

UNCERTAINTY, NON-LINEAR  
CONTAGION AND THE CREDIT QUALITY  
CHANNEL: AN APPLICATION TO  
THE SPANISH INTERBANK MARKET

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# UNCERTAINTY, NON-LINEAR CONTAGION AND THE CREDIT QUALITY CHANNEL: AN APPLICATION TO THE SPANISH INTERBANK MARKET (\*)

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## Abstract

Using granular data from the Spanish Central Credit Register, we study the contagion of financial distress via the credit quality channel in the Spanish interbank market. We propose a non-linear contagion mechanism dependent on banks' balance-sheet structure (specifically, their leverage ratios). Moreover, we explicitly model uncertainty in lenders' assessments of the probability of default of their borrowers, thus incorporating agents' lack of complete information and heterogeneous expectations in their assessment of future outcomes. We perform multiple simulations across a wide range of possible levels of stress in the system, and we focus on disentangling the effects of these two key model components by comparing the results of our model with those of a linear and deterministic counterpart. We find that non-linear contagion leads to substantially larger losses than its linear counterpart for a wide range of intermediate levels of stress in the system, while its effects become negligible for very low and very high stress levels. Regarding uncertainty, we find that its effects, while smaller than those of non-linear contagion, are nonetheless relevant and most important around levels of stress at which different parts of the system become unstable. Interestingly, losses can be amplified or mitigated with respect to the deterministic case depending on the specific level of stress considered. Finally, the interaction between these two model components – non-linear contagion and uncertainty – alters the area where uncertainty matters.

**Keywords:** financial networks, systemic risk, financial contagion, macroprudential policy, stress testing.

**JEL classification:** C63, D80, D85, G01, G17, L14.

## Resumen

Utilizando datos granulares de la Central de Información de Riesgos del Banco de España, estudiamos uno de los canales de contagio a través de los cuales se pueden transmitir las tensiones en los mercados financieros —el canal de la calidad crediticia—, enfocándonos en el mercado interbancario español. Proponemos un mecanismo de contagio no lineal, que depende de la estructura del balance bancario (específicamente, de la ratio de apalancamiento). Adicionalmente, modelizamos explícitamente la incertidumbre que rodea los análisis de los prestamistas acerca de la probabilidad de incumplimiento de sus prestatarios, incorporando así la ausencia de información completa en los mercados y las expectativas heterogéneas de los agentes en su evaluación de resultados futuros. Realizamos múltiples simulaciones que reflejan una amplia gama de posibles grados de tensión del sistema bancario y buscamos desentrañar los efectos de estos dos componentes clave del modelo al comparar nuestros resultados con los que se obtendrían empleando un modelo lineal y determinista. De esta forma, encontramos que, bajo el supuesto de no linealidad, las pérdidas son sustancialmente mayores para amplios grados intermedios de tensión del sistema, mientras que sus efectos se vuelven insignificantes para grados de tensión tanto muy bajos como muy altos. Asimismo, encontramos que incorporar los efectos de la incertidumbre genera un impacto menor relativo al supuesto de no linealidad, pero aun así relevante y más marcado en torno a los grados de tensión en los que diferentes partes del sistema se vuelven inestables. Curiosamente, las pérdidas pueden verse amplificadas o mitigadas con respecto al caso determinista, dependiendo del grado específico de tensión considerado. Finalmente, la interacción entre ambos componentes del modelo —contagio no lineal e incertidumbre— modifica el área para la que la incertidumbre tiene un impacto relevante.

**Palabras clave:** redes financieras, riesgo sistémico, contagio financiero, política macroprudencial, pruebas de esfuerzo.

**Códigos JEL:** C63, D80, D85, G01, G17, L14.

## 1. Introduction

Interbank markets are an essential component of modern financial systems. In particular, they are important for the liquidity management of credit institutions, allowing for an efficient transfer of liquidity from surplus banks to those in need of more liquidity (Acharya et al., 2012; Bianchi and Bigio, 2014). Furthermore, they are also relevant for risk sharing, allowing banks to diversify risks and reduce their exposure to their own idiosyncratic shocks (Allen and Gale, 2000; Freixas et al., 2000). Finally, interbank markets also play a role in the transmission of monetary policy, which, due to market imperfections, has a stronger impact on banks with less liquid balance sheets (Ehrmann and Worms, 2004; Freixas and Jorge, 2008). However, during crises, rising counterparty credit risk and liquidity hoarding can lead to highly stressed or almost frozen interbank markets, thus preventing efficient risk sharing and liquidity management (Afonso et al., 2011). In fact, the same exposures and interlinkages which help diversify risks are also the channel through which the contagion of financial distress, or even default, can occur (Iyer and Peydró, 2011). Moreover, an event that is not systemic at its origin can become systemic due to contagion and amplification via financial interlinkages (Freixas et al., 2000). Thus, interbank markets are crucial for understanding systemic risk and monitoring financial stability.

Due to the focus on the interlinkages between entities, it is natural to think of the interbank market in terms of a network, where the credit institutions are represented by the nodes of the network and their bilateral exposures by the edges. In this sense, since the beginning of the 2000s there has been a growing literature on financial networks and systemic risk, focused on understanding the propagation and amplification of shocks through the network of financial interlinkages (Bougheas and Kirman, 2015; Glasserman and Young, 2016; Caccioli et al., 2018; Jackson and Pernoud, 2021). One of the main goals of this literature has been to identify the specific structural features of the network at the origin of systemic risk. This trend was further bolstered by the Global Financial Crisis of 2007-2008, after which a number of policy-makers and regulators have been calling for a network-based rethink of the financial system, thereby suggesting to move beyond understanding it as a collection of individual entities and towards a view that incorporates also the connections and interactions between these entities (Haldane, 2009a,b; Haldane and May, 2011; BCBS, 2015). This type of analysis allows to capture, first of all, second-round effects, that is, indirect interactions between entities not necessarily directly connected, which escapes any attempt at studying the system as a collection of independent entities or as a collection of disconnected bilateral contracts. Secondly, by its very nature, it incorporates the amplification due to entities sitting on multiple contagion paths within the network. Finally, it allows for a truly systemic view, with a deeper understanding of which entities are central or peripheral given the whole structure of the network of interconnections. An important finding of this strand of research is that taking into account the structure of risk-bearing interlinkages is essential for assessing contagion potential in interbank markets, as it unveils impacts that cannot be captured by total volumes of exposures or liabilities, metrics commonly used as proxies for interconnectedness in regulatory exercises, such as the identification of systemic institutions (BCBS, 2013; EBA, 2014, 2020).

There are multiple ways in which shocks can spread through the network of interbank exposures. For instance, creditor banks will suffer losses in case their debtor banks default and are unable to meet some or all of their repayment commitments. Another example is the withdrawal of short-term liquidity by

creditor banks, whether because they suddenly need that liquidity or because of a loss of confidence in the corresponding debtor. Finally, a further example is given by pre-default write-downs of creditors' interbank assets due to adverse movements in the credit quality of the corresponding debtors, which constitutes the so-called *credit quality channel*, also known as distress contagion or ex-ante valuation of interbank assets (Battiston et al., 2012; Fink et al., 2016; Barucca et al., 2020). In this latter channel, which is the main focus of this paper, a negative shock to the equity of a debtor leads its creditors to increase their assessment of its probability of default and, as a consequence, to write down the corresponding interbank claim they hold, reflecting their updated expectation of the value of this claim at maturity. The extent of these creditor losses depends on their perception regarding the degree of deterioration of the balance sheet of the debtor, as well as on the specific way in which they reflect this deterioration into the value of the corresponding claims. Importantly, this channel allows for the contagion of financial distress even before the materialisation of a default.

The starting point for our analysis is a well-known model of financial contagion via the credit quality channel introduced by Battiston et al. (2012) as DebtRank. In this original framework, creditors are assumed to react linearly to negative movements in the valuation of the equity of their borrowers and these reactions are assumed to be independent of the initial balance sheet structure of the borrower. In reality, however, it seems more plausible that the size of the losses experienced by the borrower significantly affects the reaction of the corresponding creditors, with smaller losses being almost completely disregarded and larger losses leading to substantial overreactions. Furthermore, it also seems more reasonable that creditor reactions would greatly depend on the initial capitalisation of the borrower, in such a way that for poorly capitalised borrowers even small equity losses lead to strong creditor reactions while better capitalised entities are able to withstand larger losses before triggering any significant creditor reaction.

In the original DebtRank model, creditors are also assumed to form homogeneous and deterministic expectations about the probability of default of their borrowers. The implicit assumption is that they have access to complete and perfect information about the current financial position of these borrowers and that they all use the exact same model to derive, from that information, an assessment of their probability of default. In reality, however, actors participating in financial markets operate under incomplete information and bounded rationality. They do not have a full understanding of the entire system where they conduct business and often rely on partial evidence and conventional indicators in assessing the state of the system and forming expectations about the future (Farmer, 2002; Lo, 2005). These expectations impact agents' estimates of future outcomes such as the likelihood that certain debts will not be repaid. In fact, accounting standards require financial institutions to incorporate estimates regarding the creditworthiness of their counterparties in the value of their exposures and financial regulators set minimum capital requirements based on similar assessments.<sup>1</sup> Given the lack of complete and perfect information and the absence of a single and universally accepted model, there are multiple ways in which banks can estimate the future ability of their counterparties to honour their contracts and model relevant risk parameters, such as default probabilities or recovery rates. While financial regulations may include lower bounds for some of these parameters, they generally do not require the use of any specific model. Given the inherent uncertainty regarding economic outcomes in

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<sup>1</sup> To the extent that risk weighted assets are based on considerations regarding risk parameters, whether internally modelled or set by regulators.



general, it seems appropriate to explicitly model estimates of these risk parameters for a given borrower as stochastic variables, with a distribution of probability around a reasonable mean and heterogeneous draws for different creditors.

In this paper, we propose to account for these two behavioural features —non-linear creditor reactions and uncertainty about the creditworthiness of debtors— by introducing the following two extensions to the original DebtRank model. First, we introduce a non-linear contagion mechanism dependent on the balance-sheet structure of the borrower. In particular, apart from debtor equity losses, creditors also take into account the leverage ratio of these debtors when assessing their probability of default. In this way, equity losses are translated into increases of the probability of default of a given borrower depending on the relation between its total assets and liabilities, becoming more pronounced as this leverage approaches a critical threshold. Secondly, we introduce a further modification of the contagion mechanism taking into account an explicit level of uncertainty into creditors’ assessments of the probability of default of their borrowers. In particular, we add a stochastic component around the assessment derived from the original DebtRank model.

Using granular data from the Spanish Credit Register, we study potential contagion losses via the credit quality channel in the Spanish domestic interbank market.<sup>2</sup> To the best of our knowledge, this is the first application of a network-based model of distress contagion to the case of the Spanish banking system. We carry out this analysis taking into account varying degrees of financial stress in the banking system —measured by the system-wide leverage ratio— and explore by how much, under the different assumptions mentioned above, total system losses are amplified or dampened with respect to the results of the original (linear and deterministic) DebtRank model. In this sense, we find the non-linear contagion mechanism proposed to have the strongest effect, leading to substantially larger losses, as compared to its linear counterpart, for a wide range of intermediate levels of stress in the system, including levels not far from the one actually observed in the data. Therefore, ignoring non-linear creditor reactions risks dramatically underestimating total system losses for almost any reasonable level of stress above the current one. While uncertainty is found to have a weaker effect and only for higher levels of stress, its influence is nonetheless relevant, particularly around levels of stress at which certain parts of the network become unstable —in the sense of amplifying shocks until at least a bankruptcy occurs. Thus, from a regulatory and supervisory perspective, neglecting creditor uncertainty risks underestimating losses precisely at those moments when it is most important to have a precise assessment of potential contagion losses. Importantly, when both non-linear contagion and uncertainty are present, the influence of the latter is found to start from lower levels of stress, much closer to current market conditions. Therefore, in that case, even a small increase in system-wide stress from its current level can bring the system to the region where uncertainty has an important impact on losses. These results highlight the importance of taking into account both proposed features —non-linear reactions to counterparty losses dependent on counterparty leverage and uncertainty about the effect of these losses on counterparty default risk— when assessing potential contagion losses via the credit quality channel.

