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Thomas Bernardin and Hyun Woong Park

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Liquidity, Bank Runs, and Fire Sales under Local Thinking

Thomas Bernardin*

Hyun Woong Park[†]

Abstract

In this paper, we examine the implications on banking crises when markets are populated by agents that neglect tail risks and form expectations conditioned on a favorable subset of all possible states of the economy. We find that optimal bank liquidity is lower than would be the case when banks are guided by rational expectations, and, consequently, the banking system is more vulnerable to adverse shocks, which leads to bank runs. Asset pledgeability of surviving banks is also affected so that their capacity to raise external funds for purchasing assets of distressed banks is weakened. Further, we examine the case when asset returns are correlated through securitization. In this case adverse shocks will be felt uniformly across the banking sector and banks that survive with the help of a public liquidity backstop will become risk-averse and reluctant to purchase distressed assets. We also explore a government funded asset purchase program, that is implemented with an asset price target.

Keywords: bank runs, fire sales, bank liquidity, banking crises, local thinking

JEL Classification: G01, G12, G21, G32

*St. Olaf College, Department of Economics; bernar1@stolaf.edu

[†]Denison University, Department of Economics; parkhw@denison.edu

1 Introduction

One common factor in financial crises is the market’s misperception of uncertain events. Instead of carefully considering all possible outcomes of portfolio decisions, investors tend to neglect events that are very unlikely to occur. These low probability events are in many cases the worst–case scenarios. A consequence of this is overly optimistic behavior of agents in financial markets which leads to over–valuation of securities. When things are normal, risky investments based on the misperceptions are not questioned and therefore continue uninterrupted. In times of stress, however, the market quickly realizes the misperception and abruptly reacts even to small hints of bad news; a flight to safety and free fall of asset prices ensue.

The most recent example is the 2007–2009 financial crisis, which closely followed this narrative; when banks originated mortgage loans to subprime borrowers and when investors purchased subprime mortgage loan–related securities, the possibility of the subprime borrowers’ debt service failure was neglected as very unlikely; this optimism allowed the creation of voluminous risky structured securities which continued until news about mortgage delinquency started to materialize. Investor reaction to these now toxic securities was severe, and led to bank runs in short–term money markets.¹

In this paper, we examine a financial market and banking system populated by agents that are characterized by such misperceptions. For this, we adopt the model of local thinking presented in Gennaioli and Shleifer (2010) and Gennaioli et al. (2012). The model assumes that agents have a limited ability to make inferences about uncertainties and therefore neglect tail risks with the lowest probabilities. This leads to decisions and behavior shaped by excessive optimism and pessimism. The local thinking framework was developed in the spirit of Daniel Kahneman and Amos Tversky’s papers (Kahneman and Tversky, 1972, Tversky and Kahneman, 1974) that demonstrate significant deviations from the Bayesian theory of judgment under uncertainty. Its implication is also akin to Hyman Minsky’s financial

¹See Gorton and Metrick (2012) for a run on repo, Schmidt et al. (2016) for a run on money market mutual funds, and Covitz et al. (2013) for a run on asset–backed commercial paper.

instability hypothesis (Minsky, 1992), according to which an extended period of tranquility and stability makes the memory of past crises fade from agents' minds.²

The main purpose of this paper is to present a model that demonstrates how banks make portfolio decisions and manage their balance sheet when both banks and investors, who provide funds to the bank, are subject to local thinking. The agents in the model ignore the state of the economy that occurs with the lowest probability, which is usually the worst-case scenario. As a consequence, both banks and investors are overly optimistic. Banks switch their portfolio composition more toward risky assets and away from liquid reserves, while investors are willing to fund the banks' risky investment strategy with their deposits.

During normal periods, banks' risky portfolio choices and illiquid balance sheet positions proceed without interruption, and so too do the investors' decisions to hold the liabilities of these banks. When local thinking agents observe bad news, however, they abruptly revise downward their evaluations about their risky investments. As a consequence, depending on the severity of the news, investors may run on the banks. The banks cannot fully accommodate the withdrawal demand since the bank's liquidity holding is characteristically insufficient.

Consequently, the bank has to liquidate its assets even at fire sale prices in order to obtain liquidity to meet the demand for withdrawals. Banks that survived the bad news or that are protected by the government backstop appear as potential buyers in the asset market. However, since these surviving banks are also local thinkers, the bad news negatively affects them as well, by devaluing their assets and thus weakening their balance sheet status. Therefore, the demand for the distressed assets can be weak due to liquidity shortages. As a consequence, an asset price depreciation follows.

Based on this narrative, our model derives several interesting analytical results regarding the relationship between bank liquidity, bank runs, and fire sales. First, the optimal liquidity holding of a local thinking bank is negatively correlated with the degree to which the bank overvalues its risky investment by neglecting tail risks. It follows as a corollary that a local

²Bhattacharya et al. (2015) presents a formal model of Minsky's hypothesis that generates a leverage cycle.

thinking bank tends to hold a smaller amount of liquid assets than a bank with rational expectations. We also derive the condition for a bank run to take place under local thinking. The result turns out to be quite intuitive; a bank run will occur when the bank's expected payoff, after the bad news, is smaller than its debt obligations to its creditors. What is more important is that the amount of optimal liquidity of the local thinking bank is such that when the bank experiences a run, it suffers from liquidity shortage and hence is forced to liquidate its assets possibly in a fire sale.

In this context, we identify two channels through which local thinking behavior affects fire sale asset prices. For this, we follow Holmstrom and Tirole (1998) in explaining the illiquidity of a risky asset with long-term maturity by limited pledgeability. That is, due to moral hazard at the bank level, some portion of the bank's future cash flow has to be promised to a bank manager in order to guarantee that she exerts effort in monitoring the performance of the risky asset. Hence, not all the future cash flow, but only part of it, can be pledged to raise extra funds in capital markets. In this circumstance, the local thinking behavior affects asset prices through lowering the availability of liquidity in the asset market first by lowering the expected future cash flow of the bank that survived the bad news and, second by lowering the share of the bank's expected future cash flow that can be pledged for borrowing. This result is similar to Acharya et al. (2011), where only the first channel is identified.

Further, in comparison to the related literature that in many cases features homogeneous banks and homogeneous risky assets, our model introduces heterogeneity in the banking system and the risky asset class. As a first approximation, we assume two different types of banks — one with government insurance and the other without it — and two different types of risky assets with different risk–return profiles; each type of bank investing only in a single type of risky asset. This setting allows us to distinguish between the case where the two risky assets' returns exhibit a weak or no correlation and the case where they exhibit a strong correlation. When the asset returns are weakly correlated, the bad news affects only a single type of risky asset, and if that turns out to be the asset purchased by the banks without government protection, those banks can possibly experience a run and be forced to

liquidate assets in a fire sale; the other banks that survived the bad news become potential buyers of the distressed assets.

On the other hand, when the asset returns are strongly positively correlated via, for instance, securitization, the bad news will most probably affect both types of risky assets through contagious spread of underlying assets' risks to asset-backed securities. In this case, the financial position of both types of banks deteriorates to the extent that the conditions for a bank run are met. However, it is only those banks without public backstop that are forced to sell their assets even at a fire sale price. Nonetheless, the banks that survive the bad news due to the protection from the government insurance will be reluctant to bid for the liquidated asset and take on risks, since their balance sheets are also affected by the bad news to a degree similar to the distressed banks.

This leads us to explore asset purchases by the government as an outside buyer that uses the balance sheet of the central bank to prop up the asset market. As a result, the size of the central bank balance sheet exhibits a sudden increase, followed by an asset price appreciation. We consider the case where, when it injects public liquidity through an asset purchase program, the government sets an asset price target. Since the target is set according to a policy agenda for the broader economy, it is possible that the asset price target deviates from the fundamental value.

Along this line of thinking, we derive the volume of the public liquidity injection required to stabilize the asset price around the exogenously given target. The results of the model show that, in times of stress, more public liquidity is required to achieve the asset price target when the number of banks that operate without government protection is larger and when the liquidity preference of the surviving banks is stronger. In particular, the same is true when the local thinking effect is stronger, i.e. when the market tends to overvalue the assets by ignoring the worst-possible scenarios.

Our paper contributes to the literature on bank runs and fire sales by exploring these issues under Gennaioli et al. (2012)'s model of local thinking. First, in the existing literature, bank runs occur either due to coordination problems among depositors (see, e.g., Diamond and Dybvig (1983) and Rochet and Vives (2004)) or due to asymmetric information among

depositors concerning bank fundamentals (see, e.g., Chari and Jagannathan (1988) and Jacklin and Bhattacharya (1988)).³ While these factors can be made compatible with banking crises in our paper, the bank run modeled below is triggered first and foremost by the fact that depositors as well as banks are local thinkers. All of these papers, including our own, have some form of bounded rationality at the heart of the explanation of bank runs. And the fact that a bank run may or may not occur in our model is similar to the multiple equilibria result of Diamond and Dybvig (1983). However, whereas the latter result depends on depositors' confidence, which determines whether they will panic or not, in our model that depends on how damaging the neglected tail risks turn out to be after the bad news, which in turn ultimately depends on the true risk–return profile of bank assets.

Second, Shleifer and Vishny (2011) provide an extensive literature review on fire sales. One of the important themes in the literature related to our paper is whether a buyer of distressed assets is an insider or an outsider. (see Shleifer and Vishny (1992) and Williamson (1988)). In the case of an industry–wide adverse shock with inside potential buyers being also under stress, assets of a distressed firm will have to be sold to outside buyers, i.e. firms from other industries, which lack technology to use the distressed assets in their best use; hence the asset value is driven down. Whereas most of the papers in this strand of the literature focus on asset liquidation by nonfinancial firms, our paper examines asset liquidation by banks.

Acharya et al. (2011) is closely related to our paper. Both papers deal with bank asset liquidation and, particularly, adopt decreasing returns to scale for risky asset returns. Our paper is differentiated in the reason banks hold liquidity. Acharya et al. (2011), on the one hand, focuses on the ‘strategic’ motive of obtaining capital gains by buying distressed assets at a price below their value when an opportunity arises. In this way, the authors show that bank liquidity is counter–cyclical.

However, while banks acquiring other distressed banks were widely observed during the recent crisis, it was also the case that banks were reluctant to take on further risks by buying

³Allen and Gale (1998) and (Allen and Gale, 2004) show that bank runs can be socially useful as they allow depositors to share risks of aggregate uncertainty through the banking system.

distressed assets as well as taking over the banks that owned them, which prompted the government numerous government interventions.⁴ Recent empirical studies such as Berrospide (2012), Mutu and Corovei (2013), Acharya and Merrouche (2013) show that the rise in banking sector liquid assets during the recent financial crisis was mainly driven by a precautionary motive. That during the 2007–2009 financial meltdown the banks turned risk-averse, rather than seeking strategic gains, is also consistent with the above-mentioned theme from the corporate finance literature on an industry-wide adverse shock, which puts a wider pool of firms in stress; i.e., even if some firms survived the shock, they would be reluctant to purchase failed firms’ assets. For this reason, instead of the strategic motive for banks’ holding liquidity, we focus on the precautionary motive. And we show that when local thinking banks hold liquidity mainly to remain solvent and to protect themselves from liquidity shocks, their liquidity holding is smaller than the liquidity holdings of banks with rational expectations. Local thinking behavior contributes to asset price depreciation in fire sales by affecting the balance sheets of both liquidating banks and surviving banks.

