Modelling Returns and Volatilities During Financial Crises: a Time Varying Coefficient Approach

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Abstract

We examine how the most prevalent stochastic properties of key financial time series have been affected during the recent financial crises. In particular we focus on changes associated with the remarkable economic events of the last two decades in the mean and volatility dynamics, including the underlying volatility persistence and volatility spillovers structure. Using daily data from several key stock market indices we find that stock market returns exhibit time varying persistence in their corresponding conditional variances. Furthermore, the results of our bivariate GARCH models show the existence of time varying correlations as well as time varying shock and volatility spillovers between the returns of FTSE and DAX, and those of NIKKEI and Hang Seng, which became more prominent during the recent financial crisis. Our theoretical considerations on the time varying model which provides the platform upon which we integrate our multifaceted empirical approaches are also of independent interest. In particular, we provide the general solution for low order time varying specifications, which is a long standing research topic. This enables us to characterize these models by deriving, first, their multistep ahead predictors, second, the first two time varying unconditional moments, and third, their covariance structure.

Keywords: financial crisis, stochastic difference equations, structural breaks, time varying coefficients, volatility spillovers.

JEL Classifications: C53; C58; G15.

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[*]The order of the authors' names reflect their contribution. M. Karanasos and A. Paraskevopoulos are joint first authors, having defined the theoretical and empirical models, and having derived the theoretical results (Sections 2 and 3). F. Menla Ali is second author, having estimated the various univariate and bivariate models in Sections 5 and 6. M. Karoglou, by applying Karoglou (2010) tests, incorporated the break detection procedure into the empirical analysis (Section 5.1). S. Yfanti derived the autocorrelations in Section 3.1.2 and the unconditional variances in Section 3.2.1.

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1 Introduction

The global financial crisis of 2007-08 and the European sovereign-debt crisis that took place immediately afterwards are at the heart of the research interests of practitioners, academics, and policy makers alike. Given the widespread fear of an international systemic financial collapse at the time it is no wonder that the currently on-going heated discussion on the actual causes and effects of these crises is the precursor to the development of the necessary tools and policies for dealing with similar phenomena in the future.

The inevitable step in undertaking such an enormous task is to map, as accurately as possible, the 'impact' of these crises onto what are currently considered the main stochastic properties of the underlying financial time series. In this way, informed discussions on the causes and effects of these crises can take place and thus more accurately specify the set of features that have to characterize the necessary tools and policies to address them. This paper aspires to provide a platform upon which changes in the main statistical properties of financial time series due to economic crises can be measured.

In particular, we focus on the recent financial crises and examine how the mean and volatility dynamics, including the underlying volatility persistence and volatility spillovers structure, have been affected by these crises. With this aim we make use of several modern econometric approaches for univariate and multivariate time series modelling, which we also condition on the possibility of breaks in the mean and/or volatility dynamics taking place. Moreover, we unify these approaches by introducing a set of theoretical considerations for time varying (TV) AR-GARCH models, which are also of independent interest. In particular, we make three broad contributions to the existing literature.

First, we present and utilize some new theoretical results on time varying AR and/or asymmetric GARCH (AGARCH) models. We limit our analysis to low order specifications to save space and also since it is well documented that low order AR models for stock returns often emerge in practice. We show the applicability of these general results to one important case: that of abrupt breaks, which we make particular use of in our empirical investigation. Our models produce time varying unconditional variances in the spirit of Engle and Rangel (2008) and Baillie and Morana (2009). TV-GARCH specifications have recently gained popularity for modelling structural breaks in the volatility process (see, for example, Frijns et al., 2011, and Bauwens, et al., 2014). Despite nearly half a century of research work and the widely recognized importance of time varying models, until recently there was a lack of a general theory that can be employed to explore their time series properties systematically. Granger in some of his last contributions highlighted the importance of the topic (see, Granger 2007, and 2008). The stumbling block to the development of such a theory was the lack of a method that can be used to solve time varying difference equations of order two or higher. Paraskevopoulos et al. (2013) have developed such

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a general theory (see also Paraskevopoulos and Karanasos, 2013). The starting point of the solution method that we present below is to represent the linear time varying difference equation of order two as an infinite system of linear equations. The coefficient matrix of such an infinite system is row finite. The solution to such infinite systems is based on an extension of the classic Gauss elimination, called Infinite Gaussian elimination (see Paraskevopoulos, 2012, 2014). Our method is a natural extension of the first order solution formula. It also includes the linear difference equation with constant coefficients (see, for example, Karanasos, 1998, 2001) as a special case. We simultaneously compute not only the general solution but its homogeneous and particular parts as well. The coefficients in these solutions are expressed as determinants of tridiagonal matrices. This allows us to provide a thorough description of time varying models by deriving, first, multistep ahead forecasts, the associated forecast error and the mean square error, and second, the first two time varying unconditional moments of the process and its covariance structure.

Second, we use a battery of tests to identify the number and estimate the timing of breaks both in the mean and volatility dynamics. Following our theoretical results and prompted by Morana and Beltratti (2004) amongst others who acknowledge that misleading inference on the persistence of the volatility process may be caused by unaccounted structural breaks[1], we implement these break tests in the univariate context also to determine changes in the persistence of volatility. The special attention we pay to this issue is well justified, especially within the finance literature given that it is well-established that the proper detection of breaks is pivotal for a variety of financial applications, particularly in risk measurement, asset allocation and option pricing. Kim and Kon (1999) emphasize the importance of incorporating some break detection procedure into the existing financial modelling paradigms when they call attention to the fact that "*... Public announcements of corporate investment and financial decisions that imply a change in the firm's expected return and risk will be impounded in stock prices immediately in an efficient market. The announcements of relevant macroeconomic information will affect the return and risk of all securities, and hence, portfolios (indexes). Since relevant information that changes the risk structure is randomly released with some time interval (not at every moment) in sequence, these information events translate into sequential discrete structural shifts (or change-points) for the mean and/or variance parameter(s) in the time series of security returns.*"

Third, we employ the bivariate unrestricted extended dynamic conditional correlation (UEDCC) AGARCH process to analyze the volatility transmission structure, applied to stock market returns. The model is based on the dynamic conditional correlation of Engle (2002) allowing for volatility spillovers effects by imposing the unrestricted extended conditional correlation (dynamic or constant) GARCH

[1] A detailed literature review on this issue is available upon request.

specification of Conrad and Karanasos (2010). The most recent applications of the model can be found in Conrad et al. (2010), Rittler (2012), Karanasos and Zeng (2013) and Conrad and Karanasos (2013). However, we extend it by allowing shock and volatility spillovers parameters to shift across abrupt breaks as well as across two regimes of stock returns, positive (increases in the stock market) and negative (declines in the stock market) (see also Karanasos et al., 2013). Recently, following our work, Caporale et al. (2014) adopted our UEDCC framework but they do not allow for breaks in the shock and volatility spillovers. The extant literature on modelling returns and volatilities is extensive, and it has evolved in several directions. One line of literature has focused on return correlations and comovements or what is known as contagion among different markets or assets (e.g., Caporale et. al., 2005; Rodriguez, 2007, among others), while another line of the literature has focused on volatility spillovers among the markets (e.g., Baele, 2005; Asgharian and Nossman, 2011, among others). The model adopted in this paper is flexible enough to capture contagion effects as well as to identify the volatility spillovers associated with the structural changes and exact movements of each market (e.g., upward or downward) to the other, and vice versa. Knowledge of this mechanism can provide important insights to investors by focusing their attention on structural changes in the markets as well as their trends and movements (e.g., upward or downward) in order to set appropriate portfolio management strategies.

Overall, our results suggest that stock market returns exhibit time varying persistence in their corresponding conditional variances. The results of the bivariate UEDCC-AGARCH$(1,1)$ model applied to FTSE and DAX returns, and to NIKKEI and Hang Seng returns, show the existence of dynamic correlations as well as time varying shock and volatility spillovers between the two variables in each pair. For example, the results of the bivariate FTSE and DAX returns show that the transmission of volatility from DAX to FTSE exhibited a time varying pattern across the Asian financial crisis and the announcement of the €18bn German tax cuts plan as well as the global financial crisis. As far as the NIKKEI and Hang Seng pair is concerned, the results provide evidence that these two financial markets have only been integrated during the different phases of the recent financial crisis. With regard to the regime-dependent volatility spillovers, the results suggest that declines in FTSE and DAX generate shock spillovers to each other, whereas increases in each of these market generate negative volatility spillovers to the other. Furthermore, the results show that declines in NIKKEI generate shock spillovers to Hang Seng, whilst increases in NIKKEI generate negative volatility spillovers to Hang Seng.

The remainder of this paper is as follows. Section 2 considers the AR-GARCH model with abrupt breaks in the first two conditional moments, and the time varying process, which are our two main objects of inquiry. Section 3 introduces the theoretical considerations on the time varying AR and AGARCH models. In Section 3.1 we represent the former as an infinite linear system and concentrate on the associated coefficient matrix. This representation enables us to establish an explicit formula for the general solution in terms of the determinants of tridiagonal matrices. We also obtain the statistical properties of the aforementioned models, e.g., multi-step-ahead predictors and their forecast error variances. Section 4 describes our methodology and data. Section 5 presents our empirical univariate results, and the next Section discusses the results from various bivariate models. The final Section contains the summary and our concluding remarks.

2 Abrupt Breaks

First, we introduce the notation and the AR-AGARCH model with abrupt breaks both in the conditional mean and variance. Throughout the paper we will adhere to the conventions: ($\mathbb{Z}^+$) $\mathbb{Z}$, and ($\mathbb{R}^+$) $\mathbb{R}$ stand for the sets of (positive) integers, and (positive) real numbers, respectively. To simplify our exposition we also introduce the following notation. Let $t \in \mathbb{Z}$ represents present time, and $k \in \mathbb{Z}^+$ the prediction horizon.

2.1 The Conditional Mean

In this paper we will examine an AR(2) model[2] with n abrupt breaks, $0 \leq n \leq k-1$, at times $t-k_1$, $t-k_2$, ..., $t-k_n$, where $0 = k_0 < k_1 < k_2 < \cdots < k_n < k_{n+1} = k$, $k_l \in \mathbb{Z}^+$, and k_n is finite. That is, between $t-k = t-k_{n+1}$ and the present time $t = t-k_0$ the AR process contains n structural breaks and the switch from one set of parameters to another is abrupt. In particular,

$$y_\tau = \phi_{0,l} + \phi_{1,l} y_{\tau-1} + \phi_{2,l} y_{\tau-2} + \varepsilon_\tau, \tag{1}$$

for $l = 1, \ldots, n+1$, and $\tau = t-k_{l-1}, \ldots, t-k_l+1$, where[3] $\mathbb{E}[\varepsilon_\tau \,|\mathcal{F}_{\tau-1}] = 0$ and ε_τ follows a time varying AGARCH type of process with finite variance σ^2_τ (see the next Section).[4] Within the class of AR(2) processes, this specification is quite general and allows for intercept and slope shifts as well as errors with time varying variances (see also Pesaran and Timmermann, 2005, and Pesaran et al. 2006). Each regime l is characterized by a vector of regression coefficients, $\boldsymbol{\phi}_l = (\phi_{0,l}, \phi_{1,l}, \phi_{2,l})'$, and positive and finite time varying variances, σ^2_τ, $\tau = t-k_{l-1}, \ldots, t-k_l+1$. We will term the AR(2) model with n abrupt breaks: abrupt breaks autoregressive process of order $(2; n)$, AB-AR$(2; n)$.

[2] To keep the exposition tractable and reveal its practical significance we work with low order specifications.
[3] $\{\mathcal{F}_t\}$ is a non-decreasing sequence of σ-fields $\mathcal{F}_{t-1} \subseteq \mathcal{F}_t \subseteq \mathcal{F}$.
[4] Without loss of generality we will assume that outside the prediction horizon there are no breaks. That is: regime one ($l = 1$) extends to time $\tau = \ldots, t+2, t+1$ and the $(n+1)$th regime extends to time $\tau = t-k, t-k-1, \ldots$.

2.2 The Conditional Variance

We assume that the noise term is characterized by the relation $\varepsilon_\tau = e_\tau\sqrt{h_\tau}$, where h_τ is positive with probability one and it is a measurable function of $\mathcal{F}_{t-1}$; e_τ is an i.i.d sequence with zero mean and finite second and fourth moments: $\varkappa^{(i)} = \mathbb{E}(e_\tau^{2i})$, $i = 1, 2$. In other words the conditional (on time $\tau - 1$) variance of y_τ is $\mathbb{V}ar(y_\tau \,|\mathcal{F}_{\tau-1}) = \varkappa^{(1)}h_\tau$. In what follows, without loss of generality, we will assume that $\varkappa^{(1)} = 1$.

