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Study of Manhattan's consensus degrees through an extension based on the Uniform distribution

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Abstract

An important aspect to be considered in Group Decision Making problems is the study of consensus. Since in these problems it is desirable that the final decision is widely accepted, improving the consensus degree in a fair way is a very interesting task. This paper analyses the improvement in the consensus degrees -obtained by applying Manhattan distance- when the experts' preferences are slightly modified using one of the properties of the Uniform distribution. We carry out an experimental study that shows the enhancement in different cases to which Uniform extension has been applied, with different number of both, experts and alternatives.

Keywords: Consensus, Group Decision Making, Manhattan distance, Uniform distribution.

1. Preliminaries

In a Group Decision-Making (GDM) problem, a group of elements – experts – have to choose a solution from a group of alternatives and for this, each of these so called experts expresses their opinions –preferences– [1]. Therefore, there are several individual decisions –individual preferences– to reach one only collective decision. We will be often interested in that this collective decision is reached with the highest possible level of

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agreement among the experts, that is, it is desirable that exits consensus among the experts in the final collective decision.

Understanding consensus as a complete agreement may set out some difficulties, as this total consensus is certainly difficult to achieve. For this reason, in this paper we will make use of the concept “soft consensus degree” to evaluate the level of consensus [1, 2]. Thus, the consensus degree may be understood as a function that may take values between 0 (in which case we will talk about “null consensus”) and 1 (and we will then talk about “full consensus”). Following what we stated before, it is desirable that the consensus degree shows a value close to 1. To deal with this situation we will use Fuzzy sets theory which has proven to be a useful tool, especially with fuzzy preference relations [3, 4, 5].

In order to measure the level of consensus among the experts’ preferences, various distance functions may be considered [6]. We will take into account one of the most used, the Manhattan distance function [6, 7, 8] due to its simplicity and efficiency.

Different authors have used probability theory and probability distributions in GDM problems with fuzzy relations in various situations. This is the case of the Normal distribution in interval-valued fuzzy preference relations [9], Uniform distribution on values in hesitant fuzzy elements in hesitant fuzzy preference relations [10], Spearman's coefficient in the consensus measure for fuzzy preference relations [11] or in determining consensus thresholds in linguistic group decision-making problems [12].

This paper considers a new application of the Uniform distribution, in this case in a framework of GDM problems with fuzzy preference relations in which the differences between the experts' preferences are measured by the Manhattan distance function. The aim is to analyse whether the use of this novel technique allows improving the degrees of consensus in those problems that are involved in this study.

The structure of the paper is as follows: section 2 introduces basic concepts about GDM problems with fuzzy preference relations and the new proposal. In section 3 we introduce design and conditions of the comparative study and analyse the results obtained in it. Finally, we end this paper with the Conclusion section.

2. The GDM problem with fuzzy preference relations

A set of experts, $E = \{e_1,.., e_n\}$ $(n \geq 2)$, expresses their preferences about a set of alternatives, $X = \{x_1,.., x_m\}$ $(m \geq 2)$, to reach a collective decision. The experts express their preferences using fuzzy preference relations, i.e. every expert gives his/her preference of x_i over x_j , $i,j \in \{1,2,..., m\}$ [3, 4, 5].

Definition. Let X be a set of alternatives in a GDM problem. A fuzzy preference relation on X is defined as a function $p{:}X{\times}X \rightarrow [0,1]$:

$$p\left(x_i, x_j\right) = p_{ij}, \qquad 0 \leq p_{ij} \leq 1$$

where 0 indicates minimum preference and 1 expresses maximum preference. It is common to use a matrix to denote the fuzzy preference relation, $P = (p_{ij})$, and to consider one for each experts that participate in the decision procedure.

The reciprocal property, $p_{ji} + p_{ij} = 1$ with $i,j \in \{1,2,..., m\}$, is a popular assumption when dealing with a fuzzy preference relation.