The rest of the paper is organized as follows: Section 2 introduces the basic setup of the model with asset weakly correlated or uncorrelated asset returns. Section 3 introduces local thinking framework. In section 4, optimal bank liquidity is derived and in section 5, the bank run condition is derived. Section 6 analyzes the market for assets of distressed banks, from which the fire sale asset price is obtained. In section 7, we extend the model to include asset return correlation via securitization and explore the government’s asset purchase program, that uses the central bank’s balance sheet. Section 4 concludes.

2 Basic setup

Consider a three-period economy: $t = 0, 1, 2$. There are two types of risky assets which, when invested in at $t = 0$, mature at $t = 2$ yielding a stochastic return y_i^j with probability π_i^j , where $i \in \{1, 2\}$ indicates asset types and j indicates three contingent states of the

⁴For bank merger and acquisition and government takeover during the 2007–2009 financial crisis, see Frame et al. (2015) and Kowalik et al. (2015).

economy at $t = 2$. There are three states: growth, downturn, and recession indicated by $j \in \{g(\text{growth}), d(\text{downturn}), r(\text{recession})\}$. We assume:

Assumption 1 $y_i^g > 1 > y_i^d > y_i^r$, $\pi_i^g > \pi_i^d > \pi_i^r$, $E(y_i) > 1$ $i = 1, 2$

where E is an expectation operator; $E(y_i) = \pi_i^g y_i^g + \pi_i^d y_i^d + \pi_i^r y_i^r$. The risky asset earns a positive rate of return if the economy grows, a negative rate of return if the economy experiences a downturn, and an even more negative rate of return in the case of a recession. The economy has the highest probability of experiencing growth and the lowest probability of experiencing a recession.

There are a continuum of risk-neutral banks of measure one, each of them obtaining one unit deposit from investors. A fraction of the total banks are type 1 banks, hence denoted by subscript 1 and invest in type 1 risky asset, while the rest are type 2 banks, hence denoted by subscript 2 and investing in type 2 risky asset. Let us denote the fraction of type 2 banks by n ; then the fraction of type 1 banks is $1 - n$. Type 1 bank's deposit account is protected by government insurance, but this is not the case for type 2 banks.

There are a continuum of investors of measure one, each investor having sufficient initial endowments. In contrast to the banks, the investors are infinitely risk-averse and have no screening, monitoring, or loan collecting technologies, as the banks do. Therefore, the investors invest only in bank liabilities, that pay off a non-stochastic return. When deposited at type i bank from $t = 0$ to $t = 2$, an investor earns the riskless return of q_i , for which we adopt the following assumptions.

Assumption 2 $1 < q_i < E(y_i)$, $q_1 < q_2$

The last inequality reflects risk-return trade-off; i.e. type 2 bank's liability is considered riskier than type 1 bank's liability since it is not guaranteed by the public backstop in contrast to the type 1 bank's.

This setting of each type of banks investing on a distinctive type of risky asset can be viewed as each bank-type specializing in a specific industry. In order to further simplify the analysis, we assume each bank is a local monopolist and hence the investors have no choice but simply deposit their funds at the monopolist bank in the neighborhood. This simplifies

the analysis of the investor's decision-making without affecting the model results. On the other hand, the investors can also withdraw their deposits at $t = 1$ but with zero rate of return. So the investors withdraw only when they are hit by a liquidity shock, which occurs with probability θ as in Diamond and Dybvig (1983). Due to the law of large numbers, θ can be considered the number of investors hit by the liquidity shock.

Using the deposit funds, the banks at $t = 0$ make a portfolio choice between the risky asset, denoted by a , and liquid reserves, denoted by l ; hence type i bank's portfolio being (a_i, l_i) where $a_i = 1 - l_i$. We assume the risky asset's return has decreasing returns to scale. In order to obtain closed form solutions, we follow Acharya et al. (2011) in adopting a specific functional form similar to that used in Holmstrom and Tirole (2001): eqrefdiminishing return to scale

$$y_i^j = b_i^j - \frac{a_i}{2} \quad (1)$$

From this specification the lower and upper bound of the risky asset's return can be identified; $y_i^j \in [b_i^j - \frac{1}{2}, b_i^j]$. That is, b_i^j can be interpreted as an upper bound of y_i^j . Therefore, b_i has the same probability distribution as that of y_i stated in assumption 1.

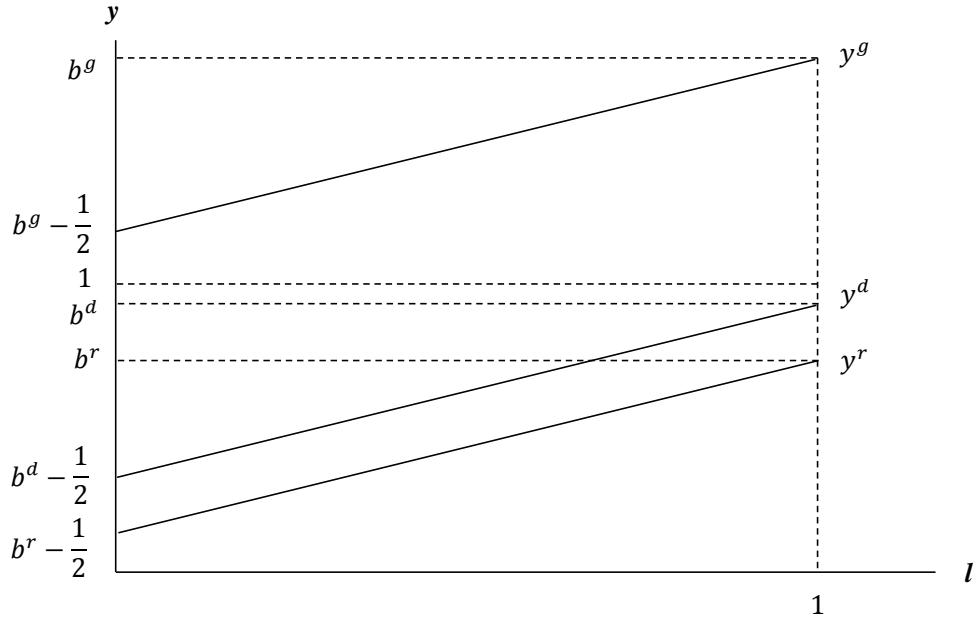
By taking the expectation operator on (1), $E(y_i)$ can be rewritten as

$$E(y_i) = E(b_i) - \frac{a_i}{2} \quad (2)$$

which clarifies that risks in y_i to be associated with risks in its upper bound. In this context, expectations about the risky assets' returns are actually expectations about their upper bound. On the other hand, the risky asset return profile given in assumption 1 along with $y_i^j \in [b_i^j - \frac{1}{2}, b_i^j]$ implies that the lower bound of y_i^g is larger than one, which therefore yields $b_i^g > \frac{3}{2}$, and the upper bound of y_i^d and y_i^r are smaller than one, i.e. $b_i^d < 1$ and $b_i^r < 1$, with $b_i^d > b_i^r$. Figure 1 displays the resulting value of the risky asset return as a function of liquidity holding.

After portfolio decisions by investors and banks are made, a signal $s \in \{\underline{s}, \bar{s}\}$ of the payoff y_i arrives in financial markets at $t = 1$. It is an aggregate, systemic signal that conveys news about both types of risky assets. The signal can be either good news \bar{s} or bad news \underline{s} about y_i . In particular, since it is a bank that invests on the risky asset, the signal is

Figure 1: The risky asset return with decreasing returns to scale.

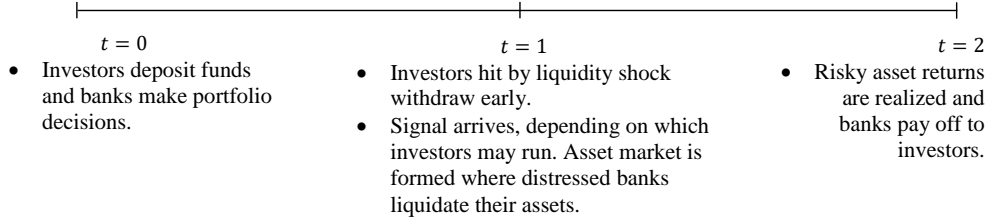


informative of whether the bank can make the promised payoffs to its investors at $t = 2$. In the case of bad news, the expected value of y_i could be too low for the bank to even return the principal to depositors, in which case the latter would be better-off withdrawing their deposits. Accordingly, by observing the signal, the depositors decide whether to run on the bank or not.

Depending on the signal and the given return profile, some banks may experience a run and thus are forced to liquidate their assets even at a fire sale price, while the others may not. The surviving banks become potential bidders for the assets liquidated by the failed banks. Since type 1 banks are protected by government insurance while type 2 banks are not, it will be type 2 banks that may experience a run, in which case it will be type 1 banks that come as a buyer in the asset market.

The timing of the model is illustrated in figure 2

Figure 2: Timing of the model



3 The model of local thinking

In characterizing the agent’s formation of expectations about risky asset returns as well as the signal that affects the reassessment of asset return expectations, we adopt the model of local thinking presented in Gennaioli and Shleifer (2010). As an alternative to rational expectations, the local thinking framework highlights judgment biases due to limited ability in forming true representations of reality. The key idea is that when agents form a statistical expectation, not all possible states of the world, but only a selected subset of them, come to mind. The selection is made according to the true probabilities of the events. That is, the states with higher probability are more likely to be represented in the agent’s inference than the states with lower probability.

More specifically, we follow Gennaioli et al. (2012) in modeling local thinking by supposing that out of the state space, $j = \{g, d, r\}$, of a risky asset’s payoff, only two most likely states are represented in the agent’s mind, while the remaining state is neglected. Then the agent forms an expectation about the risky assets’ returns, conditioned on the two selected states. That is, facing an uncertainty, a local thinker forms a conditional expectation, neglecting the least likely case which is usually the worst–possible scenario. This is in contrast to an agent with rational expectation who forms an expectation by considering the entire state space.

For instance, consider a local thinker who, at $t = 0$, forms expectations about the uncertain future concerning the returns of the risky assets. Due to assumption 1, where we have $\pi_i^g > \pi_i^d > \pi_i^r$, only the states g and d are represented in the agent’s mind while the state r is ignored. And in place of the true probability of g and d , i.e. π_i^g and π_i^d , the probability

distribution represented in the local thinker's mind is as follows:

$$\begin{aligned}
\Pr^L(y_i^g) &= \Pr(y_i^g | y_i^g, y_i^d) = \frac{\pi_i^g}{\pi_i^g + \pi_i^d} \\
\Pr^L(y_i^d) &= \Pr(y_i^d | y_i^g, y_i^d) = \frac{\pi_i^d}{\pi_i^g + \pi_i^d} \\
\Pr^L(y_i^r) &= \Pr(y_i^r | y_i^g, y_i^d) = 0
\end{aligned} \tag{3}$$

where the superscript L denotes local thinking. In comparison to the true probability distribution, $\pi_i^g > \pi_i^d > \pi_i^r$, the probability of growth and that of downturn are overestimated, while the possibility of a recession is ignored; i.e. $\Pr^L(y_i^g) > \pi_i^g$, $\Pr^L(y_i^d) > \pi_i^d$, and $\Pr^L(y_i^r) = 0$.

Accordingly, the local thinker's expectation about the risky asset's return is formed as

$$E^L(y_i) = \Pr^L(y_i^g)y_i^g + \Pr^L(y_i^d)y_i^d \tag{4}$$

which contrasts with the one formed by rational expectations, $E(y_i) = \pi_i^g y_i^g + \pi_i^d y_i^d + \pi_i^r y_i^r$. It can be easily verified that $E^L(y_i) > E(y_i)$. That is, the local thinker tends to exaggerate the expected return of the risky assets in comparison to the true expectations of rational thinkers.

