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Multi-Objective versus Single-Objective Models in Geodetic Network Optimization

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Abstract. *Configuration of a network and observation weights plays an important role in designing and establishing a geodetic network. In this paper, we consider single- and multi-objective optimization models in some numerical investigation. The results illustrate that the reliability model yields the best results in view of internal and external reliability and achievable observation precision. This result we interpret as that the reliability criterion is more sensitive to the configuration of a network than any of the other criteria. We propose re-optimization of the network in the cases where very high (non-achievable) precision is required or when some conditions are not met in the optimization process.*

Keywords: *Optimization, analytical method, geodetic network, configuration, weight.*

1 Introduction

An optimal geodetic network is a network having high precision and reliability designed according to economical considerations. The first step of the geodetic network design is zero order design (ZOD) or datum definition. The datum affects the precision of the network too. Different criteria exist for selecting the best datum (ZOD). Teunissen (1985) presented the ZOD according to the theory of generalized matrix inverses and its relations with datum and rank deficiency of the design matrix. Kuang (1996) presented different criteria for ZOD, and Eshagh (2005) suggested the minimum norm and trace of the co-factor matrix as the best criteria for datum definition.

There are two well-known ways to find the best configuration of networks, i.e., first order design (FOD). One can use either the trial and error or the analytical approaches. In the trial and error method, the objective function (OF) is computed with a proposed solution for the problem. If the suggested solution does not satisfy the OF, the solution is changed and the OF is computed again. This process is repeated until the requirement is satisfied. The analytical approaches take advantage of a mathematical algorithm and design the network in such a way that the quality requirement of the network is satisfied. A pioneer in using optimization theory was Koch (1982, 1985) who considered quadratic programming [Bazaraa and Shety, 1979] to optimize the configuration of a network. Kuang (1991, 1996) developed this approach further and considered different types of optimization methods.

Grafarend (1975) and Schmitt (1980, 1985) presented different approaches for second order design (SOD) where SOD is optimal selection of the observables weights, and Kuang (1993) presented another approach to SOD leading to maximum reliability using linear programming [see e.g. Bazaraa (1974) or Smith et al. (1983)]. According to Eshagh (2005) numerical studies, the method of Kuang (1993) yields better results in SOD than other methods. Seemkooei (2001) considered the analytical approach to FOD, SOD and also their combinations in robustness points of view.

Using the method of Kuang (1996) one can obtain optimal weights and configuration of the network in one step by different optimization algorithms and OFs. In fact, the approach proposed by Kuang (1996) to optimal design of the network is a combination of FOD and SOD. In this method the best configuration and observation precisions are determined simultaneously in an optimal way. This optimal design can be carried out using different criteria as an OF. If just one criterion exists in the OF, it is called single-objective optimization model (SOOM); if two criteria exist, it is a bi-objective optimization model (BOOM) [Mehrabi 2002], and, if we have more than two criteria, we call it multi-objective optimization model (MOOM); see e.g. Kuang (1996). A simple comparison between different SOOMs has been carried out in Eshagh and Kiamehr (2007). This comparison shows that reliability is a much better criterion than the other criteria in SOOMs. The capability of the BOOM versus SOOM was presented in Eshagh (2005).

In this paper, after a quick review of SOOM and MOOM models, we compare them in a simple simulation study. The methodology is investigated for obtaining the optimum configuration and observation weights considering the postulated reliability and precision requirements. The advantages and disadvantages of different SOOMs, such as precision, reliability and cost, as well as MOOM are presented and discussed. We investigate further the SOOM and MOOM and suggest the reliability model is better than the other SOOM and MOOM models for applying optimization of geodetic networks. The next section deals with a general review of the SOOMs and MOOMs and their mathematical models. In Section 3 we study these models numerically in a simple simulated network and compare the corresponding designs. The paper is ended by conclusions presented in Section 4.

