Intuitionistic Fuzzy Soft Set Theory and Its Application in Medical Diagnosis

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Abstract: For finding coherent and logical solution to various real life problems containing uncertainty, impreciseness and vagueness, fuzzy soft set theory is gaining importance. Later on a theoretical study of the intuitionistic fuzzy soft set was developed. The combination of intuitionistic fuzzy set and intuitionistic fuzzy soft set are more useful for application point of view in the field wherever uncertainty due to vagueness appear in more complex form.

In the present communication the concepts of fuzzy soft set and Intuitionistic fuzzy soft Set are defined as hybridization of fuzzy set and soft set theory. A new method of application of intuitionistic fuzzy soft set is studied in Medical Diagnosis following Sanchez's approach. A hypothetical case study is also discussed in brief using the proposed method.

Keywords: Fuzzy set, Soft set, Fuzzy soft set, Intuitionistic fuzzy soft set, Medical documentation, Medic diagnosis.

1. INTRODUCTION

Various real life problems in engineering, social and medical sciences, economics, management, etc. use imprecise and enormous data and mathematical principles based on uncertainty and imprecision are applied in finding their solutions. In recent years to deal with such systems in an effective way, a number of theories have been developed, like; probability, game theory, fuzzy sets, intuitionistic fuzzy sets, fuzzy soft sets, etc.

The most suitable theory for dealing with uncertainty is the theory of fuzzy sets developed by Zadeh [16] in 1965. Another one is rough set theory pioneered by Pawlak [14] in 1982, which is also a momentous technique to modelling vagueness. This theory has been successfully applied to many fields such as machine learning, data mining, data analysis, medical diagnosis, etc.

Atanassov [2] introduced the concept of intuitionistic fuzzy sets (IFS), as a generalization of fuzzy sets, which is capable of capturing the information that includes some degree of hesitation and has been applied in various fields of research. Thus another degree of freedom has been introduced into the description of intuitionistic fuzzy sets.

In 1999, Molodtsov [11] introduced a general mathematical tool known as soft set theory to handle the objects which have been defined using a very loose and hence very general set of characteristics, which was completely a new approach for modelling uncertainties due to vagueness.

In addition to defining the fundamental concepts of soft set theory, Molodtsov [11] also illustrated how soft set theory is free from parameterization insufficiency condition of fuzzy set theory, rough set theory, probability theory and game theory. Soft set theory is a universal framework, as various traditional models emerge as unique case of soft set theory.

Soft set theory has prospective for application in resolving realistic problems in economics, engineering, environment, social science, medical science and business management. The absence of any restriction on the approximate description in soft set theory makes this theory very convenient and easily applicable. Recently, Hooda and Kumari [7] have applied in data dimension reduction and medical diagnosis.

Later on, Maji *et al.* [9] studied the theory of soft sets and also promote a hybrid model known as fuzzy soft set [8], which is a combination of soft set and fuzzy set. In [7] they also studied the intuitionistic fuzzy soft set. The concept of fuzzy soft set introduced by Maji *et al.* [7] was generalized by Majumdar and Samanta [10].

Neog and Sut [12, 13] illustrated with counter examples that the axioms of contradiction and exclusion are not authenticate in case of soft sets and fuzzy soft sets if we apply the notion of complement commenced by Maji *et al.* [8, 9]. Accordingly, they put forward new definitions of complement of soft sets and fuzzy soft sets and demonstrated that all the properties of complement of a set are possessed by soft sets and fuzzy soft sets due to the proposed definition of complement.

For extreme data dimensionality reductions becomes the centre of curiosity to a significant point of study in various fields of application (refer to Gupta and Sharma [5]). A number of techniques proposed by the

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researchers and authors related to dimensionality reduction. Hooda and Kumari [6] studied dimension reduction in multivariate analysis by using fuzzy soft set.

Chen *et al.* [3] present a new definition of parameterization reduction in soft sets and compare this definition to the associated concept of attributes reduction in rough set theory. In this paper we used fuzzy soft set based approach to reduce the dimensionality of data. The proposed novel method of dimensionality reduction involves constructions of binary information table from soft sets and fuzzy soft sets in a parametric sense for dimensionality reduction.