Suppose that the probability distribution of the risky assets' return and, possibly, its order changes when a signal arrives at $t = 1$. In this case, the local thinker revises her representation of the state of the world accordingly. The states which are now represented in the agent's mind depends on the new probability distribution established after the signal, i.e. $\pi_i^j(s) = \Pr(y_i^j | s)$ for $j = g, d, r$. We assume, on the one hand,

Assumption 3 $\pi_i^g(\bar{s}) > \pi_i^d(\bar{s}) > \pi_i^r(\bar{s})$, $i = 1, 2$.

which implies that the good news \bar{s} does not modify the original probability distribution reflected in assumption 1, and hence confirms the local thinker's initial inference about the future. In this case, the status quo continues uninterrupted.

The bad signal \underline{s} , on the other hand, is assumed to modify the probability distribution of y_i^j . In order to specify this process, \underline{s} is characterized in the following way.

$$\Pr(\underline{s} | y_i^g) = 1 - \gamma_i, \quad \Pr(\underline{s} | y_i^d) = \delta_i, \quad \Pr(\underline{s} | y_i^r) = \rho_i, \quad i = 1, 2 \tag{5}$$

where $\rho_i > \delta_i > \gamma_i \geq 1/2$. That is, the bad signal, \underline{s} , is most likely to arrive in the case of a recession and is least likely to arrive in the case of growth; \underline{s} more strongly signals a recession than a downturn as reflected in $\rho_i > \delta_i$, while it reduces the probability of growth as reflected in $\Pr(\underline{s}|y_i^g) \leq 1/2$.⁵ In this regard, we assume \underline{s} reverses the order between the probabilities of downturn and recession, while the probability of growth is not changed, i.e.

$$\pi_i^g(\underline{s}) > \pi_i^r(\underline{s}) > \pi_i^d(\underline{s}), i = 1, 2. \quad (6)$$

This will be the case as long as the following assumption is adopted.

Assumption 4 $\rho_i > \hat{\rho}_i = \delta_i \frac{\pi_i^d}{\pi_i^r}, \quad i = 1, 2$

which is easily satisfied if π_i^d and π_i^r are sufficiently close to each other and we assume that they are.

With the probability distribution changed as in (6), the possibility of downturn drops out of a local thinker's mind and is replaced by that of recession in the local thinker's expectation formation. In this regard, we have

$$E^L(y_i|\underline{s}) = y_i^g \pi_i^g(\underline{s}) + y_i^r \pi_i^r(\underline{s}) \quad (7)$$

which compares to the local thinking expectation before the signal in (4). We assume the return profile and probability distribution before and after the signal in assumptions 1 and 3 are such that $E^L(y_i|\underline{s}) < E^L(y_i)$ holds.

Lastly, as noted earlier, under the assumption of diminishing return to scale specified in (1) expectations about y_i are actually expectations about its upper bound b_i as shown in equation (2). This continues to hold even after the signal. Accordingly, $E^L(y_i)$ in the case of a bad signal at $t = 1$ can also be expressed as

$$E^L(y_i|\underline{s}) = E^L(b_i|\underline{s}) - \frac{1}{2}(1 - l_i) \quad (8)$$

where

$$E^L(b_i|\underline{s}) = b_i^g \pi_i^g(\underline{s}) + b_i^r \pi_i^r(\underline{s}) \quad (9)$$

with $E^L(b_i|\underline{s}) < E^L(b_i)$

⁵Note that $\rho_i \approx 1/2$ implies that the signal is scarcely informative.

4 Banks' liquidity choice at $t = 0$

At $t = 0$, while the investors simply deposit a fixed amount of funds at the local monopolist bank in their neighborhood, the banks make a portfolio decision, between liquid and risky assets. First of all, the bank's demand for liquidity consists of two components. First is the liquidity holding to meet the withdrawal demand at $t = 1$ by the depositors hit by a liquidity shock; this is θ , the liquidity required for solvency at $t = 1$. In addition, the bank may hold extra liquidity that can be tapped when the expected return from the risky asset is smaller than what it promised to pay to the remaining depositors at $t = 2$, i.e. when $(1 - \theta)q_i > (1 - l_i)E^L(y_i)$ in the case of type i banks. Accordingly, the extra liquidity type i banks need to hold to make up this difference is

$$(1 - \theta)q_i - (1 - l_i)E_i^L(y_i) > 0 \quad (10)$$

which is the liquidity required for solvency at $t = 2$ when $E_i^L(y_i)$ is sufficiently low, i.e. $E_i^L(y_i) < \frac{(1-\theta)q_i}{1-l_i}$, such that the bank cannot meet the debt contract with their depositors. In all, the sum of the two components is the minimum total liquidity holding of type i banks at $t = 0$; hence,

$$l_i \geq \theta + (1 - \theta)q_i - (1 - l_i)E_i^L(y_i) \quad (11)$$

Consider, on the other hand, that $E_i^L(y_i)$ is sufficiently large so that the return from the risky asset is expected to be larger than the bank's debt obligations to its remaining depositors at $t = 2$; that is, $(1 - \theta)q_i < (1 - l_i)E_i^L(y_i)$. In this case, i.e. when $E_i^L(y_i) > \frac{(1-\theta)q_i}{1-l_i}$, it is only the first component that consists of the minimum liquidity holding of type i bank at $t = 0$; hence,

$$l_i \geq \theta \quad (12)$$

In sum, type i bank's liquidity holding at $t = 0$ for any possible level of $E^L(y_i)$ can be expressed as

$$l_i \geq \text{Max}[\theta, \theta + (1 - \theta)q_i - (1 - l_i)E^L(y_i)] \quad (13)$$

Now, since a bank is risk-neutral, it maximizes payoff at $t = 2$ subject to the condition of expected solvency expressed in (13), which guarantees solvency for any value of $E^L(y_i)$.

Type i bank's payoff at $t = 2$ consists of its liquidity holding at $t = 0$ and the returns from the risky asset at $t = 2$ minus the withdrawn funds of its depositors hit by a liquidity shock at $t = 1$ and what it pays out to the remaining depositors at $t = 2$. Denoting type i bank's payoff at $t = 2$ by π_i , which is a function of l_i , the bank's optimization problem is as follows.

$$\begin{aligned} \max_{l_i} \quad & \pi_i(l_i) = l_i + (1 - l_i)E^L(y_i) - \theta - (1 - \theta)q_i \\ \text{s.t.} \quad & l_i \geq \text{Max}[\theta, \theta + (1 - \theta)q_i - (1 - l_i)E_i^L(y_i)] \end{aligned} \tag{14}$$

The solution for the above system is stated in the following lemma.

Lemma 1 (Optimal bank liquidity) $l_i^* = 1 - (E^L(b_i) - 1)$, $i = 1, 2$

Two things stand out about lemma 1. First, due to assumption 1 along with the fact that b_i is an upper bound of y_i , it can be easily shown that

Corollary 1 $0 < l_i^* < 1$

That is, the optimizing local thinking bank allocates some of the deposit funds to liquidity. Second, the bank's optimal liquidity holding is a negative function of its local thinking expectation about the risky asset return, or, more precisely, about its upper bound.

Since the risky assets' returns and hence their expected values are a function of bank liquidity due to the assumption of diminishing returns to scale, the result in lemma 1 can be substituted to equations (2) and (8) in obtaining the expected return of risky assets in the case of optimal bank liquidity before and after the bad signal, respectively:

$$\begin{aligned} E^L(y_i) &= \frac{1}{2}(E^L(b_i) + 1) \\ E^L(y_i|\underline{s}) &= \frac{1}{2}(E^L(b_i|\underline{s}) + 1) \end{aligned} \tag{15}$$

Equations in (15) simply describe the relation between the risky asset's return and its upper bound and that it is positive.

As a comparison, consider a rational bank, which operates under the exact same conditions as those of a local thinking bank expect that it is not subject to local thinking. Then the optimal liquidity holding, denoted by l_{Ri}^* , of a type i rational bank can be derived in a

way similar to the local thinking banks' cases. Then, the rational bank's optimal liquidity is obtained as $l_{Ri}^* = 1 - (E(b_i) - 1)$. Notice that $E(b_i)$ is the expected value of b_i under rational expectations. Since local thinking neglects tail risks, it holds that $E(b_i) < E^L(b_i)$. Consequently, we can directly compare the optimal liquidity between the local thinking banks and the banks with rational expectation as follows.

Corollary 2 $l_i^* < l_{Ri}^*$

The main reason why a local thinking bank tends to hold less liquidity than a rational bank is because it neglects the worst-possible case scenario of the risky assets' return and thereby its expectations about the latter tends to be exaggerated, which in turn leads to a portfolio decision more towards the risky assets. A more general implication of corollary 2 is that financial markets that are governed by local thinking would tend to be less liquid, which makes them more fragile.

5 Bank run at $t = 1$

At an intermediate period $t = 1$, agents observe a signal and when it turns out that the bank is expected to fail to meet its debt obligations, the investors run in order to be the first in line to withdraw their deposits. Some of the investors are also hit by a liquidity shock and hence they will liquidate their deposit accounts anyway.

When the signal is good news, the agents' initial inferences about the return of risky assets are confirmed. The portfolio and investment decisions made at $t = 0$ carry over to the subsequent periods without disruption. Contrarily, things are different when the signal is bad news. First of all, depositors may or may not run on their banks depending on the expected return of the risky assets of the bank, revised down due to the bad signal. When it is so low that the total revenues of the bank at $t = 2$ is not expected to be large enough to pay even a unit return to its depositors, the depositors would be better-off if they liquidate their deposits early. The condition for this to take place is

$$l_i - \theta + (1 - l_i)E^L(y_i|\underline{s}) < 1 - \theta \tag{16}$$

which can be rearranged into $E_i^L(y_i|\underline{s}) < 1$, which in turn is equivalent to, using (15),

$$E^L(b_i|\underline{s}) < 1 \tag{17}$$

Inequality (17) is the condition for a run against type i bank. The result is very intuitive as it states that depositors will run on the bank when the expected return of bank asset in the case of bad news is smaller than unity, which is what they deposited at the bank.

But recall that type 1 bank's liability is backed by government insurance and therefore even when the bank run condition is met, a run on type 1 bank will not occur. Lemma 2 summarizes the condition for a run on type 1 and type 2 banks.

Lemma 2 (Bank run) *It is optimal for depositors at type i banks without public backstop to withdraw early in the case of $E^L(b_i|\underline{s}) < 1$.*

Once the bank run takes place, the amount of liquidity the bank needs is $1 - \theta$ since all of the remaining depositors, who weren't hit by the liquidity shock at $t = 1$, will withdraw. On the other hand, the actual liquidity holding of the bank at $t = 1$, after having accommodated the withdrawal demand of those depositors hit by the liquidity shock, is $l_i^* - \theta$. The difference between the two is the bank's liquidity shortage at $t = 1$ in the case of a run:

$$1 - \theta - (l_i^* - \theta) > 0 \tag{18}$$

The inequality is due to corollary 1. By implication, when the bank experiences a run, it will surely suffer from a liquidity shortage; the bank cannot accommodate the withdrawal demand of its depositors and hence is forced to liquidate its assets even at a fire sale price.

Corollary 3 *The liquidity of type i bank at $t = 1$ is smaller than what the bank might need when it experiences a run. That is, when hit by a run at $t = 1$, the liquidity shortage of each bank is positive; $1 - \theta - (l_i^* - \theta) > 0$.*

In fact, the liquidity shortage problem in the case of a bank run is not unique for a local thinking bank but a rational bank is also vulnerable as long as $0 < l_R^* < 1$, which can be easily shown in the same way as the case for the local thinking bank. That is, while the

bank run and the subsequent liquidity crisis take place due to local thinking, the liquidity crisis leading to solvency crisis does not stem from local thinking but from the standard banking of borrow short lend long and consequent maturity mismatch between assets and liabilities in bank balance sheet. However, corollary 2 implies that the liquidity shortage problem will be more intense with a local thinking bank than with a rational bank, i.e. $1 - \theta - (l_i^* - \theta) > 1 - \theta - (l_R^* - \theta)$, which makes financial markets populated by local thinking agents more vulnerable to a shock compared to a market guided by rational expectations.