2 Optimization models

An optimum geodetic network should have an acceptable precision, high reliability and low cost. A general mathematical model for network optimization can be symbolically written by the following OF [Schaffrin, 1985]:

$$\alpha_p (precision) + \alpha_r (reliability) + \alpha_c (cost)^{-1} = \max \tag{1}$$

where α_p, α_r *and* α_c are parameters (weights) related to precision, reliability and cost, respectively. It is obvious that when one of these coefficients is zero, the model is a BOOM and if two coefficients are zero it becomes a SOOM. In the following we continue with different SOOMs.

2.1 Single Objective Optimization Models (SOOM)

As mentioned before each one of the precision, reliability and cost criteria can be considered as an OF. Depending on which criterion that is regarded in the OF one can define three different SOOMs. Each SOOM can be constrained to other quality factors. For instance, the precision criterion can be considered as the OF and the optimization is defined as a process to maximize the precision subjected to reliability and cost. Also, each SOOM can be subjected to two other controlling criteria. The main purpose of the analytical approach is to improve a primary design of the network. In this approach, the best possible configuration (position shifts Δx_i, Δy_i, Δz_i) and optimum observation weights (weight shifts Δp_i) are sought. Some advantages of the analytical approach rather than other existing methods for network optimization are as follows:

- Any type of geodetic observable can be considered.
- Any condition or constraint can be considered.
- All the criteria of precision, reliability and cost can be considered simultaneously in the optimal design.
- The optimization procedure can be performed in the sense of FOD and SOD separately or simultaneously.
- This methodology can be used for the optimal design of one-, two- or three-dimensional networks.

In the following, three types of SOOMs are reviewed based on the criteria of precision, reliability and cost, respectively. We call such SOOMs precision, reliability and cost models, respectively.

2.1.1 Precision model

Precision is the simplest criterion and that is well known. The variance-covariance matrix of the unknown parameters can be written in the general form:

$$C_x = \sigma_0^2 \left[\left(A^T P A + D D^T \right)^{-1} - E \left(E^T D D^T E \right)^{-1} E^T \right], \tag{2}$$

where σ_0^2 is a priori variance factor, P is the initial weight matrix, A is the network configuration matrix or the design matrix, D is the datum matrix including translation and rotation and scale parameters and E is the basis of the null space of the configuration matrix A. The linearized form of Eq. (2) can be written:

$$C_x = C_x^0 + \sum_1^m \frac{\partial C_x}{\partial x_i} \Delta x_i + \sum_1^m \frac{\partial C_x}{\partial y_i} \Delta y_i + \sum_1^m \frac{\partial C_x}{\partial z_i} \Delta z_i + \sum_1^n \frac{\partial C_x}{\partial p_i} \Delta p_i \ , \tag{3}$$

where $\Delta x_i, \Delta y_i, \Delta z_i$ are unknown coordinate changes and Δp_i are unknown corrections to a priori weights. Equation (3) can also be represented in a vectorized matrix form as:

$$Hw = u + \varepsilon, \tag{4}$$

where

$$H_1 = \left(I_u \Theta I_u\right)^T H, \quad u_1 = \left(I_u \Theta I_u\right)^T, \quad u = \text{vec}\left(C_x\right) - \text{vec}\left(C_x^0\right), \text{ and} \tag{5}$$

$$w = \left(\Delta x_1, \Delta y_1, \Delta z_1, \ldots, \Delta x_m, \Delta y_m, \Delta z_m, \Delta p_1, \ldots, \Delta p_n\right)^T. \tag{6}$$

In the above relations, ε is the residual vector making the system of equations Eq. (4) consistent, H is the coefficient matrix of the expansion including the derivatives of C_x with respect to the vector of unknowns w, u is a known vector since the approximate C_x^0 and predetermined C_x are known, Θ is the Khatri-Rao product and vec is the converting operator of a matrix to vector; cf. e.g., Kuang (1996).