Out of several generalizations of fuzzy set theory for various objectives, the notion introduced by Atanassov [1,2] in defining intuitionistic fuzzy sets (IFSs) is interesting and useful. But Molodtsov [11] has shown that this topic suffers from some inherent difficulties due to inadequacy of parametriation tools and introduced a concept called 'Soft Set Theory' having parameterization tools for successfully dealing with various types of uncertainties.

The combination of Intuitionistic Fuzzy Set and Soft Set will be more useful in the field of applications wherever uncertainty appear. Thus, Maji *et al*. [7] have developed a theoretical study of the 'Intuitionistic Fuzzy Soft Set'(IFSS). De *et al*. [4] have studied Sanchez's [15] method of medical diagnosis using intuitionistic fizzy set. Our proposed method is an attempt to improve the results in [4] using the complement concept of IFSS to formulate a pair of medical knowledge, hereafter, called soft medical knowledge.

2. PRELIMINARIES

In this section we describe the preliminary definitions, and results which are required later in this paper.

2.1. Fuzzy Set

The concept of a fuzzy set is an extension of the concept of a crisp set. Just as a crisp set on a universal set X is defined by its characteristic function from X to {0, 1}, where a fuzzy set on a domain X is defined by its membership function from X to [0,1].

Definition 2.1: Let X is a non-empty set, (called the universal set or the universe of discourse or simply domain). Then a function $\mu_A(x): X \to [0,1]$, defines fuzzy set on X as

$$A = \left\{ \left(x_i, \mu_A\left(x_i\right) \right) : \mu_A\left(x_i\right) \in \left[0,1\right]; \forall x_i \in X \right\},$$

where $\mu_A(x_i)$ is called membership function and satisfies the following properties:

$$\mu_A\left(x_i\right) = \begin{cases} 0, \text{if } x_i \notin A \text{ and there is no ambiguity} \\ 1, \text{if } x_i \in A \text{ and there is no ambiguity} \\ 0.5, \text{there is max ambiguity whether } x_i \notin A \text{ or } x_i \in A \end{cases}$$

Representation of Fuzzy Set

There are following ways to represent fuzzy set.

(a) Fuzzy set A on X can be represented by, set of ordered pair as follows

$$A = \{(x, \mu_A(x)) : x \in X\}.$$

(b) In case, if the domain is finite fuzzy set $A = \sum \frac{\mu_A(x_i)}{x_i}.$

For example if $\mu_A(a) = 0, \mu_A(b) = 0.7, \mu_A(c) = 0.4, \mu_A(d) = 1.$

Then, fuzzy set A can be written as

$A = \{(a,0),(b,0.7),(c,0.4),(d,1)\}$ or $A = \frac{0}{a} + \frac{0.7}{b} + \frac{0.4}{c} + \frac{1}{d}$.

(c) In case, the domain is continuous $A = \int \frac{\mu_A(x)}{x}.$

(d) In case, the domain is finite and consists n-elements $x_1, x_2, x_3, \ldots\ldots x_n$. Then fuzzy set

A=

x_1	x_2	……..	x_n
0.5	0.6	……..	0.9

(e) By means of a graph

(i) The fuzzy membership function for fuzzy linguistic term “COOL” relating to temperature is as below

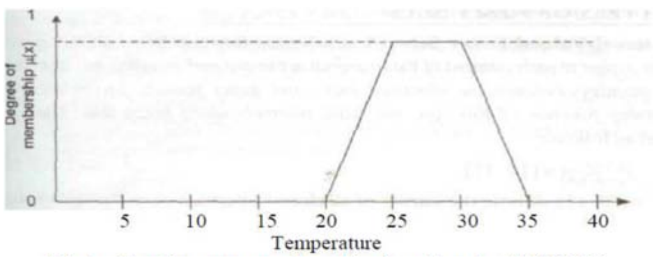

Figure 1: Continuous membership function for “COOL”.