Regarding the bank run condition in lemma 2, there are four possible scenarios as displayed in table 1. Remember that $E^L(b_i) > E^L(b_i|\underline{s})$ in all cases, that is, investors revise down the expectations about the return of risky assets in response to bad news. Depending on the severity of the revision, the bank run condition may or may not be met. Case I is the scenario where the expected returns of both types of risky assets fall only by a small degree so that the bank run condition is met for neither type of bank, while in case IV the fall of the expected returns of both types of risky asset is sufficiently large that the bank run condition is met for both types of bank. These cases represent a scenario where the returns of both types of risky asset are strongly positively correlated with each other so that in response to bad news at $t = 1$ they tend to fall by the same degree.

On the other hand, cases II and III are the scenarios where the expected return of one type of risky asset falls so strongly that the bank run condition is met for the bank, while the fall for the other type of risky asset is weak enough to avoid the bank run condition. Cases II and III represent scenarios where the returns of both types of risky asset are weakly positively correlated with each other so that in response to bad news at $t = 1$ they tend to fall by a different degree.

Another consideration is that even when the bank run condition is met for type 1 risky asset, type 1 banks do not experience a run due to the public backstop; only type 2 banks are vulnerable to a bank run. For this reason, we exclude cases I and III as uninteresting scenarios and, in the rest of the paper, focus on cases II and IV in turn. In both cases, it is only type 2 banks that experience a run and hence, due to corollary 3, have to liquidate their assets.

Table 1: Four scenarios concerning a run on the banking sector in case of \underline{s}

	$E^L(b_2 \underline{s}) \geq 1$	$E^L(b_2 \underline{s}) < 1$
$E^L(b_1 \underline{s}) \geq 1$	Case I. No bank run	Case II. Run on type 2 bank
$E^L(b_1 \underline{s}) < 1$	Case III. No bank run	Case IV. Run on type 2 bank

The difference is that in case II (weak asset correlation), as the effect of the bad signal is not so severe as to satisfy the bank run condition for type 1 bank, the bank's balance sheet status is still solid and therefore the bank can possibly become a potential buyer in the asset market. In contrast, in case IV (strong asset correlation), the effect of the bad signal is sufficiently severe for type 1 banks that they also experience a substantial reduction in the expected return of their assets but nonetheless avoid a run only because of the help of government insurance. Therefore, although they survived, type 1 banks experience a weakening of their balance sheets that make them reluctant to further assume risks by purchasing type 2 banks' distressed assets.

6 Asset market at $t = 1$

Let us first consider case II (weak asset correlation) where the bank run condition is met only for type 2 banks and therefore it is only type 2 banks that experience a run due to corollary 3. The bank is forced to liquidate its assets even at a fire sale discount. Suppose, since the type 1 bank survives bad news, it bids for the distressed assets of type 2 banks in the asset market.

There are two sources of funds for type 1 banks for asset purchases. First is the liquidity left over after having accommodated the withdrawal demand by its depositors hit by the liquidity shock at $t = 1$. Second, the bank can also raise extra funds in the capital market by pledging its future cash flows at $t = 2$. However, due to a moral hazard problem existing between the bank owner and the bank manager, the pledgeability is always limited (Holmstrom and Tirole, 1998). That is, not all the future cash flows can be pledged since

some portion of it has to be provided to the bank manager as an incentive to exert effort to monitor the performance of the risky asset. This is what makes risky assets not completely liquid. Let us see this more closely.

Limited pledgeability. Suppose that if the manager of a type 1 bank does not exert effort, the expected return of the bank's risky asset will decrease to $E^L(y_1) - \epsilon$ from $E^L(y_1)$, while the manager enjoys a benefit of $c \in (0, \epsilon)$. Therefore, in order to motivate the manager to exert effort, a share of the bank's payoffs at $t = 2$ has to be promised to the bank manager. Let us denote this share by μ . Then, the manager will exert effort only when her payoff in the case of exerting effort is larger than the payoff in the case of no effort. Since $(1 - l_1)E^L(y_1) - (1 - \theta)q_1$ is the future cash flows of type 1 bank when its manager exerts effort, the incentive compatibility is

$$\mu[(1 - l_1)E^L(y_1) - (1 - \theta)q_1] \geq \mu[(1 - l_1)(E^L(y_1) - \epsilon) - (1 - \theta)q_1] + c \quad (19)$$

Accordingly, the minimum share required to ensure that the manager exerts effort is $\underline{\mu} = \frac{c}{(1 - l_1)\epsilon}$, which, under the optimal bank liquidity expressed in lemma 1, becomes

$$\underline{\mu} = \frac{c}{(E^L(b_1) - 1)\epsilon} \quad (20)$$

It implies that the maximum share, denoted by τ , of type 1 bank's future cash flows that can be pledged to raise funds in the capital market is $\tau = 1 - \underline{\mu}$, i.e.

$$\tau = 1 - \frac{c}{(E^L(b_1) - 1)\epsilon} \quad (21)$$

In comparison to the case of a local thinking bank, consider the same maximum share of the pledgeable future cash flow for a rational bank and denote it by τ_R . Then, it can be obtained in the same way as for the local thinking bank as $\tau_R = 1 - \frac{c}{(E(b_1) - 1)\epsilon}$. Since $E^L(b_1) > E(b_1)$, we have $\tau > \tau_R$. The following counterintuitive lemma summarizes this result.

Lemma 3 *Financial markets populated by local thinking agents enable larger share of future revenues to be pledged for raising funds in the capital market compared to financial markets governed by rational expectations, i.e. $\tau > \tau_R$.*

The key mechanism underlying lemma (3) is that the local thinking mitigates the constraints generated by the moral hazard problem, between the owner and manager at the bank, by lowering $\underline{\mu}$. This can be confirmed by using the expression for $\underline{\mu}$ in equation (20); denote by $\underline{\mu}_R$ the minimum share required to ensure the bank manager to exert effort in the case of a rational bank; since $E^L(b_1) > E(b_1)$, we have $\underline{\mu} < \underline{\mu}_R$. Lastly, according to lemma (3), borrowers in the local thinking financial market tend to have larger leverage ratio and consequently the system tends to be more fragile compared to the financial market populated by rational agents.

Condition for the positive future cash flows. Raising additional funds from the capital market is possible only when the future cash flows are positive in the first place. This is the constraint type 1 bank faces when borrowing. Note that in the case of \underline{s} , the expected return of type 1 bank's risky asset is revised down to $E^L(y_1|\underline{s})$ from $E^L(y_1)$. Consequently, its expected cash flow at $t = 2$ depreciates from $(1-l_1)E^L(y_1) - (1-\theta)q_1$ to $(1-l_1)E^L(y_1|\underline{s}) - (1-\theta)q_1$, which however can still be positive as long as $E^L(y_1|\underline{s})$ is not too low such that $E^L(y_1|\underline{s}) > \frac{(1-\theta)q_1}{1-l_1}$ holds. This condition can be rewritten as

$$E^L(b_1|\underline{s}) > \frac{2(1-\theta)q_1}{E^L(b_1) - 1} - 1 \quad (22)$$

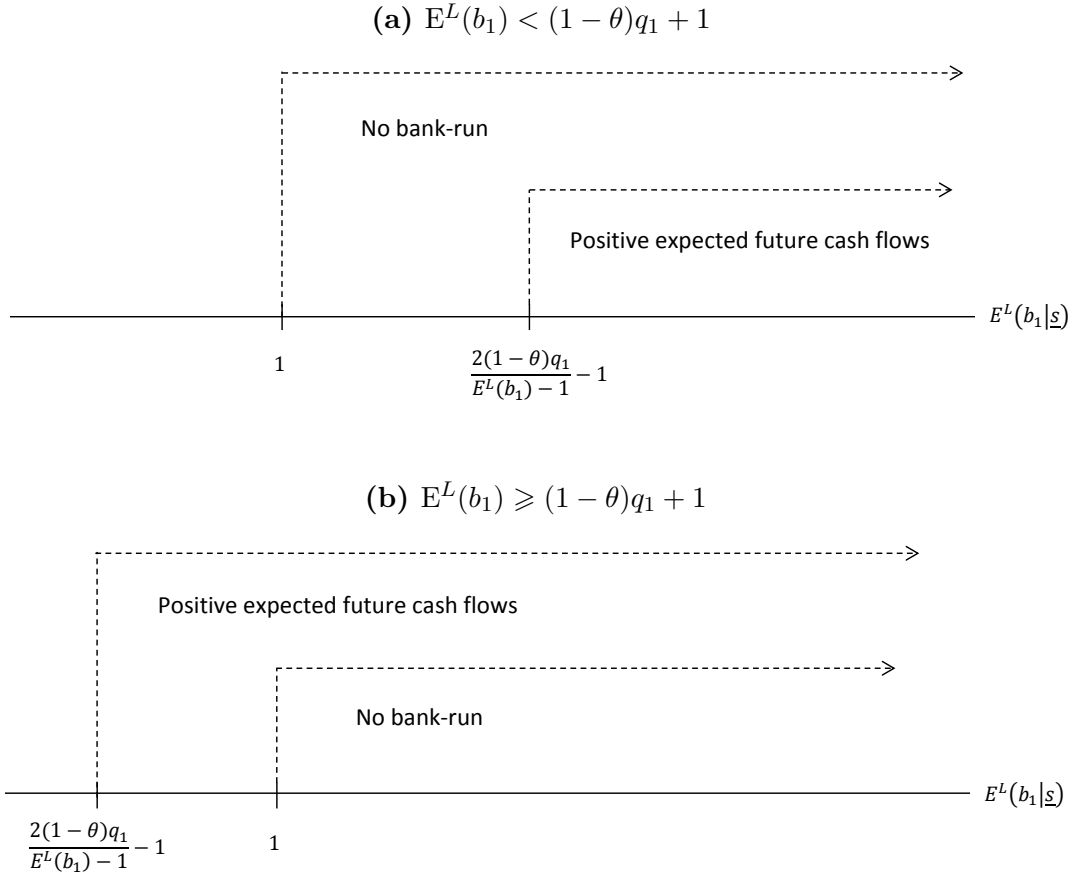
which uses the expression for the optimal liquidity holding $l_1^* = 2 - E^L(b_1)$ and (15).

The question here is whether the positive future cash flows condition, derived as in (22), is guaranteed when the no-bank run condition, $E^L(b_1|\underline{s}) \geq 1$, is met.⁶ From the comparison between two conditions, it is immediately obvious that it depends on whether $\frac{2(1-\theta)q_1}{E^L(b_1) - 1} - 1 > 1$ or $\frac{2(1-\theta)q_1}{E^L(b_1) - 1} - 1 \leq 1$. Let us consider these two in turn.

(a) When $\frac{2(1-\theta)q_1}{E^L(b_1) - 1} - 1 > 1$, which is equivalent to $E^L(b_1) < (1-\theta)q_1 + 1$: The inequality sets the upper bound of $E^L(b_1)$, implying the local thinker's expected return of type 1 risky asset is sufficiently low. The case in question is displayed in figure 3a. As long as $E^L(b_1)$ is below the upper bound, type 1 bank's no-bankrun condition at $t = 1$ does not guarantee the positiveness of its expected future cash flows at $t = 2$; the no-bank run condition is a subset

⁶Remember case II is being considered here.

