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GPS Coordinate Time Series

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Abstract In this project, I will demonstrate the process of analyzing GPS time-series data. I have used 10 years of continuous GPS measurements of four stations (ALGO, GRAZ, KOSG, and ONSA) and three different combinations of the noise component of GPS coordinate time-series, i.e., White noise, Flicker noise, and random walk noise. I use Least Squares Variance Component Estimation (LS-VCE) instead of Maximum Likelihood Estimation (MLE) because of its advantages.

Introduction Continuous GPS measurements have been used now nearly 19 years for the estimation of crustal deformation. Geophysical studies using geodetic measurements of surface displacement require both accurate estimates and errors of the parameters. This precision is often acquired by the repetition process of the least square method. Except for the significant episodic deformation, such as large earthquakes, a linear trend can be a good representative of deformation behavior. In the ideal case, it is desired that the time series have only white noise. However, in the real case, it is different. Several geodetic data sets show that there are large temporal correlations in data. The goals of noise studying are to process the coordinate time series with a stochastic model and find out the most precise solution with proper precision of the station positions and site velocities, and identify the various noise component of the stochastic model.

There are two techniques for processing the behavior of noise of geodetic time series, the power spectral method, and VCE. In this project, the second technique has been chosen. I used the least-square Variance Component Estimation Method (LS-VCE) instead of the MLE method because it has several advantages with respect to the MLE method. First is that in contrast to MLE which gives biased estimators, LS-VCE provides unbiased and minimum variance estimators, and the unbiasedness property is independent of the distribution of the data. In addition, LS-VCE is faster than MLE, and in LS-VCE; it is possible to incorporate any number of noise components in the stochastic model.

Analyzing GPS time-series

Two models have been used for processing:

1. Functional model

2. Stochastic model

Functional model:

$$E\{y(t)\} = y_0 + rt + \sum_{k=1}^{q} a_k \cos \omega_k t + b_k \sin \omega_k t$$

Design matrix:

$$a_i = [1, t_i, cos\ 2\pi t_i\ , sin\ 2\pi t_i\ , cos\ 4\pi t_i\ , sin\ 4\pi t_i\ , cos\ 2\pi \tfrac{365}{360} t_i\ , sin\ 2\pi \tfrac{365}{360} t_i, cos\ 4\pi \tfrac{365}{360} t_i, sin\ 4\pi \tfrac{365}{360} t_i, cos\ 6\pi \tfrac{365}{360} t_i, sin\ 6\pi \tfrac{365}{360} t_i\ ]$$

Where t_i expressed in terms of year (yr.)

Stochastic model As I mentioned before the GPS coordinate time-series is composed of white noise, flicker noise and random walk noise with variances σ_w^2, σ_f^2 and σ_{rw}^2 respectively. Therefore, the covariance matrix of the time-series is equal:

$$Q_y = \sigma_w^2 I + \sigma_f^2 Q_f + \sigma_{rw}^2 Q_{rw}$$

Where I is the m×m identity matrix, and Q_f and Q_{rw} are the cofactor matrices for flicker noise and random walk noise, respectively.

Note: σ_w^2, σ_f^2 and σ_{rw}^2 are unknowns.

Q$_f$:

$$q_{ij}^{(f)} = \begin{cases} \frac{9}{8} & \text{if } \tau = 0 \\ \frac{9}{8}\left(1 - \frac{log\,\tau / log\,2 + 2}{24}\right) & \text{if } \tau \neq 0 \end{cases}$$

Where $\tau = |t_j - t_i|$

Q$_{rw}$:

$$Q_{rw} = f_s^{-1} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & & 2 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & m \end{bmatrix}; f_s^{-1} = \frac{m-1}{10}$$

Three kind of combination have been used for analyzing data. First, is white noise only, second, is the combination of white noise plus flicker noise, and third, is the combination of white noise plus random walk noise.

The result of each model and its accuracy are depicted at the table below:

Site code			ALGO		GRAZ		KOSG		ONSA	
Model		C	$\sigma^\wedge$	$Q_{\sigma^\wedge}$	$\sigma^\wedge$	$Q_{\sigma^\wedge}$	$\sigma^\wedge$	$Q_{\sigma^\wedge}$	$\sigma^\wedge$	$Q_{\sigma^\wedge}$
I	WN	N	0.3931	0.0006192	0.3717	0.0006192	0.1295	0.0006192	0.3642	0.0006192
		E	0.5457	0.0006192	0.4871	0.0006192	0.3634	0.0006192	0.3983	0.0006192
		V	0.8848	0.0006192	0.93	0.0006192	0.7783	0.0006192	0.9273	0.0006192
II	WN	N	0.2986	0.000778	0.2954	0.000778	0.3701	0.000777	0.2824	0.00077
		E	0.3261	0.0000258	0.3853	0.000778	0.3101	0.000777	0.3158	0.00077
		V	0.7004	0.0007783	0.6808	0.0004220	0.5738	0.0002104	0.5839	0.000282
	FN	N	0.3706	0.0019	0.3268	0.0007782	0.3405	0.0019	0.3345	0.0019
		E	0.5787	0.0008585	0.4235	0.0019	0.2967	0.0019	0.3261	0.0019
		V	0.8112	0.0019	1.0499	0.0098	0.8872	0.0048	1.0582	0.0084
III	WN	N	0.3744	0.0006201	0.3504	0.0006201	0.3552	0.0006201	0.3405	0.0006201
		E	0.3918	0.0000179	0.4568	0.0006201	0.3598	0.0006201	0.3562	0.0006201
		V	0.8285	0.0006201	09057	0.0006201	0.7640	0.0006201	0.6915	0.0001849
	RW	N	0.1491	0.0000022	0.1538	0.0000022	0.1295	0.0000022	0.1608	0.0000022
		E	1.3162	0.0425	0.2107	0.0000022	0.0634	0.0000022	0.2216	0.0000022
		V	0.3865	0.0000022	0.2628	0.0000022	0.1848	0.0000022	2.9612	0.8864

By computing the difference in the log-likelihood values for ONSA (for each component and each error model), and comparing the values, it achieved that each model that has a larger value describes the noise better.

$$\text{Log Likelihood Function} = -\frac{m}{2}\log 2\pi - \frac{1}{2}\log\det\left(Q_y\right) - \frac{1}{2}\left(e^{\wedge T} Q_y^{-1} e^{\wedge}\right)$$

Model:	I: WN			II: WN+FN			III: WN+RW		
ONSA	East	North	Vertical	East	North	Vertical	East	North	Vertical
	-4600.2	-4600.2	-4600.2	-3312.5	-3334.8	-5574	-71098	-132250	-2305100

The results show that second model (Flicker noise plus white noise) is the preferred model model as it provides the largest values.

Conclusion It is obvious that for different noise components, the horizontal components are less noisy than the vertical component. In contrast with the white noise only, the amplitude of white noise for the second combination (white noise plus flicker noise) is about 30% smaller and for the third combination (white noise plus random walk noise) is about 20% smaller.

Note: The combination of three kinds of noise, i.e., white noise plus flicker noise plus random walk noise is not proper here, because the component of the site time series becomes negative.

Reference:

Amiri-Simkooei A.R. (2007). Least-squares variance component estimation: theory and GPS applications. PhD thesis, Delft University of Technology