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<sup>2</sup> The Spanish banking sector is an interesting case study since it is highly heterogeneous: it includes large internationally active banks, but also small and medium-size domestic banks. One entity is also designated as a “Global Systemically Important Bank” (GSIB) according to the Basel framework.

The remainder of the paper is structured as follows. Section 2 provides a deeper overview of the relevant literature and it briefly describes our contributions. Section 3 characterises the data sources used in our study, providing a few preliminary insights about the Spanish domestic interbank market. Section 4 outlines our proposed model of distress contagion with non-linear reactions to counterparty losses and uncertainty about the effect of these losses on counterparty default risk. In particular, Subsection 4.1 reviews the original DebtRank framework we take as starting point for our analysis, Subsection 4.2 presents the non-linear contagion mechanism dependent on banks' balance-sheet structure, and Subsection 4.3 focuses on our proposal for introducing an explicit level of uncertainty into this framework. Section 5 presents the main results of our analysis, starting with the original DebtRank framework in Subsection 5.1, continuing with our two proposed extensions in Subsections 5.2 and 5.3, and finishing with the interaction between these two extensions in Subsection 5.4. Finally, some concluding remarks are presented in Section 6.

## 2. Literature

Financial networks have been extensively studied in the context of assessing systemic risk stemming from interconnectedness among different financial actors (Bougheas and Kirman, 2015; Bardoscia et al., 2021; Jackson and Pernoud, 2021). Various methodologies have been developed and research in the field has been growing rapidly over the past years, studying different channels of contagion and focusing on either direct connections, e.g. via interbank lending, or indirect relationships, e.g. via overlapping portfolios, or a combination of both (Glasserman and Young, 2016; Caccioli et al., 2018; Aymanns et al., 2018).

Inspired by the seminal paper by Eisenberg and Noe (2001), a number of initial contributions focused on assessing the likelihood and the extent of default cascades owing to debtors unable to repay their debts following a shock affecting a subset of participants in the interbank market. As these contributions show, the characteristics of these default cascades depend on the structure of the network, which can amplify or mitigate shocks (Gai and Kapadia, 2010; Haldane and May, 2011; Caccioli et al., 2012; Elliott et al., 2014; Acemoglu et al., 2015; Glasserman and Young, 2015; Amini et al., 2016). Furthermore, some of these works have proposed various extensions of the initial framework, considering, for instance, an exogenous liquidation cost at default (Mommel et al., 2012; Mommel and Sachs, 2013; Rogers and Veraart, 2013; Cont et al., 2013) or the effect of the central bank within a dynamic interbank network (Georg, 2013). A key feature of this strand of research is that financial distress is transmitted from a debtor to its creditors only if the debtor defaults, otherwise the shock is completely absorbed with the debtor's own capital, with no consequence for other banks. Thus, the strand is sometimes called default contagion.

Over the years, alternative contagion channels have been considered, such as short-term liquidity withdrawal (Gai et al., 2011; Hałaj, 2020), overlapping portfolios and fire sales (Cifuentes et al., 2005; Caccioli et al., 2014, 2015; Greenwood et al., 2015; Cont and Schaanning, 2017; Poledna et al., 2021) or credit quality deterioration, also known as distress contagion (Battiston et al., 2012; Bardoscia et al., 2015; Battiston et al., 2016; Bardoscia et al., 2017b). Importantly, most of these channels allow for the contagion of financial distress even before the materialisation of a default. In particular, the credit quality channel has received a great deal of attention since the Global Financial Crisis, and it has been proposed as an explanation for the large mark-to-market losses experienced during that crisis, estimated to have been larger than those derived from outright defaults (Glasserman and Young, 2015; BCBS, 2011). While most initial studies of

the credit quality channel focused on simple heuristic rules for contagion, later works have explored rules based on structural models of credit risk (Bardoscia et al., 2019) or even empirically calibrated mechanisms more closely linked to regulatory variables (Fink et al., 2016). As shown by Barucca et al. (2020), most models of default and distress contagion can be understood as special cases of a generalised framework for the balance-sheet consistent valuation of a network of interbank claims.

A number of recent contributions have focused on effectively using these network-based methods in concrete policy-oriented applications. Many of these works have also exploited the highly granular data sets recently available to national and international financial regulators and supervisors. For example, Batiz-Zuk et al. (2016) explore different policies limiting large exposures in the Mexican interbank market with a model of default contagion. Based also on Mexican data, Poledna et al. (2015) build a multi-layer network with different types of financial contracts and use a contagion model to show that focusing on a single layer dramatically underestimates potential contagion losses. Caceres-Santos et al. (2020) apply a distress contagion model to the Bolivian interbank market and study different centrality measures of the corresponding network. Siebenbrunner (2021) tries to assess the relative importance of different contagion channels, including both direct exposures and overlapping portfolios, within the Austrian banking system. Using data for the Euro Area banking system, many recent works have sought to build a methodology for macroprudential or system-wide stress tests, generally combining different contagion channels and often including various types of financial institutions (Budnik et al., 2019; Aldasoro et al., 2020; Covi et al., 2021; Montagna et al., 2021; Roncoroni et al., 2021a). Specifically, Farmer et al. (2020) underline the risk of severely underestimating optimal capital buffers if network effects are ignored in their calibration. A multi-layer model with various contagion channels and types of institutions is used by Kleinnijenhuis et al. (2021) to assess the financial stability implications of different bail-in designs. Using the economic shock caused by the Covid-19 outbreak as an example, Sydow et al. (2021) highlight the importance of including investment funds in any such stress test, due to their role in amplifying losses through fire sales. Finally, the development of network-based climate stress test methodologies has also received a great deal of attention over the last few years (Battiston et al., 2017; Battiston and Martinez-Jaramillo, 2018; Stolbova et al., 2018; Roncoroni et al., 2021b). Focusing on transition risk, these works simulate the propagation through the financial system of various climate policy-induced shocks.

Our contributions to this literature are threefold. First, we propose a novel non-linear contagion mechanism dependent on banks' balance-sheet structure. This part of our work is closely related to the simple non-linear contagion mechanism proposed by Bardoscia et al. (2016). However, both the specific functional form we introduce and its dependence on the balance-sheet structure of the borrower clearly set our work apart. Crucially, this dependence allows us to capture differences between poorly capitalised banks and those in a stronger capital position in how much they are likely to transmit to their creditors the financial distress they experience. Secondly, we incorporate uncertainty into lenders' assessments of the probability of default of their borrowers, thereby allowing for heterogeneity in these assessments. In this regard, our work is also inspired by the different strands of literature having explored the reasons and consequences of imperfect information and heterogeneous expectations (Akerlof, 1978; Rothschild and Stiglitz, 1978; Stiglitz and Weiss, 1981; Woodford, 2001; Sims, 2003; Hommes, 2011; Coibion and Gorodnichenko, 2012, 2015). Finally, to the best of our knowledge, ours is the first study to use highly granular and virtually complete

data from the Spanish domestic interbank market within a network-based model of distress contagion.<sup>3</sup> In this sense, our work is close to similar single-country studies such as Caccioli et al. (2015), Diem et al. (2020) and Siebenbrunner (2021) for Austria, Bardoscia et al. (2017a) for the UK, Caceres-Santos et al. (2020) for Bolivia or Poledna et al. (2021) for Mexico.

### 3. Data

This section briefly describes the different data sources required for the model to represent the Spanish interbank market, thus allowing us to simulate the contagion dynamics triggered by the arrival of an exogenous shock to this specific system. In particular, we rely on granular loan-level data from the Spanish Credit Register for constructing the network of interbank exposures, as well as on other regulatory and supervisory data —stemming from mandatory reporting in the EU— for assessing the equity, total assets and total liabilities of each institution in this network.

The Spanish Credit Register (Central de Información de Riesgos, CIR) is collected by the Banco de España in its capacity as regulator and supervisor of the Spanish banking system. It contains detailed monthly loan-level information on each outstanding loan (and other financial commitments) from 9000 euros extended by any credit institutions operating in Spain to any domestic or foreign resident (for a more in-depth description of the database and its usage in the literature see Jiménez et al., 2009, 2012, 2014, 2018; Bentolila et al., 2017).<sup>4</sup> For our study, we focus on the Spanish domestic interbank market, comprising loans and debt securities for which both the lender and the borrower are credit institutions authorized to operate in Spain. In this context, the reporting threshold of 9000 euros is remarkably low, which ensures the database is almost complete, including virtually all existing domestic interbank relationships. Note, however, that given their significantly smaller size, the particularities of their balance-sheets and their dependence on their (foreign) parent companies, we have excluded branches of foreign credit institutions from our analysis.

The Spanish financial system has undergone a process of intense transformation since the Global Financial Crisis, most prominently characterised by a significant increase in market concentration through mergers and acquisitions (Cruz-García et al., 2018). Accompanying this restructuring process, another important feature of the recent decade has been the expansionary monetary policy carried out by the European Central Bank. Finally, as shown in Figure 1, a progressive shift in aggregate volume can be observed from exposures to domestic credit institutions towards exposures to foreign credit institutions. In fact, the aggregate Spanish domestic interbank exposure has gradually decreased in volume during this time, dropping by almost 60% between 2014 and 2020. Therefore, at the current juncture, the Spanish domestic interbank market is significantly smaller than in the period previous to the Global Financial Crisis, a factor that should be taken into account when interpreting the results of our analysis. With these profound transformations in mind, we have chosen to focus only on relatively recent data. In particular, we use data corresponding to Q1 2020 for all results presented in this paper.<sup>5</sup>

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<sup>3</sup> Note that the above referred studies of the Euro Area cover only a smaller share of the credit institutions operating in Spain (about 30% fewer) and their data is restricted to large exposures (those exceeding either 300 million euros or 10% of eligible capital), while our interbank exposure data is almost complete (including all exposures above 9000 euros).

<sup>4</sup> Note that this reporting threshold was set at 6000 euros until 2013, when it was updated to 9000 euros.

<sup>5</sup> In order to verify the robustness of our results, we have also worked with data corresponding to the interval between Q4 2018 and Q1 2020, obtaining results very much in line with those shown in this paper.

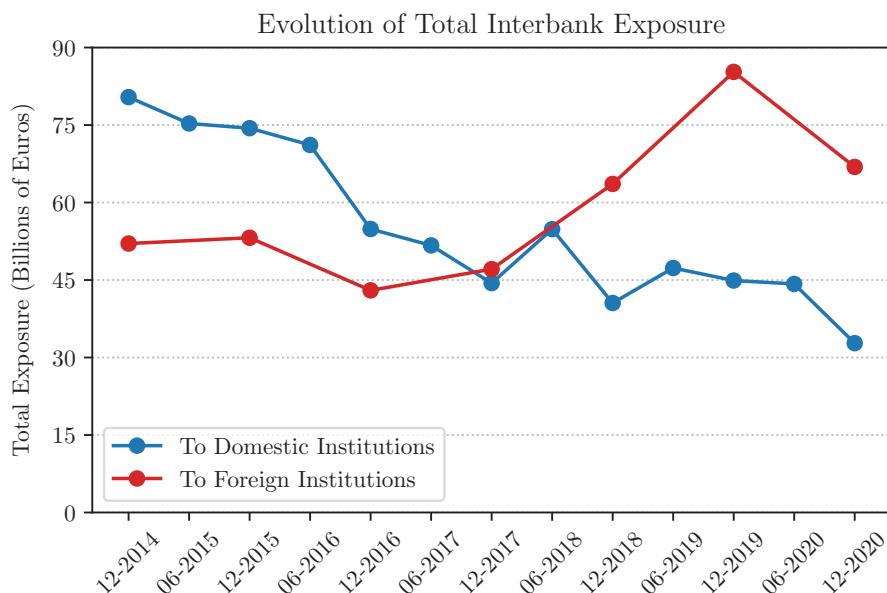


Figure 1: Evolution of aggregate exposure of financial institutions domiciled in Spain to other financial institutions domiciled in Spain and to foreign financial institutions, excluding intra-group exposures.