(ii) A membership function can also be given mathematically as

$$\mu_A(x) = \frac{1}{(1+x)^2}$$

The graph of the above function is shown below:

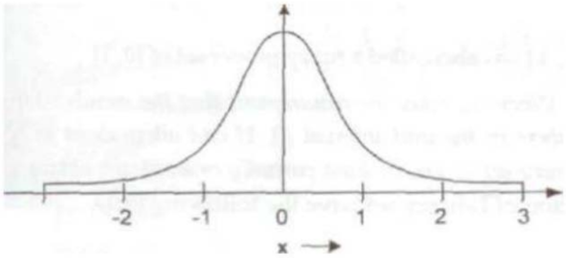

Figure 2: Continuous membership function dictated by mathematical function.

Definition 2.2. (Intuitionistic Fuzzy Set)

Let X be a universe of discourse. An intuitionistic fuzzy set A in X is an object having the form

$$A = \left\{\left\langle x, \mu_A(x), \nu_A(x)\right\rangle : x \in X\right\}, \quad (2.2)$$

where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ such that

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \ \forall x \in X.$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote the degree of membership and degree of non- membership of the element $x \in X$, respectively.

For convenience of notations, the term "intuitionistic fuzzy set" is symbolized by IFS and denoted by $IFS(X)$, the set of all the IFSs in X. For each IFS A in X, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the intuitionistic fuzzy index of an element $x \in X$ in A, which denotes hesitancy degree of x to A. It is evident that $0 \le \pi_A(x) \le 1, \ \forall x \in X$.

FSs can also be represented using the notation of IFSs. A FS A defined on X can be represented as the following IFS:

$A = \left\{\left\langle x, \mu_A(x), 1 - \mu_A(x)\right\rangle : x \in X\right\}$, with $\pi_A(x) = 0, \forall x \in X$.

The complementary set A^c of A is defined as

$$A^C = \left\{\left\langle x, \nu_A(x), \mu_A(x)\right\rangle : x \in X\right\}.$$

Example 2.1. Let *X* be the set of all states of India with elective government. Assume that the percentage of votes obtained by each state is given by $M(x)$ and let $\mu(x) = \frac{M(x)}{100}$. Let $\nu(x) = 1 - \mu(x)$ be the number which corresponds to the part of electorate who have not voted for the government. However, if $\nu(x)$ can also be defined as the number of votes given to parties or candidates outside the government, then the part of electorate who have not voted at all well have membership value equal to $1 - \mu(x) - \nu(x)$.

Thus the resulting set $\{(x, \mu(x), \nu(x)) | x \in X\}$ is called intuitionistic fuzzy set and $1 - \mu(x) - \nu(x)$ is intuitionistic index of an element $x \in X$.

FSs can also be represented using notation of IFSs. A fuzzy set A defined on *X* can be represented as the following IFS:

$A = \{x, \mu(x), 1 - \mu(x)\}$, where $\pi_A(x) = 0$ for all $x \in X$.

The complementary set A^c of A is defined as

$$A^c = \{(x, \nu_A(x), \mu_A(x)) | x \in X\}. \quad (2.2)$$

It may be noted that Atanassov [1] himself gave one example showing that fuzzy sets are intuitionistic fuzzy sets, but the converse is not true.

In his paper Atanassov [2] states that the following operations and relations are satisfied by every two IFSs A and B defined on *X*:

(a) $A \subset B$ iff $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \le \nu_B(x)$ for $x \in X$.

(b) $A = B$ iff $A \subset B$ and $B \subset A$.

(c) $A \cap B = \left\{\begin{pmatrix} x, \min(\mu_A(x), \mu_B(x)), \\ \max(\nu_A(x), \nu_B(x)) \end{pmatrix} \middle| x \in X\right\}$

(d) $A.B = \{(x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)) \mid x \in X\}$.

To prove that $A + B$ is IFS, let us consider the four non-negative numbers a,b,c,d satisfying and $0 \le c + d \le 1$. Then $a(1-c) + c \ge c \ge 0$.

It implies that
$0 \le b.d \le a + c - ac + bd \le a + c - ac + (1-a)(1-c) = 1$.

2.2. Soft Set

Soft set theory that was insinuated by Molodtsov in 1999 to deal with uncertainty in a non-parametric approach is a generalization of fuzzy set theory. Soft set theory has a productive prospective for application in various areas, some of which had been discussed by Molodtsov [4]. To deal with a collection of approximate portrayal of objects, a generalized parametric gizmo is used known as Soft set.