Moreover, we specify the parametric structure of h_τ as an AGARCH$(1, 1)$ model with m abrupt breaks, $0 \leq m \leq k-1$, at times $t-\kappa_1$, $t-\kappa_2$, ..., $t-\kappa_m$, where $0 = \kappa_0 < \kappa_1 < \kappa_2 < \cdots < \kappa_m < \kappa_{m+1} = k$, $\kappa_m \in \mathbb{Z}^+$, and κ_m is finite. That is, between $t-k = t-\kappa_{m+1}$ and the present time $t = t-\kappa_0$ the AGARCH process contains m structural breaks and the switch from one set of parameters to another is abrupt:

$$h_\tau = \omega_\ell + \alpha_\ell^*\varepsilon_{\tau-1}^2 + \beta_\ell h_{\tau-1}, \tag{2}$$

for $\ell = 1, \ldots, m+1$, and $\tau = t-\kappa_{\ell-1}, \ldots, t-\kappa_\ell + 1$; where $\alpha_\ell^* \triangleq \alpha_\ell + \gamma_\ell S_{\tau-1}^-$, with $S_{\tau-1}^- = 1$ if $e_{\tau-1} < 0$, 0 otherwise.[5] As with the AR process we will assume that outside the prediction horizon there are no breaks. Obviously, the above process nests the simple AGARCH$(1, 1)$ specification if we assume that the four coefficients are constant.

In what follows we provide a complete characterization of the main time-series properties of this model. Although in this work we will focus our attention on the AB-AR$(2; n)$-AGARCH$(1, 1; m)$ process[6] our results can easily be extended to models of higher orders (see Paraskevopoulos et al., 2013).

2.3 Time Varying Model

In the current Section we face the non-stationarity of processes with abrupt breaks head on by employing a time varying treatment. In particular, we put forward a framework for examining the AR-AGARCH specification with n and m abrupt breaks in the conditional mean and variance respectively. We begin by expressing the model as a TV-AR(2)-AGARCH$(1, 1)$ process:

$$y_t = \phi_0(t) + \phi_1(t)y_{t-1} + \phi_2(t)y_{t-2} + \varepsilon_t, \tag{3}$$

where for $l = 1, \ldots, n+1$ and $\tau = t-k_{l-1}, \ldots, t-k_l+1$, $\phi_i(\tau) \triangleq \phi_{i,l}$, $i = 0, 1, 2$, are the time varying drift and AR parameters; as before $\{\varepsilon_t, t \in \mathbb{Z}\}$ is a sequence of zero mean serially uncorrelated random variables

[5] This type of asymmetry is the so called GJR-GARCH model (named for Glosten et al., 1993). The asymmetric power ARCH process (see, among others, Karanasos and Kim, 2006; Margaronis et al., 2013) is yet another asymmetric variant. For other asymmetric GARCH models see Francq and Zakoïan (2010, chapter 10) and the references therein.

[6] That is an AR(2)-AGARCH$(1, 1)$ model with n and m abrupt breaks in the conditional mean and variance respectively.

with positive and finite time varying variances σ_t^2 $\forall$ t. Recall that we have relaxed the assumption of homoscedasticity that is likely to be violated in practice and allow ε_t to follow a TV-AGARCH$(1,1)$ type of process:

$$h_t = \omega(t) + \alpha^*(t)\varepsilon_{t-1}^2 + \beta(t)h_{t-1}, \tag{4}$$

where for $\ell = 1, \ldots, m+1$ and $\tau = t - \kappa_{\ell-1}, \ldots, t - \kappa_\ell + 1$, $\omega(\tau) \triangleq \omega_\ell$, $\alpha^*(\tau) \triangleq \alpha(\tau) + \gamma(\tau)S_{t-1}^- \triangleq \alpha_\ell^*$, and $\beta(\tau) \triangleq \beta_\ell$ are the time varying parameters of the conditional variance equation.

The TV-AGARCH$(1,1)$ formulation in eq. (4) can readily be seen to have the following representation

$$h_t = \omega(t) + c(t)h_{t-1} + \alpha^*(t)v_{t-1}, \tag{5}$$

with $c(t) \triangleq \alpha^*(t) + \beta(t) = \alpha(t) + \gamma(t)S_{t-1}^- + \beta(t)$, and for $\ell = 1, \ldots, m+1$ and $\tau = t - \kappa_{\ell-1}, \ldots, t - \kappa_\ell + 1$, $c(\tau) \triangleq c_\ell$; the 'innovation' of the conditional variance $v_t = \varepsilon_t^2 - h_t$ is, by construction, an uncorrelated term with expected value 0 and $\mathbb{E}(v_t^2) = \sigma_{vt}^2 = \widetilde{\varkappa}\mathbb{E}(h_t^2)$ (the conditions for the second unconditional moments, $\mathbb{E}(h_t^2)$, to exist for all t are available upon request), with $\widetilde{\varkappa} = \mathbb{V}ar(e_t^2) = \varkappa^{(2)} - 1$. The above equation has the linear structure of a TV-ARMA model allowing for simple computations of the linear predictions (see Section 3.2.1 below).[7]

Although in the next Section we will focus our attention on the TV-AR(2)-AGARCH$(1,1)$ model our results can easily be extended to time varying models of higher orders (see Paraskevopoulos et al., 2013).

3 Theoretical Considerations

This current Section presents some new theoretical findings for time varying models which also provide the platform upon which we unify the results we obtain from the different econometric tools. That is, we put forward a framework for examining AR models with abrupt breaks, like eq.(1), based on a workable closed form solution of stochastic time varying difference equations. In other words, we exemplify how our theoretical methodology can be used to incorporate structural changes, which in this paper we view as abrupt breaks. We also explain how we can extend our approach to the AGARCH specification with abrupt breaks in the conditional variance.

[7] As pointed out, among others, by Francq and Zakoïan (2010, p. 20) under additional assumptions (implying the second-order of h_t or ε_t^2), which in our case are available upon request, we can state that if ε_t follows a TV-AGARCH model then h_t or ε_t^2 are TV-ARMA processes as well.

3.1 The mean

In the context of eq. (3), the second-order homogeneous difference equation with time varying coefficients is written as

$$\phi_2(t)y_{t-2} + \phi_1(t)y_{t-1} - y_t = 0,\ t \geq \tau + 1 = t - k + 1. \tag{6}$$

The infinite set of equations in the above equation is equivalent to the infinite linear system whose coefficient matrix is row-finite (row-finite matrices are infinite $\mathbb{N} \times \mathbb{N}$ matrices whose rows have a finite number of nonzero elements)

$$\begin{pmatrix} \phi_2(\tau+1) & \phi_1(\tau+1) & -1 & & & & \cdots \\ & \phi_2(\tau+2) & \phi_1(\tau+2) & -1 & & & \cdots \\ & & \phi_2(\tau+3) & \phi_1(\tau+3) & -1 & & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\vdots\vdots \end{pmatrix} \begin{pmatrix} y_{\tau-1} \\ y_\tau \\ y_{\tau+1} \\ y_{\tau+2} \\ y_{\tau+3} \\ y_{\tau+4} \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \tag{7}$$

(here and in what follows empty spaces in a matrix[8] have to be replaced by zeros) or in a compact form: $\mathbf{\Phi} \cdot \mathbf{y} = \mathbf{0}.$ The equivalence of eqs. (6) and (7) follows from the fact that for an arbitrary i in $\{1, 2, 3, \ldots\}$ the ith equation of (7), as a result of the multiplication of the ith row of $\mathbf{\Phi}$ by the column of ys equated to zero, is equivalent to eq. (6), as of time $t = \tau + i$. By deleting the first column of the $\mathbf{\Phi}$ matrix and then keeping only the first k rows and columns we obtain the following square matrix:

$$\mathbf{\Phi}_{t,k} = \begin{pmatrix} \phi_1(\tau+1) & -1 & & & & \\ \phi_2(\tau+2) & \phi_1(\tau+2) & -1 & & & \\ & \phi_2(\tau+3) & \phi_1(\tau+3) & -1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & \phi_2(t-1) & \phi_1(t-1) & -1 \\ & & & & \phi_2(t) & \phi_1(t) \end{pmatrix} \tag{8}$$

(where $\tau = t - k$). Formally $\mathbf{\Phi}_{t,k}$ is a square $k \times k$ matrix whose (i, j) entry $1 \leq i, j \leq k$ is given by

[8] Matrices and vectors are denoted by upper and lower case boldface symbols, respectively. For square matrices $\mathbf{X} = [x_{ij}]_{i,j=1,\ldots,k} \in \mathbb{R}^{k \times k}$ using standard notation, $\det(\mathbf{X})$ or $|\mathbf{X}|$ denotes the determinant of matrix $\mathbf{X}$.

$$
\begin{cases}
-1 & \text{if} \qquad\qquad i = j-1, \quad \text{and} \quad 2 \le j \le k, \\
\phi_{1+d}(t-k+i) & \text{if} \quad d = 0,1, \quad i = j+d, \quad \text{and} \quad 1 \le j \le k-d, \\
0 & \text{otherwise.}
\end{cases}
$$

This is a tridiagonal or continuant matrix, that is a matrix that is both upper and lower Hessenberg matrix. We next define the bivariate function $\xi : \mathbb{Z} \times \mathbb{Z}^{+} \longmapsto \mathbb{R}$ by

$$
\xi_{t,k} = \det(\mathbf{\Phi}_{t,k}) \tag{9}
$$

coupled with the initial values $\xi_{t,0} = 1$, and $\xi_{t,-1} = 0$. $\xi_{t,k}$ for $k \ge 2$, is a determinant of a $k \times k$ matrix; each two nonzero diagonals (below the superdiagonal) of this matrix consists of the time varying coefficients $\phi_i(\cdot)$, $i = 1, 2$, from $t-k+i$ to t. That is, the number of elements of $\phi_i(\cdot)$ in the diagonals below the superdiagonal is $k-i+1$. In other words, $\xi_{t,k}$ is a k-order tridiagonal determinant. For the AB-AR$(2;n)$ process, $\xi_{t,k}$ is given by

$$
\xi_{t,k} = \det(\mathbf{\Phi}_{t,k}) = \begin{vmatrix}
\phi_{1,n+1} & -1 & & & & & & & \\
\phi_{2,n+1} & \phi_{1,n+1} & -1 & & & & & & \\
 & \ddots & \ddots & \ddots & & & & & \\
 & & \phi_{2,l} & \phi_{1,l} & -1 & & & & \\
 & & & \phi_{2,l} & \phi_{1,l} & -1 & & & \\
 & & & & \ddots & \ddots & \ddots & & \\
 & & & & & \phi_{2,1} & \phi_{1,1} & -1 \\
 & & & & & & \phi_{2,1} & \phi_{1,1}
\end{vmatrix}, \tag{10}
$$

that is, the $(i, i-1)$, and (i, i) elements in rows $i = k - k_{l-1}, \ldots, k - (k_l - 1)$, $l = 1, \ldots, n+1$, of the matrix $\mathbf{\Phi}_{t,k}$ are given by $\phi_{2,l}$ and $\phi_{1,l}$, respectively.

The general term of the general homogeneous solution of eq. (6) with two free constants (initial condition values), y_{t-k} and y_{t-k-1}, is given by

$$
y_{t,k}^{hom} = \xi_{t,k} y_{t-k} + \phi_2(t-k+1)\xi_{t,k-1} y_{t-k-1}. \tag{11}
$$

Similarly, the general particular solution, $y_{t,k}^{par}$, can be expressed as

$$
y_{t,k}^{par} = \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r}. \tag{12}
$$

The general solution of eq. (3) with free parameters y_{t-k}, y_{t-k-1} is given by the sum of the homogeneous solution plus the particular solution:

$$y_{t,k}^{gen} = y_{t,k}^{hom} + y_{t,k}^{par} = \xi_{t,k} y_{t-k} + \phi_2(t-k+1)\xi_{t,k-1} y_{t-k-1} + \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r}. \tag{13}$$

(see the Appendix and also Paraskevopoulos et al., 2013 and Karanasos et al., 2014a). In the above expression $y_{t,k}^{gen}$ is decomposed into two parts: the $y_{t,k}^{hom}$ part, which is written in terms of the two free constants (y_{t-k-i}, $i = 0, 1$); and, the $y_{t,k}^{par}$ part, which contains the time varying drift terms ($\phi_0(\cdot)$) and the error terms (εs) from time $t-k+1$ to time t. When $k = 1$, since $\xi_{t,0} = 1$ and $\xi_{t,1} = \phi_1(t)$, the above expression reduces to eq. (3). Notice also that for the model with n abrupt breaks, we have

$$\sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) = \sum_{l=1}^{n+1} \phi_{0,l} \sum_{r=k_{l-1}}^{k_l-1} \xi_{t,r}, \text{ and } \phi_2(t-k+1) = \phi_{2,n+1},$$

where $\xi_{t,r}$ is given in eq. (10). The main advantage of our TV model/methodology is that we suppose that the law of evolution of the parameters is unknown, in particular they may be stochastic (i.e., we can either have a stationary or non-stationary process) or non stochastic (e.g., periodic models serve as an example, see Karanasos et al., 2014a,b). Therefore, no restrictions are imposed on the functional forms of the time varying AR parameters. In the non stochastic case the model allows for (past/known) abrupt breaks.

