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Cadastral Triangulation: A Block Adjustment Approach for Joining Numerous Cadastral Blocks

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Abstract. *In the last decade or so, there has been a very clear transition in many countries throughout the world from a graphical cadastre and/or relatively non-accurate digital cadastre toward an accurate coordinate based legal cadastre. Aiming at defining accurately the turning points position of the cadastral sub-division based on current data without the need to re-measure the cadastral entities, motivates the development of new algorithms and approaches suitable to performing the task. Implementation on a nationwide level requires to first develop advanced mathematical algorithms and methods to process separate parcellations (cadastral blocks or mutation plans), and then additional algorithms and methods to combine the numerous separate parcellations into a cadastral continuity maintaining rigid topological compatibility.*

Practical experience, especially from the Israeli viewpoint, indicates that implementation of advanced computational techniques for processing separate cadastral blocks, is only a partial solution of the problem. An optimal joining of the separate cadastral blocks into a homogeneous seamless cadastral space constitutes a complex task due to discrepancies between the adjoining parcellations. These discrepancies, significant in terms of their magnitude and characteristics, are mainly caused by the cadastral parcellation process based on separate cadastral measuring projects on the one hand, and limited accuracy of the measuring techniques in previous decades (mainly in the first half of the 20th century) on the other hand.

The paper introduces a new algorithm based on the existing mathematical model, customary in photogrammetric mapping, aimed at connecting the adjoining photographs into blocks based on Block Adjustment by Independent Models. The proposed adjustment method (named the "Cadastral Triangulation") is executed based on the classic

Adjustment of Indirect Observations combined with the Chained Similarity Transformation. This adjustment process which is carried out by a global transformation mechanism, enables obtaining both optimal transformation parameters of all the separate parcellations, as well as optimal coordinates of the cadastral boundary turning points.

The initial results of the proposed method indicate its effectiveness in connecting the adjoining cadastral blocks, effectiveness expressed by a significant decrease of systematic and random errors compared to their pre-adjusted situation. Additionally, the proposed method enables bringing the adjusted cadastral boundary turning points maximally close to their theoretical true (and unknown) locations and, in any case, much closer than locations computed by currently practiced methods. Therefore, the proposed method may effectively be used as a primary computational algorithm for implementing a nationwide coordinate based legal cadastre.

Keywords: *coordinate based cadastre, boundary matching, least squares adjustment, block adjustment by independent models*

1 Introduction

Cadastre is a systematic method of land property registration and management that includes information about land parcels, e.g. their boundaries, areas, ownership, mortgages, pledges, etc. (Henssen, 1995, Dale & McLaughlin, 1988). Cadastre constitutes essential factor in national economy establishing strong basis to the existence of human society (Dale, 1997, Kaufmann & Steudler, 1998, Kaufmann, 1999). Good practice in land property administration, which includes development of modern cadastral systems, gives rise to a strong foundation of sustainable national development (Williamson, 2001, Bennett et al., 2008).

According to the customary principles in the Israeli cadastre - Torrens principles (Dale, 1976), parcel boundaries and areas are determined based on ground surveying, officially managed, controlled and approved by the state. Ground surveying is executed based on a national geodetic control network, and thus the X, Y planar coordinates define every parcel turning point.

Until now, the Israeli cadastre was based on block maps, field measurements books, computation files and physical markings on the ground of the parcel turning points. These turning points have legal validity whenever parcels boundary restoration is needed. Notwithstanding their legal status, the original cadastral documents suffer considerably from lack of completeness, and physical marking is mostly based on ground surveying of low accuracy (especially that carried out during the first decades of the modern cadastral era in Israel – the first half of the 20th century). As a result, the coordinates of parcel turning points are of low accuracy and there is great difficulty in integrating adjoining blocks into a spatial cadastral continuity. Additionally,

most of ground mark points no longer exist in the field due to urban development activity and construction. This situation, which is quite common to various countries all over the world, has stimulated awareness of the urgent need to transform the existing paper-based cadastre into a coordinate based cadastre, a process which would be characterized by optimally determined turning point positions and improved accuracy. The coordinate based cadastre will enable using turning points coordinates as a legal basis for parcel boundaries restoration. In order to achieve this goal it is necessary to: (i) develop an optimal method of original cadastral documents processing referring to separate parcellations (separate cadastral projects); and, (ii) develop a model enabling joining the separate parcellations into a cadastral continuity maintaining a rigid topological structure.