Figure 3: Whether the no-bank run condition, $E^L(b_1|\underline{s}) > 1$, will guarantee the positiveness of the expected future cash flows for type 1 bank



of the positive future cash flows condition. That is, although the downward revision of type 1 risky asset's expected return is not so intense as to meet the bank run condition, it is intense enough to bring the bank's expected future cash flows to a negative value. Accordingly, as can be seen from figure (3a), case (a) consists of two possible subregions of $E^L(b_1|\underline{s})$ that generate distinctive results.

(a-i) $\frac{2(1-\theta)q_1}{E^L(b_1)-1} - 1 < E^L(b_1|\underline{s})$: $E^L(b_1|\underline{s})$ is high enough to avoid a run *and* have positive expected future cash flows. The bank is able to raise extra capital from the market by pledging its expected future cash flows.

(a-ii) $1 < E^L(b_1|\underline{s}) < \frac{2(1-\theta)q_1}{E^L(b_1)-1} - 1$: $E^L(b_1|\underline{s})$ is high enough for type 1 banks to avoid

a run but is not sufficiently high to enable the bank to have positive expected future cash flows. Therefore, when purchasing type 2 bank's distressed assets, type 1 bank cannot raise extra funds from the capital market but has to rely only on its own liquidity holding.

(b) When $\frac{2(1-\theta)q_1}{E^L(b_1)-1} - 1 \leq 1$, which is equivalent to $E^L(b_1) \geq (1-\theta)q_1 + 1$. The inequality sets the lower bound of $E^L(b_1)$, implying the local thinker's expected return of type 1 risky asset is sufficiently high. The case in question is illustrated in figure (3b). As long as $E^L(b_1)$ is above the lower bound, the condition for no bank run guarantees the positiveness of expected future cash flows. But since we are here considering $E^L(b_1|\underline{s}) \geq 1$, type 1 banks in all cases will avoid a run *and* have positive expected future cash flows. Accordingly, there is only one region of $E^L(b_1|\underline{s})$ to consider, which is $E^L(b_1|\underline{s}) \geq 1$.

In all, case (a) is more plausible than case (b) for the following reason. Given the plausible values for q_1 and θ , it is likely that $(1-\theta)q_1$ is not far different from one;⁷ this renders $E^L(b_1) \geq (1-\theta)q_1 + 1$, which is case (b), an unrealistically extreme case of the local thinking overoptimism with the expected return close to 100%. Therefore, we will consider only case (a) below.

Fire sale asset prices. Now we examine how the price of the risky asset liquidated by type 2 bank is determined. In doing so, we assume the condition in case (a), displayed in figure 3a, holds, which allows the two distinctive subcases concerning whether or not the type 1 bank is able to pledge its future cash flows, thereby obtaining extra liquidity from the capital market in purchasing type 2 banks' liquidated assets. Let us denote the asset price by p^A when type 1 bank can pledge its future cash flows, and denote it by p^B when the bank cannot pledge. The fundamental value, denoted by \bar{p} , of type 2 risky asset in case of \underline{s} is $\bar{p}=E^L(y_2|\underline{s}) < 1$.⁸

(a-i) $E^L(b_1|\underline{s}) > \frac{2(1-\theta)q_1}{E^L(b_1)-1} - 1$: Type 1 bank's expected future cash flows are positive, which allows it to raise extra liquidity. When the signal observed at $t = 1$ is \underline{s} , the share, τ ,

⁷Remember that q_1 is the return on a riskless asset, which therefore should be not far larger than one, and θ is the probability of liquidity shock, which therefore should be not far larger than zero.

⁸The inequality holds since we are considering case II; see table 1.

of the future cash flows that is pledgeable is

$$\tau = 1 - \frac{c}{(E^L(b_1|\underline{s}) - 1)\epsilon} \quad (23)$$

which compares to τ before the signal as expressed in equation (21). Then the overall amount of funds, denoted by M , available to type 1 bank at $t = 1$ for the asset purchase is

$$M = l_1 - \theta + \tau[(1 - l_1)E^L(b_1|\underline{s}) - q_1(1 - \theta)] \quad (24)$$

Since the number of type 1 banks is $1 - n$, the total funds available for the asset purchase is $(1 - n)M$.

On the other hand, the nominal value of risky assets of type 2 banks is $n(1 - l_2)p^A$. Considering the optimal liquidity for type 1 and type 2 bank, i.e. l_1^* and l_2^* , p^A can be obtained from the equilibrium condition in the asset market, $(1 - n)M = n(1 - l_2)p$, as

$$p^A = \frac{(1 - n)\left[2 - E^L(b_1) - \theta + \frac{\tau}{2}\left((E^L(b_1) - 1)(E^L(b_1|\underline{s}) + 1) - 2(1 - \theta)q_1\right)\right]}{n(E^L(b_2) - 1)} \quad (25)$$

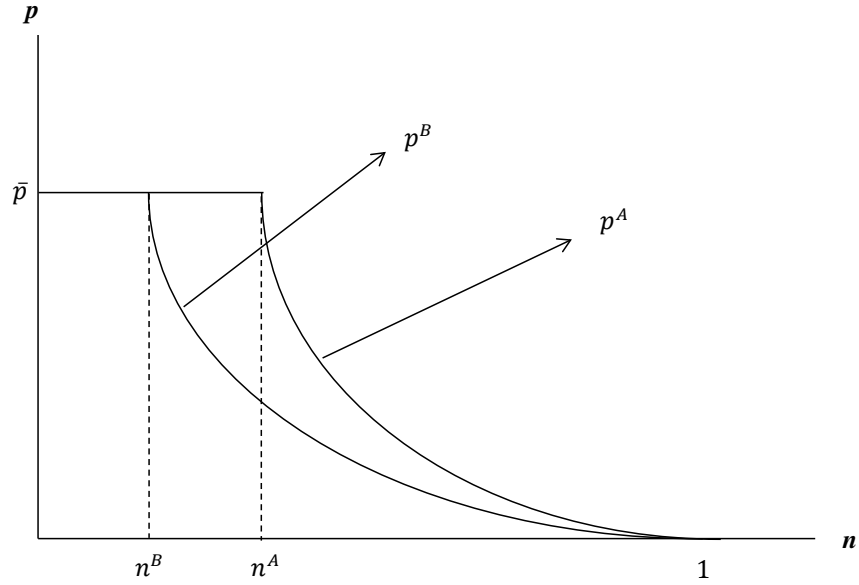
It is optimal for type 1 banks to bid for the asset only when the asset price is set less than or equal to its fundamental value, i.e. $p^A \leq \bar{p}$, in which case the expected return from the asset purchase is $\frac{\bar{p} - p^A}{p^A}$. When $p^A > \bar{p}$, it is optimal for type 1 bank not to bid for the asset.

If the number of type 2 banks is relatively small so that a sufficiently large number of type 1 banks bid for type 2 risky asset, the asset price will be established at the zero profit price, i.e. $p^A = \bar{p}$. But if the number of type 2 banks is sufficiently large, the asset price will be established at $p^A < \bar{p}$; that is, the type 2 bank has to sell its assets at a price less than their fundamental value. Since $\bar{p} = E^L(y_2|\underline{s})$, the threshold point of n , denoted by n^A , can be obtained by solving $p^A = E^L(y_2|\underline{s})$:

$$n^A = \frac{2 - E^L(b_1) - \theta + \frac{\tau}{2}\left[(E^L(b_1) - 1)(E^L(b_1|\underline{s}) + 1) - 2(1 - \theta)q_1\right]}{\frac{1}{2}(E^L(b_2) - 1)(E^L(b_2|\underline{s}) + 1) + 2 - E^L(b_1) - \theta + \frac{\tau}{2}\left[(E^L(b_1) - 1)(E^L(b_1|\underline{s}) + 1) - 2(1 - \theta)q_1\right]} \quad (26)$$

When $n \leq n^A$, the asset can be sold at its full price, i.e. $p^A = \bar{p}$, while when $n > n^A$ the asset will be sold at less than its full price, i.e. $p^A < \bar{p}$, which is the fire sale price. See figure 4. (a-ii) $1 < E^L(b_1|\underline{s}) < \frac{2(1-\theta)q_1}{E^L(b_1)-1} - 1$: $E^L(b_1|\underline{s})$ is so low that type 1 bank's expected future

Figure 4: Price function in proposition 1



cash flow is not positive, in which case the bank cannot borrow and hence its own $t = 1$ liquidity is the only source of funds for asset purchase, i.e. $M = l_1 - \theta$. On the other hand, the nominal value of the risky assets of type 2 banks is $n(1 - l_2)p^B$. Considering the optimal liquidity for type 1 and type 2 bank, i.e. l_1^* and l_2^* , the asset price, p^B , can be obtained from the equilibrium condition in the asset market, $(1 - n)(l_1 - \theta) = n(1 - l_2)p^B$, as

$$p^B = \frac{(1 - n)(2 - E^L(b_1) - \theta)}{n(E^L(b_2) - 1)} \quad (27)$$

If the number of type 2 banks is sufficiently small, type 1 banks have to pay the full price of the liquidated assets of type 2 banks; otherwise, the asset will be sold at less than the full price. The associated threshold n , denoted by n^B , is obtained from $p^B = E^L(y_2|\underline{s})$ as

$$n^B = \frac{2 - E^L(b_1) - \theta}{\frac{1}{2}(E^L(b_2) - 1)(E^L(b_2|\underline{s}) + 1) + 2 - E^L(b_1) - \theta} \quad (28)$$

That is, when $n \leq n^B$, $p^B = \bar{p}$, while when $n > n^B$, $p^A < \bar{p}$. See figure 4.

The results discussed so far regarding the asset price of case II as a function of the number of failed banks is summarized in proposition 1 and is displayed in figure 5 as well as figure 4.

Proposition 1 *The price of type 2 banks' liquidated assets are determined in the following way.*

(i) *If $E^L(b_1|\underline{s}) > \frac{2(1-\theta)q_1}{E^L(b_1)-1} - 1$ so that a type 1 bank can pledge its future cash flow and raise additional funds from the capital market for purchasing the asset,*

$$p^A = \begin{cases} \bar{p} & \text{for } n \leq n^A \\ \frac{(1-n)[2 - E^L(b_1) - \theta + \frac{\tau}{2}((E^L(b_1) - 1)(E^L(b_1|\underline{s}) + 1) - 2(1-\theta)q_1)]}{n(E^L(b_2) - 1)} & \text{for } n > n^A \end{cases}$$