3.1.1 First Moments

We turn our attention to a consideration of the time series properties of the TV-AR(2)-AGARCH(1, 1) process. Let the triplet $(\Omega, \{\mathcal{F}_t, t \in \mathbb{Z}\}, P)$ denote a complete probability space with a filtration, $\{\mathcal{F}_t\}$. L_p stands for the space of P-equivalence classes of finite complex random variables with finite p-order. Finally, $H = L_2(\Omega, \mathcal{F}_t, P)$ stands for a Hilbert space of random variables with finite first and second moments. Assuming that the drift and the two AR time varying coefficients $\phi_i(t)$, $i = 0, 1, 2$, are non stochastic and taking the conditional expectation of eq. (13) with respect to the σ field $\mathcal{F}_{t-k}$ yields the k-step-ahead optimal (in L_2-sense) linear predictor of y_t

$$\mathbb{E}(y_t \,|\mathcal{F}_{t-k}) = \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \xi_{t,k} y_{t-k} + \phi_2(t-k+1)\xi_{t,k-1} y_{t-k-1}. \tag{14}$$

In addition, the forecast error for the above k-step-ahead predictor, $\mathbb{FE}(y_t \,|\mathcal{F}_{t-k}) = y_t - \mathbb{E}[y_t \,|\mathcal{F}_{t-k}]$, is given by

$$\mathbb{FE}(y_t \,|\mathcal{F}_{t-k}\,) = \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r}, \tag{15}$$

which is a linear combination of k error terms from time $t-k+1$ to time t, where the time varying coefficients, $\xi_{t,r}$, are (for $r \geq 2$) the determinants of an $r \times r$ tridiagonal matrix ($\Phi_{t,r}$); each nonzero variable diagonal of this matrix consists of the AR time varying coefficients $\phi_i(\cdot)$, $i = 1, 2$ from time $t-r+i$ to t.

The Assumption below provides conditions that are used to obtain the equivalent of the Wold decomposition for non-stationary time varying processes with non stochastic coefficients.

Assumption 1. $\sum_{r=0}^{k} \xi_{t,r}\phi_0(t-r)$ as $k \to \infty$ converges for all t, and $\sum_{r=0}^{\infty} \sup_t(\xi_{t,r}^2\sigma_{t-r}^2) < M < \infty$, $M \in \mathbb{Z}^+$.

The challenge we face is that in the time varying models we can not invert the AR polynomial because of the presence of time dependent coefficients. We overcome this difficulty and formulate a type of time varying Wold decomposition theorem (see also Singh and Peiris, 1987; Kowalski and Szynal, 1991).

Under Assumption 1 the model in eq. (3) with non stochastic coefficients admits a second-order MA(∞) representation:

$$y_t \overset{L_2}{=} \lim_{k\to\infty} y_{t,k}^{par} \overset{L_2}{=} \sum_{r=0}^{\infty} \xi_{t,r}[\phi_0(t-r) + \varepsilon_{t-r}], \tag{16}$$

which is a unique solution of the TV-AR(2)-AGARCH(1, 1) model (3). In other words y_t is decomposed into a non random part and a zero mean random part. In particular, the time dependent first moment:

$$\mathbb{E}(y_t) = \lim_{k\to\infty} \mathbb{E}(y_t \,|\mathcal{F}_{t-k}\,) = \sum_{r=0}^{\infty} \xi_{t,r}\phi_0(t-r) \tag{17}$$

is the non random part of y_t while $\lim_{k\to\infty} \mathbb{FE}(y_t \,|\mathcal{F}_{t-k}\,) = \sum_{r=0}^{\infty} \xi_{t,r}\varepsilon_{t-r}$ is the zero mean random part.

The time varying expected value of y_t is an infinite sum of the time varying drifts where the time varying coefficients are expressed as determinants of continuant matrices (the ξs).

3.1.2 Second Moments

The current Section and Section 3.2.1 below discusses the second-order properties of the TV-AR(2)-AGARCH(1, 1) model. Next we state the results for the second moment structure.

The mean square error

$$\mathbb{V}ar[\mathbb{FE}(y_t \,|\mathcal{F}_{t-k}\,)] = \sum_{r=0}^{k-1} \xi_{t,r}^2\sigma_{t-r}^2 \tag{18}$$

is a linear combination of k variances from time $t-k+1$ to time t, with time varying coefficients (the squared ξs).

Moreover, under Assumption 1 the second time varying unconditional moment of y_t exists and it is given by

$$\mathbb{E}(y_t^2) = [\mathbb{E}(y_t)]^2 + \sum_{r=0}^{\infty} \xi_{t,r}^2 \sigma_{t-r}^2, \tag{19}$$

which is an infinite sum of the time varying unconditional variances of the errors, σ_{t-r}^2, (see Section 3.2.1 below) with time varying 'coefficients' or weights (the squared values of the ξs).

In addition, the time varying autocovariance function $\gamma_{t,k}$ is given by

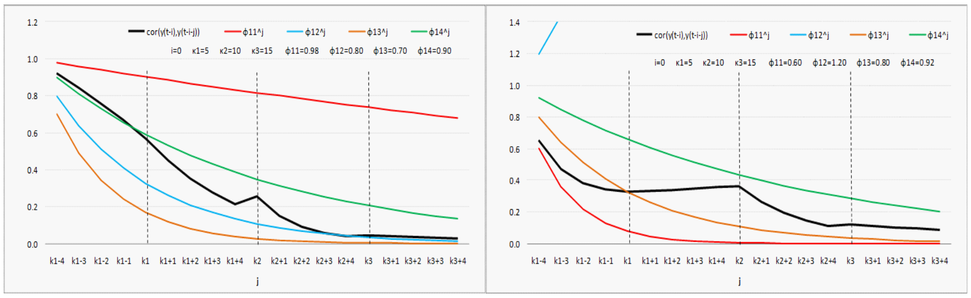
$$\begin{aligned} \gamma_{t,k} &= \mathbb{C}ov(y_t, y_{t-k}) = \sum_{r=0}^{\infty} \xi_{t,k+r} \xi_{t-k,r} \sigma_{t-k-r}^2 \\ &= \xi_{t,k} \mathbb{V}ar(y_{t-k}) + \phi_2(t-k+1)\xi_{t,k-1} \mathbb{C}ov(y_{t-k}, y_{t-k-1}), \end{aligned} \tag{20}$$

where the second equality follows from the MA(∞) representation of y_t in eq. (16), and the third one from the general solution in eq. (13), and

$$\mathbb{C}ov(y_{t-k}, y_{t-k-1}) = \sum_{r=0}^{\infty} \xi_{t-k,r+1} \xi_{t-k-1,r} \sigma_{t-k-1-r}^2.$$

For any fixed t, $\lim_{k\to\infty} \gamma_{t,k} \to 0$ when $\lim_{k\to\infty} \xi_{t,k} = 0 \; \forall \; t$. For the process with n abrupt breaks in eq. (1) $\xi_{t,k}$ is given by eq. (10).[9]

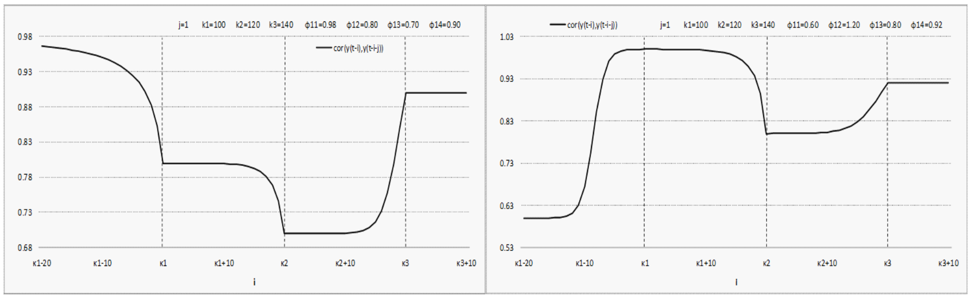
Panel A: AR(1) Model; 3 Breaks at: $t-5$, $t-10$ and $t-15$;



$\mathbb{C}or(y_t, y_{t-j}), j = 1, \ldots, 19$ $\qquad$ $\mathbb{C}or(y_t, y_{t-j}),\ j = 1, \ldots, 19$

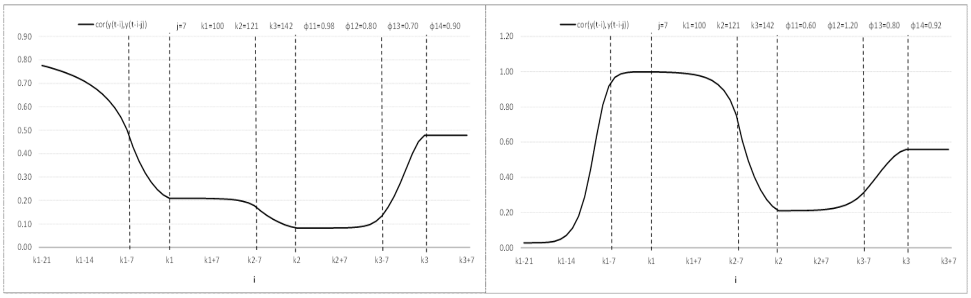
[9] Estimating the time varying parameters of forecasting models is beyond the scope of this paper (see Elliott and Timmermann, 2008, for an excellent survey on forecasting methodologies available to the applied economist).

Panel B: AR(1) Model; 3 Breaks at: $t - 100$, $t - 120$ and $t - 140$;



$\mathbb{C}or(y_{t-i}, y_{t-i-1}), i = 80, \ldots, 150$ $\mathbb{C}or(y_{t-i}, y_{t-i-1}), i = 80, \ldots, 150$

Panel C: AR(1) Model; 3 Breaks at: $t - 100$, $t - 121$ and $t - 142$;



$\mathbb{C}or(y_{t-i}, y_{t-i-7}), i = 79, \ldots, 149$ $\mathbb{C}or(y_{t-i}, y_{t-i-7}), i = 79, \ldots, 149$

Figure 1. Time Varying Autocorrelations

As an illustrative example Figure 1 shows the autocorrelations (ACR) of an AR(1) model with three breaks and homoscedastic/independent innovations. The left graph in Panel B shows the first order ACR, $\mathbb{C}or(y_{t-i},\, y_{t-i-1})$, for an AR(1) model with three breaks at times $t - k_1(= 100)$, $t - k_2(= 120)$ and $t - k_3(= 140)$, and autoregressive coefficients $\phi_{1,1} = 0.98$, $\phi_{1,2} = 0.80$, $\phi_{1,3} = 0.70$, and $\phi_{1,4} = 0.90$. The first part of the graph shows the ACR when $i < k_1 = 100$, that is, when y_{t-i} is after all three breaks: $t - i > t - k_1$ (the construction of the autocorrelations is based on eq. (20)).[10] As i increases, that is, as we are going back in time, the first order ACR decrease at an increasing rate. The second part of the graph shows the ACR when $k_1 \leq i \leq k_2 - 1$, that is, when y_{t-i} is between the first and the second break. The third part of the graph shows the ACR when $k_2 \leq i \leq k_3 - 1$. The ACR increase since after the third break the autoregressive coefficient increases from 0.70 to 0.90. Finally, for $i \geq k_3$, the first order

[10] The details are available upon request. See also the Additional Appendix, which is available on the personal webpage of the first author: http://www.mkaranasos.com/PublicationsB.htm

ACR are not affected by the three breaks and therefore are equal to $\phi_{1,4} = 0.90$, whereas when $i \to -\infty$, the ACR converge to $\phi_{1,1} = 0.98$.

Moreover, the right graph in Panel C shows the seventh order ACR (y_{t-i}, y_{t-i-7}) for an AR(1) model with three breaks at times $t - k_1(= 100)$, $t - k_2(= 121)$ and $t - k_3(= 142)$, autoregressive coefficients $\phi_{1,1} = 0.60$, $\phi_{1,2} = 1.20$, $\phi_{1,3} = 0.80$, and $\phi_{1,4} = 0.92$ and homoscedastic/independent innovations. The second part of the graph shows the ACR when $i \leq k_1 - 1$ and $k_1 + 1 \leq i + 7 \leq k_2$. The fourth part of the graph shows the ACR when $k_1 \leq i \leq k_2 - 1$ and $k_2 + 1 \leq i + 7 \leq k_3$. The sixth part of the graph shows the ACR when $k_2 \leq i \leq k_3 - 1$ and $k_3 + 1 \leq i + 7$. Notice that when $i \leq k_1 - 1$ or $k_2 \leq i \leq k_3 - 1$ the seventh order ACR increase with i whereas when $k_1 \leq i \leq k_2 - 1$ they decrease as i increases. Finally, for $i \geq k_3$, the ACR are equal to $\phi_{1,4}^7 = 0.56$, whereas when $i \to -\infty$, the ACR converge to $\phi_{1,1}^7 = 0.03$.