We approached the first task (Klebanov & Doytsher, 2008) by applying the Least Squares Adjustment as an optimal method of defining the parcel turning points position, considering all available information recorded in the original cadastral documents. The current paper presents a solution of the second task.

2 Motivation

The solution proposed in our recent work (Klebanov & Doytsher, 2008), enabled obtaining optimal positions of parcel turning points (and as a by-product, their accuracies) based on original cadastral document processing. The solution, based on the Gauss-Markov adjustment model, has assured gaining minimally possible differences between observations made during ground surveying and their values, analytically calculated from adjusted coordinates. The latter quality meets one of the essential requirements in cadastral documents processing – keeping adjusted values of observations maximally close to their measured values of legal validity. Additionally, the proposed solution enabled considering other kinds of information (geometrical and cadastral constraints) in defining the optimal position of the turning points.

Applying the aforementioned method allows obtaining the optimal position of turning points in separate cadastral blocks (or other cadastral parcellations, i.e., mutation plans). However, difficulties arise when one tries to integrate adjoining blocks. Practice shows that despite the ability to optimally process separate parcellations, their optimal connection into a homogeneous seamless space remains a very complicated task (Shmutter & Doytsher, 1992), mainly due to the accuracy problems pointed out in the previous section. Though some attempts have already been made to find local solutions (Doytsher & Gelbman, 1995, Nimre & Doytsher, 2000, Takashi et al, 2001), a comprehensive solution of the issue has not as yet been achieved.

Historically, the Israeli cadastre consists of a numerous projects based on different geodetic grids of relatively low accuracy (from several decimeters up to few meters), as well as of local grid systems without any connection to the national geodetic. Nowadays, when an accurate geodetic grid (IG05) exists in Israel, based on satellite geodesy and permanent ground stations (Steinberg & Even-Tzur, 2004), a technical possibility exists to determine point positions on a sub-decimeter level which is undoubtedly an appropriate accuracy level for a future coordinate based cadastre. Thus, the main problem in forming a homogeneous cadastral space is finding the appropriate transformation model and parameters of the numerous cadastral multi-grid based projects to be merged into a continuous seamless cadastral reality. The difficulty arises because only a small part of the parcel point ground marks have survived in the field due to urban development activity and construction. This situation prevents their direct ground re-surveying, aimed at obtaining accurate planar coordinates. Consequently, some kind of transformation is required to convert the point coordinates computed in the origin grid, to coordinates in the current updated Israel grid (IG05). As a result, new cadastral projects, based on old materials, should be accompanied by a transformation of the coordinates of parcel turning points, which have not been located in the field. The transformation issue becomes even more complicated when one tries to compute the appropriate transformation parameters for two or more adjoining cadastral parcellations aiming to obtain identical coordinates of peripheral common turning points. These identical peripheral coordinates will ensure continuous cadastral reality without gaps or overlaps between adjacent cadastral projects.

The purpose of the current work is to develop a comprehensive model of global coordinate transformation aimed at creating a homogeneous seamless cadastral space of high accuracy based on separate cadastral projects optimally pre-processed.

3 Proposed method

The proposed method adopts and follows the customary principles in photogrammetric mapping - principles of aerial triangulation (Kraus, 1993). According to these principles, the separate aerial photographs are connected into blocks by common tie points in overlapping regions of adjacent photographs. The photogrammetric model is transformed from a model space to a ground coordinate system based on pre-defined control points. A commonly used computational model, Block Adjustment by Independent Models (Mikhail et al., 2001, Linder, 2006) enables obtaining ground coordinates of a large-scale topographic area from numerous aerial photographs based only on a limited number of ground control points. The theory of Block Adjustment method and its implementations have been

widely discussed in professional literature (Jacobsen, 1998, Triggs et al., 2000, Cothren, 2005, Baz et al., 2007).