In fact, Eq. (4) presents a simple fitting to a predetermined variance-covariance matrix (of required precision). The position and weight vector w changes until the best possible fit to the required precision is met. The unknown w can be determined by using simple quadratic programming. In this case we may minimize the following relation:

$$\left\| Hw - u \right\|_{L_2} \quad \rightarrow \quad \min \tag{7a}$$

subject to:

$$H_1 w - u_1 \leq 0, \tag{7b}$$

$$\left\| r_{00} + R_{11} w \right\| \geq r_m, \tag{7c}$$

$$\left\| C_{00} + C_{11} w \right\| \geq c_m, \tag{7d}$$

$$\begin{bmatrix} D^T & 0 \end{bmatrix} w = 0 \quad \text{and} \tag{7e}$$

$$A_{00} w \leq b_{00}, \tag{7f}$$

where r_m, c_m are pre-defined redundancy number and weight boundary value. The matrices of H_1, u_1, r_{00}, R_{11}, C_{00} and C_{11} are related to precision, reliability and cost. $\| \; \|_{L_2}$ stands for L_2-norm. For the details of computing these criteria, the interested readers are referred to Kuang (1996). Equations (7e) and (7f) are the datum constraints and shift limitation of unknowns; see also Kuang (1996).

2.1.2 Reliability model

Reliability can also be considered as an OF to be maximized. Since in linear programming [Bazaraa 1974] the OF is minimized, this OF has to be considered with a minus sign. Such an OF is generally written as

$$\hat{v} = \hat{l} - l = \left[A\left(A^T PA + DD^T\right)^{-1} A^T P - I \right] l = -Rl, \tag{8}$$

where $\hat{v}$ is the residual vector of the system of observation equations. It differs from ε presented for the precision model. The Taylor expansion of Eq. (8) is:

$$R = R^0 + \sum_1^m \frac{\partial R}{\partial x_i}\Delta x_i + \sum_1^m \frac{\partial R}{\partial y_i}\Delta y_i + \sum_1^m \frac{\partial R}{\partial z_i}\Delta z_i + \sum_1^n \frac{\partial R}{\partial p_i}\Delta p_i, \tag{9}$$

where R^0 is the redundancy matrix obtained from approximate a priori positions. Since we need just the diagonal elements of the redundancy matrix for our maximization problem, we can write the reliability criterion as

$$\left\| -\left(r_{00} + R_{11}w\right)_i \right\|_{L_\infty} \to \min , \tag{10}$$

where

$$r_{00} = \left(I_n \Theta I_n\right)^T r^0 \tag{11}$$

$$R_{11} = \left(I_n \Theta I_n\right)^T R_1, \quad r^0 = \text{vec}\left(R^0\right), \tag{12}$$

$$R_1 = \left[\text{vec}\left(\frac{\partial R}{\partial x_1}\right) \text{vec}\left(\frac{\partial R}{\partial y_1}\right) \text{vec}\left(\frac{\partial R}{\partial z_1}\right) \text{vec}\left(\frac{\partial R}{\partial x_m}\right) \right. \tag{13}$$

$$\left. \text{vec}\left(\frac{\partial R}{\partial y_m}\right) \text{vec}\left(\frac{\partial R}{\partial z_m}\right) \text{vec}\left(\frac{\partial R}{\partial p_1}\right) \text{vec}\left(\frac{\partial R}{\partial p_n}\right) \right],$$

and $\| \; \|_{L_\infty}$ is the L_∞-norm. The above maximization problem can be summarized as:

$$\left\| -\left(r_{00} + R_{11}w\right)_i \right\|_{L_\infty} \to \min \tag{14}$$

subject to:

$$H_1 w - u_1 \le 0, \tag{14a}$$

$$\gamma^T c_{00} + \gamma^T C_{11} w \le c_m , \tag{14b}$$

$$\begin{bmatrix} D^T & 0 \end{bmatrix} w = 0 \quad \text{and} \tag{14c}$$

$$A_{00} w \le b_{00}. \tag{14d}$$

This problem can be solved by either linear programming or minimax programming. γ is a $(3m + n)$ constant vector and the other parameters and constraints that have been introduced in the previous section. For details the reader is referred to Kuang (1996).