3.2 The Conditional Variance

In order to simplify the description of the analysis of this Section we will introduce the following notation. As before t represents the present time and k the prediction horizon. We define the bivariate function $\varsigma : \mathbb{Z} \times \mathbb{Z}^+ \longmapsto \mathbb{R}$ by

$$\varsigma_{t,k} = \prod_{j=0}^{k-1} c(t-j), \tag{21}$$

coupled with the initial values $\varsigma_{t,0} = 1$, and $\varsigma_{t,-1} = 0$ where $c(\cdot)$ has been defined above (see eq. (5)). In other words $\varsigma_{t,1} = c(t)$, and $\varsigma_{t,k}$ for $k \geq 2$ is a product of k terms which consist of the time varying coefficients $c(\cdot)$ from time $t - k + 1$ to time t. For the GARCH process with m abrupt breaks in eq. (2) we have

$$\varsigma_{t,k} = \prod_{\ell=0}^{m} c_{\ell+1}^{\kappa_{\ell+1}-\kappa_\ell}. \tag{22}$$

Next, we define

$$g_{t,r+1} = \varsigma_{t,r} \alpha^*(t-r), \ r \geq 0, \tag{23}$$

where $\alpha^*(t)$ has been defined in eq. (4). Notice that when $r = 0$, $g_{t,1} = \alpha^*(t)$, since $\varsigma_{t,0} = 1$.

Since the TV-AGARCH(1,1) model can be interpreted as a 'TV-ARMA(1,1)' process, it follows directly from the results in Section 3.1 that the general solution of eq. (5) with free constant (initial condition value) h_{t-k}, is given by

$$h_{t,k}^{gen} = h_{t,k}^{hom} + h_{t,k}^{par} = \varsigma_{t,k} h_{t-k} + \sum_{r=0}^{k-1} \varsigma_{t,r} \omega(t-r) + \sum_{r=1}^{k} g_{t,r} v_{t-r}, \tag{24}$$

where $\varsigma_{t,r}$ and $g_{t,r}$ have been defined in eqs. (21) and (23) respectively. In the above expression h_t^{gen}

is decomposed into two parts: the $h_{t,k}^{hom}$ part, which is written in terms of the free constant (h_{t-k}); and the $h_{t,k}^{par}$ part, which contains the time varying drift terms, $\omega(\cdot)$, and the uncorrelated terms (vs). Notice that in eq. (24) $h_{t,k}^{gen}$ is expressed in terms of diagonal determinants (the ςs and therefore the gs).

Next consider the case of a GARCH(1, 1) model with constant coefficients. Since for this model $\alpha(t) \triangleq a$, and $c(t) \triangleq c \triangleq \alpha + b$, for all t, then $\varsigma_{t,k}$ reduces to c^k and $g_{t,k}$ becomes $c^{k-1}a$, for $k \in \mathbb{Z}^+$ (see, for example, Karanasos, 1999).

3.2.1 Time Varying Unconditional Variances

In this Section in order to provide a thorough description of the TV-AGARCH(1, 1) process given by eq. (4) we derive, first its multistep ahead predictor, the associated forecast error and the mean square error, and second, the first unconditional moment of this process (the second unconditional moment and the covariance structure are available upon request).

The k-step-ahead predictor of h_t, $\mathbb{E}(h_t \,|\mathcal{F}_{t-k-1})$, is readily seen to be[11]

$$\mathbb{E}(h_t \,|\mathcal{F}_{t-k-1}) = \sum_{r=0}^{k-1} \overline{\varsigma}_{t,r}\omega(t-r) + \overline{\varsigma}_{t,k} h_{t-k}, \tag{25}$$

where, for $r \geq 1$, $\overline{\varsigma}_{t,r} = \mathbb{E}(\varsigma_{t,r})$.[12] In addition, the forecast error for the above k-step-ahead predictor, $\mathbb{FE}(h_t \,|\mathcal{F}_{t-k-1})$, is given by

$$\mathbb{FE}(h_t \,|\mathcal{F}_{t-k-1}) = \sum_{r=1}^{k} g_{t,r} v_{t-r}. \tag{26}$$

Notice that this predictor is expressed in terms of k uncorrelated terms (the vs) from time $t-k$ to time $t-1$, where the 'coefficients' have the form of diagonal determinants (the ςs). The mean square error is given by

$$\mathbb{V}ar(h_t \,|\mathcal{F}_{t-k-1}) = \mathbb{V}ar[\mathbb{FE}(h_t \,|\mathcal{F}_{t-k-1})] = \widetilde{\varkappa} \sum_{r=1}^{k} \overline{g}_{t,r}^2 \mathbb{E}(h_{t-r}^2), \tag{27}$$

where $\overline{g}_{t,r}^2 = \mathbb{E}(g_{t,r}^2)$ for $r \geq 1$.[13] This is expressed in terms of k second moments, $\mathbb{E}(h_{t-r}^2)$, from time $t-k$ to time $t-1$, where the coefficients are the expectations of the squared coefficients of the multistep ahead predictor multiplied by $\widetilde{\varkappa}$. Moreover, the definition of the uncorrelated term v_t implies that $\mathbb{E}(\varepsilon_t^2 \,|\mathcal{F}_{t-k-1}) = \mathbb{E}(h_t \,|\mathcal{F}_{t-k-1})$, $\mathbb{FE}(\varepsilon_t^2 \,|\mathcal{F}_{t-k-1}) = v_t + \mathbb{FE}(h_t \,|\mathcal{F}_{t-k-1})$. The associated mean squared

[11] For the issue of temporal aggregation and a discussion of the wider class of weak GARCH processes see Bollerslev and Ghysels (1996) and Ghysels and Osborn (2001, pp. 195-197).

[12] $\mathbb{E}(\varsigma_{t,r}) = \mathbb{E}[\prod_{j=0}^{r-1} c(t-j)] = \prod_{j=0}^{r-1} \overline{c}(t-j)$ with $\overline{c}(t) \triangleq \mathbb{E}[c(t)] = \alpha(t) + \beta(t) + \frac{\gamma(t)}{2}$. For the process with m abrupt breaks: $\mathbb{E}(\varsigma_{t,r}) = \prod_{\ell=0}^{m} \overline{c}_{\ell+1}^{\kappa_{\ell+1}-\kappa_\ell}$.

[13] $\mathbb{E}(g_{t,r+1}^2) = \mathbb{E}(\varsigma_{t,r}^2)[\alpha^2(t-r) + \gamma^2(t-r)/2 + \alpha(t-r)\gamma(t-r)]$ and, for $r \geq 1$, $\mathbb{E}(\varsigma_{t,r}^2) = \prod_{j=0}^{r-1} \mathbb{E}[c^2(t-j)]$, with $\mathbb{E}[c^2(t)] = [\alpha(t) + \beta(t)]^2 + \gamma^2(t)/2 + [\alpha(t) + \beta(t)]\gamma(t)$.

error is given by $\mathbb{V}ar[\mathbb{FE}(\varepsilon_t^2 \,|\mathcal{F}_{t-k-1})] = \widetilde{\varkappa}\mathbb{E}(h_t^2) + \mathbb{V}ar[\mathbb{FE}(h_t \,|\mathcal{F}_{t-k-1})] = \widetilde{\varkappa}\sum_{r=0}^{k} \overline{g}_{t,r}^2 \mathbb{E}(h_{t-r}^2)$.

Next to obtain the first unconditional moment of h_t, for all t, we impose the conditions that: $\sum_{r=0}^{k} \overline{\varsigma}_{t,r}\omega(t-r)$ as $k \to \infty$ is positive and converges, and

$$\widetilde{\varkappa}\sum\nolimits_{r=1}^{\infty} \sup_t[\overline{g}_{t,r}^2 \mathbb{E}(h_{t-r}^2)] < M < \infty,\ M \in \mathbb{Z}^+, \tag{28}$$

which guarantees that, for all t, the model in eq. (5) admits the second-order MA(∞) representation:

$$h_{t,\infty}^{gen} = \lim_{k\to\infty} h_{t,k}^{par} \overset{L_2}{=} \sum_{r=0}^{\infty} \overline{\varsigma}_{t,r}\omega(t-r) + \sum_{r=1}^{\infty} g_{t,r} v_{t-r}, \tag{29}$$

which is a unique solution of the TV-AGARCH(1,1) model in eq. (4). The above result states that $\{h_{t,k}^{par}$, $t \in \mathbb{Z}^+\}$ (defined in eq. (24)) L_2 converges as $k \to \infty$ if and only if $\sum_{r=0}^{k} \overline{\varsigma}_{t,r}\omega(t-r)$ as $k \to \infty$ converges and $\sum_{r=1}^{k} g_{t,r} v_{t-r}$ converges a.s., and thus under the aforementioned conditions $h_{t,\infty}^{gen} \overset{L_2}{=} \lim_{k\to\infty} h_{t,k}^{par}$ satisfies eq. (24).

Moreover, the first time varying unconditional moment of h_t, $\mathbb{E}(h_t) = \sigma_t^2$, is the limit of the $(k+1)$-step-ahead predictor of h_t, $\mathbb{E}(h_t \,|\mathcal{F}_{t-k-1})$, as $k \to \infty$:

$$\mathbb{E}(h_t) = \lim_{k\to\infty} \mathbb{E}(h_t \,|\mathcal{F}_{t-k-1}) = \sum_{r=0}^{\infty} \overline{\varsigma}_{t,r}\omega(t-r). \tag{30}$$

Notice that the first moment is time varying. The expected value of the conditional variance, that is the unconditional variance of the error, is an infinite sum of the time varying drifts where the coefficients (the $\overline{\varsigma}$s) are expressed as expectations of diagonal determinants. Finally, for the process with m abrupt breaks in eq. (2), for $i \leq \kappa_1$ we have (if and only if $\overline{c}_{m+1} < 1$):

$$\mathbb{E}(h_{t-i}) = \frac{1-\overline{c}_1^{\kappa_1 - i}}{1-\overline{c}_1}\omega_1 + \sum_{\ell=2}^{m} \widetilde{c}_\ell \frac{1-\overline{c}_\ell^{\kappa_\ell - \kappa_{\ell-1}}}{1-\overline{c}_\ell}\omega_\ell + \widetilde{c}_{m+1}\frac{1}{1-\overline{c}_{m+1}}\omega_{m+1}, \tag{31}$$

with

$$\widetilde{c}_\ell = \overline{c}_1^{\kappa_1 - i}\prod\nolimits_{j=2}^{\ell-1}(\overline{c}_j^{\kappa_j - \kappa_{j-1}}),$$

where we use the convention $\prod_{r=i}^{j}(\cdot) = 1$ for $j < i$, and the ωs and the cs are defined in eqs. (4) and (5) respectively. Notice that if and only if $\overline{c}_1 < 1$ the above expression as $i \to -\infty$ becomes: $\mathbb{E}(h_{t-i}) = \frac{\omega_1}{1-\overline{c}_1}$ since $\widetilde{c}_\ell = \overline{c}_1^{\kappa_1 - i} = 0$ for all ℓ. Finally, when $i > \kappa_m$, that is when we are before all the breaks, then if and only if $\overline{c}_{m+1} < 1$: $\mathbb{E}(h_{t-i}) = \frac{\omega_{m+1}}{1-\overline{c}_{m+1}}$.

4 Methodology and Data

This Section outlines the methodology we have employed to study the different properties of the stochastic processes during the various financial crises and offers an overview of the data employed. First, we describe the univariate models we have estimated. Then we mention the break identification method which we have adopted.