The proposed method – Cadastral Triangulation (CT) Method – refers to separate cadastral projects, optimally pre-processed, determined in various local coordinate systems (origin grids). The peripheral common turning points belonging to adjoining cadastral parcellations play the role of tie points. Points measured at the time when the cadastral projects were carried out and remained in the field, and can now be re-measured in the current grid system (IG05), serve as control points. The latter points gained the name of "authentic" points. It should be mentioned that due to the rapid urban development, not many authentic points can be presently located in the field, and these are not necessarily dispersed in an optimal manner, especially in reference to the old cadastral projects. As these authentic points have two sets of coordinates – in the origin grid from analytical pre-processing of the old measurements, and in the new grid (IG05) from modern re-surveying – the authentic points play the role of the control points in the photogrammetric aerial triangulation and a central role in the global transformation of the cadastral projects.

The proposed CT method, based on Block Adjustment by Independent Models, treats the separate cadastral projects, aiming to create a homogeneous seamless space by applying the global transformation mechanism. The proposed method uses the Chained Similarity Transformation (Kraus, 1993) as a model of global adjustment aimed at achieving a minimum sum of weighted squared residuals of peripheral common tie points and authentic control point coordinates by applying the Least Squares Adjustment. We considered similarity transformation to be the most appropriate type of cadastral transformation, as its conformal qualities have legal importance. We also considered its general case, using four transformation parameters for each parcellation.

Additional research can be considered regarding the choice of optimal number of transformation parameters and the effect of the number of authentic points and their dispersion on the final results.

Mathematical model

The linearized mathematical model of planar similarity transformation has the following expression (Kraus, 1993):

$$\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} Y_o \\ X_o \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} \quad (1)$$

where

Y_i, X_i - point coordinates in target grid

Y_o, X_o - point coordinates in origin grid

a, b - parameters of scale m and rotation K calculated as $a = m * \cos K$,
 $b = m * \sin K$

c, d - shift parameters

Due to pre-processing of the separate parcellations, the point coordinates in the origin grid are always known both for tie and control points. Transformation parameters of separate parcellations are always unknown. Point coordinates in the target grid are unknown for tie points and known for control points.

The implementation of the aforementioned model (1) in the process of Chained Similarity Transformation, which involves numerous points and parcellations, enables computing the estimates of position residuals $\hat{\varepsilon}_i$ of point i based on estimates of adjusted transformation parameters of the separate parcellation j to which point i belongs:

$$\begin{aligned}\hat{\varepsilon}_{yi} &= \hat{a}^j * Y_{oi} - \hat{b}^j * X_{oi} + \hat{c}^j - \tilde{Y}_{ii} \\ \hat{\varepsilon}_{xi} &= \hat{a}^j * X_{oi} + \hat{b}^j * Y_{oi} + \hat{d}^j - \tilde{X}_{ii}\end{aligned}\quad (2)$$

where $\hat{}$ - stands for Least Squares Estimator

Note: $\tilde{Y}_{ii}, \tilde{X}_{ii}$ stands for: (i) coordinate estimates of tie points in target grid ($\hat{Y}_{ii}, \hat{X}_{ii}$); or, (ii) known coordinates of authentic points in target grid (Y_{ii}, X_{ii}).

As mentioned before, the objective of the proposed CT method is to obtain during a global adjustment process such estimates of transformation parameters of the original cadastral parcellations (treated as independent models) that would minimize point position residuals (2) while applying LS Adjustment:

$$\sum p_i \hat{\varepsilon}_i^2 = \min \quad (3)$$

where p_i are the weights of points coordinates computed inversely to their accuracies

Equations (1) determine the functional relations between known observations and unknown adjustment parameters, which according to the model of Adjustment of Indirect Observations, may be expressed in general form as:

$$y = F(\beta) \tag{4}$$

where observations y are the coordinates of authentic control points computed in target grid on the basis of modern ground surveying, and unknowns β are parcellation transformation parameters and peripheral tie points coordinates in the target grid.