All exposures are considered at the level of the consolidated domestic group, i.e. intra-group exposures are excluded from the analysis, and all exposures between firms of the same group and third parties are considered as stemming from the same, consolidated institution. Furthermore, all exposures from a given institution  $i$  to another one  $j$  are collected under a single, aggregate exposure. Importantly, for every pair of institutions  $i$  and  $j$ , this aggregation is performed independently for each direction, that is, exposures from  $i$  to  $j$  are not netted out with those from  $j$  to  $i$ . After consolidation, the network is comprised of 73 banks, among which 26 are exclusive lenders, 4 are exclusive borrowers and 43 are both lenders and borrowers. Moreover, there are 454 exposures among these banks, out of which 336 are unidirectional and 118 are arranged in 59 pairs of bidirectional relationships. The resulting frequency distribution of exposure sizes is shown in Figure 2, where the strong similarity to a log-normal distribution (shown in red) hints at well-defined moments. A summary of these moments is provided in Table 1.

Mean	$48.2 \cdot 10^6$
Median	$4 \cdot 10^6$
Standard deviation	$166.4 \cdot 10^6$
Skewness	7.2
Kurtosis	65.9

Table 1: Summary statistics

A final consideration to underline about the exposure data is related to the corresponding distribution of maturities, shown in Figure 3. As opposed to the usual view of the interbank market as an extremely short-term exchange, what we observe in this figure is that recent Spanish domestic interbank exposures seem to be characterised by a broad distribution of maturities, with a large fraction of the volume with medium and long maturities. In fact, only 30% of the exposure volume has maturities up to 3 months. If we look at maturities above a year, we find that these account for more than half of the volume, reaching almost

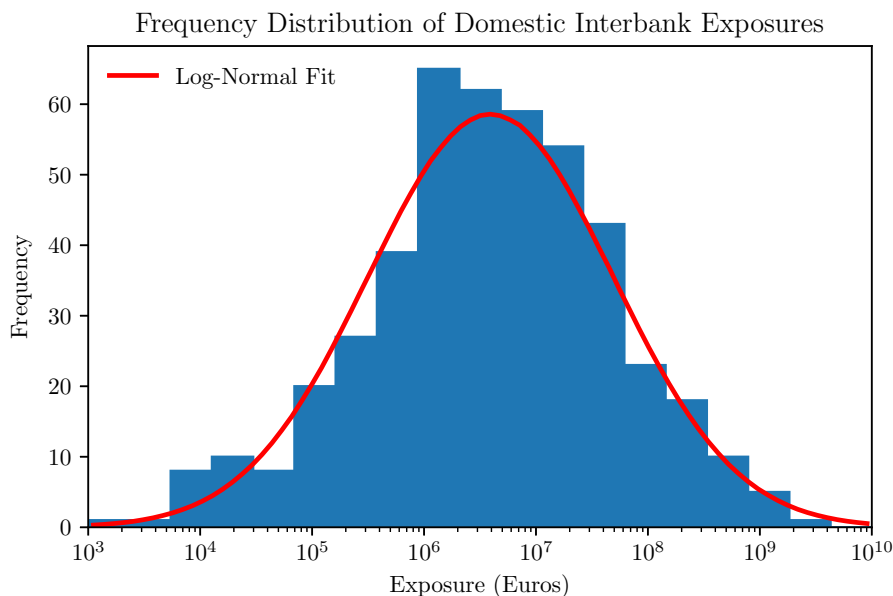


Figure 2: Frequency distribution of exposure sizes between financial institutions domiciled in Spain, excluding intra-group exposures, as of Q1 2020. The red line shows the frequency distribution resulting from a log-normal fit to the data.

55% of it. Finally, we find that exposures with very long maturity —above 5 years— actually account for almost as much volume as exposures with very short maturity —up to a year—, with almost 26% and 30% of the volume respectively. This is relevant for the model we use and the contagion channel we focus on, as this channel, based on credit quality deteriorations, makes more sense when thinking of longer maturities, while liquidity concerns are more appropriate for shorter maturities.

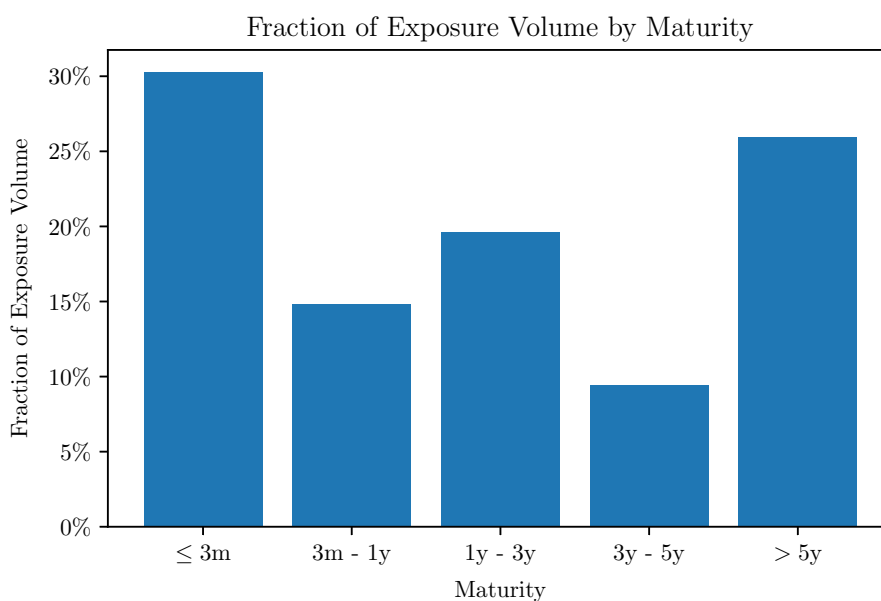


Figure 3: Frequency distribution of the maturity of exposures between financial institutions domiciled in Spain, excluding intra-group exposures, as of Q1 2020.

As mentioned above, we also use data on each bank’s equity, total assets and total liabilities, as collected by the Banco de España in its role as bank supervisor. Before merging both databases, we apply to this balance-sheet data the same consolidation of domestic banking groups as we did to the interbank exposure data.<sup>6</sup> With respect to the data on banks’ equity or own funds, we use a form of equity that includes social capital and reserves, and is thus close to the accounting notion of capital.

## 4. Model

In this section we describe the basic features of the original DebtRank model (Battiston et al., 2012) as well as the extensions we propose to study the effects of non-linear reactions to counterparty losses and uncertainty about these losses or about their effect on counterparty default risk. In presenting the original model, we follow the formalism developed by Bardoscia et al. (2015), which is a particular case of the more general framework for financial network valuation introduced by Barucca et al. (2020). Regarding the two extensions proposed, we present them here as independent developments and thus in separate subsections. However, their interaction will also be considered for the results presented in Section 5, apart from their individual effects.

### 4.1. Original DebtRank model

The DebtRank framework was originally proposed by Battiston et al. (2012) to model the transmission of financial distress via the credit quality channel (Fink et al., 2016). This original proposal was later generalised and derived from basic accounting principles by Bardoscia et al. (2015). In order to summarise here the main features of the model, we will follow this latter formalism.

In essence, the model focuses on a set of financial institutions interconnected by financial contracts. Some of these institutions are initially hit by an exogenous shock leading to a devaluation of their external assets, and thus to a decrease in their equity. This reduction of their capital buffers, in turn, weakens their capacity to withstand further shocks, thereby increasing their probability of default. As a result, the credit quality of their creditors’ portfolios deteriorates due to higher expected credit losses, which are to be accounted for by a devaluation of the corresponding interbank assets on the balance sheets of these creditors, according to applicable accounting standards.<sup>7</sup> Finally, these asset devaluations are reflected in a decrease in the equity of these creditors, which in turn affects the creditors of these creditors, thereby starting a new wave of contagion. In this context, the DebtRank model constitutes a consistent method for estimating losses due to financial contagion in a network of interbank exposures after a shock hits the system and before the maturity of the corresponding contracts.

In a more formal way, and as is common in the literature on financial contagion (Glasserman and Young, 2016; Bardoscia et al., 2021), a financial system composed of  $n$  institutions (hereafter “banks”) can be represented as a directed network of interconnected balance sheets, such that the edges of the network correspond to bilateral exposures between the banks. In particular, let  $L_{ij}$  denote the book value of the

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<sup>6</sup> In particular, we perform the consolidation by aggregating all domestic subsidiaries of banking groups operating in Spain.

<sup>7</sup> Note that the model considers the effect of credit quality deteriorations only in terms of increasing expected losses—that is, via the devaluation of the corresponding interbank assets—and not in terms of increasing unexpected losses—that is, via higher regulatory charges in the form of risk weights. In any case, both effects act in the same direction, that of driving down the affected bank’s capital ratio, defined as capital over risk-weighted assets (Fink et al., 2016).

interbank liability of bank  $i$  towards bank  $j$ , with the book value of the corresponding interbank asset of bank  $j$  written as  $A_{ji}$ . Note that, for consistency, we have  $A_{ji} = L_{ij}$ . Apart from these interbank assets and liabilities, banks also have external assets (e.g. loans to the real sector) and liabilities (e.g. deposits), whose book value is denoted by  $A_i^e$  and  $L_i^e$ , respectively. Thus, the book value of bank  $i$ 's total assets is  $A_i = A_i^e + \sum_{j=1}^n A_{ij}$ , while that of its total liabilities is  $L_i = L_i^e + \sum_{j=1}^n L_{ij}$ . Finally, applying the basic balance sheet identity, the book value of the equity  $E_i$  of bank  $i$  can be written as

$$E_i = A_i^e - L_i^e + \sum_{j=1}^n A_{ij} - \sum_{j=1}^n L_{ij}, \quad (1)$$

where  $\sum_{j=1}^n A_{ij}$  and  $\sum_{j=1}^n L_{ij}$  are, respectively, the total interbank assets and liabilities of this bank.<sup>8</sup>

In order to move beyond book values and obtain a valuation of its equity at any point in time before the maturity of its assets and liabilities, a bank needs to perform a valuation of its assets, considering their expected value at maturity. Importantly, the framework assumes no debt renegotiation takes place before maturity, and thus no valuation of liabilities is needed, as they always keep their book value. This assumption, common in the literature on financial contagion before default (Caccioli et al., 2018; Bardoscia et al., 2019; Barucca et al., 2020; Bardoscia et al., 2021), simply translates the fact that banks cannot unilaterally reduce the value of their obligations based on their own creditworthiness or the shocks they receive. Such a discount on their liabilities could only materialise via a debt renegotiation process, which is assumed to happen only at or after maturity, and is thus irrelevant for a valuation of the bank's equity before maturity. On the asset side, on the contrary, a fair valuation of a bank's equity requires incorporating up-to-date information about the expected value of these assets at maturity. In particular, the framework considers a mark-to-market valuation of interbank assets, such that banks incorporate any new information about the creditworthiness of their debtors into the value of the corresponding interbank assets. Formally, this is done by introducing the interbank valuation function  $\mathbb{V}_{ij}$ , such that the valuation that bank  $i$  makes of its exposure to bank  $j$  at time  $t$  can be written as  $A_{ij}(t) = A_{ij}\mathbb{V}_{ij}$ . In this way, the interbank valuation function takes values in the interval  $[0, 1]$  and has the role of a discount factor, equal to one if the interbank loan is expected to be fully repaid and zero if the lender does not expect to recover anything. While the interbank valuation function can, in principle, depend on any variable and piece of information used by bank  $i$  to assess how much of its exposure to bank  $j$  it expects to recover at maturity, the DebtRank framework focuses only on the role played by the borrower's equity valuation. Writing this dependence explicitly, we have  $A_{ij}(t) = A_{ij}\mathbb{V}_{ij}(E_j(t))$ . Regarding external assets, the model assumes their valuation remains constant in time except for the initial exogenous shock, which is introduced as a relative devaluation of a bank's external assets. Thus,  $A_i^e(t) = \alpha A_i^e$  for any  $t > 0$  if bank  $i$  is among those receiving the initial exogenous shock, while  $A_i^e(t) = A_i^e$  otherwise. For simplicity, this variable is kept as  $A_i^e(t)$  for now. Finally, in order to obtain the valuation of the equity of bank  $i$  at time  $t$ , one just needs to replace the book values of the assets with their respective valuations in Eq. (1),

$$E_i(t) = A_i^e(t) - L_i^e + \sum_{j=1}^n A_{ij}\mathbb{V}_{ij}(E_j(t)) - \sum_{j=1}^n L_{ij}. \quad (2)$$

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<sup>8</sup> Note that banks cannot have exposures towards themselves, and thus all diagonal elements of both matrices are equal to zero, i.e.  $A_{ii} = L_{ii} = 0$  for all  $i$ .