4.1 Univariate Modelling

Let stock returns be denoted by $r_t = (\log p_t - \log p_{t-1}) \times 100$, where p_t is the stock price index, and define its mean equation as:

$$r_t = \mu + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \varepsilon_t, \tag{32}$$

where $\varepsilon_t \mid \mathcal{F}_{t-1} \sim N(0,\, h_t)$, that is the innovation is conditionally normal with zero mean and variance h_t.[14] Next, the dynamic structure of the conditional variance is specified as an AGARCH$(1,1)$ process of Glosten et al. (1993) (the asymmetric power ARCH could also be employed, as in Karanasos and Kim, 2006). In order to examine the impact of the breaks on the persistence of the conditional variances, the following equation is specified as follows:

$$\begin{aligned} h_t &= \omega + \sum_{i=1}^{7} \omega_i D_i + \alpha \varepsilon_{t-1}^2 + \sum_{i=1}^{7} \alpha_i D_i \varepsilon_{t-1}^2 + \gamma S_{t-1}^{-} \varepsilon_{t-1}^2 + \sum_{i=1}^{7} \gamma_i D_i S_{t-1}^{-} \varepsilon_{t-1}^2 \\ &\quad + \beta h_{t-1} + \sum_{i=1}^{7} \beta_i D_i h_{t-1}, \end{aligned} \tag{33}$$

where $S_{t-1}^{-} = 1$ if $e_{t-1} < 0$, and 0 otherwise. Note that failure to reject $H_0 : \gamma = 0$ and $\gamma_i = 0$, $i = 1, \ldots, 7$, implies that the conditional variance follows a symmetric GARCH$(1,1)$ process. Furthermore, the second order conditions require that $\overline{c} < 1$ and $\overline{c} + \sum_{i=1}^{7} \overline{c}_i < 1$.[15] The breakdates $i = 1, ..., 7$ are given in Table 1, and D_i are dummy variables defined as 0 in the period before each break and one after the break.[16] We also consider a simple GARCH$(1,1)$ model which allows the dynamics of the conditional variances to switch across positive and negative stock returns. This is given by

[14] Since mainly structural breaks in the variance are found statistically significant (see Section 5.1 below) we do not include any dummies in the mean. Moreover, low order AR specifications capture the serial correlation in stock returns.

[15] $\overline{c} \triangleq \alpha + \beta + \frac{\gamma}{2}$ and $\overline{c}_i \triangleq \alpha_i + \beta_i + \gamma_i/2$.

[16] The relation between the parameters in eq. (33) and the ones in eq. (2) is given by, i.e., for the ωs: $\omega + \sum_{i=1}^{m+1-\ell} \omega_i = \omega_\ell$, $\ell = 1, \ldots, m+1$, where the ωs in the right hand side are the ones in eq. (2).

$$h_t = \omega + \omega^- D^-_{t-1} + \alpha\varepsilon^2_{t-1} + \alpha^- D^-_{t-1}\varepsilon^2_{t-1} + \beta h_{t-1} + \beta^- D^-_{t-1} h_{t-1}. \quad (34)$$

where $D^-_{t-1} = 1$ if $r_{t-1} < 0$, 0 otherwise.[17] This is an example of a TV-AGARCH model with stochastic coefficients.

4.2 Data and Breaks Overview

We use daily data that span the period 1-1-1988 30-6-2010 for the stock market indices, obtained from Thomson DataStream. To account for the possibility of breaks in the mean and/or volatility dynamics we use a set of non-parametric data-driven methods to identify the number and timing of the potential structural breaks. In particular, we adopt the two-stage Nominating-Awarding procedure of Karoglou (2010) to identify breaks that might be associated either to structural changes in the mean and/or volatility dynamics or to latent non-linearities that may manifest themselves as dramatic changes in the mean and/or volatility dynamics and might bias our analysis.[18] Alternatively, we could choose the break points by employing the methodologies in Kim and Kon (1999), Bai and Perron (2003) and Lavielle and Moulines (2000) (see, for example, Karanasos and Kartsaklas, 2009, and Campos et al., 2012).

5 Empirical Analysis

This Section presents the empirical results we obtain from the different econometric tools. First, we present the breaks that we have identified and discuss the possible economic events that may be associated with them. Then we focus on the stock market returns and condition our analysis based on these breaks to discuss first the findings from the univariate modelling and then from the bivariate one (presented in Section 6).

5.1 Estimated Breaks

After applying the Nominating-Awarding procedure on stock market returns we find that the stochastic behaviour of all indices yields about three to seven breaks during the sample period, roughly one every two to four years on average. The predominant feature of the underlying segments is that mainly changes in variance are found statistically significant. Finally, there are several breakdates that are either identical in all series or very close to one another, which apparently signify economic events with a global impact.

[17] We estimate another specification with $\alpha^+ D^+_{t-1}$, $\beta^+ D^+_{t-1}$, and $\omega^+ D^+_{t-1}$, instead of $\alpha^- D^-_{t-1}$, $\beta^- D^-_{t-1}$, and $\omega^- D^-_{t-1}$, where $D^+_{t-1} = 1$ if $r_{t-1} > 0$, 0 otherwise. The results (not reported) are very similar.

[18] The details of the two stages in the Nominating-Awarding procedure and a summary of the statistical properties of stock market returns are available upon request.

It appears that dates for the extraordinary events of the Asian financial crisis of 1997, the global financial crisis of 2007–08 and the European sovereign-debt crisis that followed are clearly identified in all stock return series with very little or no variability (see Table 1). Other less spectacular events, such as the Russian financial crisis of 1998, the Japanese asset price bubble of 1986-1991 or the UK's withdrawal from the European Exchange Rate Mechanism (ERM), can also be associated with the breakdates that have been identified in some series.[19]

5.2 Univariate Results

The quasi-maximum likelihood estimates of the AGARCH$(1,1)$ model allowing the drifts (the ωs) as well as the 'dynamics of the conditional variance' (the αs, βs and γs) to switch across the considered breaks, as in eq. (33), are reported in Table 2.[20] The estimated models are shown to be well-specified: there is no linear or nonlinear dependence in the residuals in all cases, at the 5% level. Note that the insignificant parameters are excluded. The impact of the breaks on the ω is insignificant in all eight cases. However, there exists a significant impact of the breaks on the 'dynamic structure of the conditional variance' for all stock returns (irrespective of whether a symmetric GARCH$(1,1)$ or an AGARCH $(1,1)$ model is considered). More specifically, while the ARCH parameter shows time varying features across a single break in the cases of S&P and DAX, for CAC and Hang Seng it is shifted across two breaks and for STRAITS it is shifted across three breaks (see the α_i coefficients). With regard to the GARCH parameter, CAC and NIKKEI show time varying parameters for only one break, but S&P, TSE, and FTSE across two breaks. Furthermore, the GARCH parameter shows a time varying pattern across three breaks in the case of DAX and across five breaks in the case of STRAITS.

Interestingly, the asymmetry parameter also displays significant time variation over the considered breaks. Specifically, the TSE, DAX, and Hang Seng cases are significantly shifted for one break, whereas S&P, CAC, and FTSE show a time varying pattern across three breaks, and STRAITS for two breaks (see the γ_i coefficients in Table 2). Furthermore, the results are shown to be robust by considering the dynamics of a GARCH$(1,1)$ process to switch across positive and negative stock returns (see Table 3). Clearly, the ARCH and GARCH parameters show time dependence across positive and negative returns in all cases (see the α^-, and β^- coefficients).

Overall, Table 4 shows that the persistence of the conditional variances of stock returns varies over the

[19] A detailed account of the possible associations that can be drawn between each breakdate for stock returns and a major economic event that took place at or around the breakdate period either in the world or in each respective economy is available upon request, as is a summary of the descriptive statistics of each segment.

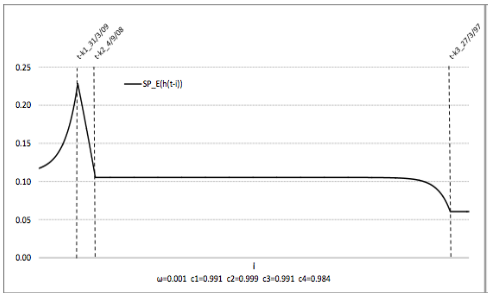
[20] The quasi-maximum likelihood estimates of the standard AGARCH$(1,1)$ model are available upon request. The results of the symmetric GARCH $(1,1)$ model allowing the dynamics of the conditional variance to switch across the considered breaks are reported in Paraskevopoulos et al. (2013).

considered breaks in all cases by considering the AGARCH $(1,1)$ models. The persistence is measured by $\overline{c}_\ell = \alpha_\ell + \beta_\ell + \gamma_\ell/2$, $\ell = 1,\ldots,m+1$ (these are the $\overline{c}$s used in eq. (31) as well), and, for example, $\beta_\ell = \underbrace{\beta + \sum_{i=1}^{m+1-\ell}\beta_i}_{\text{Eq. (33)}}$.[21]

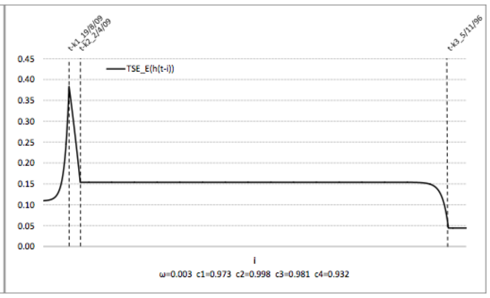
The cases which are shown to have been impacted strongly by the breaks are those of TSE, DAX, Hang Seng, NIKKEI and STRAITS. In particular, the persistence of the conditional variance of TSE increases from 0.93 to 0.98 after the break in 1996, remains 0.98 during the recent financial crisis and then increases to near unity after the European sovereign-debt crisis. With regard to the persistence of the conditional variance of DAX, it appears to be unaffected by German reunification, its highest value is 0.98 during the Asian financial crisis, its lowest value is 0.94 after the break associated with the announcement of the €18bn tax cuts plan in Germany (17/06/03), it increases to 0.97 on the onset of the recent financial crisis and remains there during the sovereign-debt crisis. The results also suggest that the persistence of the conditional variance of Hang Seng declines from 0.97 to 0.92 (its lowest value) after the savings deposits were removed in July 2001, increases to 0.99 during the recent financial crisis in 2007/2008, and finally it declines to 0.94 after the European sovereign-debt crisis. Furthermore, the corresponding persistence of STRAITS increases from 0.87 to near unity (0.99) after the Asian financial crisis. However, such persistence declines after the break in June 2000 to 0.91, remains the same through the unexpected economic recession in Singapore in 2001 before bounding back to 0.97 at the onset of the global financial crisis, and then exhibits a sharp decline to 0.88 during the European sovereign-debt crisis. Surprisingly, the persistence of the conditional variance of NIKKEI increases from 0.90 to approximately 0.98 during the asset price bubble in Japan over the period 1986-1991 and remains unaffected afterwards. For example, the impact of the Asian financial crisis as well as that of the recent financial crisis are shown to be limited, which may be due to the fact that Japan has been immune to such crises.

The persistence of the conditional variances by allowing the GARCH $(1,1)$ process to switch across positive and negative returns also shows a time varying pattern (see Table 5). In particular, it is shown that the persistence of the conditional variances stemming from positive returns is lower than those of the negative counterparts. More specifically, positive returns are shown to lower the persistence of the conditional variances in most of the cases to around 0.90 whereas the persistence of the negative returns is close to unity (0.99).

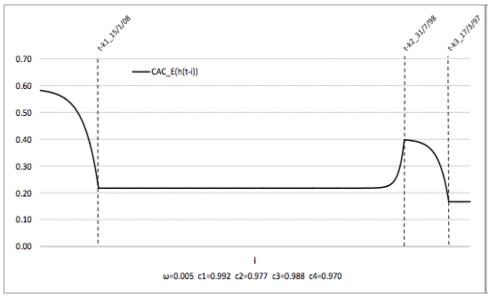
[21] The plot of the time varying-piecewise persistence of the conditional variances of stock returns against the persistence generated from the standard AGARCH$(1,1)$ models is available upon request.