Using the experts' individual preference matrices we obtain a collective preference matrix by aggregating all the individual preferences matrices. The aggregation operation by a quantifier guided Ordered Weighted Averaging (OWA) operator is carried out as [13]:

$$p_{ij}^{c} = \phi_Q\left(p_{ij}^{1},\ldots,p_{ij}^{n}\right) = \sum_{k=1}^{n} w_k \cdot p_{ij}^{\sigma(k)}$$

with p^c_{ij} collective preference, p^k_{ij} individual preference for each expert $e_k \in E$, σ a function that allows a permutation operation: $p_{ij}^{\sigma(k)} \geq p_{ij}^{\sigma(k+1)}, \quad \forall k \in \{1,2,...,n-1\}$, and Q a fuzzy linguistic quantifier of fuzzy majority that allows the calculation of the weights vector, $W = [w_1,\ldots, w_n]$.

The notion of fuzzy majority and its alternative representations have been discussed in the literature [1].

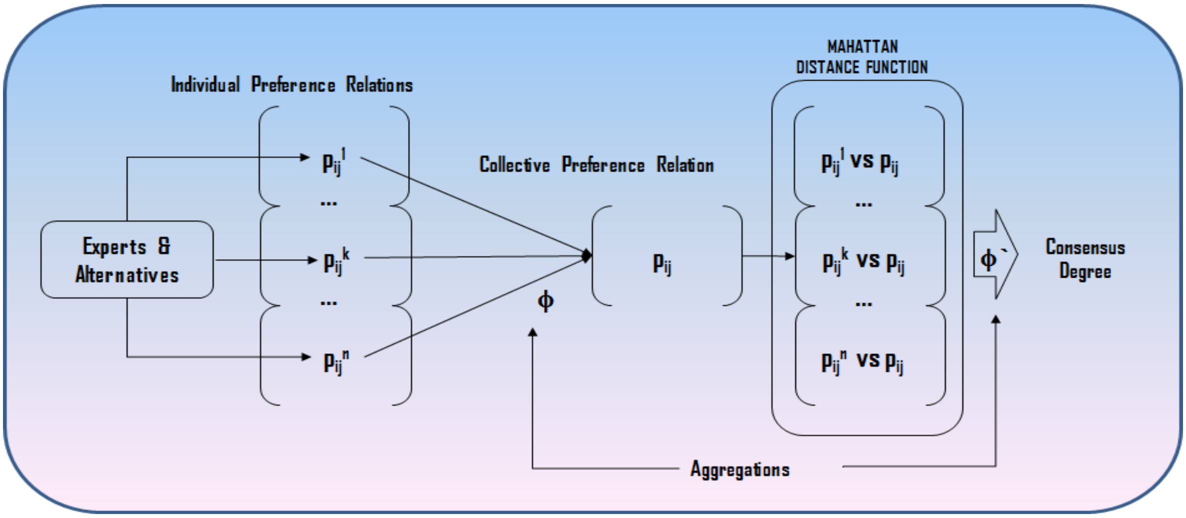

Fig. 1. Consensus model with distance functions and aggregation operators.

2.1. The Consensus Question

To measure the level of consensus between individual and collective preferences, it is necessary to evaluate the distance among their preferences. Various distance functions may be used to get this goal [6]. In this paper we consider the Manhattan distance function [7, 8].

Definition. Let $A = \{a_1,...,a_n\}$ and $B = \{b_1,...,b_n\}$ be two vectors of real numbers. The Manhattan distance function is defined as:

$$d(A,B) = \sum_{j=1}^{n} \left| a_j - b_j \right|$$

Given their equivalence, it is possible and customary to consider the proximity -similarity- among the experts´ preferences instead of taking into account their separation -distance-. Accordingly, in this paper we use the function $s = 1 - d$ to quantify the similarity between the different preferences [7,8].

Figure 1 shows the consensus model described above.

2.2. The Uniform extension

This paper proposes to modify the fuzzy preference relation of each expert by extendind their preferences using one of the properties of the uniform distribution.

It is known [14] that the discrete uniform distribution on a finite number of points assigns the same probability to each of them. The continuous uniform distribution translates this idea to the continuous case, where the sample space of the uniform random variable has an unconuntable number of points and the events of interest are subsets of the outcome space that have a well defined measure -length in one dimension, area in the plane and so on-. This distribution assigns to any subset of the outcome space a probability proportional to the size -measure- of the set, regardless of its shape or position.

Focusing on this last idea, we introduce the concept of uniform extension as follows.