Each approximate portrayal has two parts a predicate and an approximate value sets. Since the initial portrayal of the object has an approximate nature, therefore, there is no need to introduce the notion of exact solution. The soft set theory is very handy and simply valid in performance due to the nonexistence of any restrictions on the approximate descriptions. With the aid of words and sentences, real number, function, mapping and so on; any parameter can be operate that we desire.

Definitions 2.3. Let $X$is an initial universe set and $E$is the set of parameters and A ©E. Then the pair (F, A) is called a soft set (over X) if F is a mapping of A into the power set of X, i.e., $F: A \rightarrow P(X)$. Obviously, for a given universe X a soft set is a parameterized family of subsets over X

Example 2.1. Let X= {c_1,c_2,c_3} be the set of three cars and E={e_1=costly, e_2=metallic color, e_3=cheap}be set of parameters. Let A = {e_1,e_2}, then(F,A)= {F(e_1)= {**c_1**,c_2,c_3}, F(e_2)={c_1,c_2}} is the crisp soft set over X which describes the " attractiveness of cars" which Mr. S is going to buy.

Table 1: Example of Soft Set

X/A	e_1	e_2
c_1	1	1
c_2	1	1
c_3	1	0

Example 2.2. Let $X = \{m_1, m_2, \dots, m_5\}$ be the sets of mobiles under consideration and E be the set of parameters, i.e., A=E, where
$E = \{e_1 = expensive, e_2 = good\ quality, e_3 = cheap, e_4 = stylish, e_5 = latest\}$.
Then the soft set (F, E) describes the attractiveness of the mobiles as

$(F, E) = \{\{expensive = m_1, m_2, m_5\}, \{good\ quality = m_3, m_5\}, \{cheap = m_3, m_4, m_5\}, \{stylish = m_1, m_2, \dots, m_5\}, \{latest = m_1, m_3, m_4, m_5\}\}$.

This soft set representation is shown in the Table **1** form as above:

Table 2: Attractiveness of the Mobiles Soft Set

X/E	e_1	e_2	e_3	e_4	e_5
m_1	1	0	0	1	1
m_2	1	0	0	1	0
m_3	0	1	1	1	1
m_4	0	0	1	1	1
m_5	1	1	1	1	1

2.3. Fuzzy Soft Set

By hybridization of fuzzy sets and soft sets, Maji *et al.* [6] defined fuzzy soft sets. Actually, the concept of fuzzy soft set is an extension of crisp soft set. The fuzziness or vagueness deals with uncertainty inherent in the dimensionality reduction and decision making problems like medical diagnosis. The definition of fuzzy soft set is given and illustrated by examples.

Definition 2.4. Let X be a universal set and F(X) be the set of all fuzzy subsets of X. Let E a set of parameters and A © E, a pair (F, A) is called fuzzy soft set, where F is a mapping from A to F(X). Thus, a fuzzy soft set (F, A) over X can be represented by the set of ordered pairs

$$(F, A) = \{(p, F_A(p)) : p \in P, F_A(p) \in F(X)\}$$

Example 2.3. Let $X = \{b_1, b_2, b_3\}$ be the set of three bikes represented as set of object and $P = \{\text{costly}(p_1), \text{colour}(p_2), \text{getup}(p_3)\}$ be the set of parameters, where $A = \{p_1, p_2\} \subset P$ .Then $\mathrm{F}_A = \{\mathrm{F}_A(p_1) = \{b_1/0.5, b_2/0.7, b_3/0.4\}, \mathrm{F}_A(p_2) = \{b_1/0.6, b_2/0.3, b_3/0.8\}\}$ is the fuzzy soft set over X which describes the "attractiveness of the bikes".