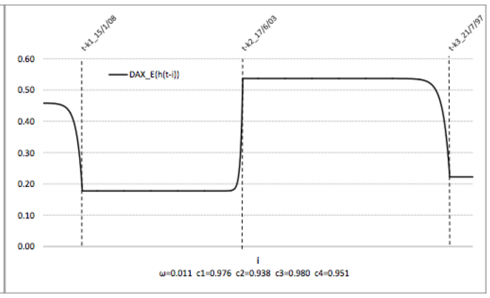
S&P Index



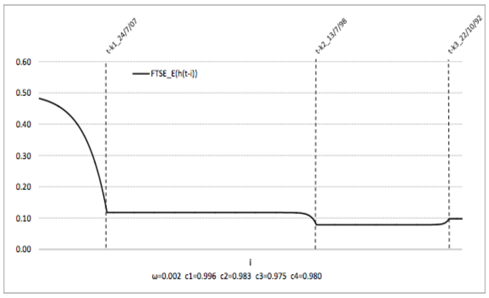
TSE Index



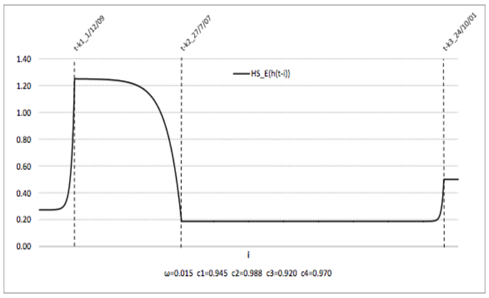
CAC Index



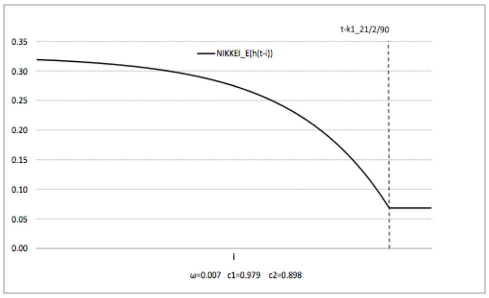
DAX Index



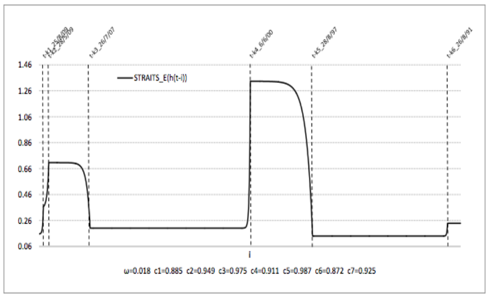
FTSE Index



HS Index



NIKKEI Index



STRAITS Index

Figure 2. Unconditional Variances (Stock Returns)

AGARCH(1, 1) model allowing for abrupt breaks in the variance

Figure 2 shows the estimated time varying unconditional variances for the eight stock index returns. For the S&P the first part of the graph shows the unconditional variances when $i < k_1$, that is, when h_{t-i} is after all three breaks ($t - k_3$(=03/97), $t - k_2$(=09/08) and $t - k_1$(=03/09)) (we construct the time varying unconditional variances using the formula in eq. (31)). When $i \to -\infty$, the unconditional variances converge to $\omega/(1-\bar{c}_1) = 0.001/(1-0.990) = 0.100$. As i increases, that is, as we are going back in time, the unconditional variances increase at an increasing rate. The second part of the graph shows the unconditional variances when $k_1 \leq i \leq k_2 - 1$, that is, when h_{t-i} is between the first and the second break. Higher values of i are associated with lower unconditional variances. When $i = k_1$, the unconditional variance is $[(1-\bar{c}_2^{k_2-k_1})/(1-\bar{c}_2)+\bar{c}_2^{k_2-k_1}(1-\bar{c}_3^{k_3-k_2})/(1-\bar{c}_3)+\bar{c}_2^{k_2-k_1}\bar{c}_3^{k_3-k_2}/(1-\bar{c}_4)]\omega = 0.228$ (see eq. (31) and the $\bar{c}$s in the first column of Table 4). The third part of the graph shows the unconditional variances when $k_2 \leq i \leq k_3 - 1$. When $i = k_2$, the unconditional variance is $[(1-\bar{c}_3^{k_3-k_2})/(1-\bar{c}_3)+\bar{c}_3^{k_3-k_2}/(1-\bar{c}_4)]\omega = 0.105$. Finally, for $i \geq k_3$, the unconditional variances are not affected by the three breaks and therefore are equal to $\omega/(1-\bar{c}_4) = 0.061$.

Similarly, for the DAX the first part of the graph shows the unconditional variances when $i < k_1$, that is, when h_{t-i} is after all three breaks ($t - k_3$(=07/97), $t - k_2$(=06/03) and $t - k_3$(=01/08)). When $i \to -\infty$, the unconditional variances converge to $\omega/(1-\bar{c}_1) = 0.011/(1-0.976) = 0.458$. As i increases, that is, as we are going back in time, the unconditional variances decrease at an increasing rate. The second part of the graph shows the unconditional variances when $k_1 \leq i \leq k_2 - 1$ ($\mathbb{E}(h_{t-k_1}) = 0.177$). Higher values of i are associated with higher unconditional variances. The third part of the graph shows the unconditional variances when $k_2 \leq i \leq k_3 - 1$. They are decreasing with i. Finally, for $i \geq k_3$, the unconditional variances are not affected by the three breaks and therefore are equal to $\omega/(1-\bar{c}_4) = 0.222$.

For the NIKKEI the first part of the graph shows the unconditional variances when $i < k_1$, that is, when h_{t-i} is after the only break ($t - k_1$(=02/90)). When $i \to -\infty$, the unconditional variances converge to $\omega/(1-\bar{c}_1) = 0.326$. As i increases the unconditional variances decrease at an increasing rate. In addition, for $i \geq k_1$, the unconditional variances are not affected by the break and therefore are equal to $\omega/(1-\bar{c}_2) = 0.068$.

Finally, STRAITS exhibits the highest number of breaks, that is six. The first part of the graph shows the unconditional variances when $i < k_1$, that is, when h_{t-i} is after all six breaks ($t - k_6$(=08/91), $t - k_5$(=08/97), $t - k_4$(=06/00), $t - k_3$(=07/07), $t - k_2$(=05/09), $t - k_1$(=08/09)). As i increases, that is, as we are going back in time, the unconditional variances increase at an increasing rate. When $i \to -\infty$, the unconditional variances converge to $\omega/(1-\bar{c}_1) = 0.157$. The second part of the graph shows the unconditional variances when $k_1 \leq i \leq k_2 - 1$. Higher values of i are associated with higher unconditional variances. The third part of the graph shows the unconditional variances when $k_2 \leq i \leq k_3 - 1$. They are decreasing with i. For the fourth and sixth part the unconditional variances increase with i whereas for the fifth part they decrease with i. Finally, for $i \geq k_6$, the unconditional variances are not affected by the six breaks and therefore are equal to $\omega/(1-\bar{c}_7) = 0.238$.

6 Bivariate Models

In this Section we use a bivariate extension of the univariate formulation of Section 4.. In particular, we use a bivariate model to simultaneously estimate the conditional means, variances, and covariances of stock returns. Let $\mathbf{y}_t = (y_{1,t}, y_{2,t})'$ represent the 2×1 vector with the two returns. $\mathcal{F}_{t-1} = \sigma(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots)$ is the filtration generated by the information available up through time $t-1$. We estimate the following bivariate AR(2)-AGARCH(1, 1) model

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}_1 \mathbf{y}_{t-1} + \boldsymbol{\Phi}_2 \mathbf{y}_{t-2} + \boldsymbol{\varepsilon}_t, \tag{35}$$

where $\boldsymbol{\mu} = [\mu_i]_{i=1,2}$ is a 2×1 vector of drifts and $\boldsymbol{\Phi}_l = [\phi_{ij}^{(l)}]_{i,j=1,2}$, $l = 1, 2$, is a 2×2 matrix of autoregressive parameters. We assume that the roots of $\left|\mathbf{I} - \sum_{l=1}^{2} \boldsymbol{\Phi}_l L^l\right|$ (where $\mathbf{I}$ is the 2×2 identity matrix) lie outside the unit circle.

Let $\mathbf{h}_t = (h_{1,t}, h_{2,t})'$ denote the 2×1 vector of $\mathcal{F}_{t-1}$ measurable conditional variances. The residual vector is defined as $\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t})' = [\mathbf{e}_t \odot \mathbf{q}_t^{\wedge -1/2}] \odot \mathbf{h}_t^{\wedge 1/2}$, where the symbols $\odot$ and $^\wedge$ denote the Hadamard product and the elementwise exponentiation respectively. The stochastic vector $\mathbf{e}_t = (e_{1,t}, e_{2,t})'$ is assumed to be independently and identically distributed (*i.i.d.*) with mean zero, conditional variance vector $\mathbf{q}_t = (q_{11,t}, q_{22,t})'$, and 2×2 conditional correlation matrix $\mathbf{R}_t = diag\{\mathbf{Q}_t\}^{-1/2} \mathbf{Q}_t diag\{\mathbf{Q}_t\}^{-1/2}$ with diagonal elements equal to one and off-diagonal elements absolutely less than one. A typical element of $\mathbf{R}_t$ takes the form $\rho_{ij,t} = q_{ij,t} / \sqrt{q_{ii,t}\, q_{jj,t}}$ for $i, j = 1, 2$. The conditional covariance matrix $\mathbf{Q}_t = [q_{ij,t}]_{i,j=1,2}$ is specified as in Engle (2002)

$$\mathbf{Q}_t = (1 - \alpha_D - \beta_D)\bar{Q} + \alpha_D \mathbf{e}_{t-1}\mathbf{e}_{t-1}' + \beta_D \mathbf{Q}_{t-1}, \tag{36}$$

where $\bar{Q}$ is the unconditional covariance matrix of $\mathbf{e}_t$, and α_D and β_D are non-negative scalars fulfilling $\alpha_D + \beta_D < 1$.

Following Conrad and Karanasos (2010) and Rittler (2012), we impose the UEDCC-AGARCH(1, 1) structure on the conditional variances (multivariate fractionally integrated APARCH models could also be used, as in Conrad et al., 2011 or Karanasos et al., 2014), and we also amend it by allowing the shock and volatility spillovers parameters to be time varying:

$$\mathbf{h}_t \quad = \quad \boldsymbol{\omega}+\mathbf{A}^{*}\boldsymbol{\varepsilon}_{t-1}^{\wedge 2}+\sum_{l=1}^{n}\mathbf{A}_l D_l \boldsymbol{\varepsilon}_{t-1}^{\wedge 2}+\mathbf{B}\mathbf{h}_{t-1}+\sum_{l=1}^{n}\mathbf{B}_l D_l \mathbf{h}_{t-1}, \tag{37}$$

where $\boldsymbol{\omega}=[\omega_i]_{i=1,2}$, $\mathbf{A}=[\alpha_{ij}]_{i,j=1,2}$, $\mathbf{B}=[\beta_{ij}]_{i,j=1,2}$; $\mathbf{A}_l$, $l=1,\ldots,n$ (and $n=0,1,\ldots,7$) is a cross diagonal matrix with nonzero elements α_{ij}^{l}, $i,j=1,2$, $i\neq j$, and $\mathbf{B}_l$, is a cross diagonal matrix with nonzero elements β_{ij}^{l}, $i,j=1,2$, $i\neq j$; $\mathbf{A}^{*}=\mathbf{A}+\mathbf{\Gamma}\mathbf{S}_{t-1}$, $\mathbf{\Gamma}$ is a diagonal matrix with elements γ_{ii}, $i=1,2$, and $\mathbf{S}_{t-1}$ is a diagonal matrix with elements $S_{i,t-1}^{-}=1$ if $e_{i,t-1}<0$, 0 otherwise. The model without the breaks for the shock and volatility spillovers, that is $\mathbf{h}_t=\boldsymbol{\omega}+\mathbf{A}^{*}\boldsymbol{\varepsilon}_{t-1}^{\wedge 2}+\mathbf{B}\mathbf{h}_{t-1}$, is minimal in the sense of Jeantheau (1998, Definition 3.3) and invertible (see Assumption 2 in Conrad and Karanasos, 2010). The invertibility condition implies that the inverse roots of $|\mathbf{I}-\mathbf{B}L|$, denoted by φ_1 and φ_2, lie inside the unit circle. Following Conrad and Karanasos (2010) we also impose the four conditions which are necessary and sufficient for $\mathbf{h}_t\succ 0$ for all t: (i) $(1-b_{22})\omega_1+b_{12}\omega_2>0$ and $(1-b_{11})\omega_2+b_{21}\omega_1>0$, (ii) φ_1 is real and $\varphi_1>|\varphi_2|$, (iii) $\mathbf{A}^{*}\succeq 0$ and (iv) $[\mathbf{B}-\max(\varphi_2,0)\mathbf{I}]\mathbf{A}^{*}\succ 0$, where the symbol $\succ$ denotes the elementwise inequality operator. Note that these constraints do not place any *a priori* restrictions on the signs of the coefficients in the $\mathbf{B}$ matrix. In particular, these constraints imply that negative volatility spillovers are possible. When the conditional correlations are constant, the model reduces to the UECCC-GARCH$(1,1)$ specification of Conrad and Karanasos (2010).

Finally, we also amend the UEDCC-AGARCH$(1,1)$ model by allowing shocks and volatility spillovers to vary across positive and negative returns:

$$\mathbf{h}_t=\boldsymbol{\omega}+\mathbf{A}^{*}\boldsymbol{\varepsilon}_{t-1}^{\wedge 2}+\mathbf{B}^{*}\mathbf{h}_{t-1},$$

where $\mathbf{A}^{*}=\mathbf{A}+\mathbf{\Gamma}\mathbf{S}_{t-1}+\mathbf{A}^{-}\mathbf{D}_{t-1}^{-}$ and $\mathbf{B}^{*}=\mathbf{B}+\mathbf{B}^{+}\mathbf{D}_{t-1}^{+}$; $\mathbf{A}^{-}(\mathbf{B}^{+})$ is a cross diagonal matrix with nonzero elements $\alpha_{ij}^{-}(\beta_{ij}^{+})$, $i,j=1,2$, $i\neq j$; $\mathbf{D}_t^{-}(\mathbf{D}_t^{+})$ are 2×1 vectors with elements $d_{it}^{-}(d_{it}^{+})$, $i=1,2$, where $d_{it}^{-}(d_{it}^{+})$ is one if $r_{jt}<0$ $(r_{jt}>0)$ and zero otherwise, $j=1,2$, $j\neq i$.