2.1.3 Cost model

Cost may also be considered as the OF to be minimized. In the analytical design approach the number of repetitions of each individual observation can be a criterion for the cost. Such an OF is generally written as the target function for optimal cost in the network optimization:

$$\|P\|_{L_\infty} \to \min . \tag{15}$$

In the analytical approach by specifying approximate weights P_i^0, we look for the best possible improvements (ΔP_i) for these weights as:

$$P_i = P_i^0 + \Delta P_i , \tag{16}$$

where the weight improvement ΔP_i is given by:

$$\Delta P = \sum_1^n \frac{\partial P}{\partial p_i} \Delta P_i. \tag{17}$$

After linearization of the OF of Eq. (15) by using a Taylor series expansion we obtain the minimization problem of the cost:

$$\gamma^T c_{00} + \gamma^T C_{11} w \quad \rightarrow \quad \min \tag{18a}$$

$$c_{00} = \left(I_n \Theta I_n\right)^T \operatorname{vec}\left(P^0\right) \text{ and} \tag{18b}$$

$$C_{11} = \left[\left(I_n \Theta I_n\right)^T \operatorname{vec}\left(\frac{\partial P}{\partial p_1}\right) \left(I_n \Theta I_n\right)^T \operatorname{vec}\left(\frac{\partial P}{\partial p_n}\right) \right],$$

subject to:

$$H_1 w - u_1 \leq 0, \tag{18c}$$

$$(r_{00} + R_{11} w) \geq r_m , \tag{18d}$$

$$\begin{bmatrix} D^T & 0 \end{bmatrix} w = 0 \text{, and} \tag{18e}$$

$$A_{00} w \leq b_{00} . \tag{18f}$$

Similarly, all parameters and constraints have been introduced in the previous section. Such model can be solved by using linear programming algorithms like simplex. For more details about linear programming the interested reader is referred to e.g. Baazara and Shety (1979).

2.2 Multi-objective optimization model (MOOM)

As mentioned before, all SOOMs attempt to minimize or maximize a single OF describing the cost, precision or reliability. In practice there are constraints between each of the above criteria leading to inconsistencies in the optimization of the SOOM. To avoid such problems and lack of a unique minimum solution we can represent all OFs in one OF model [Kuang, 1996]:

$$\left[\frac{\|Hw - u\|}{\|\operatorname{vec}(C_s)\|} + \frac{\|R_{11} w - (r_m - r_{00})\|}{\|r_m\|} + \frac{\|\gamma^T C_{11} w - (c_m - \gamma^T c_{00})\|}{\|c_m\|} \right] \rightarrow \min \tag{19}$$

subject to

$$\begin{bmatrix} D^T & 0 \end{bmatrix} w = 0 \text{, and} \tag{20}$$

$$A_{00} w \leq b_{00} . \tag{21}$$

All parameters have been defined in the previous parts. Considering the L_2-norm, Eq. (19) can be written as

$$w^T H_0^T H_0 w - 2 u_0^T H_0 w + u_0^T u_0 \quad \rightarrow \quad \min, \tag{22}$$

where

$$H_0 = \begin{bmatrix} \dfrac{H}{\sqrt{u_s^T u_s}} \\ \dfrac{R_{11}}{\sqrt{r_m^T r_m}} \\ \dfrac{\gamma^T C_{11}}{c_m} \end{bmatrix} \text{ and } u_0 = \begin{bmatrix} \dfrac{u}{\sqrt{\left[\mathrm{vec}(C_s)\right]^T \left[\mathrm{vec}(C_s)\right]}} \\ \dfrac{(r_m - r_{00})}{\sqrt{r_m^T r_m}} \\ \dfrac{(c_m - \gamma^T C_{00})}{c_m} \end{bmatrix}, \tag{23}$$

$$A_{11} w \le b_{11}, \tag{24}$$

$$A_{11} = \begin{bmatrix} H_1 \\ -R_{11} \end{bmatrix} \text{ and } \mathrm{b}_{11} = \begin{bmatrix} \mathrm{u}_1 \\ \mathrm{r}_{00} - \mathrm{r}_{\mathrm{m}} \end{bmatrix}. \tag{25}$$

Here w is the vector of unknown parameters as before, and u_0 and H_0 are known matrices.