Example 2.4. In example 2.2 of soft set considered above if m_2 has stylish then it will not be possible to express it with only the two numbers 0 and 1. In that case we can characterize it by a membership function instead of the crisp number 0 and 1 that associated with each element a real number in the interval [0, 1]. The fuzzy soft set can then be described as

$$\begin{aligned} \mathrm{F}_E = \Big\{ \mathrm{F}_E(e_1) &= \left\{\frac{m_1}{0.36}, \frac{m_2}{0.24}, \frac{m_3}{0}, \frac{m_4}{0}, \frac{m_5}{0.6}\right\}, \mathrm{F}_E(e_2) \\ &= \left\{\frac{m_1}{0}, \frac{m_2}{0}, \frac{m_3}{0.32}, \frac{m_4}{0}, \frac{m_5}{0.40}\right\}, \mathrm{F}_E(e_3) \\ &= \left\{\frac{m_1}{0}, \frac{m_2}{0}, \frac{m_3}{0.48}, \frac{m_4}{0.36}, \frac{m_5}{0.6}\right\}, \mathrm{F}_E(e_4) \\ &= \left\{\frac{m_1}{0.6}, \frac{m_2}{0.4}, \frac{m_3}{0.8}, \frac{m_4}{0.6}, \frac{m_5}{1}\right\}, \mathrm{F}_E(e_5) \\ &= \left\{\frac{m_1}{0.48}, \frac{m_2}{0}, \frac{m_3}{0.64}, \frac{m_4}{0.48}, \frac{m_5}{0.8}\right\}\Big\} \end{aligned}$$

The tabular representation of this fuzzy soft set F_E is as shown below:

Table 3: Example of Fuzzy Soft Set

X/E	e_1	e_2	e_3	e_4	e_5
m_1	0.36	0.00	0.00	0.60	0.48
m_2	0.24	0.00	0.00	0.40	0.00
m_3	0.00	0.32	0.48	0.80	0.64
m_4	0.00	0.00	0.36	0.60	0.48
m_5	0.60	0.40	0.60	1.00	0.80

3. INTUITIONISTIC FUZZY SOFT SET

Definition 3.1. Let X be a universal set, E a set of parameters and A$\subseteq$E. Let I(X) denote the set of all intuitionistic fuzzy subsets of X. Then a pair (F,A) is called an intuitionistic fuzzy soft set(IFSS) over X, where F is a mapping from A to I(X).

Definition 3.2. Let (F,A) and (G,B) be two IFSSs over a common universe X, then

(i) (F,A) $\subset$ (G,B), if A $\subset$ B and for all e $\in$ A, then F(e) is an intuitionistic fuzzy subset of G(e)

(ii) (F,A)=(G,B), if (F,A) $\subset$(G,B) and (G,B) contains (F,A)

(iii) The complement of an IFSS (F,A) denoted by $(F,A)^c$ and is defined by

$(F,A)^c = (F^c, \neg A)$, where $F^c: \neg A \to I(X)$ is a mapping given by

$$F^c(\alpha) = intuitionistic\ fuzzy\ setof\ X\ of\ F(\alpha), \forall\ \alpha\ \epsilon \neg A$$

(iv) (F,A) is said to be a null IFSS, denoted by ϕ if $\forall$ $e \in$ A, F(e)=null intuitionistic fuzzy set of X.

(v) An IFSS set (F,A) is said to be absolute IFSS over X, denoted by $\tilde{A}$, if $\forall$ $e \in$ A, F(e)=X.

(vi) Union of two IFSSs (F,A) and (G,B) is an IFSS, denoted by

(H,C)=(F,A) $\cup$ (G,B), if C=A u B and $\forall$ e € C, H(e)
= F(e), if $e \in$A - B,
= G(e), if $e \in$ B-A,
=F(e) $\cup$ G(e), if $e \in$ A $\cap$ B .

(vii) Intersection of two IFSSs (F,A) and (G,B) is an IFSS, denoted by

(H,C)=(F,A)$\cap$ (G,B), if C=A $\cap$ B and $\forall$ e ϵ C, H(e)= F(e) $\cap$ G(e).