6.1 Bivariate Results

Example 1: FTSE-DAX

Table 6 reports the results of the UEDCC-AGARCH$(1,1)$ model between the returns on FTSE and DAX allowing shock and volatility spillover parameters to shift across the breaks in order to analyze the time varying volatility transmission structure between the two variables.[22] As is evident from Table 6, the results suggest the existence of strong conditional heteroscedasticity in the two variables. The ARCH as well as the asymmetry parameters of the two variables are positive and significant, indicating the existence of asymmetric responses in the two variables. In addition, rejection of the model with constant conditional correlation, using Tse's (2000) test, indicates the time varying conditional correlation between the two financial markets. Figure 3 displays the evolution of the time varying conditional correlation between the two variables over the sample period.

Furthermore, the results suggest that there is evidence of shock spillovers as well as negative volatility spillovers from DAX to FTSE (the α_{12} and β_{12} coefficients are significant at the 1% and 10% levels, respectively).[23] With regard to the impact of the breaks on the volatility transmission structure, it is shown that both shock and volatility spillovers between the two variables change over time. The most significant changes include the impact of the fourth break in DAX (15/01/2008), which corresponds to the global financial crisis, in which it shifts the shock spillovers parameter from DAX to FTSE (the α_{12}^4 coefficient is significant at the 1% level). Also, volatility spillovers from DAX to FTSE are shown to be shifted after the second (21/07/1997) and the third break (17/06/2003), corresponding to the Asian financial crisis and the announcement of the €18bn German tax cuts plan, respectively (see the β_{12}^2 and β_{12}^3 coefficients in Table 6).

These results are consistent with the time varying conditional correlations. The average time varying conditional correlation for the period before the break 15/01/2008 is 0.58 compared to the period after the break of 0.89. This also applies for the break 21/07/1997 (17/06/2003) with an average time varying correlation of 0.43 (0.52) for the period before the break and 0.75 (0.82) for the period after the break. Overall these findings are indicative of the existence of contagion between DAX and FTSE during the turbulent periods of the two financial crises.

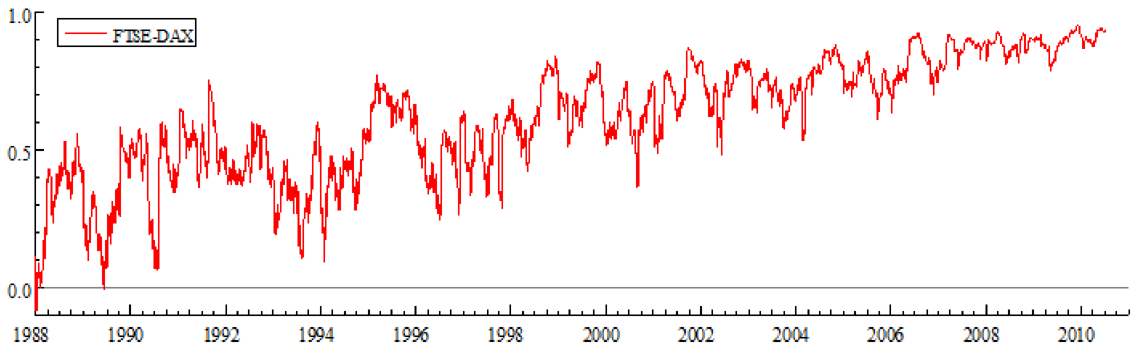


Figure 3. Evolution of the dynamic conditional correlation between FTSE and DAX returns.

[22] For an application on the returns of commodity metal futures see Karanasos et al. (2013).

[23] The results for the conventional UEDCC-GARCH(1, 1) process are available upon request. For this model the stationarity condition of Engle (2002) is satisfied over time.

Another way to look at the structure of the volatility spillovers between DAX and FTSE is to allow volatility (and shock) spillover parameters to shift across two regimes of stock returns: positive (increases in the stock market) and negative (declines in the stock market) returns. The results, displayed in Table 7, suggest that declines in each market generate shock spillovers to the other (the coefficients α_{12}^{-} and α_{21}^{-} are positive and significant), whilst increases in each market generate negative volatility spillovers to the other (the coefficients β_{12}^{+} and β_{21}^{+} are negative and significant).

Example 2: NIKKEI-Hang Seng

Next, we consider the structure of the volatility spillovers between the returns on NIKKEI and Hang Seng to provide an example about the dynamic linkages between the Asian financial markets. The estimated bivariate model, reported in Table 8, suggests the existence of strong conditional heteroscedasticity. There is evidence of asymmetric effects of the two variables as the ARCH and asymmetry parameters (the α and the γ coefficients) are positive and significant. Furthermore, the model with constant conditional correlation is rejected according to Tse's (2000) test, hence the correlation between the two variables is time varying. This is also confirmed by Figure 4, which shows the evolution of the time varying correlation between the two variables.

With regard to the linkages between the two variables, the results show the existence of shock spillovers from Hang Seng to NIKKEI after the third (05/05/2009) and the fourth break (01/12/2009), which correspond to the different phases of the European sovereign-debt crisis. Also, while Hang Seng generates negative volatility spillovers to NIKKEI after the third break in the former (05/05/2009), there are positive volatility spillovers from NIKKEI to Hang Seng after the second break (04/01/2008) in the former, which corresponds to the global financial crisis. These findings indicate the superiority of the time varying spillover model over the conventional one. In contrast to the conventional model, allowing for breaks shows that the two financial markets have been integrated during the global financial crisis.[24]

With regard to the time varying conditional correlations, the average time varying conditional correlation for the period before the breaks 04/01/2008, 05/05/2009, and 01/12/2009 are respectively $0.40, 0.41$, and 0.415 compared to the period after the breaks of $0.60, 0.58$, and 0.585, respectively. These results are consistent with those of volatility spillovers in which these two types of markets have become more dependent during the recent financial crisis.

[24] The results from the conventional bivariate UEDCC-AGARCH$(1,1)$ process indicate that there is no evidence of volatility spillovers between the two financial markets (they are available upon request). For this model the stationarity condition of Engle (2002) is fulfilled.

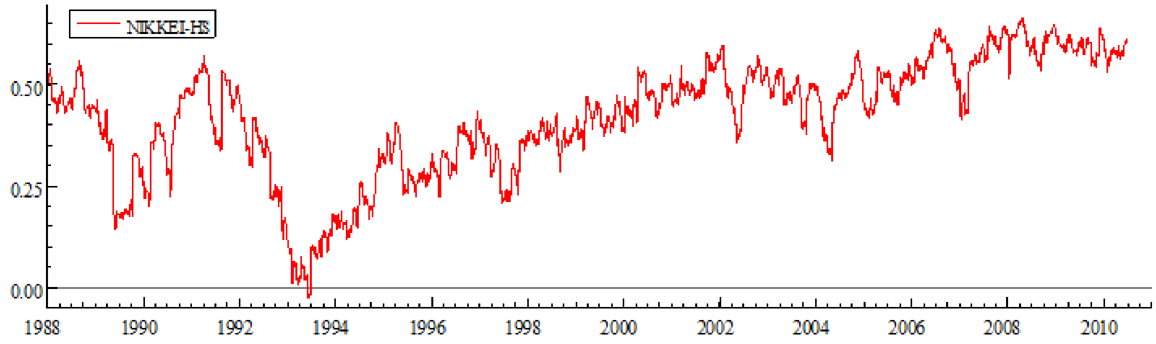


Figure 4. Evolution of the dynamic conditional correlation between NIKKEI and HS returns.

Finally, allowing the volatility spillover structure to shift across two different regimes, that is, positive and negative returns, also shows the existence of time varying volatility spillovers between the two variables. Specifically, the results, displayed in Table 9, suggest that declines in NIKKEI generate shock spillovers to Hang Seng (the estimated α_{21}^{-} coefficient is positive and significant), whilst increases in NIKKEI generate negative volatility spillovers to Hang Seng (the estimated β_{21}^{+} coefficient is negative and significant).

7 Summary and Conclusions

In this paper, we have introduced a platform to examine empirically the link between financial crises and the principal time series properties of the underlying series. We have also adopted several models, both univariate and bivariate, to examine how the mean and volatility dynamics, including the volatility persistence and volatility spillovers structure of stock market returns have changed due to the recent financial crises and conditioned our analysis on non-parametrically identified breaks. Overall, our findings are consistent with the intuitively familiar albeit empirically hard-to-prove time varying nature of asset market linkages induced by economic events and suggest the existence of limited diversification opportunities for investors, especially during turbulent periods.

In particular, with respect to the mean and volatility dynamics our findings suggest that in general the financial crises clearly affect more the (un)conditional variances. Also, the results of the volatility persistence are clear-cut and suggest that they exhibit substantial time variation. This time variation applies to all stock market returns irrespective of whether we allow for structural changes or positive and negative changes in the underlying market. As far as the direction of this time variation during financial crises is concerned the jury is still out, but there is little doubt that the financial crises are the primary driving force behind the profound changes in the unconditional variances.

Finally, with respect to the existence of dynamic correlations as well as time varying shock and volatility spillovers our findings are also conclusive. Specifically, they suggest that in the cases we examine there is an increase in conditional correlations, occurring at different phases of the various financial crises, hence providing evidence as to the existence of contagion during these periods. Such a finding is comparable to those of other studies using only conditional correlation analysis to examine the existence of contagion during the various financial crises. The results also suggest the existence of regime dependent volatility spillovers in all cases we examine by using two regimes of returns, positive and negative. Given that this is to our knowledge the first attempt to take into account the joint effect of dynamic correlations, volatility spillovers and structural breaks in the mean and/or volatility dynamics, these findings are of particular interest to those seeking refuge from financial crises.

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Table 1

The break points (Stock Returns)								
Break	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
1	**27/03/97**	**05/11/96**	**17/03/97**	27/08/91	**22/10/92**	**24/10/01**	**21/02/90**	**26/08/91**
2	**04/09/08**	15/01/08	**31/07/98**	**21/07/97**	**13/07/98**	**27/07/07**	04/01/08	**28/08/97**
3	**31/03/09**	**02/04/09**	**15/01/08**	**17/06/03**	**24/07/07**	05/05/09	03/04/09	**06/06/00**
4	16/07/09	**19/08/09**	03/04/09	**15/01/08**	06/04/09	**01/12/09**		**26/07/07**
5	27/04/10		27/04/10	03/04/09	27/04/10			**28/05/09**
6								**25/08/09**
7								28/04/10

Notes: The dates in bold indicate breakdates for which, in the univariate estimation (see Table 2), at least one dummy variable is significant, i.e, for the S&P index for the 04/09/08 breakdate β_2 and γ_2 are significant. The underlined dates indicate breakdates for which, in the bivariate estimation (see Tables 6 and 8), at least one dummy variable is significant, i.e., for the NIKKEI-Hang Seng bivariate model, for the 01/12/09 breakdate α_{12}^4 is significant.