3 Numerical studies

A simple two dimensional geodetic network is considered for our numerical investigations. The network configuration is presented in Figure 1. This network consists of 5 points having initial coordinates and all possible distances and angles as well as their initial weights are considered in the optimization problem.

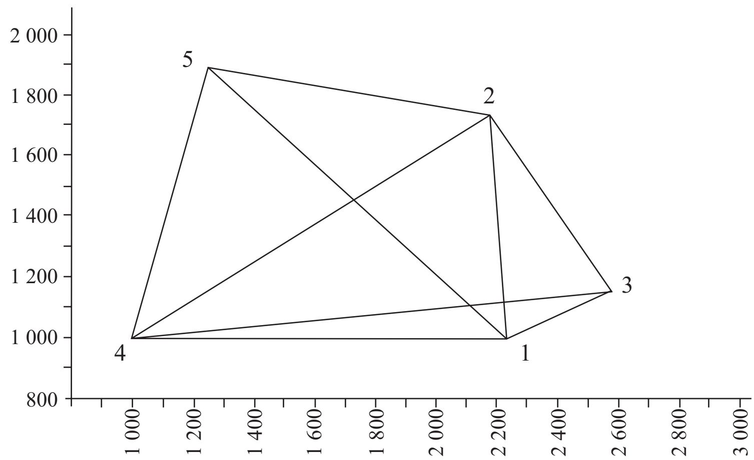

Figure 1. *The Network Configuration.*

Now, we should find the best configuration of the network as well as the best weights of observations according to the required quality. The standard error of 5 mm was considered as required precision of the position of net points. The network should be optimized in such a way that the redundancy of all observations is larger than r_m = 0.6. The configuration of the network is allowed to change up to 3 m around each point during optimization process ($-3 \leq \Delta x_i, \Delta y_i, \Delta z_i \leq 3$). The optimized observation weights are supposed to be positive and smaller than $1/(5\text{ mm})^2$ for the distances and 1/(1.5 sec) (in radians) for angles with the initial values $P^0_{l_i} = 1/(7\text{mm})^2$ and $P^0_{\alpha_i} = 1/(2\sec)^2$ (in radians), respectively. Before starting the optimization process let us determine the best datum of this network. As a criterion for finding the best datum for this network we have considered the minimum trace of the co-factor matrix. The best datum is defined when point 4 and the direction from point 4 to 2 are kept fixed.

Three SOOMs of precision, reliability and cost are considered as Model I, II and III, respectively. A MOOM is also utilized to obtain the position shifts and optimal observation weights in the network. The position shifts, or, in other words, the network configuration vs. approximate configuration is presented in Table 1.

Table 1. *The obtained shift values for optimization models. Unit: metre.*

	Precision model		**Reliability model**		**Cost model**		**MOOM**	
Points	**Δx**	**Δy**	**Δx**	**Δy**	**Δx**	**Δy**	**Δx**	**Δy**
1	1.909	0.306	3.148	1.467	1.519	1.596	0.600	1.200
2	–0.459	–0.287	1.450	0.925	1.718	1.074	0.0009	0.0006
3	0.371	–1.054	2.044	2.502	1.813	1.517	1.800	0.600
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	–5.632	–0.736	0.994	2.701	1.5161	1.649	1.200	0.600

It illustrates large position shifts in the network optimized by SOOMs, while these position shifts are considerably smaller when the network is optimized by the MOOM. The optimal standard errors of distances and angles after optimization are shown in Tables 2 and 3, respectively.