(viii) AND(**^**) operation of two IFSSs : If (F,A) and (G,B) are two IFSSs then

(F,A) AND (G,B) is an IFSS, denoted by (H,A x B)= (F,A) /^ (G,B), where

$$H(\alpha,\beta) = F(\alpha)\tilde{\cap}G(\beta),\ \ \forall \alpha \in A \text{ and } \forall \beta \in B .$$

(ix) OR (v) operation of two IFSSs : If (F,A) and (G,B) are two IFSSs then (F,A)OR(G,B) is an IFSS, denoted by (0,A x B)= (F,A) v (G,B),

where $O(\alpha,\beta) = F(\alpha)\tilde{\cup}G(\beta), \forall \alpha \in A \text{ and } \forall \beta \in B$.

Example 3.1. Let X=$\{c_1, c_2, c_3\}$be the set of three cars and E={costly(e_l), metallic colour(e_2), cheap(e_3)) be the set of parameters. Consider two intuitionistic fuzzy soft sets (F,A) and (G,B), where

A={ e_l, e_2} $\subset$ E and B={ e_l, e_2,e_3} $\subset$ E and (F,A)={F(e_1)={c_l /(.6,.3), c_2 /(.4,.5), c_3 /(.3,.6)}, F(e_2)={ c_l /(.6,.2), c_2/(.7,.2), c_3/(.5,).4)} and (G,B)= {G(e_l)= {c_l /(.6,.3), c_2 /(.4,.5), c_3 /(.3,.6)},G(e_2)= {c_1/(.6,.4),c_2/(.7,.2),c_3/(.5,.5)), G(e_3)={ c_l / (.2,.6), c_2 / (.4,.4), c_3 / (.5,.3)}}.

Then

(i) $(F,A)^c$= {F(⌐e_1)=**{c_1/(.3,.6),c_2/(.4,.4),c_3/(.6,**,F(⌐e_2) = {(c_1/(.2,.6),c_2/(.2,.7),c_3/(.4, .5))

(ii) (F,A) $\subset$(G,B) .

(iii) (H,C)=(F,A)$\tilde{\cup}$ (G,B)={H(e_l)={c_l/(.6,.3),c_2/(.4,.5),c_3/(.3,.6)}**,** H(e_2)={c_1/(.6,.2),c_2/(.7,.2),c_3/(.5,),.4)}, H(e_3)={ c_1/(.2,.6),c_2/(.4,.4), c_3/(.5,.3)}

(iv) (H,C)=(F,A) $\cap$ (G,B)={H(e_l)= **{** c_l /(6,.3), c_2 /(.4.4), c_3/(.3,.6)}, H(e_2)= {c_l **/(.6,.2),** c_2**/(.7,.2),** c_3/(.5,}.4)}.

Proposition 3.1. If (F,A) and (G,B) are two soft sets (or fuzzy soft sets or intuitionistic fuzzy soft sets), then(i) (i) $(i)(F,A) \cup (F,A) = (F,A)$

$$(ii)\ (F,A) \cap (F,A) = (F,A)$$

$$(F,A) \cup \emptyset = \emptyset$$

$$(F,A) \cap \emptyset = \emptyset$$

$(F,A) \cap \overline{A = \bar{A}},\ \ \bar{A}$ is the absolute soft set(or absolute fuzzy soft set / IFSS).

$$(F,A) \cap \bar{A} = \bar{A}$$

$$((F,A) \cup (G,B))^c = (F,A)^c \cup (G,B)^c$$

$$((F,A) \cap (G,B))^c = (F,A)^c \cap (G,B)^{c,}$$

$$((F,A) \vee (G,B))^c = (F,A)^c \wedge (G,B)^c$$

$$((F,A) \wedge (G,B))^c = (F,A)^c \vee (G,B)^c$$

4. APPLICATION OF INTUITIONISTIC FUZZY SOFT SET IN MEDICAL DIAGNOSIS PROBLEM

1n this section we extend Sanchez's approach for medical diagnosis using intuitionistic fuzzy soft sets and exhibit the technique with a hypothetical case study.

Suppose S is a set of symptoms related to sickness, D is a set of diseases and P is a set of patients. Construct an intuitionistic fuzzy soft set (F,D) over S, where F is a mapping F:D $\rightarrow$P(S).A relation matrix say, R_1 is constructed from the intuitionistic fuzzy soft set (F,D) and name it symptom-disease matrix. Similarly its complement $(F,D)^c$ gives another relation matrix, say R_2, called non symptom-disease matrix. Analogous to Sanchez's notion of 'Medical knowledge' we refer to each of the matrices R_1 and R_2 as 'Intuitionistic Soft Medical Knowledge'.