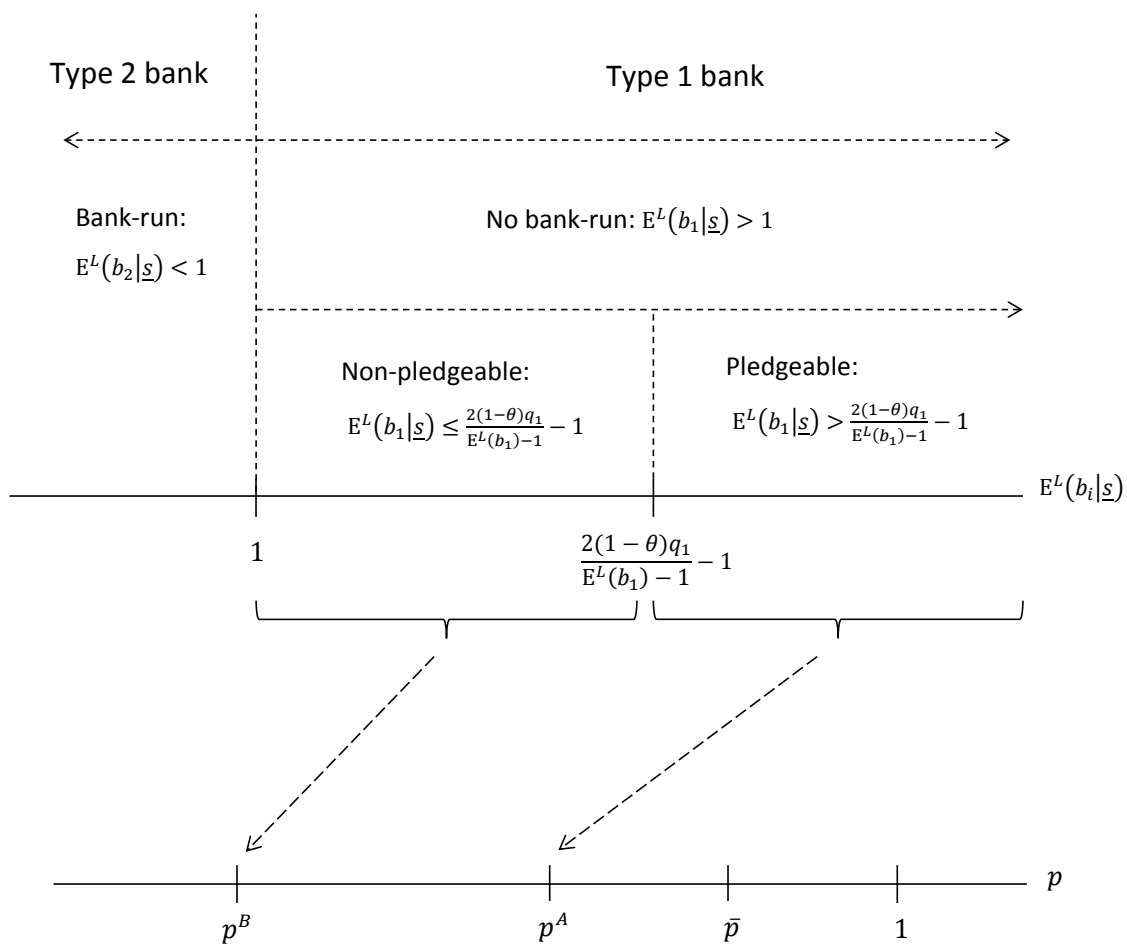
(ii) *If $E^L(b_1|\underline{s}) \leq \frac{2(1-\theta)q_1}{E^L(b_1)-1} - 1$ so that a type 1 bank cannot pledge its future cash flow and therefore has to rely on its own liquidity for purchasing the asset,*

$$p^B = \begin{cases} \bar{p} & \text{for } n \leq n^B \\ \frac{(1-n)(2 - E^L(b_1) - \theta)}{n(E^L(b_2) - 1)} & \text{for } n > n^B \end{cases}$$

The fourth term in the square bracket in the numerator of p_A is type 1 bank's external funds raised in the capital market — hence, strictly positive — remembering that τ is the pledgeable share of the bank's future cash flows while the future cash flow itself is expressed by the rest of the fourth term. It can be seen that this term does not exist in the expression for p_B because p_B is the asset price corresponding to the situation where type 1 banks cannot raise funds from the capital market. Consequently, we can easily confirm that $p^A > p^B$ and $n^A > n^B$; verify that $\tau = 0$ leads to $p^A = p^B$ and $n^A = n^B$

Concerning case II — $E^L(b_1|\underline{s}) \geq 1$ and $E^L(b_2|\underline{s}) < 1$ — as analyzed so far, there are two channels through which the local thinking affects the asset price. First, in response to a bad signal, the expected return of type 2 bank's risky asset is revised significantly downward so that the bank experiences a run and hence is forced to liquidate its assets. Second, while the downward revision of the expected return of type 1 bank's risky asset is not so significant as to subject the bank to a run, it still has a negative impact on the bank's borrowing capacity; when the revised expected return of the risky asset is lower, the extra liquidity the bank can raise in the capital market becomes smaller, or in an extreme case it cannot borrow at all; in any case, the asset market will suffer from a liquidity shortage. In sum, once type 2

Figure 5: Conditions of a bank run and determination of fire sale asset prices under local thinking



banks are forced to liquidate their assets, local thinking exerts downward pressure on the asset price by weakening type 1 bank's borrowing capacity.

In order to see this distinct local thinking effect on the asset price formally, we can conduct a comparative static analysis of p^A with respect to $E^L(b_1|\underline{s})$. One caveat is that the comparative statics needs to be performed carefully considering that $E^L(b_1|\underline{s})$ is endogenous to $E^L(b_1)$. For clarification, let us first reproduce the expressions for $E^L(b_1)$ and $E^L(b_1|\underline{s})$:

$$\begin{aligned} E^L(b_1) &= b^g \Pr^L(b^g) + b^d \Pr^L(b^d) \\ E^L(b_1|\underline{s}) &= b^g \Pr^L(b^g) + b^r \Pr^L(b^r) \end{aligned}$$

Remember that $E^L(b_1)$ is measured with the worst-possible case scenario, a recession, neglected. Thus when the neglected risk turns out to be a more severe one, i.e. b^r being lower, the revised expectation about the risky asset after the bad signal will be weaker and therefore $E^L(b_1|\underline{s})$ will be smaller, while the expectation before the signal, i.e. $E^L(b_1)$, remains the same. This is what we have in mind when conducting the comparative static analysis of p^A with respect to $E^L(b_1|\underline{s})$.

By examining the expression of p^A in proposition 1 and that of τ in (23), it can be easily verified that $E^L(b_1|\underline{s})$ affects the asset price in two ways. First, the lower the $E^L(b_1|\underline{s})$, the smaller the type 1 bank's expected future cash flow; let the latter be denoted by C , then we have $\frac{\partial C}{\partial E^L(b_1|\underline{s})} > 0$. Second, the lower the $E^L(b_1|\underline{s})$, the smaller the share of the pledgeable future cash flows of type 1 banks; $\frac{\partial \tau}{\partial E^L(b_1|\underline{s})} > 0$. In both cases, a decrease in $E^L(b_1|\underline{s})$ due to a lower b^r undermines type 1 bank's borrowing capacity. This result on the impact of the severity of the local thinking effect on the asset price is stated in the following corollary.

Corollary 4 *In the case of a bad signal \underline{s} , when the worst-possible case scenario neglected by the local thinking agent turns out to be worse, as reflected in a lower b^r and consequently in lower $E^L(b_1|\underline{s})$, it will make the asset price lower, making the fire sale worse, in two ways:*

- (i) *By reducing the bidding bank's expected future cash flows, i.e. $\frac{\partial C}{\partial E^L(b_1|\underline{s})} > 0$, where $C = \frac{1}{2}(E^L(b_1) - 1)(E^L(b_1|\underline{s}) + 1) - (1 - \theta)q_1$, the expected future cash flows;*
- (ii) *by reducing the share, τ , of the bidding bank's expected future cash flow, that can be pledged in the capital market for borrowing, i.e. $\frac{\partial \tau}{\partial E^L(b_1|\underline{s})} > 0$.*

In sum, a lower $E^L(b_1|\underline{s})$ due to a lower b^r leads to a lower p^A through the two mechanisms described in corollary 4; once $E^L(b_1|\underline{s})$ is lowered all the way to $\frac{2(1-\theta)q_1}{E^L(b_1)-1} - 1$, at which point type 1 bank's expected future cash flow is zero, the asset price is established at p^B .

7 Asset returns correlation

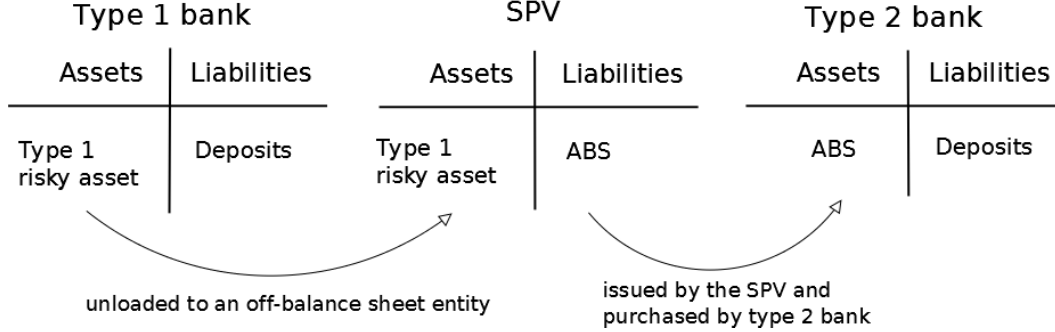
The benchmark model analyzed thus far especially in the previous section has focused on case II where the two risky assets are weakly correlated so that bad news significantly brings down the expected return of type 2 risky asset while the downward revision of type 1 risky asset's expected return is only mild. Consequently, type 1 banks survive the bad signal and hence bid for the distressed assets of type 2 banks. Now, let us consider case IV where the two risky assets are strongly correlated with each other such that in the case of a bad signal type 1 risky asset's expected return is also severely affected. In this case it is reluctant to purchase the distressed assets even though it does not experience a run thanks to the government protection.

A possible institutional setting for the strong asset returns correlation is securitization of bank assets. A type 1 bank, instead of keeping type 1 risky assets on its balance sheet, now packages and unloads them to an off-balance sheet entity such as a special purpose vehicle (SPV); the SPV issues asset-backed securities (ABS) in order to finance its holdings of type 1 risky asset bundles; it is type 2 banks that buy the ABSs. In comparison to the benchmark model, type 2 risky asset is now replaced by the ABS as an investment vehicle for type 2 banks.⁹ The balance sheet schema of the securitization is demonstrated in figure 6.

Although type 1 bank unloads the risky asset to the SPV, it continues to be the entity that collects earnings from type 1 risky asset, but it passes them through to the SPV. Therefore, the SPV's payoff at $t = 2$ is y_1 , which is the return of type 1 risky asset. On the other hand, since the SPV obtains funds by issuing the ABS, the return of the ABS is what the SPV pays out to its creditor, type 2 bank. Let us denote the ABS return by y_x . We further introduce the spread between y_1 and y_x as the source of the SPV's profits and denote

⁹For this reason, we will replace the subscript 2 on the notations for type 2 bank by the subscript x .

Figure 6: Securitization



it by w . Accordingly, we have $y_x = y_1 - w$ and, due to assumption 1, the return profile and probability distribution of the ABS is as follows.

$$y_x^g (= y_1^g - w) > 1 > y_x^d (= y_1^d - w) > y_x^r (= y_1^r - w) \quad \text{and} \quad \pi_x^g (= \pi_1^g) > \pi_x^d (= \pi_1^d) > \pi_x^r (= \pi_1^r) \quad (29)$$

We continue to assume the decreasing return to scale as in (1) and therefore the ABS return can be expressed as

$$y_x = b_1 - \frac{1 - l_1}{2} - w \quad (30)$$

Similar to the diminishing return to scale specification of y_i in (1), the specification in (30) defines the lower and upper bound of y_x as $y_x \in [b_1 - w - \frac{1}{2}, b_1 - w]$, where the upper bound of y_x denoted by b_x can be defined as $b_x = b_1 - w$. And the expectation of b_x is easily obtained as

$$E(b_x) = E(b_1) - w \quad (31)$$

In all, the state-contingent payoffs for the ABS are smaller than those for type 1 risky asset by w , while the probability distribution of the ABS return is the same as the probability distribution of the return of type 1 risky asset. Denoting the variation of each asset by σ_i^2 , $i \in \{1, x\}$, the risk-return profile of type 1 risky asset and the ABS can be compared in the following way.

$$E(y_x) = E(y_1) - w \quad \text{and} \quad \sigma_x^2 = \sigma_1^2 \quad (32)$$

While the expected return of the ABS is lower than the expected return of type 1 risky asset by w , their variation is the same.

