$

and

$$\begin{aligned} &\sum_{j=1,j\neq i}^{N}\sum_{h=1,h\neq i}^{N} a_{ij,s}a_{ih,s}\bar{\Gamma}\mathbb{E}\left\{e_{j,s}e^T_{h,s}\right\}\bar{\Gamma}^T \\ =&\frac{1}{2}\sum_{j=1,j\neq i}^{N}\sum_{h=1,h\neq i}^{N} a_{ij,s}a_{ih,s}\bar{\Gamma}\mathbb{E}\left\{e_{j,s}e^T_{h,s} + e_{h,s}e^T_{j,s}\right\}\bar{\Gamma}^T \\ \le&\frac{1}{2}\sum_{j=1,j\neq i}^{N}\sum_{h=1,h\neq i}^{N} a_{ij,s}a_{ih,s}\bar{\Gamma}\mathbb{E}\left\{e_{j,s}e^T_{j,s} + e_{h,s}e^T_{h,s}\right\}\bar{\Gamma}^T \\ =& \sum_{j=1,j\neq i}^{N} a_{ij,s}\sum_{j=1,j\neq i}^{N} a_{ij,s}\bar{\Gamma}\mathbb{E}\left\{e_{j,s}e^T_{j,s}\right\}\bar{\Gamma}^T. \end{aligned} \quad (68)$$

Furthermore, we have

$$\begin{aligned} &\Theta_{i,s}\mathbb{E}\left\{\bar{x}^T_{i,s}E^T\Psi_{i,s}E\bar{x}_{i,s}\right\} \\ =&\Theta_{i,s}\mathrm{tr}\left\{E\mathbb{E}\left\{\left(e_{i,s}+\hat{x}_{i,s}\right)\left(e_{i,s}+\hat{x}_{i,s}\right)^T\right\}E^T\Psi_{i,s}\right\} \\ \le&\Theta_{i,s}\mathrm{tr}\left\{E\left[(1+\varepsilon_2)P_{i,s} + \left(1+\varepsilon_2^{-1}\right)\hat{x}_{i,s}\hat{x}^T_{i,s}\right]E^T\Psi_{i,s}\right\} \\ =&\left(1+\varepsilon_2^{-1}\right)\Theta_{i,s}\mathrm{tr}\left\{E\hat{x}_{i,s}\hat{x}^T_{i,s}E^T\Psi_{i,s}\right\} + (1+\varepsilon_2)\Theta_{i,s} \\ &\times \mathrm{tr}\{EP_{i,s}E^T\Psi_{i,s}\}. \end{aligned} \quad (69)$$

Substituting (67)-(69) into (27) yields

$$\begin{aligned} \bar{R}^{[\gamma]}_{i,s+1} \le& \left(1 + \varepsilon_1\sum_{j=1,j\neq i}^{N} a_{ij,s}\right)\left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)P_{i,s} \\ &\times \left(\bar{A}_{i,s} + a_{ii,s}\bar{\Gamma}\right)^T + \left(\varepsilon_1^{-1} + \sum_{j=1,j\neq i}^{N} a_{ij,s}\right) \\ &\times \sum_{j=1,j\neq i}^{N} a_{ij,s}\bar{\Gamma}P_{j,s}\bar{\Gamma}^T + \left(1+\varepsilon_2^{-1}\right)\Theta_{i,s}\mathrm{tr}\{E \\ &\times \hat{x}_{i,s}\hat{x}^T_{i,s}E^T\Psi_{i,s}\} + (1+\varepsilon_2)\Theta_{i,s}\mathrm{tr}\{EP_{i,s}E^T \\ &\times \Psi_{i,s}\} + W_{i,s}. \end{aligned} \quad (70)$$

Considering the assumption that $P_{i,\kappa} \le \bar{P}_{i,\kappa}$ for all the nodes and every previous time step, we have $R^{[\gamma]}_{i,s+1} \le \bar{R}^{[\gamma]}_{i,s+1}$. Based on (30) and (31), it follows directly that $Q^{[\gamma]}_{i,s} \le \bar{Q}^{[\gamma]}_{i,s}$ and $P^{[\gamma]}_{i,s} \le \bar{P}^{[\gamma]}_{i,s}$, respectively.