Again we construct another intuitionistic soft set (F_1,S) over P, where F_1 is a mapping given by F_l:S $\rightarrow$

P(P). This intuitionistic fyzzy soft set gives another relation matrix Q called patient-symptom matrix. Then we obtain two new relation matrices T_1 = Q o R_l and T_2=Q0R_2, called symptom-patient matrix and non symptom-patient matrix respectively, in which the membership values are given by

$$\mu_{T_1}\left(p_i,d_k\right)=\vee\left\{\mu_Q\left(p_i,e_j\right)\vee\mu_{R_1}\left(e_j,d_k\right)\right\}$$

$$\mu_{T_2}\left(p_i,d_k\right)=\vee\left\{\mu_Q\left(p_i,e_j\right)\vee\mu_{R_2}\left(e_j,d_k\right)\right\}$$

and the non-membership function given by

$$\nu_{T_1}\left(p_i,d_k\right)=\vee\left\{\nu_Q\left(p_i,e_j\right)\vee\nu_{R_1}\left(e_j,d_k\right)\right\}$$

$$\nu_{T_2}\left(p_i,d_k\right)=\vee\left\{\nu_Q\left(p_i,e_j\right)\vee\nu_{R_2}\left(e_j,d_k\right)\right\},$$

where v = max and ^ = min

If $\pi_{T_1}=\left(1-\mu_{T_1}-\nu_{T_1}\right)$ and $\pi_{T_2}=\left(1-\mu_{T_2}-\nu_{T_2}\right)$ are the hesitation parts with respect to

T $_1$ and T_2 respectively then we calculate If $S_{T_1}=\left(\mu_{T_1}-\nu_{T_1}\pi_{T_1}\right)$ and $S_{T_2}=\left(\mu_{T_2}-\nu_{T_2}\pi_{T_2}\right)$

which we call as diagnosis score for and against the disease respectively.

Next, if $\max_j\left\{S_{T_1}\left(p_1,d_j\right)-S_{T_2}\left(p_1,d_j\right)\right\}$ occurs for exactly (p_1 d_k) only, then we conclude that the acceptable diagnostic hypothesis for patient p_1, is the disease d_k. In case there is a tie, the process has to be repeated for patient p_1 by reassessing the symptoms.

4.1. Algorithm

For algorithm the following steps are suggested:

1. Input the IFSSs (F,D) and $(F,D)^c$ over the sets S of symptoms, where D is the set of diseases. Also write the soft medical knowledge $R_1$and R_2representing the relation matrices of the IFSS (F,D) and $(F,D)^c$ respectively.

2. Input the IFSS (FI,S) over the set P of patients and write its relation matrix Q.

3. Compute the relation matrices T_1=Q o R_1 and T_2=Q***0*** R_2.

4. Compute the diagnosis scores $S_{T1 \text{ and }}S_{T2}$

5. Find $S_k=\max_j\left\{S_{T_1}\left(p_1,d_j\right)-S_{T_2}\left(p_1,d_j\right)\right\}$ and conclude that the patient p_1 is suffering from Disease d_k.

6. If S_k has more than one value. Then go to step one and repeat the process by reassessing the symptoms for the patients.

4.2. A Case Study

Suppose there are three patients p_l, p_2 and p_3 in a hospital with symptoms temperature, headache, cough and stomach problem. Let the possible diseases relating to the above symptoms be viral fever and malaria .We consider the set S={ $e_{l,}$ $e_{2,}$ $e_{3,}$ e_4) as universal set, where $e_{l,}$ $e_{2,}$ e_3 and e_4 represent the symptoms temperature, headache,cough and stomach problem respectively and the set D={d_l,d_2)where d $_l$ and d_2 represent the parameters viral fever and malaria respectively.

Suppose that

F(d_l)={e_l/(.9,e_2/(.4,.5),e_3/(.5,.3),e_4/(.2,.7)},

F(d_2)= e_1/(.6,.2),e_2/(.5,.3),e_3/(.2,.6),e_4/(.8,.1)}..1)}.