Using this equity valuation, bank  $i$  is considered as default if at any point its total liabilities are equal to or exceed its total assets,<sup>9</sup> that is, if  $E_i(t) \leq 0$ .

Interpreting Eq. (2) as a set of fixed point equations for the vector of equities  $\mathbf{E}(t)$ , Barucca et al. (2020) have shown that:

1. it can be solved iteratively by choosing an arbitrary starting point for the vector of equities and repeatedly applying Eq. (2) to compute the subsequent iterations until convergence,
2. provided that the interbank valuation function  $\mathbb{V}_{ij}$  is a non-decreasing function of the equity  $E_j(t)$  and continuous from above, there is a solution simultaneously optimal for all banks, i.e. a solution which simultaneously minimizes all individual and total losses,
3. by choosing as a starting point for the iteration the vector of equities corresponding to all interbank assets being valued at their book values, the system converges to this optimal solution.

The condition on the interbank valuation function being a non-decreasing function of the equity of the borrower is well in line with the non-speculative character of interbank credit contracts, such that the valuation of a given contract cannot increase because the equity of the borrower decreases.

The specific definition of the interbank valuation function  $\mathbb{V}_{ij}(E_j(t))$  is the key part of the model, as it determines the way in which contagion happens. Following Bardoscia et al. (2017b, 2019) and Barucca et al. (2020), this function can be written as a sum of two contributions corresponding to the two possibilities at maturity: that the borrower bank  $j$  has not defaulted, and thus it is able to repay its obligations in full, and that it has defaulted, in which case its creditors will only recover a fraction of what they are owed by  $j$ . More formally, one can write

$$\mathbb{V}_{ij}(E_j(t)) = (1 - p_j^D(E_j(t))) + \rho p_j^D(E_j(t)), \quad (3)$$

where  $p_j^D(E_j(t))$  is the probability that bank  $j$  defaults between the current time  $t$  and the maturity of the interbank contract, with an explicit dependence on its current equity, and  $\rho$  is an exogenously determined recovery rate in case of default, thus accounting for the bankruptcy costs involved in the liquidation process.<sup>10,11</sup> Note that, if  $\rho = 1$ , then the interbank valuation function becomes trivially  $\mathbb{V}_{ij}(E_j(t)) = 1$  and no devaluation of interbank assets takes place. This is a direct consequence of the fact that, if lenders expect to recover the full book value of their claims even in case of default of their borrowers, then there is no credit quality deterioration and thus no contagion of financial distress. In other words, financial distress can only propagate via the credit quality channel if some loss is expected in case of default (Veraart, 2020).

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<sup>9</sup> While this is a common assumption in the literature (Glasserman and Young, 2016; Caccioli et al., 2018), it should be stressed that it is in fact only a proxy for a real default event. More realistic default criteria could reflect, for example, minimum regulatory requirements.

<sup>10</sup> In reality, recovery rates are a combination of two factors. The first one is related to the actual equity shortfall at default, i.e. by how much the liabilities of the defaulted bank exceed its assets, and it can be modelled as endogenous to the contagion process under study (Bardoscia et al., 2019; Barucca et al., 2020; Veraart, 2020). The second one has to do with the financial frictions and costs associated with the liquidation of the assets of the defaulted bank and it is thus exogenous to the contagion process. For simplicity, the framework presented here deals only with a single exogenous recovery rate (Bardoscia et al., 2017b).

<sup>11</sup> For an estimation of the distribution of real recovery rates see Memmel et al. (2012), for a model of default contagion in interbank networks where heterogeneous recovery rates are drawn from an estimated distribution see Memmel and Sachs (2013). Finally, some credit institutions include in their annual Prudential Relevance Report an internal assessment of the expected recovery rate for their aggregate exposure to other credit institutions.

Moreover, as  $\rho$  moves from 1 to 0 the contagion of distress becomes more and more important, and so does the total loss experienced by the system. Following the literature (Battiston et al., 2012; Bardoscia et al., 2015; Caccioli et al., 2018; Bardoscia et al., 2021), and given our interest in comparing losses between versions of the model rather than in the specific values of these losses, we will focus on the case  $\rho = 0$  for all results to be presented below.

Turning now to the functional form of the probability of default  $p_j^D(E_j(t))$ , it is useful to express it rather as a function of the relative equity loss of bank  $j$  at time  $t$ , defined as

$$h_j(t) = \frac{E_j - E_j(t)}{E_j}, \quad (4)$$

that is, as how much equity bank  $j$  has lost up to time  $t$  as a fraction of its initial or book value equity. Expressed in this way, the probability of default  $p_j^D(h_j(t))$  must be a function that (i) maps the interval  $[0, 1]$  into itself, (ii) leads to a zero probability of default in case of no losses, that is,  $p_j^D(0) = 0$ , (iii) leads to a probability of default equal to one in case all equity has been lost, that is,  $p_j^D(1) = 1$ , and (iv) is a non-decreasing function of  $h_j(t)$ , such that  $\mathbb{V}_{ij}$  is a non-decreasing function of the equity  $E_j(t)$ . In this context, the DebtRank model assumes the most simple functional form fulfilling these requirements, that is, the identity function

$$p_j^D(h_j(t)) = h_j(t). \quad (5)$$

In this way, relative changes in the valuation of the equity of a borrower translate into proportional changes in its probability of default, as assessed by its creditors. Note that we are dealing here with the probability that a borrower defaults due to changes in its financial position occurred after the origination of the contract, assuming any previous probability of default has been compensated in the terms of the contract, for example, with the interest rate or the collateral demanded.

#### 4.2. Non-linear contagion

In the original DebtRank framework, presented above, the probability of default of a given borrower, as assessed by its creditors, is assumed to behave linearly with relative (negative) changes in the valuation of its equity. In particular, the relationship between both variables is assumed to be an identity [see Eq. (5)], that is, creditors assess the probability of default of a given borrower to be equal to the relative equity loss experienced by this borrower. When combined with the interbank valuation function in Eq. (3), this implies that relative losses in the valuation of the equity of a borrower translate into proportional relative devaluations in the corresponding interbank assets of its creditors, with  $(1 - \rho)$  as the coefficient of proportionality.

In reality, it seems more plausible that the reactions of creditors to the relative equity losses experienced by their borrowers are different depending, on the one hand, on the size of these losses and, on the other hand, on the initial capitalisation of these borrowers, as measured, for example, by their leverage ratio — defined here as total equity over total assets. In particular, for a given initial leverage ratio, small additional equity losses should have a negligible impact on the probability of default of banks close to their full initial equity, while they can have a dramatic impact on the probability of default of banks whose equity has already experienced large losses and are thus in a more vulnerable financial position. Finally, for banks whose equity has been almost completely wiped out and are thus already close to defaulting, small additional losses are

again expected to lead to only marginal increases of the corresponding probability, since their default was already considered as almost certain. In other words, the same additional loss of equity has far more impact after having already suffered an important relative decrease of equity than before, even though this impact decreases again as the default starts to be considered as almost certain. In this sense, we can think of the probability of default as having two regions of sub-linear growth with the relative equity loss separated by a region of super-linear growth, where most of the increase takes place. Furthermore, we would expect that, for poorly capitalised banks, even small losses of equity could be enough for them to enter the region of super-linear growth, while better capitalised entities will be able to withstand larger losses before reaching this region of super-linear growth.

Bardoscia et al. (2016) have proposed a phenomenological, non-linear functional form for the probability of default  $p_j^D$  as a function of the relative equity loss  $h_j$ , capturing the effect of the loss size described above. However, their specification does not consider the impact of the initial capitalisation, and thus a given relative equity loss has the same effect regardless of the initial capitalisation of the affected bank. In order to capture both of these effects, we propose an alternative non-linear s-shaped specification. S-shaped functional forms, such as the logistic function, have been widely used in a number of areas in economics, including the diffusion of innovations (Bass, 1969; Mahajan et al., 1990), business cycle dynamics (Perez, 2002) and the modelling of default probabilities of financial institutions (Martin, 1977; Ohlson, 1980). Typically, these functional forms are used to model a smooth transition between two states; in our case, between a state of low and a state of high probability of default. In particular, we propose the form

$$p_j^D(h_j) = \frac{1}{1 + \left( \frac{h_j}{(1-h_j)} \frac{(1-\mu_j)}{\mu_j} \right)^{-\beta}}, \quad (6)$$

where, consistent with other s-shaped functions,  $\mu_j$  is a parameter controlling the mid-point of the transition, and thus the area of the domain around which most of the change takes place,  $\beta$  is a parameter controlling how smooth or abrupt the transition is<sup>12</sup> (see Figure 4), and, for sake of notational simplicity, we have dropped the dependence on the time  $t$ . As will be described below, it is precisely via the calibration of the parameter  $\mu_j$  that we will introduce a relationship with the initial capitalisation of bank  $j$ . Importantly, and as opposed to most of the s-shaped functions commonly used in the literature, the proposed form has the correct domain and codomain, both of them equal to the interval  $[0, 1]$ . Furthermore, it fulfils the requirements that  $p_j^D(0) = 0$ , meaning that the probability of default of borrowers having experienced no loss is equal to zero, and that  $p_j^D(1) = 1$ , meaning that the probability of default of borrowers having lost all their equity is equal to one.<sup>13</sup> Finally, as in the logistic function, for  $\beta \rightarrow \infty$  we recover a step function with a threshold set by  $\mu_j$ . For both  $\mu_j = 0$  and  $\mu_j = 1$  we also recover step functions regardless of the value of  $\beta$ . On the contrary, for  $\beta = 1$  and  $\mu_j = 0.5$  we recover the original (linear) DebtRank model. Interestingly, for  $\beta = 1$  and  $\mu_j > 0.5$  we obtain fully convex functional forms similar to those proposed by Bardoscia et al. (2016).

<sup>12</sup> Note that we must have  $\beta \geq 1$  for the function to have a meaningful economic interpretation. Otherwise, the beginning of the function would be concave, meaning that initial losses would have a larger impact on the probability of default than later losses.

<sup>13</sup> Note that the logistic function, even if shifted to be centred at  $1/2$ , is not a valid form for this purpose, as it is defined in the domain  $(-\infty, +\infty)$  and it has  $p_j^D(0) > 0$ , leading to a spurious contagion of distress in the absence of any loss to the borrower, and  $p_j^D(1) < 1$ , leading to borrowers being considered as not defaulted even if they already are.