Table 2

The estimated univariate AGARCH (1,1) allowing for breaks in the variance								
	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
μ	0.012^a (0.004)	0.011^a (0.003)	0.010^c (0.006)	0.019^a (0.005)	0.009^b (0.004)	0.019^a (0.005)	0.006 (0.005)	0.010^b (0.005)
ϕ_1		0.129^a (0.013)				0.079^a (0.014)		0.124^a (0.016)
ω	0.001^c (0.0002)	0.003^a (0.0007)	0.005^a (0.0004)	0.011^a (0.0006)	0.002^a (0.0003)	0.015^a (0.003)	0.007^a (0.001)	0.018^a (0.004)
α	0.018^a (0.006)	0.012^c (0.007)	0.006^b (0.003)	0.031^a (0.006)	0.013^a (0.004)	0.039^a (0.007)	0.019^a (0.005)	0.018^c (0.010)
α_1	-0.039^a (0.008)					-0.050^a (0.011)		0.059^a (0.013)
α_2			0.011^c (0.006)			0.068^a (0.014)		
α_3			-0.044^a (0.016)	-0.050^a (0.011)				
β	0.954^a (0.002)	0.906^a (0.016)	0.936^a (0.003)	0.861^a (0.002)	0.952^a (0.001)	0.866^a (0.013)	0.820^a (0.026)	0.854^a (0.011)
β_1					-0.019^a (0.002)		0.081^a (0.021)	-0.112^a (0.029)
β_2	-0.048^a (0.009)		-0.031^a (0.003)	0.029^a (0.007)	-0.019^a (0.006)			0.115^a (0.029)
β_3	0.039^a (0.015)	0.017^c (0.009)		-0.029^b (0.012)				-0.076^a (0.018)
β_4		-0.025^c (0.013)		0.038^a (0.006)				0.137^a (0.029)
γ	0.023^c (0.012)	0.028^a (0.009)	0.056^a (0.004)	0.117^a (0.023)	0.029^a (0.006)	0.130^a (0.021)	0.117^a (0.013)	0.105^a (0.017)
γ_1	0.092^a (0.014)	0.097^a (0.023)	0.035^a (0.007)		0.028^a (0.005)			
γ_2	0.113^a (0.027)		0.019^b (0.009)		0.055^a (0.016)			
γ_3	-0.094^a (0.029)		0.117^a (0.038)	0.075^c (0.043)	0.026^b (0.012)			
$LogL$	−2921.3	−1837.5	−4374.3	−4469.8	−2904.1	−5231.4	−4764.1	−3957.7
$LB(5)$	8.343 [0.138]	2.316 [0.128]	10.870 [0.054]	5.170 [0.395]	9.745 [0.082]	2.928 [0.231]	2.555 [0.768]	3.303 [0.069]
$LB^2(5)$	1.947 [0.856]	0.759 [0.979]	3.953 [0.556]	5.524 [0.354]	4.192 [0.522]	4.105 [0.534]	8.992 [0.109]	1.635 [0.897]

Notes: Robust-standard errors are used in parentheses. $LB(5)$ and $LB^2(5)$ are Ljung-Box tests for serial correlations of five lags on the standardized and squared standardized residuals, respectively (p-values reported in brackets). Insignificant parameters are excluded. ^{a}, ^{b}, and ^{c} indicate significance at the 1%, 5%, and 10% levels, respectively. For the Hang Seng index ϕ_3 and γ_4 are significant, and for the STRAITS index $\alpha_4, \alpha_6, \beta_6, \gamma_5$, and γ_6 are also significant.

Table 3

The estimated univariate GARCH (1, 1) models allowing for different persistence across positive and negative returns: $h_t = \omega + \omega^- D_{t-1}^- + \alpha\varepsilon_{t-1}^2 + \alpha^- D_{t-1}^-\varepsilon_{t-1}^2 + \beta h_{t-1} + \beta^- D_{t-1}^- h_{t-1}$

	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
μ	0.036^a (0.005)	0.023^a (0.004)	0.044^a (0.007)	0.054^a (0.008)	0.032^a (0.004)	0.051^a (0.007)	0.034^a (0.007)	0.027^a (0.004)
ϕ_1		0.114^a (0.012)				0.069^a (0.013)		0.112^a (0.011)
ω	0.002^a (0.0008)	0.002^a (0.0006)	0.007^a (0.001)	0.008^a (0.002)	0.002^a (0.0005)	0.009^a (0.002)	0.004^a (0.0008)	0.006^a (0.002)
α	0.054^a (0.005)	0.062^a (0.012)	0.070^a (0.008)	0.091^a (0.018)	0.066^a (0.006)	0.088^a (0.011)	0.065^a (0.008)	0.051^a (0.015)
α^-		0.033^c (0.017)				0.033^c (0.020)	0.025^c (0.015)	0.104^a (0.021)
β	0.837^a (0.023)	0.861^a (0.027)	0.822^a (0.023)	0.779^a (0.039)	0.832^a (0.014)	0.815^a (0.025)	0.842^a (0.016)	0.883^a (0.023)
β^-	0.208^a (0.034)	0.106^a (0.024)	0.181^a (0.029)	0.233^a (0.043)	0.187^a (0.023)	0.141^a (0.037)	0.157^a (0.027)	
$LogL$	-2941.2	-1865.7	-4388.4	-4478.8	-2903.4	-5260.7	-4799.1	-4048.6
$LB(5)$	9.526 [0.089]	1.674 [0.195]	3.256 [0.071]	4.464 [0.484]	8.031 [0.154]	4.521 [0.104]	2.180 [0.823]	3.650 [0.056]
$LB^2(5)$	2.398 [0.791]	0.573 [0.989]	4.237 [0.515]	5.340 [0.375]	5.428 [0.365]	4.998 [0.416]	8.430 [0.134]	2.385 [0.793]

Notes: See notes of Table 2. The ϕ_3 coefficient was significant for the CAC and Hang Seng indices.

Table 4

The persistence of the AGARCH (1,1) models

The persistence of the standard AGARCH (1,1) models

S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
0.986	0.986	0.978	0.979	0.985	0.976	0.990	0.990

The persistence of the AGARCH (1,1) allowing for breaks in the variance

Break	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
0	$(\overline{c}_4 =)0.983$	0.932	0.970	$(\overline{c}_4 =)0.950$	0.979	0.970	0.897	0.924
1	$(\overline{c}_3 =)0.990$	0.980	0.987		0.974	0.920	0.978	0.871
2	$(\overline{c}_2 =)0.998$		0.976	$(\overline{c}_3 =)0.979$	0.982	0.988		0.986
3	$(\overline{c}_1 =)0.990$	0.997	0.990	$(\overline{c}_2 =)0.937$	0.995			0.910
4		0.972		$(\overline{c}_1 =)0.976$		0.945		0.974
5								0.948
6								0.884

Notes: Break 0 covers the period preceding all breaks, while break 1 covers the period between break 1 and 2, and break 2 covers the period between break 2 and 3, and so on (see Table 1 for the dates of the breaks). When the value of the persistence is left blank for a break, it indicates that such persistence has not changed during the period covered by such a break. The persistence is measured by $\overline{c}_\ell = \alpha_\ell + \beta_\ell + \gamma_\ell/2$, $\ell = 1, \ldots, m+1$, and, for example, $\beta_\ell = \underbrace{\beta + \sum_{i=1}^{m+1-\ell} \beta_i}_{\text{Eq. (33)}}$. That is $\overline{c}_{m+1}$ is the persistence before all breaks, and $\overline{c}_1$ is the persistence after all the breaks.

Table 5

The persistence of the GARCH (1,1) allowing for different persistence across positive and negative returns								
Break	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
r	0.986	0.986	0.978	0.979	0.985	0.976	0.990	0.990
r^+	0.891	0.923	0.892	0.870	0.898	0.903	0.907	0.934
r^-	0.995	0.992	0.982	0.986	0.991	0.990	0.998	0.986

Notes: r denotes the persistence generated from returns, that is from the standard AGARCH model whilst $r^+(r^-)$ corresponds to the persistence generated from positive (negative) returns.

Table 6

Coefficient Estimates of Bivariate UEDCC-AGARCH Models Allowing for Shifts in Volatility Spillovers between FTSE and DAX					
Conditional Variance Equation					
ω_1	0.003^a (0.0006)	γ_{11}	0.078^a (0.016)	β_{12}^3	-0.007^a (0.002)
ω_2	0.004^a (0.001)	γ_{22}	0.082^a (0.022)	α_D	0.044^a (0.010)
α_{11}	0.016^b (0.007)	α_{12}	0.010^a (0.003)	β_D	0.952^a (0.011)
α_{22}	0.033^a (0.009)	α_{12}^4	0.011^a (0.004)		
β_{11}	0.921^a (0.014)	β_{12}	-0.007^c (0.003)		
β_{22}	0.912^a (0.015)	β_{12}^2	0.003^a (0.001)		
$LogL$	-5427.03				
$Q(5)$	27.970 [0.110]	$Q^2(5)$	9.427 [0.977]		

Notes: Robust-standard errors are used in parentheses, 1= FTSE, 2=DAX. $Q(5)$ and $Q^2(5)$ are the multivariate Hosking (1981) tests for serial correlation of five lags on the standardized and squared standardized residuals, respectively (p-values are reported in brackets). $\alpha_{12}(\beta_{12})$ indicates shock (volatility) spillovers from DAX to FTSE, while $\alpha_{12}^l(\beta_{12}^l)$ indicates the shift in shock (volatility) spillovers for the break l (see Table 1) from DAX to FTSE. Insignificant parameters are excluded.^{a} , ^{b} and ^{c} indicate significance at the 1%, 5%, and 10% levels, respectively. Tse's (2000) test for constant conditional correlation: 20.41.

Table 7

Coefficient Estimates of Bivariate UEDCC-AGARCH Models Allowing for Different Spillovers Across Positive and Negative Returns (FTSE-DAX)

Conditional Variance Equation

ω_1	0.002^a (0.0005)	γ_{11}	0.058^a (0.012)	α_D	0.043^a (0.010)
ω_2	0.004^a (0.001)	γ_{22}	0.060^a (0.016)	β_D	0.954^a (0.011)
α_{11}	0.030^a (0.008)	α_{12}^-	0.019^a (0.005)		
α_{22}	0.027^a (0.008)	β_{12}^+	-0.014^a (0.004)		
β_{11}	0.926^a (0.012)	α_{21}^-	0.042^a (0.015)		
β_{22}	0.928^a (0.012)	β_{21}^+	-0.036^a (0.016)		
$LogL$	-5430.26				
$Q(5)$	26.965 [0.136]	$Q^2(5)$	9.533 [0.975]		

Notes: Robust-standard errors are used in parentheses, 1= FTSE, 2=DAX. $Q(5)$ and $Q^2(5)$ are the multivariate Hosking (1981) tests for serial correlation of five lags on the standardized and squared standardized residuals, respectively (p-values reported in brackets). $\alpha_{12}^-(\beta_{12}^+)$ indicates the shock (volatility) spillovers from DAX to FTSE generated by negative(positive) returns in DAX.. $\alpha_{21}^-(\beta_{21}^+)$ reports shock (volatility) spillovers form FTSE to DAX generated by negative(positive) returns in FTSE. Insignificant parameters are excluded.

^{a} indicates significance at the 1% level.

Table 8

Coefficient Estimates of Bivariate UEDCC-AGARCH Models Allowing for Shifts in Volatility Spillovers between NIKKEI and Hang Seng

Conditional Variance Equation

ω_1	0.003^a (0.0008)	γ_{11}	0.094^a (0.012)	α_D	0.015^a (0.005)
ω_2	0.009^a (0.002)	γ_{22}	0.081^a (0.021)	β_D	0.982^a (0.006)
α_{11}	0.024^a (0.004)	α_{12}^3	0.050^a (0.017)		
α_{22}	0.050^a (0.007)	α_{12}^4	0.025^b (0.011)		
β_{11}	0.920^a (0.007)	β_{12}^3	-0.046^a (0.015)		
β_{22}	0.885^a (0.015)	β_{21}^2	0.016^c (0.009)		
$LogL$	-9413.42	Tse's test:	10.10		
$Q(5)$	22.122 [0.333]	$Q^2(5)$	13.594 [0.850]		

Notes: Robust-standard errors are used in the parentheses, 1= NIKKEI, 2=Hang Seng. $Q(5)$ and $Q^2(5)$ are the multivariate Hosking (1981) tests for serial correlation of five lags on the standardized and squared standardized residuals, respectively (p-values are reported in brackets). direction. $\alpha_{12}^l(\beta_{12}^l)$ indicates shift in shock (volatility) spillovers for the break l (see Table 1) from Hang Seng to NIKKEI, whilst β_{21}^l reports the shift in volatility spillovers for the break l in the reverse direction. Insignificant parameters are excluded. ^{a} , ^{b} and ^{c} indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 9

Coefficient Estimates of Bivariate UEDCC-AGARCH Models Allowing for Different Spillovers Across Positive and Negative Returns (NIKKEI-Hang Seng)

Conditional Variance Equation

ω_1	0.003^a (0.0009)	β_{11}	0.917^a (0.007)	α_{21}^-	0.017^a (0.009)
ω_2	0.008^a (0.002)	β_{22}	0.897^a (0.013)	β_{21}^+	-0.018^a (0.008)
α_{11}	0.027^a (0.005)	γ_{11}	0.099^a (0.015)	α_D	0.016^a (0.007)
α_{22}	0.052^a (0.007)	γ_{22}	0.065^a (0.019)	β_D	0.980^a (0.010)
$LogL$	-9414.61				
$Q(5)$	22.918 [0.292]	$Q^2(5)$	9.534 [0.975]		

Notes: Robust-standard errors are used in parentheses, 1= NIKKEI, 2=Hang Seng. $Q(5)$ and $Q^2(5)$ are the multivariate Hosking (1981) tests for serial correlation of five lags on the standardized and squared standardized residuals, respectively (p-values are reported in brackets). $\alpha_{21}^-(\beta_{21}^+)$ reports shock (volatility) spillovers from NIKKEI to Hang Seng generated by negative(positive) returns in NIKKEI. Insignificant parameters are excluded. ^{a} indicates significance at the 1% level.