As an adjustment model of equations (4), we use the Gauss-Markov model (Koch, 1999), which ensures obtaining the best unbiased solution regarding the estimates of unknown parameters and adjusted tie points coordinates, provided condition (3) with residuals (2), computed in general form as:

$$\hat{\varepsilon} = X\hat{\beta} - y \tag{5}$$

where $X = \frac{\partial F(\beta)}{\partial \beta}$ - Jacobian matrix of partial derivatives of equations (2);

$F(\beta)$ - the right side of equations (2).

Note: observations vector y includes known coordinates for authentic points in target grid (Y_i, X_i) and zeros – for tie points.

Following are the fragments of Jacobian matrix X , observations vector y and transposed vector of unknowns β^T for point i belonging to parcellation j :

For authentic point:

$$(\beta^j)^T = [a^j \quad b^j \quad c^j \quad d^j]$$

$$[X_i^j] = \begin{bmatrix} Y_{oi} & -X_{oi} & 1 & 0 \\ X_{oi} & Y_{oi} & 0 & 1 \end{bmatrix}, [y_i] = \begin{bmatrix} Y_{ii} \\ X_{ii} \end{bmatrix}$$

For tie point:

$$(\beta^j)^T = [a^j \quad b^j \quad c^j \quad d^j \quad Y_{ii} \quad X_{ii}]$$

$$[X_i^j] = \begin{bmatrix} Y_{oi} & -X_{oi} & 1 & 0 & -1 & 0 \\ X_{oi} & Y_{oi} & 0 & 1 & 0 & -1 \end{bmatrix}, [y_i] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The aforementioned model (4) is linear ($\frac{\partial F(\beta)}{\partial \beta} = const$); therefore, the solution for vector of unknowns β is obtained during one iteration according to following expression (Koch, 1999):

$$\hat{\beta} = (X^T P X)^{-1} X^T P y \quad (6)$$

P - is the weight matrix defined inversely to the accuracy estimates of points coordinates computed during pre-processing (Klebanov & Doytsher, 2008) in the origin grid. Matrix P has a block diagonal structure composed of sub-matrices referring to separate parcellations:

$$P^j = (\hat{\Sigma}_x^j)^{-1} = (\hat{\sigma}_{0x}^{j^2} (N_x^j)^{-1})^{-1} = \frac{N_x^j}{\hat{\sigma}_{0x}^{j^2}}$$

where for j parcellation

Σ_x^j - covariance matrix of points coordinates estimates in origin grid

$\hat{\sigma}_{0x}^{j^2}$ - unit variance estimate

N_x^j - normal matrix

Covariance matrix of unknown parameters estimates is computed as

$$\hat{\Sigma}_\beta = \frac{\hat{\varepsilon}^T P \hat{\varepsilon}}{r} (X^T P X)^{-1}, \text{ where } r \text{ is the system redundancy.}$$

4 Simulation of proposed method

In order to be able to analyze the results and the accuracy of the proposed CT method, it was tested on synthetic data. The simulation of the synthetic sample was composed as an array of 300 rectangular cells (10x30) (Figure 1a). Every cell had the dimensions of 200x300 meters and thus, the entire synthetic sample covered the area of 2x9 km. Every single cell represented a separate cadastral block (or separate parcellation). The initial array was used as an "ideal" situation where a complete topological accordance of boundaries between adjoining cadastral parcellations exists (Figure 1b). Additionally, some nodes (61 turning points), evenly distributed on the synthetic sample, were chosen as the "authentic" points (marked on Figure 1 with a double circle) with the coordinates computed in a target geodetic grid based on newly performed ground surveying.

Afterwards, the "true" nodal coordinates of initial situation were "spoiled" (distorted) by applying a normal (Gauss) distribution error mechanism. The distortion process was performed in two steps. In the first step, every cell was independently shifted, rotated and scaled by applying normally distributed conformal transformation parameters: $\beta_1 \rightarrow (\beta_0, \Sigma_\beta)$, where $\beta_0^T = [a_0 = m_0 \quad b_0 = K_0 \quad c_0 \quad d_0] = [1 \quad 0 \quad 0 \quad 0]$ and standard deviations σ_β : $\sigma_m = 0.0003$, $\sigma_K = 10^{\min}$, $\sigma_c = \sigma_d = 0.3 \text{ meter}$ (Figure 1c).