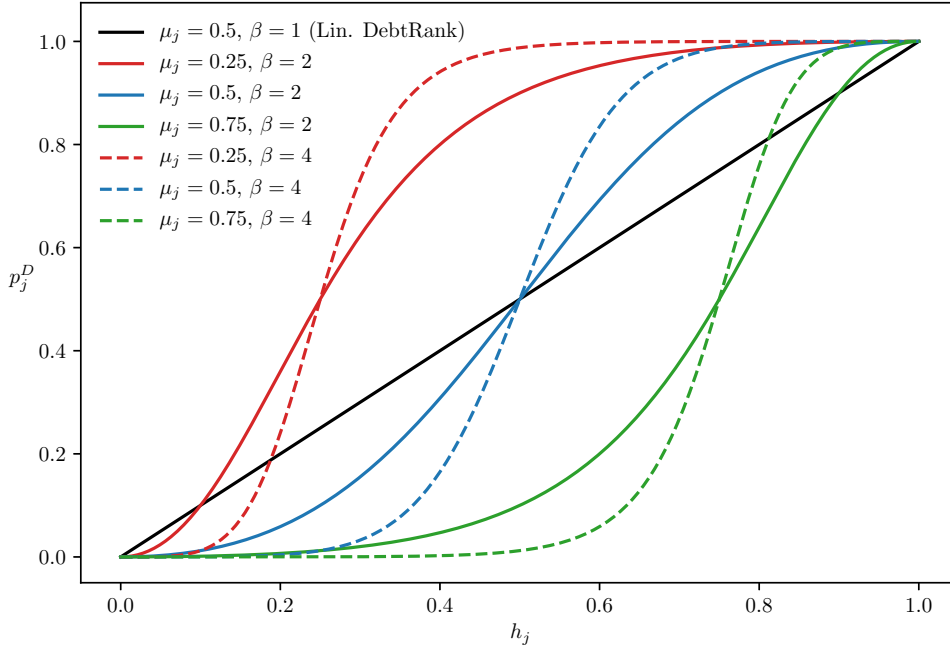


Figure 4: Non-linear s-shaped functional form for different parameter values: Probability of default  $p_j^D$  as a function of the relative equity loss  $h_j$  for different values of the parameters  $\mu_j$  and  $\beta$ . The non-linear form proposed includes the original (linear) DebtRank model as a special case.

As described above, the parameter  $\mu_j$  has a clear financial meaning, as it sets the value of the relative equity loss  $h_j$  at which the probability of default  $p_j^D$  reaches 0.5. Furthermore, given that most of the increase in the probability of default  $p_j^D$  takes place around  $h_j \sim \mu_j$ , with the width of this region set by the parameter  $\beta$ , we can interpret  $\mu_j$  as a soft threshold. In this way, relative equity losses below (above) this threshold have a negligible (strong) impact on the probability of default. We take advantage of this feature in order to make the functional form proposed above dependent on the initial leverage ratio of the borrower bank  $j$ , whose probability of default is being assessed, bearing in mind that this initial ratio is nothing but the book value of its equity divided by the book value of its total assets,  $E_j/A_j$ . In particular, instead of setting  $\mu_j$  directly by choosing the threshold value of the relative equity loss  $h_j$  at which the probability of default  $p_j^D$  should reach 0.5, as its definition would suggest, we first rewrite  $h_j$  in terms of the leverage ratio and its initial or book value and then choose a threshold value of the leverage ratio at which  $p_j^D$  should reach 0.5 (see Appendix A for a derivation). In this way, we obtain

$$\mu_j = \frac{\frac{E_j}{A_j} - R^*}{\frac{E_j}{A_j} (1 - R^*)}, \quad (7)$$

where  $E_j$  is the book value of bank  $j$ 's equity,  $A_j$  stands for the book value of its total assets and  $R^*$  is the threshold value of the leverage ratio.<sup>14</sup> With this transformation, the parameter that we now need to set,  $R^*$ , has an even clearer financial meaning, as it refers to a variable —the leverage ratio— commonly used to assess the solvency position of financial institutions. In particular, for all results to be presented below,

<sup>14</sup> Note that the set of parameters  $\mu_j$  will remain fixed throughout a given simulation, as they only depend on the parameter  $R^*$  and the initial leverage ratio of bank  $j$ .

we choose  $R^*$  to be the 10th percentile of the distribution of leverage ratios in our data,  $R^* = 6\%$ . Finally, by making  $\mu_j$  depend on the distance between the initial leverage ratio and its threshold value, we have managed to transform the model in such a way that lower initial ratios will lead to quicker increments of the probability of default while better capitalised banks would suffer only slow increases of this probability.

### 4.3. Uncertainty

In the original DebtRank framework, presented in Subsection 4.1, banks perform a valuation of their interbank assets before maturity, and thus with an implicit uncertainty around the possible default of their borrowers and the corresponding losses. However, they are assumed to have complete and perfect information about the current financial position of their borrowers, in the sense that they can all accurately observe the relative equity loss experienced by these borrowers. Furthermore, they are all assumed to use the same model to process this information and derive a probability of default, the underlying assumption being that there is only one possible model. As a consequence, banks have homogeneous and deterministic expectations, and thus all lenders of a given borrower make the exact same (deterministic) assessment of its probability of default.

In reality, banks are confronted with a situation of incomplete and imperfect information, which leads them to form heterogeneous expectations about the probability of default of their borrowers. In the context under consideration, the incompleteness and imperfection of the information available to lenders can lead to heterogeneous expectations in three different ways. First, different lenders might have access to different sets of information about the current financial position of a given borrower, a type of information failure close in spirit to the seminal works by Akerlof (1978), Rothschild and Stiglitz (1978) and Stiglitz and Weiss (1981). Secondly, the information available to lenders might be noisy or polluted by observational errors related, for example, to limits in their ability to process such information or rational inattention, as proposed by Woodford (2001) and Sims (2003). In both of these cases, lenders would arrive at different assessments of the probability of default of the borrower even if they were to use the exact same model to form their expectations. Thirdly, a general context of incomplete information would prevent banks from having a perfect model of the economy, thus leading them to develop different (imperfect) models with which to form their expectations, an idea similar in spirit to those explored by Hommes (2011) and Coibion and Gorodnichenko (2012, 2015). In this case, lenders would arrive at different assessments of the probability of default of a given borrower even if they had access to the exact same information about its financial position. Our modelling approach does not intend to distinguish between these different paths to heterogeneous expectations. That is, we focus on the uncertainty and the diversity of assessments of the probability of default of a given borrower, regardless of whether this heterogeneity is due to lenders using different models, them having access to different sets of information, their information being noisy or a combination of these.

More formally, we incorporate a stochastic component into each lender's assessment of the probability of default of each one of its borrowers, thereby explicitly capturing the uncertainty experienced by the former regarding the future financial position of the latter. Since this stochastic component is a characteristic of each relationship between a lender  $i$  and a borrower  $j$ , we can replace the probability of default of the original DebtRank model  $p_j^D(h_j(t))$ , dependent only on the borrower  $j$ , with a new probability of default

$p_{ij}^D(h_j(t))$ , dependent on both the lender  $i$  and the borrower  $j$ . In this way, we can rewrite the interbank valuation function as

$$\mathbb{V}_{ij}(h_j(t)) = (1 - p_{ij}^D(h_j(t))) + \rho p_{ij}^D(h_j(t)). \quad (8)$$

Specifically, we model the stochastic component as a Gaussian noise added to the probability of default of the original model,

$$p_{ij}^D(h_j(t)) = p_j^D(h_j(t)) + \sigma X_{ij}, \quad (9)$$

where  $X_{ij}$  is a random number drawn from a truncated standard normal distribution and  $\sigma$  is a parameter related to the standard deviation of the stochastic component. The reason for the truncation is to keep the new probability of default within the correct interval for a probability,  $[0, 1]$ .<sup>15</sup> To this end, an additional constraint is required on the parameter  $\sigma$ , which will be discussed below. Note that the random numbers  $X_{ij}$  are drawn at the beginning of each simulation and kept fixed throughout the process, such that they represent a persistent bias or error characterising each lender-borrower relationship.<sup>16</sup> Nevertheless, since they are symmetrically distributed around zero, there is no systematic bias, and thus banks are, on average, correct—in the sense of matching the probability of default corresponding to the original (deterministic) DebtRank model. Furthermore, results will be averaged over a number of realisations of the stochastic process.

Let us now focus on the parameter  $\sigma$  in Eq. (9). As mentioned above, this parameter controls the standard deviation of the new default probabilities  $p_{ij}^D$ , which is equal to the product of  $\sigma$  times the standard deviation of the random numbers  $X_{ij}$ .<sup>17</sup> Having a fixed  $\sigma$  would imply that the uncertainty experienced by its creditors about the probability of default of a given borrower is independent of its financial position. From an economic point of view, however, it is reasonable to think that this level of uncertainty will be very small in situations where the borrower has experienced losses so large as to make default almost certain, as well as in situations where its losses are so small as to make the probability of default vanishingly small. In other words, assessments of the probability of default of a given borrower will tend to converge as its financial situation becomes clearer, whether in the sense of a default or the opposite. On the contrary, in intermediate situations, where losses are large but it is unclear whether they are large enough to compromise the future solvency of the borrower, the level of uncertainty will be higher. As a consequence, creditors' assessments of the corresponding probability of default will tend to diverge. In order to capture this behaviour in a stylised way, we model  $\sigma$  as a symmetric triangular function of the original DebtRank probability of default  $p_j^D$ ,

$$\sigma(p_j^D) = \begin{cases} \frac{p_j^D}{3}, & \text{if } p_j^D \leq \frac{1}{2}, \\ \frac{1 - p_j^D}{3}, & \text{otherwise,} \end{cases} \quad (10)$$

<sup>15</sup> In particular, we choose to truncate the standard normal distribution with symmetric bounds at a distance of 3 standard deviations from the mean, thus keeping 99.73% of the original probability mass.

<sup>16</sup> Apart from this specification, where the stochastic component is introduced as a quenched or frozen noise characterising each relationship between a creditor and a borrower, we have also studied the case of a noise attached to each creditor, the case of a noise attached to each borrower, as well as annealed versions of all of these noises. In all cases, the results of the model are broadly equivalent to those presented below in Section 5.

<sup>17</sup> Note that the random numbers  $X_{ij}$  are drawn from a truncated standard normal distribution, and thus their standard deviation will be smaller than 1, that is, smaller than the standard deviation corresponding to the underlying standard normal distribution.

where the maximum of the function has been calibrated so as to keep the new probabilities of default  $p_{ij}^D$  within the interval  $[0, 1]$ , bearing in mind also the truncation of the random numbers  $X_{ij}$ . Figure 5 illustrates the interaction between the original DebtRank probabilities  $p_j^D$  and the new probabilities with uncertainty  $p_{ij}^D$  by showing several probability density functions of  $p_{ij}^D$  corresponding to different values of  $p_j^D$ .

## 5. Results

We present here results for the two extensions described above (non-linear contagion in Subsection 5.2 and uncertainty in Subsection 5.3), as well as the interaction between them (Subsection 5.4). Since this presentation will be based on a comparison with results for the original DebtRank model, we include a brief initial subsection with the general picture resulting from applying this original model to our data (Subsection 5.1). To the best of our knowledge, this is the first application of the DebtRank framework to Spanish interbank data.