Table 2. *The optimal standard errors for distance observations after optimization. I, II, III, and M stand for precision, reliability, cost and MOOM, respectively. Unit: metre.*

Distance		**Optimal standard error of observations**			
from	**to**	**I**	**II**	**III**	**M**
1	3	0.004	0.003	0.012	0.003
1	5	0.003	0.003	0.005	0.005
2	3	0.003	0.005	0.012	0.002
2	4	0.003	0.003	0.008	0.003
2	5	0.003	0.003	0.012	0.003
3	4	0.002	0.002	0.012	0.003
3	5	0.003	0.003	0.012	0.003
4	5	0.012	0.004	0.012	0.003
1	2	0.003	0.003	0.012	0.003
1	4	0.003	0.001	0.012	0.003

Table 3. *The optimal standard errors for angles observations. I, II, III, and M stand for precision, reliability, cost and MOOM, respectively. Unit: sec.*

Angles			Optimal standard errors (sec)			
st	**from**	**to**	**I**	**II**	**III**	**M**
1	4	5	0.58	0.89	1.63	0.89
1	4	3	0.9	0.64	1.63	1.63
2	3	4	0.69	0.55	1.63	0.89
2	5	3	1.15	0.66	1.63	0.57
3	1	4	1.60	0.76	1.63	0.84
3	1	5	0.78	0.98	1.63	N/A
3	5	2	0.67	0.65	1.63	N/A
4	5	1	0.68	0.89	1.63	0.69
4	5	2	0.90	0.51	1.63	0.68
4	5	3	0.62	0.86	1.63	0.89
5	3	4	0.58	0.65	1.63	0.68
5	1	4	0.87	0.58	1.63	0.51
5	2	1	0.67	0.89	1.63	0.51

According to the initial precision of distances (5 mm), one can observe that all optimal standard errors of the distances are satisfied in the precision model except for one observable. The reliability model seems to have good capability to preserve the required accuracy. As could be expected, the cost model is not as good with respect to accuracy as the models I and II. These results agree with those obtained by Eshagh (2005) and Eshagh and Kiamehr (2007). As we will see in Table 8 and Figure 2, there is no proper internal and external reliability for the cost model. The MOOM seems to be the most flexible model, and, as one may observe, all standard errors of the positions are smaller than the requested accuracy (5 mm) in Table 7.

In Table 3 the standard errors of the angles are illustrated. The requested accuracy for the angles is 1 arc second. The precision model shows good agreements of the optimal precision with required accuracy except for two observables. A very bad result is seen in the cost model. As expected, the reliability model presents very good agreement with the considered accuracy. However, one observable has larger standard error than the requested accuracy in the multi-objective model. The optimal weights for some observations are unsuitable as they are too large and therefore the observation variances are zero or close to zero. We know that observation weights are always assumed to be positive numbers. Now, after performing the optimization procedure, it is possible to see small weights for some observations yielding large standard deviations. On the other hand, it is of interest to have small weights to exclude unnecessary observations from optimization procedure. We use minimum and maximum bounds for the observable precision but some constraints are inconsistent even if we define lower and upper bounds for them. The cost constraint is inconsistent with reliability and precision constraints, and then it violates some of the constraints. According to Table 3 we can neglect the angles 315 and 352 which have inappropriate variances and weights. Having eliminated those observations we can perform a new optimization process excluding the mentioned angles. The results are shows in Tables 4–6.

Table 4. *The new optimal position shifts obtained after eliminating inappropriate observations. Unit: metre.*

Position shifts		
	MOOM	
Points	**Δx**	**Δy**
1	2.958	1.242
2	2.958	1.848
3	–1.158	1.758
4	0.000	0.000
5	–1.158	1.758

Table 5. *The new optimal standard errors for distance observations after eliminating insignificant observations. Unit: metre.*

σ (m)		
from	**to**	**MOOM**
1	3	0.002
1	5	0.003
2	3	0.003
2	4	0.002
2	5	0.003
3	4	0.002
3	5	0.005
4	5	0.005
1	2	0.012
1	4	0.003