This step simulated the accuracy problem of geodetic grid, which was very common in old cadastral projects. The values of standard deviations were chosen according to the statistically obtained accuracies of geodetic control points coordinates. In the second step, the cell nodes were independently

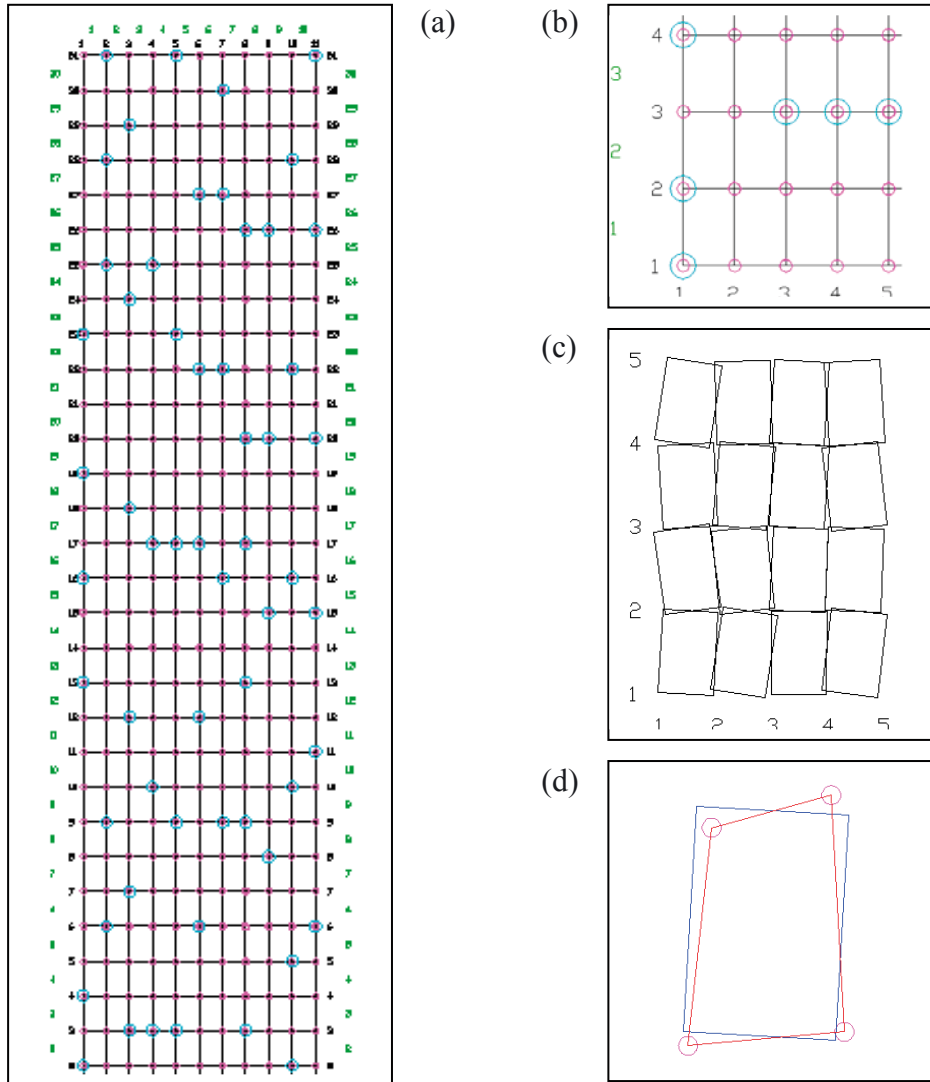


Figure 1. Simulation of proposed method: a) general chart; b) fragment of "ideal" situation; c) fragment of distorted situation; d) scheme of distortion

moved from the transformed values in the first step position Y_1, X_1 (Figure 1d) by shift values normally distributed: $[Y_2 \ X_2]^T \rightarrow ([Y_1 \ X_1]^T, \Sigma_{dY,dX})$ with standard deviations $\sigma_{dY} = \sigma_{dX} = 0.12m$ which are double the standard deviations obtained in simulated pre-processing (Klebanov & Doytsher, 2008). This step simulated the accuracy problem of ground surveying (direct field observations) in carrying out cadastral projects. After the two aforementioned distortion steps, the final standard deviation of cell nodes position was obtained, computed according to error propagation (Anderson & Mikhail, 1998), as equal to $\sigma_Y = \sigma_X = 0.49m$, which is close to commonly encountered differences between turning points coordinates belonging to adjoining old cadastral projects.