Now we have proved that $\bar{P}^{[\gamma]}_{i,s}$ is an upper bound of $P^{[\gamma]}_{i,s}$, and it is to be shown that $R_{i,s+1}$ in (36) can minimize $\mathrm{tr}\left\{\bar{P}^{[\gamma]}_{i,s}\right\}$. According to (30) and the condition (19), the following cost function is to be minimized:

$$\begin{aligned} V^{[\gamma]}_{i,s+1} =& \mathrm{tr}\left\{\bar{P}^{[\gamma]}_{i,s}\right\} - 2\mathrm{tr}\big\{(R_{i,s+1}\bar{C}_{i,s+1}X_{i,s+1}B_{i,s}U_{i,s} \\ &- I)\Lambda^T_{i,s+1}\big\}, \end{aligned} \quad (71)$$

where $\Lambda_{i,s+1}$ is the Lagrange multiplier. Then, we have

$$\frac{\partial V^{[\gamma]}_{i,s+1}}{\partial R_{i,s+1}} = 2R_{i,s+1}\bar{Q}^{[\gamma]}_{i,s+1} - 2\Lambda_{i,s+1}U^T_{i,s}B^T_{i,s}X^T_{i,s+1}\bar{C}^T_{i,s+1}. \quad (72)$$

Considering the constraint (19) and setting

$$\frac{\partial V^{[\gamma]}_{i,s+1}}{\partial R_{i,s+1}} = 0, \quad (73)$$

the following equation can be established:

$$\left[R_{i,s+1},\Lambda_{i,s+1}\right]\Phi_{i,s+1} = \hat{F}, \quad (74)$$

where $\Phi_{i,s+1}$ and $\hat{F}$ are given in (33) and (35), respectively. When (32) holds, it is obvious that (74) is solvable and the solution satisfies

$$\begin{aligned} \left[R_{i,s+1},\Lambda_{i,s+1}\right] =& \hat{F}\Phi^{\dagger}_{i,s+1} + \Xi\left(I - \Phi_{i,s+1}\Phi^{\dagger}_{i,s+1}\right) \\ =& \tilde{F}_{i,s+1}. \end{aligned} \quad (75)$$

Moreover, we have

$$\frac{\partial^2 V^{[\gamma]}_{i,s+1}}{\partial R_{i,s+1}\partial R^T_{i,s+1}} = 2\bar{Q}^{[\gamma]}_{i,s+1} \ge 0. \quad (76)$$

Therefore, $R_{i,s+1}$ in (36) can minimize the cost function $V^{[\gamma]}_{i,s+1}$, and this concludes the proof.

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Appendix D
Proof of Theorem 3

Considering (57) and (64), we have

$$e_{i,s+1} = \left(I - S_{i,s}R_{i,s+1}\bar{C}_{i,s+1}\right)\eta_{i,s+1} + S_{i,s}R_{i,s+1} \times \left(\bar{C}_{i,s+1}KF - I\right)v_{i,s+1} - KFv_{i,s+1}. \quad (77)$$

From (65) and the fact $R_{i,s+1}^{[\gamma]} \le \bar{R}_{i,s+1}^{[\gamma]}$, it follows that

$$\begin{aligned} P_{i,s+1} \le& \left(I - S_{i,s}R_{i,s+1}\bar{C}_{i,s+1}\right)X_{i,s+1}\bar{R}_{i,s+1}^{[\gamma]}X_{i,s+1}^T\big(I \\ &- S_{i,s}R_{i,s+1}\bar{C}_{i,s+1}\big)^T + \big[S_{i,s}R_{i,s+1}\big(\bar{C}_{i,s+1} \\ &\times KF - I\big) - KF\big]V_{i,s+1}\big[S_{i,s}R_{i,s+1} \\ &\times \big(\bar{C}_{i,s+1}KF - I\big) - KF\big] \\ =&\bar{P}_{i,s+1}, \end{aligned} \quad (78)$$

and the proof is now complete.