Table 6. *Optimal standard errors of the angles after eliminating insignificant observations. Unit: sec.*

Angles			**MOOM**
st	**from**	**to**	
1	4	5	0.61
1	4	3	0.68
2	3	4	0.58
2	5	3	0.68
3	1	4	0.51
4	5	1	0.89
4	5	2	0.68
4	5	3	0.57
5	3	4	0.57
5	1	4	0.89
5	2	1	0.51

Table 7. *The standard errors for three optimization models, I, II, III, and M stand for precision, reliability, cost and MOOM, respectively.*

Points	**σ (Achieved) (mm)**				**Precision required**
	I	**II**	**III**	**M**	
1	1	0.3	1	0.8	5
	3	2	5	2.8	5
2	2	1	3	1	5
	3	2	5	2	5
3	1	0.5	1	0.8	5
	4	2	5	2.9	5
4	0	0	0	0	5
	0	0	0	0	5
5	2	1	3	1.9	5
	1	1	4	1.8	5
trace	17	9.8	25	14	

Table 4 illustrates the position shifts after eliminating the insignificant observations (the observations having very low weights from the optimization process) in re-optimization. We may conclude that the elimination of observations causes larger position shifts in the network configuration. Tables 5 and 6 show the optimal standard errors of the distance and angle observables. This study shows that the configuration of the network changes considerably to satisfy the required precision. One can interpret these results as compensation of accuracy by changing the configuration.

Table 7 shows the square roots of the diagonal elements of the variance-covariance matrix of the positions. The MOOM is the best after the reliability model. We experienced that the convergence of the optimization process based on MOOM takes more time versus the SOOMs. There is no inconsistency problem in optimization by MOOM, because all criteria are presented in one OF. Furthermore, for optimization of the network by using a SOOM we can vary the norm for minimizing or maximizing each single OF. All models satisfy the required precision (5 mm) in Table 7, but we have mentioned that reliability model has best internal and external reliability among all models. The cost model is inconsistent with precision and reliability conditions inferior vs. other models. Also Table 7 shows that the reliability model yields a smaller trace of the variance-covariance matrix than the other models and it agrees with precision and reliability constraints. Similarly we have the same situation as the precision model, but the OF is not similar to reliability and cost models. In the precision model the network is fitted to the required precision in a least squares sense (based on L2 norm). In other words, we obtain the best fit to the desired precision of network by changing the network configuration and observations precision. In this case, the standard error of the positions is close to the desired one. Putting a precision constraint forces the optimization to deliver smaller error for the positions than the required one. However, in the reliability model the L_∞ -norm is used so that it maximizes the minimum reliability of the network in the optimization procedure. In such

Table 8. *Internal reliability for the three optimization models I, II, III, and M stand for precision, reliability, cost and MOOM, respectively.*

Points		Internal Reliability on the distances (mm)			
from	to	I	II	III	M
1	4	0.006	0.002	0.006	0.005
1	5	0.006	0.003	0.003	0.006
2	3	0.007	0.005	0.009	0.007
3	4	0.006	0.002	0.007	0.005
2	4	0.007	0.004	0.016	0.007
2	5	0.007	0.004	0.014	0.006
1	3	0.007	0.003	0.008	0.005
1	2	0.007	0.004	0.01	0.01
3	5	0.006	0.003	0.007	0.007
4	5	0.012	0.006	0.01	0.01

a case, there is no fitting to the residuals and the OF of reliability is maximized directly. The reliability model is quite consistent with precision condition as by increasing of reliability, the network tries to accept further observations. Thus, the position errors obtained from the least squares method (precision model) are decreasing when number of observations increase and reliability criterion satisfies the precision requirements inherently. However there is no lower bound for the position errors in the reliability model, and they are minimized as much as possible while in the precision model the errors are delivered smaller but close to the required position errors. This is why the trace of variance-covariance matrix and position shifts due to the reliability model are smaller and larger than the precision model, respectively.