Following the distortion process, the synthetic sample was processed according to the CT method as proposed in section 03 with the original "ideal" coordinates of the points used as the known coordinates of authentic points in the target grid. Cell transformation parameters and unknown nodes coordinates were computed according to (6). Aiming to achieve consistent results, the proposed method was processed 10 times, changing each time the simulative dataset by reactivating the aforementioned distortion process each time.

The accuracy evaluation results of these 10 runs computed before and after the adjustment are depicted in Table 1.

Table 1. Point position accuracy computed by the proposed method

Accuracy factor	(1) Before adjustment	(2) After adjustment	Improvement Ratio (1) vs. (2)
Sum of variances $\sum p_i \hat{\varepsilon}_i^2$ (sq. meters)	631.22	8.55	74
MSE $\sigma_0 = \sqrt{\frac{\sum p_i \hat{\varepsilon}_i^2}{r}}$, (meters)	0.51	0.08	6
Max residuals $ \varepsilon_i $, (meters)	1.75	0.19	9

Notes: (i) Variances before adjustment (after distortion) were computed as squared differences between distorted points positions and their average position; variances after adjustment were computed according to (5).

(ii) r is the system redundancy computed as the difference between the number of all point appearances in all parcellations and the number of unknowns.

In order to analyze the stability of the solutions, the results of the 10 runs were statistically evaluated and the standard deviations of the parameters in Table 1 (Sum of variances, MSE, Max residuals) were computed. These standard deviations results are depicted in Table 2.

Table 2. Stability results (standard deviations of runnings accuracy factors)

Accuracy factor	Before adjustment	After adjustment
MSE of sum of variances (sq. meters)	32.31	0.39
MSE of MSEs (meters)	0.01	0.00
MSE of Max residuals (meters)	0.12	0.01

As can be seen from Tables 1 and 2, the proposed CT method enabled reducing drastically the discrepancies between adjoining cells and brings them to the level that approaches the requirements of a legal coordinate based cadastre (sub-decimeter level). Moreover, the stability of the different simulations is very high, which points out the validity and correctness of the proposed method.

Table 3. Differences between "true" coordinates and coordinates computed according to the proposed and the "average" methods

Accuracy factor	(1) "True" coordinates vs. adjusted coordinates		(2) "True" coordinates vs. "average" coordinates		Improvement Ratio (2) vs. (1)	
	Y	X	Y	X	Y	X
Average (meters)	0.00	0.00	-0.01	0.01	-	-
MSE (meters)	0.11	0.10	0.38	0.29	4	3
Max difference (meters)	0.36	0.33	1.03	1.02	3	3

It should be mentioned that the currently widely used in practice method of boundary matching between adjoining parcellations is: (i) computing average point position (by means of plain or weighted average) between the appropriate adjoining points; or, (ii) accepting one of parcellations as an

“anchor” and attaching the adjoining parcellations to it. The first method provides reduced points position errors compared to the second one; accordingly, it was chosen as the method for comparison with the proposed CT method.

Adjusted points coordinates computed according to the proposed CT method and coordinates computed by “average” method of adjoining boundaries matching were compared with initial "true" coordinates. For comparison purposes, the differences between: (i) "true" coordinates and adjusted coordinates; and, (ii) "true" coordinates and "average" coordinates, were computed. The comparison results are depicted in Table 3.

In order to analyze the stability of the solutions, the results of the 10 runs have been statistically evaluated and the standard deviations of the parameters in Table 3 (average, MSE, max differences) were computed. These standard deviations results are depicted in Table 4.