Appendix E
Proof of Theorem 4

Firstly, we need to investigate the boundedness of $\mathcal{D}_{1,s}$ and $\mathcal{D}_{2,s}$ according to Assumption 1. From (19) and (49), it follows that

$$\|R_{i,s+1}\| \le \frac{1}{\underline{r}_i} \triangleq \bar{r}_i. \quad (79)$$

Based on (18), we have

$$\|X_{i,s+1}\| \le \sqrt{1 + \bar{c}_i^2} \triangleq \bar{x}_i. \quad (80)$$

Substituting (80) and (49) into (22) yields

$$\|S_{i,s}\| \le \bar{x}_i\bar{b}_i\bar{u}_i \triangleq \bar{s}_i. \quad (81)$$

From (79)-(81), it follows that

$$\|\mathcal{D}_{1,s}\| \le \max_{i=1,\dots,N}\left[\left(1 + \bar{s}_i\bar{r}_i\bar{c}_i\right)\bar{x}_i\right] \triangleq \bar{d}_1, \quad (82)$$

and

$$\|\mathcal{D}_{2,s}\| \le \max_{i=1,\dots,N}\left[\bar{s}_i\bar{r}_i\left(\bar{c}_i + 1\right)\right] + 1 \triangleq \bar{d}_2. \quad (83)$$

Now we can analyze the boundedness of the state/fault estimation errors. Denote $\tilde{P}_s \triangleq \operatorname{diag}\left\{\bar{P}_{1,s}, \dots, \bar{P}_{N,s}\right\}$. Considering (47), we have

$$\begin{aligned} \tilde{P}_{s+1} =& \mathcal{A}_{1,s}\mathcal{D}_{1,s}\tilde{A}_s\tilde{P}_s\tilde{A}_s^T\mathcal{D}_{1,s}^T + \mathcal{A}_{2,s}\mathcal{A}_{3,s}\tilde{\Gamma}_1\tilde{P}_s\tilde{\Gamma}_2^T \\ &+ \mathcal{D}_{1,s}\tilde{W}_s\mathcal{D}_{1,s}^T + \mathcal{D}_{2,s}\tilde{V}_s\mathcal{D}_{2,s}^T. \end{aligned} \quad (84)$$

It is readily accessible that

$$\left\|\tilde{P}_{s+1}\right\| \le \rho\left\|\tilde{P}_s\right\| + \bar{\delta}, \quad (85)$$

where

$$\begin{aligned} \rho =& \|\mathcal{A}_{1,s}\|\left\|\mathcal{D}_{1,s}\tilde{A}_s\right\|^2 + \left\|\mathcal{A}_{2,s}\mathcal{A}_{3,s}\tilde{\Gamma}_1\right\|\left\|\tilde{\Gamma}_2\right\|, \\ \bar{\delta} =& \max_{i=1,\dots,N}\bar{d}_1\bar{w}_i + \max_{i=1,\dots,N}\bar{d}_2\bar{v}_i. \end{aligned}$$

According to (49) and (50), we have $\rho < 1$ and $\bar{\delta}$ is uniformly bounded. Naturally, $\left\|\tilde{P}_s\right\|$ is convergent with a bounded initial condition. Since $\operatorname{tr}\left\{\tilde{P}_s\right\}$ has proved to be an upper bound of $\sum_{i=1}^{N}\mathbb{E}\left\{e_{i,s}^Te_{i,s}\right\}$, it can be asserted that the state estimation error is bounded in mean square.

Based on (29)-(31), it can be easily seen that the upper bound of the fault estimation error covariance is convergent if $\left\|\tilde{P}_s\right\|$ is convergent. It follows directly that the fault estimation error is bounded in mean square, and the proof is complete now.

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