In this simple setting, the two assets' returns exhibit perfectly positive correlation, i.e. their correlation coefficient being unity. The important point is that because of the securitization the risk involved in the underlying asset, i.e. in type 1 risky asset, is transferred to the other asset class in the system, i.e. the ABS; thus the risks of the ABS track the risks of type 1 risky asset.

We assume w small enough for the return of the ABS to be profitable so that type 2 banks take positive positions in ABS. For this, we adopt

Assumption 5 $\omega < E^L(b_1) - 1$

In the case of securitization, there are two additional things to consider in comparison to the benchmark model without securitization. First, since type 2 banks now invest in a different asset, i.e. ABS, their liquidity holding will change. Second, type 1 banks obtain additional liquidity as proceeds from selling part of their risky assets; note that they can sell by the amount that matches the volume of type 2 banks' investment on ABS. Suppose type 1 banks use the proceeds to make additional investments in type 1 risky assets. The question is whether it will use the entire proceeds or only part of them.

Note that a type 1 bank is making an additional investment not by issuing additional debt but by tapping fresh liquidity additionally acquired through securitization. Also note that the existing liquidity holding, before securitization, was determined in the first round of portfolio decision, as discussed in the benchmark model, in a way that avoids insolvency in the subsequent periods. Hence, one might think that although more investments are made, no more liquidity holding is required for solvency. That would be true if the risky asset return in $t = 2$ remains the same despite the additional investments, which however is not the case due to the diminishing return to scale. As type 1 banks makes additional investments in the type 1 risky asset, the asset return becomes lower. Hence, the existing liquidity holding, the volume of which was determined according to the initial asset return, becomes insufficient to guarantee solvency in $t = 2$. This implies that type 1 banks cannot use all their new liquidity but only part of it in making an extra investment in type 1 risky assets; the rest has to be held as a liquidity buffer for solvency in the subsequent periods.

In all, when securitization and hence asset return correlation are introduced, we have to examine type 2 bank's portfolio decision between liquidity and ABS and type 1 bank's second round of portfolio decision with the fresh liquidity from securitization.

7.1 Banks' liquidity choice at $t = 0$

First, type 2 bank's liquidity choice in the case of a portfolio decision between liquidity and ABS can be examined in the same way as in the case without securitization in the benchmark model, which has resulted in optimal liquidity expressed in lemma 1. Using this result, the optimal liquidity of type 2 bank and consequently its investment in ABS, each denoted by l_x and a_x respectively, can be determined as

$$\begin{aligned} l_x^* &= 1 - (\mathbb{E}^L(b_x) - 1) = 2 + w - \mathbb{E}^L(b_1) \\ a_x^* &= \mathbb{E}^L(b_x) - 1 = \mathbb{E}^L(b_1) - 1 - w \end{aligned} \tag{33}$$

where the relation in (31) is used for the second equality in each case. Confirm that the budget constraint $l_x + a_x = 1$ holds. Moreover, we have $a_x^* > 0$ due to assumption 5, which restricts profit margin for the SPV to be sufficiently small so that the investment in the ABS is profitable.

As for type 1 banks, their first round of optimal portfolio decisions before securitization is the same as that analyzed in the benchmark model where the resulting portfolio of the banks was $(a_1^*, l_1^*) = (\mathbb{E}^L(b_1) - 1, 2 - \mathbb{E}^L(b_1))$. Out of the outstanding risky assets of $\mathbb{E}^L(b_1) - 1$, the bank securitizes part of them by the amount of $\mathbb{E}^L(b_x) - 1$, which is identical to the type 2 bank's demand for ABS. Hence, type 1 bank's proceeds from the securitization is $\mathbb{E}^L(b_x) - 1$ and the remaining risky assets in the bank balance sheet are $\mathbb{E}^L(b_1) - \mathbb{E}^L(b_x)$.

Next, out of the new liquidity of $\mathbb{E}^L(b_x) - 1$, the type 1 bank makes the second round of optimal portfolio decisions between type 1 risky assets and liquidity, each of which is denoted by \tilde{a}_1 and \tilde{l}_1 , respectively. Hence, the type 1 bank's budget constraint for the second round of investment is

$$\tilde{a}_1 + \tilde{l}_1 = \mathbb{E}^L(b_x) - 1 \tag{34}$$

The type 1 bank's second round of optimization problem is the same as that for its first round of optimization problem, expressed in the system in (14), except two modifications required. First, in place of l_1 in the problem, we need to have $l_1^* + \tilde{l}_1$, where the first term is taken as given and the second term is the unknown variable in question. That is, when the bank decides how much out of the proceeds from securitization need to be additionally invested in the risky asset, it takes the existing liquidity holding taken as given.

Second, as for $E^L(y_i)$ in the optimization problem, we had $E^L(y_1) = E^L(b_1) - \frac{a_1}{2}$ in the first round of portfolio decision. Note that the second term on the right-hand side captures the effect of diminishing return to scale. As mentioned earlier, this second term will change as type 1 banks now are able to make extra investments on type 1 risky assets by tapping the proceeds from securitization. Reflecting this, let us use I_1 to denote total investment on type 1 risky asset. Accordingly, the expression for $E^L(y_1)$ is revised into

$$E^L(y_1) = E^L(b_1) - \frac{I_1}{2} \quad (35)$$

And we know that, by definition,

$$I_1 = a_1^* + \tilde{a}_1 = a_1^* + E^L(b_x) - 1 - \tilde{l}_1 \quad (36)$$

where the second equation uses the budget constraint (34).

On the other hand, type 1 risky assets held in type 1 bank's balance sheet is different from I_1 as some of the latter is securitized. Let us denote the outstanding balance of type 1 risky assets by A_1 , which compares to a_1 of the benchmark model; then we have $A_1 < I_1$. As mentioned earlier, the risky assets remaining on the bank balance sheet immediately after securitization is $E^L(b_1) - E^L(b_x)$ and additional investment on type 1 risky asset using the proceed from securitization is \tilde{a}_1 . Hence, the outstanding balance of type 1 risky asset is

$$A_1 = E^L(b_1) - E^L(b_x) + \tilde{a}_1 = E^L(b_1) - 1 - \tilde{l}_1 \quad (37)$$

In all, type 1 bank's optimization problem in the second round of portfolio decision is

$$\begin{aligned} \max_{\tilde{l}_1} \quad & \pi_1(\tilde{l}_1) = l_1^* + \tilde{l}_1 + A_1 E^L(y_1) - \theta - (1 - \theta)q_1 \\ \text{s.t.} \quad & l_1^* + \tilde{l}_1 \geq \text{Max}[\theta, \theta + (1 - \theta)q_1 - A_1 E^L(y_1)] \end{aligned} \quad (38)$$

The solution for the above optimization problem is $\tilde{l}_1^* = \frac{1}{2}(E^L(b_1) - 1 - \omega)$, which is type 1 bank's optimal liquidity in the second round of portfolio decision after securitization. Total liquidity holding, $l_1^* + \tilde{l}_1^*$, of the type 1 bank after securitization and the second round of portfolio decision making is stated in the following lemma.

Lemma 4 $l_1^* + \tilde{l}_1^* = \frac{1}{2}(3 - E^L(b_1) - \omega)$

Similar to the result in lemma 1, which demonstrates the optimal liquidity of each type of bank in the benchmark model without securitization, the total optimal liquidity that results from the two rounds of portfolio decision in the case of securitization is negatively related to the expected return $E^L(b_1)$. That is, when the risky asset is expected to generate larger return, the optimal bank liquidity will be smaller, thereby making the bank more vulnerable to external shocks. More important, the same logic generates the optimal liquidity of a rational bank in the similar context of securitization to be $l_{R1}^* + \tilde{l}_{R1}^* = \frac{1}{2}(3 - E(b_1) - \omega)$. Since $E^L(b_1) > E(b_1)$, it can be easily shown that the total optimal liquidity of the rational bank is larger than that of the local thinking bank; this is summarized in the following corollary.

Corollary 5 $l_1^* + \tilde{l}_1^* < l_{R1}^* + \tilde{l}_{R1}^*$

which is the securitization counterpart of corollary 2.

7.2 Bank run at $t = 1$

The bank run condition in the benchmark model without securitization and hence no asset return correlation is expressed in (16), which is reproduced below.

$$l_i - \theta + (1 - l_i)E^L(y_i|\underline{s}) < 1 - \theta$$

remembering $E^L(y_1|\underline{s}) = E^L(b_1|\underline{s}) - \frac{a_1}{2}$ as in (8), which reflects the diminishing return to scale. The left-hand side is type i bank's total payoff at $t = 2$ in the case of a bad signal, while the right-hand side is the bank's $t = 2$ obligations to its depositors. When securitization is introduced, the same modifications, discussed in section 7.1, above need to be considered; a

change in the liquidity holding l_i and, consequently, a change in the expected asset return $E^L(y_i|\underline{s})$ due to the diminishing returns to scale.

More specifically, as the type 1 bank now makes an additional round of portfolio decision between the type 1 risky asset and liquidity by using the proceeds from securitization, l_1 has to be replaced by $l_1 + \tilde{l}_1$ and, consequently, as total investment in the type 1 risky asset is now $I_1 = a_1 + \tilde{a}_1$ instead of simply a_1 , $E^L(y_1|\underline{s}) = E^L(b_1|\underline{s}) - \frac{a_1}{2}$ has to be replaced by $E^L(y_1|\underline{s}) = E^L(b_1|\underline{s}) - \frac{I_1}{2}$. Accordingly, the above-reproduced bank run condition for type 1 bank is modified into

$$l_1 + \tilde{l}_1 - \theta + (1 - l_1 - \tilde{l}_1) \left(E^L(b_1|\underline{s}) - \frac{I_1}{2} \right) < 1 - \theta \quad (39)$$

By the same logic, since the type 2 bank now invests in ABS, not in the type 2 asset, l_2 is to be replaced by l_x . In addition, since the expected return of ABS is correlated to that of the type 1 risky asset with the spread of w as noted in (32), $E^L(y_2|\underline{s}) = E^L(b_2|\underline{s}) - \frac{a_2}{2}$ is to be replaced by $E^L(y_x|\underline{s}) = E^L(y_1|\underline{s}) - w$. Accordingly, the above-reproduced bank run condition for type 2 bank is modified into

$$l_x - \theta + (1 - l_x) \left(E^L(y_1|\underline{s}) - w \right) < 1 - \theta \quad (40)$$

Now, by using lemmas 1 and 4 and equation (36), the bank run conditions in (39) and (40), respectively, can be summarized as follows.

Lemma 5 *In the case of asset return correlation via securitization, when bad news arrives at $t = 1$, it is optimal for depositors at type 1 banks and depositors at type 2 banks, respectively, to withdraw early in each of the following conditions.*

$$E^L(b_1|\underline{s}) < \frac{1}{4} \left(3E^L(b_1) - \omega + 1 \right) \quad (41)$$

$$E^L(b_1|\underline{s}) < \frac{1}{4} \left(3E^L(b_1) + 3\omega + 1 \right) \quad (42)$$

By comparing the two inequalities in lemma 5, it can be shown that the bank run condition for type 1 bank is a subset of the bank run condition for type 2 bank since $\frac{1}{4} \left(3E^L(b_1) - \omega + 1 \right) < \frac{1}{4} \left(3E^L(b_1) + 3\omega + 1 \right)$. This observation leads to the following corollary.