Table 4. Stability results (standard deviations of runnings accuracy factors)

Accuracy factor	(1) "True" coordinates vs. adjusted coordinates		(2) "True" coordinates vs. "average" coordinates	
	Y	X	Y	X
MSE of Averages (meters)	0.02	0.01	0.02	0.03
MSE of MSEs (meters)	0.01	0.01	0.01	0.01
MSE of Max differences (meters)	0.08	0.06	0.17	0.17

As can be seen from Tables 3 and 4, in both methods the coordinate differences converge to zero, meaning that both adjusted and “average” coordinates converge to their “true” values with a minor advantage to proposed method. However, the dispersion of points around their "true" position in the proposed CT method is much closer (3-4 times) than in the average" method. This means that in a real situation, after applying the proposed adjustment process, specific parcels turning points would be defined quite close (on sub-decimeter accuracy level) to their true (and unknown) positions and, in any case, much closer than by currently practiced methods. Moreover, the stability of the different simulations is very high, which indicates the validity and the correctness of the proposed method.

An additional test has been performed on a synthetic dataset with the simulative array reduced to 100 parcels that accordingly included less authentic points still evenly distributed on synthetic sample. The test was

performed with a gradually reduced number of authentic points – from 23 to 6 points.

Table 5. Differences between "true" coordinates and coordinates computed according to proposed and "average" methods for a reduced number of authentic points

Accuracy factor	(1) "True" coordinates vs. adjusted coordinates		(2) "True" coordinates vs. "average" coordinates		Improvement Ratio (2) vs. (1)	
	Y	X	Y	X	Y	X
23 authentic points						
Average (meters)	-0.01	0.00	-0.04	0.03	-	-
MSE (meters)	0.14	0.14	0.44	0.34	3	2
12 authentic points						
Average (meters)	0.02	0.00	-0.01	0.01	-	-
MSE (meters)	0.17	0.20	0.43	0.34	3	2
6 authentic points						
Average (meters)	-0.04	-0.06	0.04	-0.02	-	-
MSE (meters)	0.32	0.32	0.42	0.33	1	1

As can be seen, reducing the number of authentic points led to decreasing the accuracy of the proposed CT method compared to the simple "average" method. Generally, thinning out authentic points caused a convergence of both methods to produce almost similar results regarding dispersion of points around their "true" position. Nonetheless, during a detailed analysis of obtained results it was noticed that within the surrounding regions close to the authentic control points there remains a strong improvement of the proposed CT method compared to the "average" method, an improvement that gradually diminishes as one moves away from the authentic points. This indicates a possibility of defining the authentic point influence regions where applying the proposed CT method might be effective. In a real situation, cadastral space partitioning on separate influence regions would enable transforming the nationwide task of connecting the adjoining parcellations to local (regional) solutions of reduced extent.

The method proposed in this paper has been tested on a synthetic dataset aiming to confirm the converging of turning point adjusted coordinates to their initial true values which are unknown in a real situation. In addition to

the theoretical mathematical developments of the CT method, using simulative synthetic data has the advantage of being able to analyze thoroughly the meaning of the different mathematical steps and the different parameters being used. In a second phase of the research, the proposed approach is tested on real data of cadastral projects, and the results will be reported next year (Klebanov & Doytsher, 2009).

5 Conclusion and future work

Applying the proposed CT method to global adjustment of adjoining parcellations enabled us to: (i) convert separate cadastral projects prepared in different origin grids into cadastral continuity in a uniform geodetic target grid; (ii) reduce considerably position discrepancies between adjoining cadastral parcellations; and, (iii) increase position accuracy of parcel boundary turning points compared to the existing boundaries matching method. Applying the proposed method ensured obtaining parcel turning points positions much nearer their "true" positions than those obtained by applying the existing matching methods.

An additional study should be made to analyze: (i) the optimal number of transformation parameters referring to separate parcellations during global transformation and adjoining boundaries adjustment; and, (ii) optimal dispersion of authentic points in the adjusted area and their number. These issues will probably drastically affect the computation of optimal transformation parameters and turning point positions and their accuracy.

Applying the proposed CT method on the nationwide level, including calculation of tens of thousands of transformation parameters and millions of peripheral turning point coordinates, is not a simple task in respect to the required computational resources. Therefore, an additional study should be carried out aimed at simplifying the proposed solution, and improving the adjustment model in order to achieve the optimal algorithm and increase the effectiveness of the entire computational process.

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