Definition (Uniform extension). Let p^k_{ij} be the preference of alternative x_i over alternative x_j for expert e_k. The

Uniform extension of p^k_{ij} in u is defined as:

$$[\max(0, p_{ij}^k - u), \min(p_{ij}^k + u, 1)] \subset [0,1] \quad \text{with} \quad u \in (0,1)$$

As an example, let us consider the case of an expert –expert number 5, e_5– who prefers alternative 2 over alternative 3 in 0.7, $p^5_{23} = 0.7$, and let us assume $u = 0.05$. We propose to substitute the preference 0.7 for the preference [0.65, 0.75] where any of the values of such interval has the same opportunity of being chosen, i.e. any value in the range is equally valid.

In this situation the consensus model is modified as represented in Figure 2.

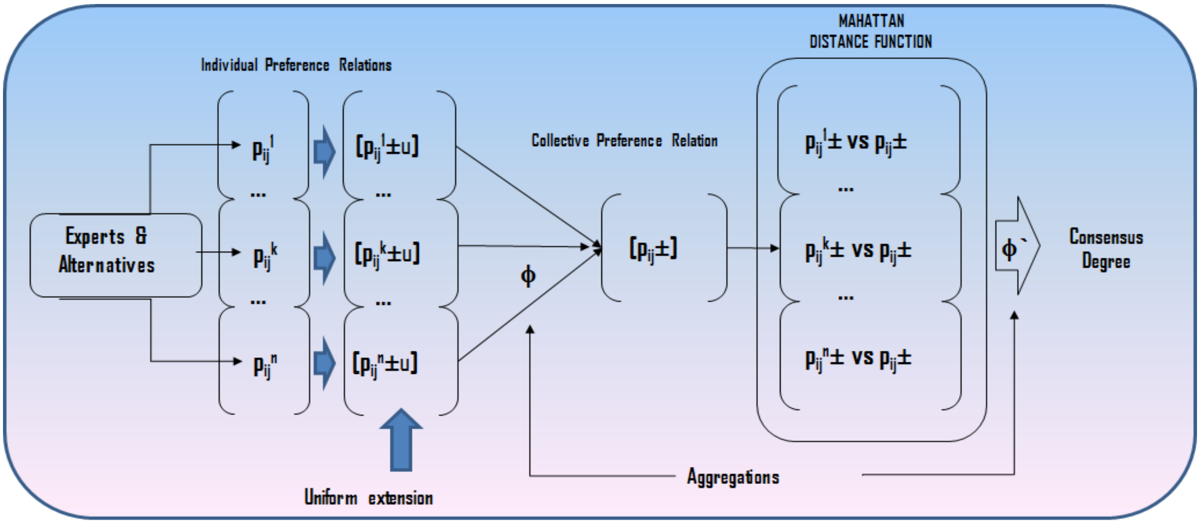

Fig. 2. Consensus model with Uniform extension.

3. Comparative Study

At this point, we carry out a strategy similar to the one we already used in our previous papers [7, 8], which consists of comparing values of consensus degrees for randomly generated GDM problems with/without Uniform extension. We take into account several values for experts and alternatives. To calculate consensus degrees we use the Manhattan distance. This process is shown in Figure 3.

We generate 100 random GDM problems with a specific number of experts (2,3,4) and a fixed number of alternatives (3,4,5). The OWA operator is the Average and the value for the Uniform extension is $u = 0.1$.

This study attempts to answer two questions:

- Is the consensus degree higher for Uniform extension?
- If so, how much higher is it?

Table 1. If the Uniform extension is applied, is the consensus degree higher?

Alt/Exp	2	3	4
3	yes /99%	yes /100%	yes /100%
4	yes /100%	yes /100%	yes /100%
5	yes /100%	yes /100%	yes /100%

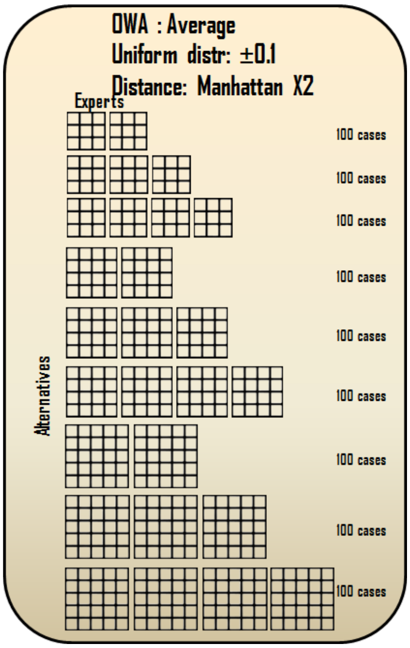

Fig. 3. Study environment.