In order to study the propagation of financial distress across the network of financial institutions, an initial shock must be introduced in the system. Since the main focus of this paper is on uncovering the effects of the extensions proposed across a wide range of system configurations, we are not interested here in a realistic, scenario-based stress test of the system. On the contrary, for the purpose of comparing results of the extensions with those of the original model, we are interested here in relative magnitudes, and thus it is enough for us to simulate broadly reasonable but ad-hoc shocks. Following most of the literature on distress propagation before default (Caccioli et al., 2018), we introduce the initial shock as a relative devaluation of a bank's external assets. Specifically, all results presented here correspond to an initial shock of a 0.5% devaluation of external assets. Furthermore, in order to measure the impact of each bank on the system, as well as its vulnerability to the system, we run as many simulations as banks there are in the network, with

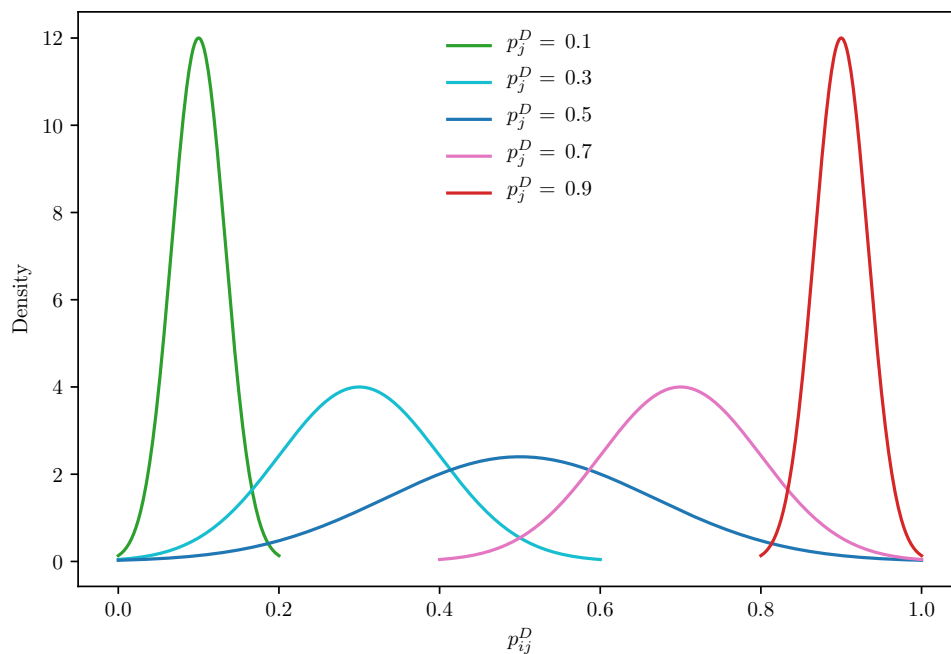


Figure 5: Probability density functions of  $p_{ij}^D$  for different values of  $p_j^D$ : The probability of default with uncertainty  $p_{ij}^D$  behaves as a (truncated) Gaussian random variable with mean equal to the probability of default without uncertainty  $p_j^D$  and standard deviation set by Eq. (10), again as a function of  $p_j^D$ .

a single, different bank receiving the initial shock in each of them. This allows us to define the *impact* of a bank  $i$  as the relative equity loss suffered by the total system when this bank is shocked,

$$H_i = \frac{\sum_{j=1}^n (E_j - E_j^i(T))}{\sum_{j=1}^n E_j}, \quad (11)$$

where  $E_j^i(T)$  is the final (after system convergence) equity of bank  $j$  in the simulation in which bank  $i$  receives the initial shock. Analogously, the *vulnerability* of a bank  $i$  can be defined as the relative equity loss suffered by this bank averaged over all these simulations, i.e. as the mean or expected relative equity loss suffered by bank  $i$  if a random bank of the system receives the initial shock,

$$V_i = \frac{1}{n} \sum_{j=1}^n \frac{E_i - E_i^j(T)}{E_i}. \quad (12)$$

When considering the extension of the model with uncertainty, results are further averaged over 1000 different realisations of the stochastic component. All of these measures can be decomposed into a direct and a contagion components, the former including only the direct losses caused by the shock on the affected bank and the later only those losses due to the propagation of the shock through the network.

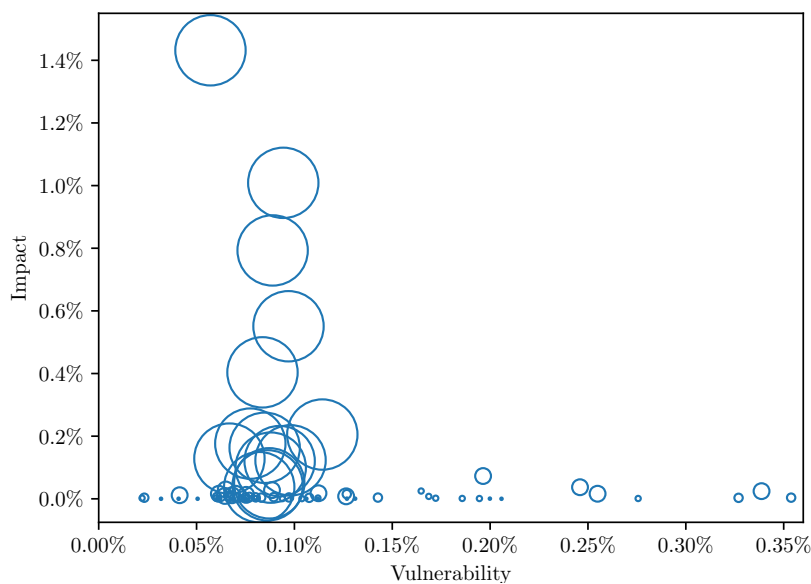
For the purpose of comparing the results of both extensions, we perform a series of experiments with different values of the total leverage ratio of the system, thus systematically exploring a wide range of levels of stress the system can be subject to. High leverage, corresponding to low leverage ratios, makes institutions more vulnerable to shocks and can also make new issuances (of debt and capital) more expensive; it is thus plausible to consider that, all other things being equal, a more leveraged system is more fragile or stressed. Importantly, the leverage ratio of the system is modified with neither altering the structure of the network of exposures nor changing the nominal size of the initial shock, by adjusting, for each bank, the sizes of its external liabilities and its equity with respect to each other, thus keeping their sum constant. In other words, the leverage ratio of the systems is increased by converting, for each bank, a fraction of its external liabilities into equity, and decreased by the reverse transformation. In this way, both external and interbank assets, as well as interbank liabilities, remain untouched, thereby keeping both the network and the nominal size of the initial shock unchanged. In order to have a single metric with which to compare results across different values of the leverage ratio of the system, we aggregate the impacts of all banks into a single average impact,  $H = \frac{1}{n} \sum_{i=1}^n H_i$ , which is then a measure of the mean or expected total system relative equity loss if a random bank receives the initial shock. For simplicity, we will hereafter refer to this metric as the *total system mean loss*.

### 5.1. Results with original DebtRank model

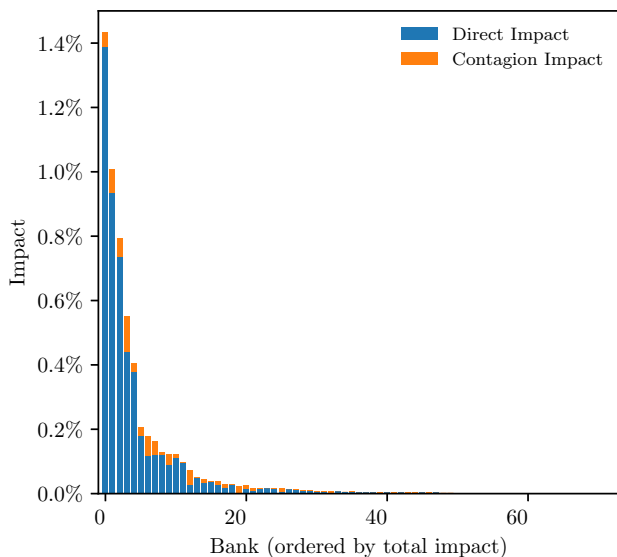
Let us start with a general overview of the main results obtained by applying the original DebtRank model to our data, which we present in the different panels of Figure 6. As we will show in this subsection, in terms of the credit quality channel analysed here, the system in its current state appears to be in a generally stable and robust condition. While all results presented here correspond to Q1 2020, this main conclusion remains across the time window analysed for robustness, from Q4 2018 to Q1 2020.



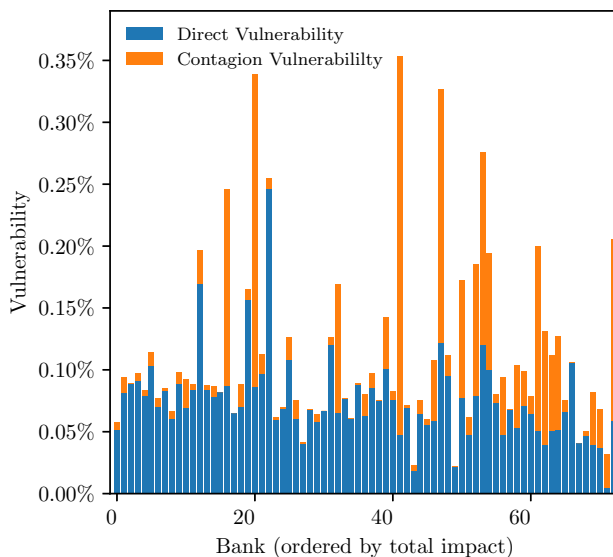
A first picture of the stability of the system can be obtained by looking at panel 6a, where we show the relationship between impact and vulnerability for each bank in the network. There, we can observe that (i) most banks have zero or close to zero impact, with only a handful of them reaching significant values, (ii) while all banks have a positive vulnerability, values are small for most of them, and (iii) most importantly, banks with a relatively strong impact appear to be characterised by a relatively small vulnerability, and vice versa. From the point of view of systemic risk, this latter aspect is of utmost relevance, as it means that the most dangerous banks —those which can cause the largest losses to the system— are also the most robust ones —those which are less likely to experience financial distress.



(a) Impact vs vulnerability.



(b) Direct and contagion impact per bank



(c) Direct and contagion vulnerability per bank

Figure 6: Impact and vulnerability for banks active in the Spanish interbank market in Q1 2020. Panel a shows a scatter plot of impact vs vulnerability for each bank, where the size of each bubble is proportional to the average total assets of banks in the corresponding quintile. Panels b and c show, for each bank, the direct (blue) and contagion (orange) components of, respectively, impact and vulnerability, with banks ordered in both panels according to their total impact.

Another aspect of the stability of the system is highlighted in panel 6b, which shows the direct and the contagion components of the impact of each bank. It is easy to see there that most of the impact is actually driven by the direct effects of the initial shock and not by the network contagion process. This means the system is able to withstand the shocks simulated without large loss amplification due to network contagion. When looking at the vulnerability of individual entities, however, as depicted in panel 6c, we do observe, for some of them, an important amplification of their individual losses due to network contagion. Nevertheless, in line with what was shown in panel 6a, both the direct and the network components of the vulnerability seem to be mostly uncorrelated with total impact, as we can infer from the disordered arrangement of bar heights, bearing in mind that banks are organised in reverse order according to their total impact. Therefore, from a systemic point of view, this increased vulnerability of some banks due to network contagion does not constitute a problem.

Finally, as will be shown in the following subsections, provided that the system remains in its current system-wide level of stress (leverage ratio), this picture of general stability seems to be robust also to the extensions proposed in this paper. Results, however, start to diverge as stress increases in the system.

## 5.2. Results with non-linear contagion

In this subsection, we report on the results of considering lenders' non-linear reactions and their dependence on borrowers' balance-sheet structure, modelled as described in Subsection 4.2. When exploring the effects of this non-linear contagion mechanism on the dynamics of the model and its results, we are interested not only on its relevance under current market conditions, but also on its wider repercussions across different stress scenarios the system could be subject to. In this sense, we study here a wide range of values of the total system leverage ratio, stretching from a situation of severe capital depletion ( $\sim 0.1\%$ ) to one of capital over-abundance ( $\sim 99\%$ ), always with respect to total assets. Furthermore, we are specifically interested in understanding how the inclusion of this non-linearity modifies the resulting dynamics in comparison with the original DebtRank model at an aggregate level, that is, focusing on total system losses. Figure 7 illustrates such a comparison, with the vertical axis representing the total system mean loss corresponding to each of the two models and the horizontal axis covering the broad range of system-wide leverage ratios studied.