The absolute error ellipses for all five network points after performing optimization by SOOMs and MOOM are presented in Table 9.

Table 9. *Absolute error ellipses for all optimization models. a and b are the semi-major and the minor axes of the error ellipses. Unit: mm.*

Points	Before optimization		After optimization							
	a	b	a				b			
			I	II	III	M	I	II	III	M
1	29	18	3	2	5	3	1	0.3	1	0.8
2	38	28	4	2	5	2	1	1	1	0.8
3	30	25	4	2	5	3	1	0.4	1	*0.7*
4	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
5	*27*	*18*	*2*	*1*	*4*	*2*	*1*	*0.7*	*3*	*2*

The effect of the largest undetectable gross error on the estimated positions obtained in a least squares adjustment are presented in Figure 2 for the *x*- and *y*-parameters. It is obvious that the reliability model yields the smallest external reliability.

The figure illustrates the external reliability of each position component of the five network points. As would be expected, a network having high internal reliability, as presented in Table 8 delivers small external reliability, too.

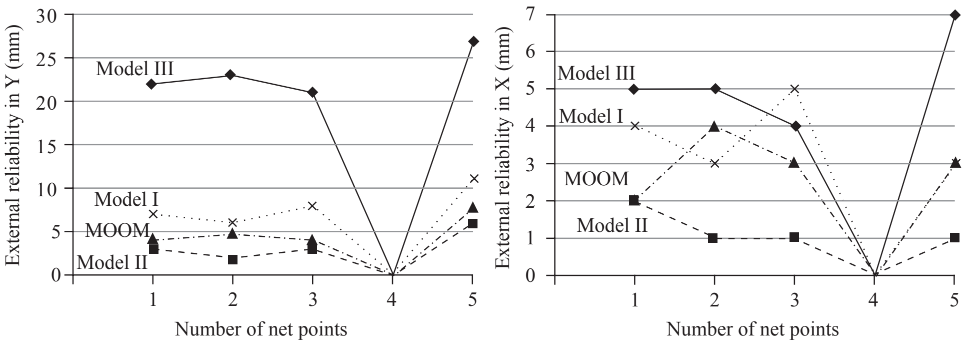

Figure 2. *The comparison of external reliabilities for optimization models.*

4 Conclusions

Before any measurement campaign is started, the geodesist should know the goal and requirements of the geodetic network to be designed. As is obvious, the highest precision and reliability of a geodetic network are expected, if all the observations are measured with highest accuracy. Since time and cost limitations do not allow extreme quality of a network, an optimum survey planning has to be made to achieve some prescribed design criteria with minimum effort. The main purpose is therefore to select the best datum, configuration and suitable precision for observations for satisfying client criteria. In this paper single- and multi-objective optimization models were reviewed. The models were applied in a simple geodetic network to illustrate their performance. Numerical results show that the best SOOM is Model II (reliability model), maximizing the internal reliability. On the other hand, the reliability model also yields the best results in precision. The numerical results show that the reliability model delivers larger configuration changes than the other SOOMs. It means that the initial configuration was not well fitted to the reliability model. It could also suggest that reliability is a more sensitive criterion to configuration than any of the other OFs. In some cases contradictions between constraints exist, and some of the constraints may not be met. However, as we showed, such a problem seldom happens in the reliability model. One way for overcoming such problems is to use the MOOM, by which the constraints would be fulfilled simultaneously in the best way. The analytical solution of the geodetic network is one of the best methods to design a network in such a way that it becomes an optimum network. Sometimes in the optimization process some observations get unrealistic weights corresponding to very high precision and some of the requirements are not met. In such a case re-optimization of the network is suggested after deleting those observables. In our numerical studies the MOOM yielded such results and we re-optimized the network. It is interesting to see that after eliminating the observables having unrealistic weights and re-optimizing the network, we got larger position changes with respect to previous optimization. Also we found that optimization based on MOOM does not converge as fast as the reliability model.

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