Table 2 and its graphic representation in figure 4 allow us to answer the second question. Table 2 shows the growth in the values of the consensus degree (in percentage) when the uniform extension is applied. The results corroborate those obtained in Table 1 and lay bare the fact that the use of Uniform Extension notably improves the values of the degrees of consensus. It is also observed that for a given value of experts, the improvement obtained with different alternatives does not change too much except in the case of 2 experts and 5 alternatives.

Table 2. Growth in the consensus degree resultant from applying Uniform extension (percentage)

Alt/Exp	2	3	4
3	16,25%	17,37%	18,96%
4	16,28%	17,59%	18,45%
5	14,41%	17,43%	18,61%

The analysis reveals that the levels of consensus increase with the use of Uniform extension as the number of experts increases. Thus, the average of growth for 3 alternatives is 17.53% (Table 3) but, according to the number of experts, it ranges between 16.25% for 2 experts and 18.96% for 4 experts more than a 2.5% of difference. In addition, we can see that the growth is around 17% (17.23%, Table 3) for the three cases according to the alternatives, but it is not so for the growth according to the number of experts that, as said before, grows according to the number of experts. Thus, a growth of more than 1% is appreciable for each increase of one unit in the number of experts: 15.65%, 17.43% and 18.61%, which seems to indicate that there exists a regularity behaviour in these growths. This regularity behaviour is observable in the graph of Figure 4.

Table 3. Growth average values in the consensus degree according to the number of experts and alternatives (percentage)

Alternatives		Experts		
3	17,53%	2	15,65%	
4	17,44%	3	17,43%	
5	16,72%	4	18,61%	17,23%

The ranking according to the growth in the consensus degree from highest to lowest percentages is (Experts/Alternatives): 4/3, 4/5, 4/2, 3/4, 3/5, 3/3, 2/4, 2/3 and 2/5. As seen above, the number of expert classifies but not so the number of alternatives.

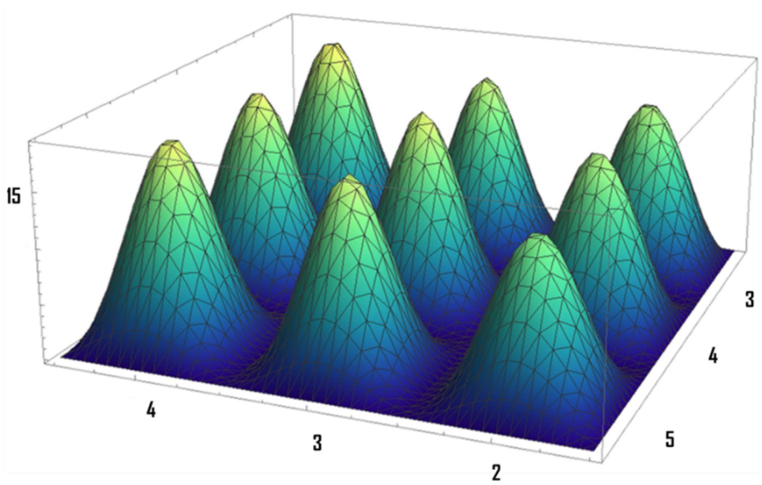

Fig. 4. Consensus degree in percentages.

4. Conclusion

In this paper we have introduce a new tool to improve consensus degrees by slightly modifying the preferences of each of the experts in certain situations of group decision making, the so called Uniform extension. All preferences are modified in the same way, avoiding discrimination among individual preferences depending on how they separate from the collective preference. The distances between individual and collective preferences have been calculated by using the Manhattan distance. A comparative study of consensus degrees with/without the Uniform extension is carried out with different number of experts and alternatives. To manage the distance function, the average operator has been used as an aggregator operator.

The results of the comparative study reveals two interesting issues. First, the consensus degree is higher when the Uniform extension is used than when it is not. Second, the obtained improvements reach almost 20%. It is also observed that the number of experts seems to directly affect the percentage of improvement, while the number of alternatives does not influence this percentage.

Future research directions could address this problem by considering different distance functions following one of our current work lines and showing a theoretical point of view by conducting in-depth research on it.

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