The first and most important observation from Figure 7 is that the losses suffered by the system under both contagion mechanisms diverge importantly for a wide range of system-wide stress levels around the centre of the studied spectrum, with the non-linear model leading to significantly higher losses. In particular, this divergence can be clearly observed for leverage ratios between approximately 0.8% and 8%, with losses derived from the non-linear model being about an order of magnitude larger than those corresponding to the linear case —specifically, up to 13 times larger. Under increasingly stressed conditions, as the system-wide leverage ratio moves below 0.8%, more and more exposures within the network become highly leveraged and thus more prone to amplification regardless of which of the two contagion mechanisms is used. As a consequence, losses become very similar between both models. For higher leverage ratios —above 8%— the losses derived from both models seem to converge again, as balance-sheets are so strong that there is barely any contagion and thus losses are mostly driven by their direct component, which is, by definition, the same in both cases. It should be noted, however, that the contagion component of losses, while becoming negligibly small with respect to the direct component for both models for increasing leverage ratios above

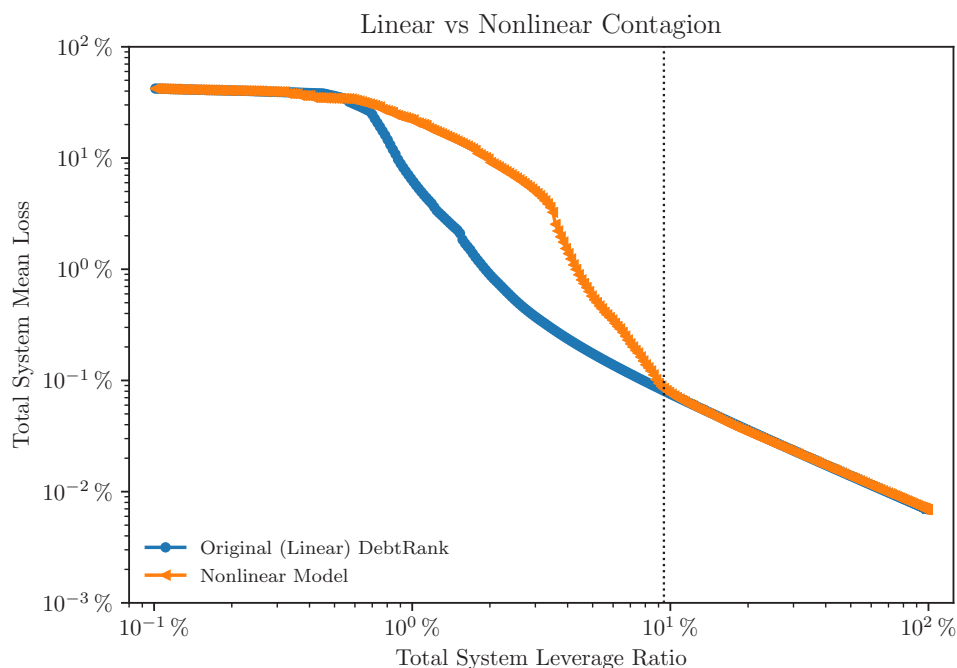


Figure 7: Effect of non-linear contagion: Total system mean loss for both the non-linear model (orange) and the original (linear) DebtRank model (blue). The dotted black vertical line marks the actual leverage ratio of the system in our data.

8%, does decrease faster in the non-linear case. A final point to note about this figure is that the area where non-linear contagion has a strong effect on the results is just below current market conditions (marked with a dotted black vertical line). In other words, even a small deterioration of leverage ratios would suffice to make non-linear contagion crucial in accounting for potential losses via the credit quality channel.

### 5.3. Results with uncertainty

Let us now turn our attention to the results of introducing uncertainty in lenders' assessments of the probability of default of their borrowers, modelled as described in Subsection 4.3. As in the previous subsection, we're interested here in the aggregate effects of including this uncertainty, that is, on whether it increases or decreases the resulting total system losses with respect to the deterministic case. Figure 8 illustrates such a comparison, with the vertical axis representing the total system mean loss corresponding to the model with uncertainty relative to the original (deterministic) DebtRank model. That is, values above (below) 100% imply an increase (decrease) in losses due to uncertainty. Regarding the horizontal axis, we present results for the same wide range of system-wide leverage ratios as in the previous subsection, from slightly below 0.1% to slightly above 99%. In order to clarify the origin of the behaviours observed, the figure includes also a decomposition of this total system result into individual bank contributions.

A first observation from Figure 8 is that, under current market conditions (marked with a dotted black vertical line), uncertainty seems to have, on average, a negligible impact on the results. More broadly, uncertainty seems to have a mostly negligible effect, on average, when the banking system is either under very low levels of stress (high leverage ratios) or under very high levels of stress (low leverage ratios). However, for intermediate levels of stress (intermediate leverage ratios) the average results of both models diverge significantly from each other, with important fluctuations in the sense of these differences, ranging

Deterministic Model vs Model with Uncertainty

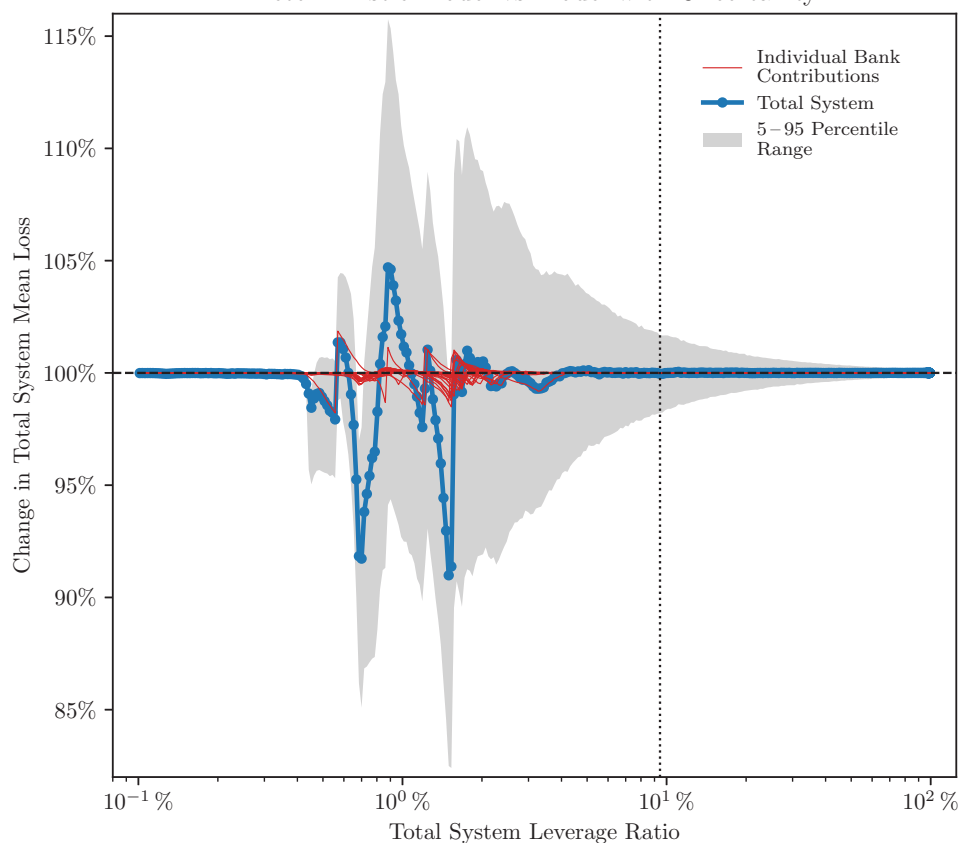


Figure 8: Effect of uncertainty: The blue line represents the change in total system mean loss, that is, the total system mean loss of the model with uncertainty relative to the original (deterministic) DebtRank case. The grey shadow indicates the 5–95 percentile range for this variable. The red lines show the contribution of each bank to this change, i.e. the blue line is a weighted sum of the red lines. The dashed black horizontal line marks the line of equality, when both models lead to the same results. The dotted black vertical line marks the actual leverage ratio of the system in our data.

from an increase in average losses of almost 5% to a decrease of around 9%. Importantly, when we move away from average results and focus on the 5–95 percentile range (grey shadow in Figure 8), we can see that the potential increase in losses more than triples with respect to the average result, reaching almost 16%, while their potential decrease roughly doubles, up to almost 18%. Moreover, a noticeable increase (decrease) in losses can be observed for the 95<sup>th</sup> (5<sup>th</sup>) percentile even around current market conditions. It is important to stress here that, from a systemic risk perspective, these worst-case increases in losses are probably of more relevance than the average results. Interestingly, looking at individual bank contributions to the average result, we can identify some of the mentioned fluctuations as being driven by changes in the impact of a single bank, and thus characterised in Figure 8 by a single red line (individual contribution) overlapping with the blue one (total system) around the area where this blue line crosses the 100% mark. This is the case, for example, of the first large fluctuation from the left, around leverage ratio 0.6%. On the contrary, other fluctuations are driven by a combination of changes in the impact of many banks, as is the case of the largest fluctuations between leverage ratio 1% and 2%.

In order to explain the origin of the fluctuations observed in Figure 8, let us start by focusing on the original (deterministic) model and proceed from higher to lower leverage ratios. In this way, at specific points in the deterioration of leverage ratios, certain thresholds are reached at which certain parts of the

network switch from dampening to amplifying shocks, thereby becoming unstable structures.<sup>18</sup> When this happens, there is an abrupt (though possibly small) increase in losses, which tends to be accompanied by an increase in bankruptcies too. That is, for the original (deterministic) DebtRank model, the increase in losses as leverage ratios deteriorate is not smooth and gradual, but instead characterised by small discontinuities at those points where different structures within the network make a transition towards instability or the amplification of shocks. Turning now our attention to the model with uncertainty, one should bear in mind that different realisations of the stochastic process will lead, for a given part of the network, to a slightly different situation in terms of how close or far it is to its instability threshold, and whether this threshold is crossed or not. In this sense, for a given value of the system leverage ratio close to but above one of the thresholds of the deterministic model, some realisations of the stochastic process will already bring the system to an unstable configuration, thus leading to larger losses as compared to the deterministic case, while in other realisations the system will remain stable, thus leading to losses comparable to the deterministic model. In a similar way, for values of the system leverage ratio close to but below one of the transition thresholds, when the deterministic model is locked into an unstable configuration, some realisations of the stochastic process will allow the model with uncertainty to escape from that instability, thus leading to smaller losses. Moreover, at each point in which the deterministic model crosses one its thresholds, the effect of the uncertainty abruptly swings between positive and negative, that is, between increasing and decreasing total losses (see oscillations in Figure 8). Finally, when system-wide leverage is very high, the whole network is very stable and far from any transition to instability, while, when it is very low, all possible structures within the network have already made the transition to instability and the system is far from any of the corresponding thresholds. Thus, in both cases, the stochastic component is not strong enough to change the stability of the system, with positive and negative fluctuations of the stochastic component compensating each other and leading the results of both models to be very similar.

These results highlight the need to incorporate uncertainty and heterogeneous biases when modelling lenders' assessments of the probability of default of their borrowers, as they can have a significant impact in the contagion and amplification of losses through the network of interbank exposures. This effect is particularly important around levels of system-wide stress where some subset of the network becomes unstable. One interpretation of this is that, as the system transitions from a state of general stability to one where vulnerabilities are more obvious —whether general or affecting only some institutions—, future outcomes become more blurred than usual. In this transition, contrasting narratives (e.g. whether or not a speculative bubble could be on the verge of collapse) can lead to diverging valuations among lenders. These, in turn, can push the system to a different equilibrium, characterised by amplified or mitigated losses when compared to a homogeneous and deterministic model. Importantly, our results show that these effects would become relevant around this type of transition, that is, precisely when decisions relying on an accurate analysis of the situation are more likely to be needed.

#### *5.4. Interaction between uncertainty and non-linear contagion*

As described in the previous two subsections, the non-linear contagion mechanism (described in Subsection 4.2) has a much stronger impact on the results of the model than the linear mechanism with uncertainty

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<sup>18</sup> Note that the amplification of shocks in the network is related to the multiple paths through which a given shock reaches a given bank and, most importantly, to exposures being highly leveraged (exposure value over the equity of the borrower).

(described in Subsection 4.3), leading to a substantially larger variation —increase— in losses. In fact, when comparing the results of a model with both non-linear contagion and uncertainty with those of the original DebtRank model (linear and deterministic), the difference in total system mean losses is fundamentally driven by the non-linearity, and thus such a comparison would look remarkably similar to that shown in Figure 7. A more interesting question is whether the inclusion of uncertainty on top of a non-linear model modifies its effects as compared to including it on top of the original, linear model. In order to answer this question, we show in Figure 9 the total system mean loss corresponding to the non-linear model with uncertainty relative to the deterministic non-linear model. In other words, Figure 9 is equivalent to Figure 8 but, instead of comparing two linear models —with uncertainty and deterministic—, it compares two non-linear models —again, with uncertainty and deterministic. As before, results are shown for a wide range system-wide leverage ratios covering different levels of stress in the system.