Corollary 6 *If the bank run condition is met for type 1 bank — which securitizes part of its assets — the bank run condition of type 2 bank — which purchases ABS — will also be met.*

It implies that a trouble in one asset class is spread to another asset class. A contagious effect emerges through securitization.

7.3 Asset market at $t = 1$

Now, let us consider the case where the expected return of type 1 risky asset after bad news is so low that the bank run condition, in (41), for type 1 bank is met and consequently that for type 2 bank, expressed in (42), is also satisfied due to corollary 6. We continue to suppose that only type 1 banks are protected by government insurance for their deposits. Hence, although the bank run condition is satisfied for both types of banks, it will be only type 2 banks that experience a run and are forced to liquidate their assets, the ABS, even at a fire sale discount.

Although the type 1 bank is the one that invested in the underlying asset for the ABS, in this simple model of two types of bank we take the type 1 bank as a bidder in the market for the assets of the type 2 bank similarly to the benchmark model since we are continuing to assume that type 1 bank is protected by the public guarantee and hence does not go bankrupt. The difference from the benchmark model, however, is that it is not only the type 2 bank that is in trouble but also the type 1 bank is also severely affected by the bad signal as its asset's return, i.e. type 1 risky asset return, is also revised down substantially; in fact, the trouble is originating from the type 1 bank's balance sheet as it takes position in the underlying asset, whose risks are spread to the other segment of the system.

Therefore, although the type 1 bank does not experience a run and hence does not go bankrupt, its financial position is weak in case of the bad signal. In this context, it is highly possible that the type 1 bank's liquidity preference will be very high, being reluctant to spend all of its liquidity in purchasing the liquidated assets, as was the case in the benchmark model, but only a share of it. Let us denote this share by $1 - \lambda$, where λ is a measure of the bank's

liquidity preference when it is hit by the bad signal shock at $t = 1$.

Furthermore, when the type 1 bank is using only a portion of its liquidity in bidding for the distressed assets due to the heightened liquidity preference $\lambda > 0$, the bank is unlikely to pledge its future cash flows to raise additional funds in the asset purchase; hence $\tau = 0$.

In this context, the only funding source type 1 bank can tap to purchase distressed assets is its own liquidity at $t = 1$ which is expressed in lemma (4). Since there are, at $t = 1$, θ depositors who are hit by liquidity shocks and thus withdraw early, the liquidity eventually available for asset purchase is $l_1^* + \tilde{l}_1^* - \theta$:

$$\frac{1}{2}(3 - E^L(b_1) - \omega) - \theta \quad (43)$$

Considering the number of type 1 banks n and their liquidity preference λ , total volume of funds used for asset purchase is

$$M = (1 - n)(1 - \lambda) \left[\frac{1}{2}(3 - E^L(b_1) - \omega) - \theta \right] \quad (44)$$

On the other hand, the volume of distressed ABS is $E^L(b_x) - 1 = E^L(b_1) - \omega - 1$. Denoting the price of the distressed ABS by p^X , the total nominal value of ABS is

$$n(E^L(b_1) - \omega - 1)p^X \quad (45)$$

The price of the distressed asset is obtained as the solution for the asset market equilibrium condition, $M = n(E^L(b_1) - \omega - 1)p^X$:

$$p^X = \frac{(1 - n)(1 - \lambda)(3 - E^L(b_1) - \omega - 2\theta)}{2n(E^L(b_1) - \omega - 1)} \quad (46)$$

The fundamental value of the ABS in case of a bad signal is $\bar{p}^X = E^L(b_x|\underline{s})$. Hence, when the ABS is purchased at p^X , its return will be $\frac{\bar{p}^X - p^X}{p^X}$. Type 1 banks will bid for the asset only when the asset price is set below its fundamental value, i.e. $p^X \leq \bar{p}^X$, which generates non-negative return. When $p^X > \bar{p}^X$, it is optimal for type 1 bank not to bid for the asset. If there are a small number of type 2 banks so that a sufficiently large number of type 1 banks bid for the distressed ABS, the ABS price will be bid up towards its upper limit at $p^X = \bar{p}^X$,

in which case the expected return from the asset purchase is zero. But if the number of type 2 banks is sufficiently large, the asset price will be established at $p^X < \bar{p}^X$; that is, the type 2 bank will have to sell its assets at a fire sale price. Accordingly, the threshold point of n , denoted by n^X , can be obtained from $p^X = E^L(y_x|\underline{s})$ as

$$n^X = \frac{(1 - \lambda) \left[\frac{3}{2} - \frac{1}{2} E^L(b_1) - \frac{1}{2} \omega - \theta \right]}{\left(E^L(b_1) - \omega - 1 \right) \left[E^L(b_1|\underline{s}) - \frac{3}{4} \left(E^L(b_1) - 1 - \omega \right) \right] + (1 - \lambda) \left[\frac{3}{2} - \frac{1}{2} E^L(b_1) - \frac{1}{2} \omega - \theta \right]} \quad (47)$$

The ABS price as a function of the number of type 2 banks is summarized in the following proposition.

Proposition 2 *The price of type 2 banks' liquidated assets, ABS, is determined in the following way.*

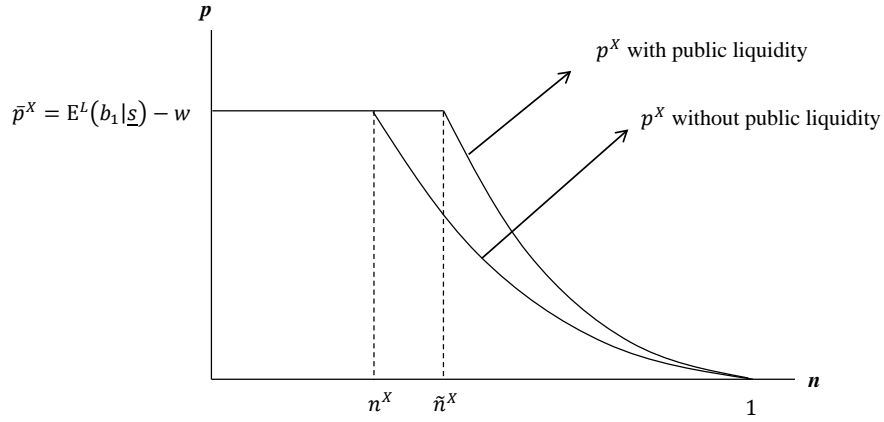
$$p^X = \begin{cases} \bar{p}^X & \text{for } n \leq n^X \\ \frac{(1 - n)(1 - \lambda) \left(3 - E^L(b_1) - \omega - 2\theta \right)}{2n \left(E^L(b_1) - \omega - 1 \right)} & \text{for } n > n^X \end{cases}$$

If $\lambda = 0$, the analysis will be similar to the one in the benchmark model with weak asset return correlation. However, since type 1 banks' assets are also severely affected, due to the bad signal, to the extent that it would have experienced a run were it not for the government guarantee. Hence, it is very likely that its liquidity preference is high and hence $\lambda \gg 0$. In which case, the asset price depreciation can be significant; it can be verified that λ approaching one brings p^X closer to zero.

In this case let us consider the likely appearance of the government in the asset market as an outside buyer to prop up the asset price. Suppose the government injects liquidity by purchasing the troubled assets. Let us denote the amount of the public liquidity by F . Then the total liquidity available M will rise by F . As a consequence, the asset price will rise to

$$p^X = \frac{(1 - n)(1 - \lambda) \left(3 - E^L(b_1) - \omega - 2\theta \right) + F}{2n \left(E^L(b_1) - \omega - 1 \right)} \quad (48)$$

Figure 7: ABS Price appreciation with public liquidity



Then the threshold level of the number of type 2 bank, denoted by \tilde{n}^X , at which the ABS will trade at its fundamental value, is yielded as

$$\tilde{n}^X = \frac{(1 - \lambda) \left[\frac{3}{2} - \frac{1}{2} E^L(b_1) - \frac{1}{2} \omega - \theta \right] + F}{\left(E^L(b_1) - \omega - 1 \right) \left[E^L(b_1 | \underline{s}) - \frac{3}{4} \left(E^L(b_1) - 1 - \omega \right) \right] + (1 - \lambda) \left[\frac{3}{2} - \frac{1}{2} E^L(b_1) - \frac{1}{2} \omega - \theta \right]} \quad (49)$$

It is readily confirmed that $\tilde{n}^X > n^X$. The comparison between the ABS price with and without the public liquidity is illustrated in figure 7.

In comparison to the analysis in the previous section, especially figure 5, we can see that the public liquidity plays the similar role played by external funds raised in capital market in bolstering asset prices. That is, it increases the threshold number of type 2 banks, beyond which the asset price falls below its fundamental value; as a consequence, with public liquidity, the asset price is higher than otherwise for any number of type 2 banks.

Then how is F determined? Suppose the government, when implementing the asset purchase program, sets a target asset price, which is a negative function of target interest rate due to present valuation asset pricing; hence if a lower interest rate target implies a higher asset price target. Let us denote the asset price target by p^T . By equating p^T to the expression in 48, we can obtain the expression for F as

$$F = p^T 2n \left(E^L(b_1) - \omega - 1 \right) - (1 - n)(1 - \lambda) \left(3 - E^L(b_1) - \omega - 2\theta \right) \quad (50)$$

From equation (50), we can identify several factors that affect the required volume of public liquidity injection. This is summarized in the following proposition.

Proposition 3 $\frac{\partial F}{\partial p^T} > 0$, $\frac{\partial F}{\partial n} > 0$, $\frac{\partial F}{\partial \lambda} > 0$, $\frac{\partial F}{\partial \theta} > 0$, $\frac{\partial F}{\partial E^L(b_1)} > 0$

The first four results are quite intuitive. Other things being equal, a larger volume of public liquidity injection is required i) the larger the asset price target, ii) the larger the number of banks without government insurance, iii) the stronger the liquidity preference of potential buyers in asset markets, and iv) the stronger the liquidity shock. More interesting is the last result which states that the stronger the local thinking effect also leads to a larger F . Two underlying mechanisms can be identified. First, accordingly to lemma 4, larger $E^L(b_1)$ implies smaller liquidity holdings of type 1 banks; the internal funding sources type 1 bank can tap to purchase distressed asset is weak, which in turn requires larger public liquidity injections to prop up asset markets. Second, larger $E^L(b_1)$ implies larger investment in ABS by type 2 banks; hence in times of stress, the volume of liquidated assets will be larger, which in turn will also require larger public liquidity injection.

8 Conclusion

The main contribution of this paper has been to extend the model of local thinking to study bank runs and fire sales, which are key phenomena of financial disruptions and banking crises. More specifically, we have shown that local thinking behavior affects bank runs and fire sales in two ways. First, it lowers optimal bank liquidity compared to the case in which banks have rational expectations. Consequently the banking system becomes fragile and vulnerable to liquidity shocks. Second, even when a local thinking bank survives bad news, its balance sheet status is weaker than would have been the case if it had rational expectations; as a result, the bank may not be able to pledge its future cash flows to raise additional liquidity for asset purchases; hence, there is further downward pressure on asset prices.

We have also examined the injection of public liquidity in the case where the risky asset returns are strongly positively correlated due to securitization so that adverse shocks are

felt uniformly across the banking sector. In this case, even the surviving banks become risk-averse and pile up liquidity. An important policy question can be raised on the size of the public liquidity injection. As a first approximation, we have considered a government asset purchase program that sets the asset price target. This target is established in relation to a policy agenda for the broader economy. As a policy guide for stabilizing asset markets, the determination of public liquidity merits further study. Some of the approaches that can address this issue include welfare analysis of different forms of public liquidity injection policy, and empirical analysis of the relation between the size of central bank balance sheet and asset price indexes.

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