The intuitionistic fuzzy soft set (F,D) is a parameterized family (F(d_l), F(d_2)) of all intuitionistic fuzzy sets over the set S and are determined from expert medical documentation.

Thus the fuzzy soft set (F,D) gives an approximate description of the intuitionistic soft medical knowledge of the two diseases and their symptoms. This intuitionistic fizzy soft set (F,D) and its complement $(F,D)^c$ are represented by two relation Matrices R_1 and R_2, called symptom-disease matrix and non symptom-disease matrix respectively and are given by

$$R_1=\begin{array}{c} \\ e_1\\ e_2\\ e_3\\ e_4\end{array}\begin{array}{c}\begin{array}{cc} d_1 & d_2\end{array}\\ \begin{bmatrix}(.9,.1) & (.9,.1)\\ (.4,.5) & (.5,.3)\\ (.5,.3) & (.2,.6)\\ (.2,.7) & (.8,.1)\end{bmatrix}\end{array} \text{ and } R_2=\begin{array}{c} \\ e_1\\ e_2\\ e_3\\ e_4\end{array}\begin{array}{c}\begin{array}{cc} d_1 & d_2\end{array}\\ \begin{bmatrix}(.1,.9) & (.2,.6)\\ (.5,.4) & (.3,.5)\\ (.3,.5) & (.6,.2)\\ (.7,.2) & (.1,.8)\end{bmatrix}\end{array}$$

Again, we take P={ p_l, p_2, p_3) as the universal set where p_l, p_2, and p_3 represent patients respectively and S={ $e_{l,}$ $e_{2,}$ $e_{3,}$ e_4) as the set of parameters. Suppose that

F_l(e_l)= **{**p_1/(.8,.2),p_2/(.7,.1),p_3/(.4,.5l)}, F_1(e_2)={p_1/ (.4,.5),p_2/(.3,.6),p_3/(.5,.5)},

F_l(e_3)= {p_1/(.6,.3),p_2/(.4,.5),p_3/(4,.6)} and F_l (e4)= {p_1/(3,.4), p_2/(.6,.3), p_3/(.7,.2)}.

The intuitionistic fuzzy soft set (F_l,S) is another parametrized family of all intuitionistic fuzzy sets and gives a collection of approximate description of the patient-symptoms in the hospital. This intuitionistic fuzzy soft sets (F_l,S) represents a relation matrix Q called patient-symptom matrix given by

$$Q=\begin{array}{c} \\ P1\\ P2\\ P3\end{array}\begin{array}{c}\begin{array}{cccc} e_1 & e_2 & e_3 & e_4\end{array}\\ \begin{bmatrix}(.8,.2) & (.4,.5) & (.6,.3) & (.3,.4)\\ (.7,.1) & (.3,.6) & (.4,.5) & (.6,.3)\\ (.4,.5) & (.5,.5) & (.4,.6) & (.7,.2)\end{bmatrix}\end{array}$$

Then combining the relation matrices R_1 and R_2 separately with Q we get two matrices T_1 and T_2 called patient-disease and patient-non disease matrices respectively, given by

$$T_1 = Q \circ R_1 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{bmatrix} (.8,.2) & (.6,.2) \\ (.7,.1) & (.6,.2) \\ (.4,.5) & (.7,.2) \end{bmatrix} \end{array},$$

$$T_2 = Q \circ R_2 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{bmatrix} (.4,.4) & (.6,.3) \\ (.6,.3) & (.4,.5) \\ (.7,.2) & (.4,.5) \end{bmatrix} \end{array}$$

Now we calculate

S_{T1} - S_{T2}	d_1	d_2
P_1	.48	-.01
P_2	.11	.21
P_3	-.33	.33

Now, it is clear that the patient pl is suffering from the disease dl and patients p_2 and p_3 are both suffering from disease d_2.

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The work done by the authors is really very good and best suited for publication in your esteemed journal in your esteemed journal. I think it is among the best application of Fuzzy Soft Set. I recommend for the favour of publication.

CONFLICT OF INTEREST

There is no conflict of interest of authors.

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