While the general effects of uncertainty observed in Figure 9 for the non-linear model are similar to those shown in Figure 8 for the linear model, the specific patterns and numerical values are certainly different. Regarding the similarities, we can observe in both cases an almost negligible effect of uncertainty for very low and very high leverage ratios, an important effect for intermediate leverage ratios and strong fluctuations in

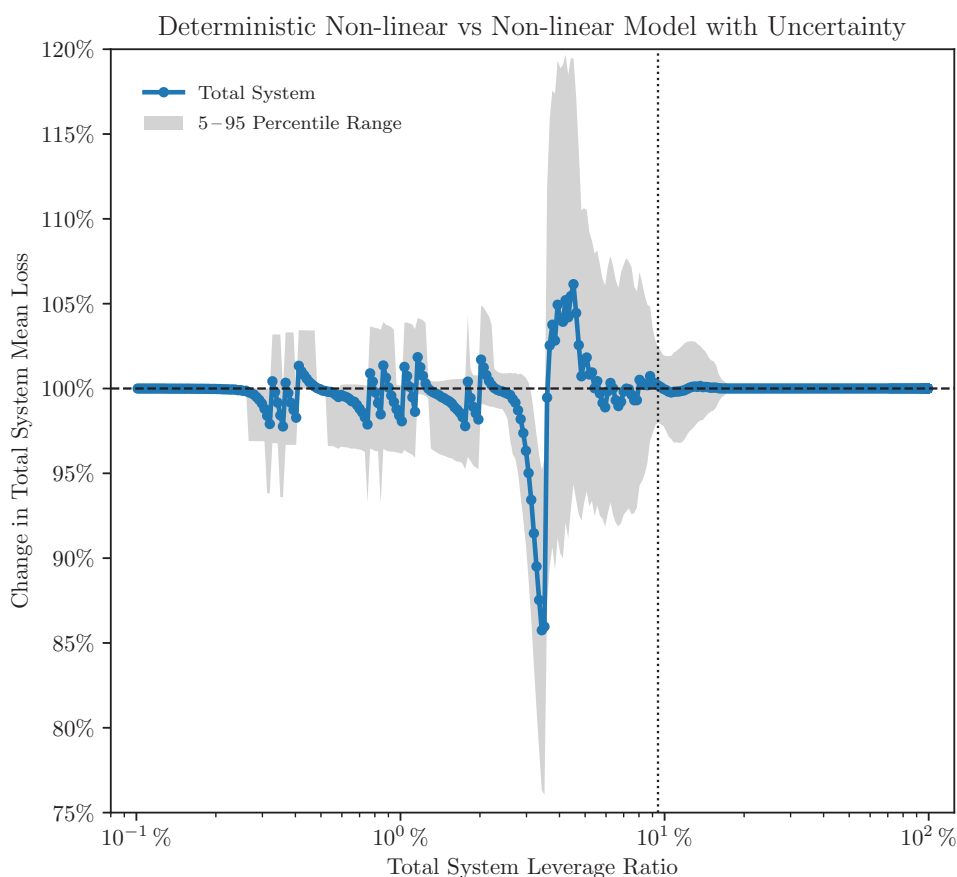


Figure 9: Effect of uncertainty over non-linear model: The blue line represents the change in total system mean loss, that is, the total system mean loss of the model with uncertainty and non-linear contagion relative to the deterministic non-linear case. The grey shadow indicates the 5–95 percentile range for this variable. The dashed black horizontal line marks the line of equality, when both models lead to the same results. The dotted black vertical line marks the actual leverage ratio of the system in our data.

the sense of the effect for this intermediate leverage region. Note that the origin of these fluctuations is the same as in the linear case, that is, the fact that the curve of total system mean losses of the deterministic model is characterised by small discontinuities, as different thresholds are reached which make different parts of the network unstable (shock-amplifying), while the curve for the model with uncertainty, being an average over realisations of the stochastic process, has a smoother behaviour and is thus able to interpolate across these discontinuities. On the contrary, the most important difference between the linear and the non-linear models in terms of the effect of uncertainty is that the leverage ratio at which this effect starts being important —when moving from higher to lower values— is much higher in the non-linear case, and thus much closer to current market conditions (marked with a dotted black vertical line in both figures). In particular, this leverage ratio at which uncertainty starts having a strong impact moves from roughly 1.5% in the linear case presented in Figure 8 to approximately 5% in the non-linear case discussed here. It is worth noting the relevance of the latter point, as it means that in the presence of non-linear contagion even a small deterioration of leverage ratios would be enough to make the inclusion of uncertainty fundamental for correctly assessing potential losses via the credit quality channel. Finally, also the magnitude of the change in average losses produced by the inclusion of uncertainty is larger in the non-linear case studied here, where this change moves between  $-14\%$  and  $+6\%$ , than in the linear equivalent presented above, where it moved between  $-9\%$  and  $+5\%$ . Again, from a systemic risk perspective, it is usually more informative to look at measures of worst-case losses. To this end, the 5–95 percentile range is also included in Figure 9. In particular, we can see that the potential increase in losses more than triples with respect to the average result, reaching almost 20%, while their potential decrease grows up to 24%. Moreover, as in the linear case, a non-negligible increase (decrease) in losses can be observed for the 95<sup>th</sup> (5<sup>th</sup>) percentile even around current market conditions.

## 6. Conclusions

In this paper, we propose two extensions to a widely used model for assessing the contagion of financial distress in a network of interconnected credit institutions, known as DebtRank. These extensions are based on considerations regarding the non-linear character of agents' reactions to counterparty losses, the uncertain nature of economic activity and agents' idiosyncratic biases in interpreting available information surrounding such economic activity. In the original DebtRank framework, however, the probability of default of a given borrower, as assessed by its creditors, is assumed to behave linearly with relative (negative) changes in the valuation of its equity. At the same time, banks are assumed to form deterministic and homogeneous expectations regarding their borrowers' probability of default.

Moving beyond this original, linear and deterministic DebtRank framework, we propose a model of distress contagion with non-linear reactions to counterparty losses and uncertainty about the effect of these losses on counterparty default risk. In particular, we argue that it seems more plausible that creditors would assess the likelihood of a borrower defaulting based not only on the size of the relative equity loss this borrower has experienced, but also its initial capitalisation. From this perspective, we propose an extension to the original model in the form of a non-linear specification for the relationship between the probability of default and the relative equity loss, thereby capturing the aforementioned dynamics. We also argue that

expectations are probably more heterogeneous in practice, due to the incomplete and imperfect information available to lenders' when assessing the probability of default of their borrowers, as well as their idiosyncratic biases when interpreting that information. In order to account for this characteristic, we propose a second extension of the original model, incorporating an explicit level of uncertainty into lenders' assessments. When exploring the effects of these two extensions, we consider different levels of stress in the system, which we approximate by modifying banks' total leverage.

We find that both of these extensions have a significant impact on the total losses experienced by the system. Regarding the non-linear contagion mechanism proposed, we find that its effects are very important for a wide range of intermediate stress levels, below but close to the current one as observed in the data, leading to substantially larger losses than in the linear case (up to 13 times larger). Regarding uncertainty, we find that its effects, while weaker than those of non-linear contagion, are nonetheless important around specific levels of stress at which different parts of the system switch from stability to instability. In particular, losses are, on average, almost 5% larger around those levels of stress, and up to 16% larger when focusing on the 95<sup>th</sup> percentile, a more relevant measure of potential losses from a systemic risk perspective. For both mechanisms, their influence seems to be negligible under very low and very high levels of stress in the system. Finally, the interaction between non-linear contagion and uncertainty has the effect of decreasing the stress levels around which uncertainty matters, bringing them much closer to current market conditions.

These results indicate that explicitly modelling these two features —non-linear reactions and uncertainty— can offer valuable insights regarding shock transmission in interbank markets, both in times of stress and in periods where, as the current one, capital levels are higher and the system could be considered as more robust. In particular, our results with non-linear contagion underline the risk of dramatically underestimating system-wide losses if lenders' are assumed to react linearly to counterparty losses. Furthermore, we have shown how uncertainty is most relevant at specific levels of stress when the system, or a part thereof, becomes unstable. In this sense, uncertainty becomes relevant precisely at the point when it is most important to have an accurate assessment of contagion risks and when policy decisions are likely to be needed. Moreover, our results indicate that the interaction between both features makes uncertainty strongly influential for even lower levels of stress, in such a way that even a small deterioration of capital levels from their current values can bring the system to the region where uncertainty has an important impact on losses.

While we have focused here on the credit quality channel, we expect our findings to be relevant also for other contagion channels, particularly for those where creditor assessments of counterparty default risk play a role, such as short-term liquidity withdrawal triggered by a loss of confidence in the counterparty. For other contagion channels, such as overlapping portfolios and liquidity withdrawal triggered by the lender's own liquidity needs, an understanding of the specific role played by uncertainty and non-linear reactions would require further research. A caveat of our research is that we focus on a reduced set of financial instruments, namely loans and debt securities. As previous research has shown, this could lead to a dramatic underestimation of potential contagion losses. Thus, the inclusion of further network layers representing different financial instruments, such as derivative exposures, constitutes a promising avenue for future research. Given the relevance of losses given default for the contagion of financial distress via the credit quality channel, a better understanding of collateral arrangements would be crucial for a correct appraisal of potential contagion losses.



## Appendix A. Calibration of $\mu_j$

Our goal is to set a fixed  $\mu_j$  value for each bank  $j$  dependent on its initial (or book value) leverage ratio  $R_j$ . By definition of the functional form, the parameter  $\mu_j$  is the value of the relative equity loss  $h_j$  at which the probability of default  $p_j^D(h_j)$  reaches 1/2, that is,

$$p_j^D(h_j = \mu_j) = 1/2. \quad (\text{A.1})$$

However, in order to differentiate entities with high and with low leverage ratios, we would prefer to set a condition directly on the leverage ratio  $R_j$  instead of on the relative equity loss  $h_j$ . That is, we do not want to decide directly on a value of  $h_j$  at which  $p_j^D$  is to become 1/2, but rather on a value of  $R_j$  at which this should happen, which we then can use to find the desired value of  $h_j$  and thus  $\mu_j$ . To this end, we just need to find an expression of  $h_j$  as a function of  $R_j$ .

Let us start with the definition of the leverage ratio,

$$R_j(t) = \frac{E_j(t)}{A_j(t)}, \quad (\text{A.2})$$

where  $E_j(t)$  is the equity of bank  $j$  at time  $t$  and  $A_j(t)$  its total assets at that same time. We can rewrite both equity and total assets as functions of their initial (book) values and the relative equity loss  $h_j(t)$  as

$$E_j(t) = E_j(1 - h_j(t)) \quad (\text{A.3})$$

and

$$A_j(t) = A_j - E_j h_j(t), \quad (\text{A.4})$$

where we note that  $E_j h_j(t)$  is simply the nominal amount lost. Thus, the leverage ratio stands as

$$R_j(t) = E_j(1 - h_j(t))/A_j - E_j h_j(t). \quad (\text{A.5})$$

We can now invert this relationship, solving for  $h_j(t)$

$$h_j(t) = \frac{\frac{E_j}{A_j} - R_j(t)}{\frac{E_j}{A_j}(1 - R_j(t))}, \quad (\text{A.6})$$

where we can already see that the current relative equity loss  $h_j(t)$  is a function of the initial and the current leverage ratios, respectively,  $E_j/A_j$  and  $R_j(t)$ . Finally, we just need to choose a value of the leverage ratio  $R_j$  at which we would like the probability of default  $p_j^D$  to reach 1/2, which we will call  $R^*$ , and introduce it in this relationship in order to find the corresponding value of the relative equity loss  $h_j^*$ , which will then, by definition, be the value of  $\mu_j$  we were looking for,

$$\mu_j = \frac{\frac{E_j}{A_j} - R^*}{\frac{E_j}{A_j}(1 - R^*)}. \quad (\text{A.7})$$

Note that, while we have chosen to define  $R^*$  such that it does not depend on the specific bank  $j$ , the corresponding relative equity loss  $h_j^*$ , and thus also  $\mu_j$ , do depend on  $j$ . In this way, we have defined a fixed value of  $\mu_j$  for each bank  $j$  as a function of its initial leverage ratio and a threshold value of the leverage ratio  $R^*$  which we assume exogenously set and common to all banks.

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