Analyzing and Designing Control System for an Inverted Pendulum on a Cart

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Abstract

It is a collection of MATLAB functions and scripts, and SIMULINK models, useful for analyzing Inverted Pendulum System and designing Control System for it. Automatic control is a growing field of study in the field of Mechanical Engineering. This covers the proportional, integral and derivative (PID). The principal reason for its popularity is its nonlinear and unstable control. The reports begin with an outline of research into inverted pendulum design system and along with mathematical model formation. This will present introduction and review of the system. Here one dimensional inverted pendulum is analyzed for simulating in MATLAB environment. Control of Inverted Pendulum is a Control Engineering project based on the flight simulation of rocket or missile during the initial stages of flight. The aim of this study is to stabilize the Inverted Pendulum such that the position of the carriage on the track is controlled quickly and accurately so that the pendulum is always erected in its inverted position during such movements.

Keywords: MATLAB, Inverted pendulum, PID Controller, Simulation

Introduction

An inverted pendulum is a pendulum which has its center of mass above its pivot point (Said,L., Latifa, B.,, 2012). It is often implemented with the pivot point mounted on a cart that can move horizontally and may be called a cart and pole. Most applications limit the pendulum to 1 degree of freedom by affixing the pole to an axis of rotation. Whereas a normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright; this can be done either by applying a torque at the pivot point, by moving the pivot point horizontally as part of a feedback system, changing the rate of rotation of a mass mounted on the pendulum on an axis parallel to the pivot axis and thereby generating a net torque on the pendulum, or by oscillating the pivot point vertically. A simple demonstration of moving the pivot point in a feedback system is achieved by balancing an upturned broomstick on the end of one's finger. The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies. Can anyone balance a ruler upright on the palm of his hand? If he concentrates, he can just barely manage it by constantly reacting to the small wobbles of the ruler (Irza M. A., Mahboob I., Hussain C.,, 2001). This challenge is analogous to a classic problem in the field of control systems design: stabilizing an upside-down ("inverted") pendulum.

Simulation is the imitation of the operation of a real-world process or system over time (Banks J., Carson J., Nelson B., Nicaol D., 2001). The act of simulating something first requires that a model be developed; this model represents the key characteristics or behaviors/functions of the selected physical or abstract system or process. The model represents the system itself, whereas the simulation represents the operation of the system over time. The inverted pendulum is among the most difficult systems to control in the field of control engineering. Due to its importance in the field of control engineering, it has been a task of choice to be assigned to control engineering students to analyze its model and propose a linear compensator according to the control law. Being an unstable system, it creates a problem in case of controlling (O., 2012).The reasons for selecting the Inverted Pendulum as the system are:

•It is the most easily available system

• It is a nonlinear system, which can be treated to be linear, without much error (Maravall D., Zhou C., Alonso J., 2005).

• Provides good practice for prospective control engineering.

Theory

The system involves cart, able to move backwards and forwards. And a pendulum hinged to the cart at the bottom of its length such that the pendulum can move in the plane as the cart moves. That is, the pendulum mounted on the cart is free to fall along the cart's axis of rotation. The system is to be controlled so that the pendulum remains balanced and upright. If the pendulum starts off-center, it will begin to fall. The pendulum will move to opposite direction of the cart movement. It is a complicated control system because any change to a part will cause change to another part. We only take feedback from the angle of the pendulum relative to vertical axis other than state of being carriage position, carriage velocity and pendulum angular velocity. The cart undergoes linear translation and the link is unstable at the inverted position. So, briefly the inverted pendulum is made up of a cart and a pendulum. The goal of the controller is to move the cart to its commanded position causing the pendulum without tip over. In open loop the system is unstable. This is a SIMO output system.

Basic block diagram for the feedback control system:

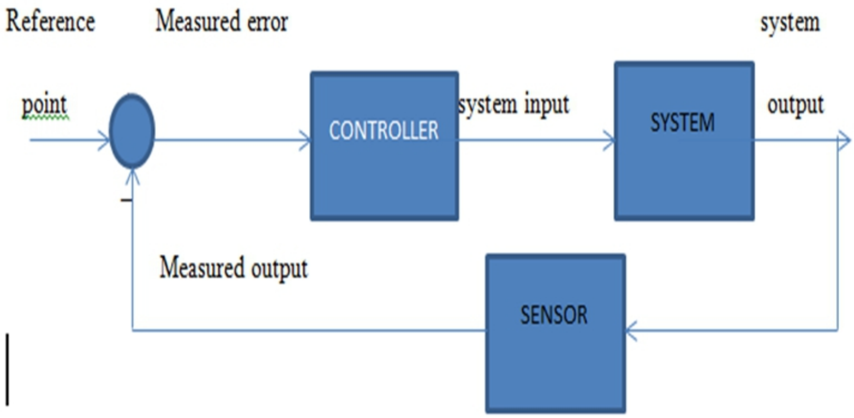


Figure-01: Feedback control system

Analysis Of Inverted Pendulum With Cart System

The inverted pendulum on a cart is representative of a class of system that includes stabilization of a rocket during launch. The position of the cart is P, the angle of rod is θ, the force input to the cart is F, the cart mass is M, the mass of the bob is m, the length of the rod is L, the coordinate of the bob is (P_2, Z_2).

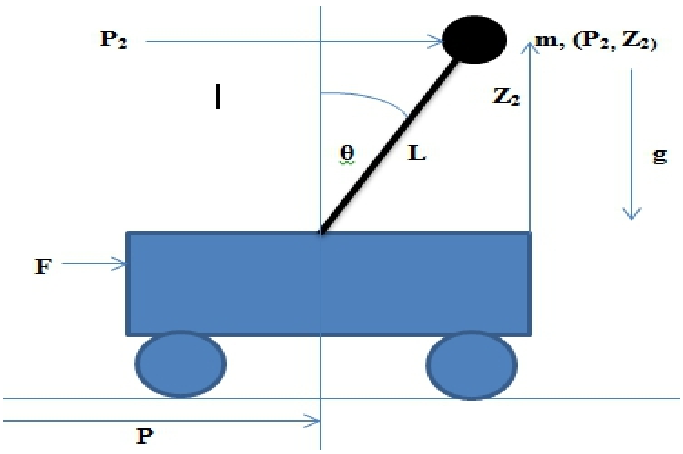


Figure-02: Inverted pendulum on a cart

In the following, the differential equations which describe the dynamics of the inverted pendulum using lagrangian's equation because these deal with the scalar energy functions rather than vector forces and acceleration in case of Newtonian approach, thus it minimizes error (Ogata, 2002).

The partial differential equations (F., 1994) yield:

$(M+m)*\ddot{P} + m*L*\ddot{\theta}*\cos\theta - m*L*\dot{\theta}^2*\sin\theta = F$ ………………(1)

$M*L*\ddot{P}*\cos\theta + (I+m*L^2)*\ddot{\theta} + m*g*L*\sin\theta = 0$ ………...…… (2)

$(M+m)\ddot{p} - mL\ddot{\emptyset} = u$……………………......................... (3)

$(I+mL^2)\ddot{\emptyset} + mL\ddot{p} = mgL\emptyset$…………..……….......................(4)

If friction force is considered the equation converts to:

$(M+m)\ddot{p} - mL\ddot{\emptyset} + b\dot{p} = u$………………………………….(5)

$(I+mL^2)\ddot{\emptyset} - mL\ddot{p} = mgL\emptyset$……………………………….(6)

To obtain the transfer functions of the linearized system equations, the Laplace transform of the system equations assuming zero initial conditions has been taken. The resulting transfer function for pendulum position becomes:

$$P_{pend}(s) = \frac{\emptyset(s)}{u(s)} = \frac{\frac{mLs^2}{q}}{s^4 + \frac{b(I+mL^2)s^3}{q} - \frac{(M+m)mgls^2}{q} - \frac{bmgLs}{q}} \text{ (rad/N)} \quad \ldots(7),$$

where, $q=[(M+m)(I+mL^2)-(mL)^2]$

Again for transfer function for cart position as follow:

$$P_{cart}(s) = \frac{p(s)}{u(s)} = \frac{(I+ml^2)s^2 - gml}{s^4 + \frac{b(I+mL^2)s^3}{q} - \frac{(M+m)mgls^2}{q} - \frac{bmgLs}{q}} \text{ (m/N)} \quad \ldots(8)$$

For this example, assuming the following quantities:

Mass of the cart, (M) = 0.5 kg, Mass of the pendulum, (m) = 0.2 kg, Coefficient of friction for cart, (b) = 0.101N/m/sec,

Length to pendulum center of mass, (l)= 0.3 m , Mass moment of inertia of the pendulum, (I)= 0.006 kg.m^2, Force applied on the cart = F (N) , Cart position coordinate = x (m), Initial Pendulum angle from vertical downward = theta

For the PID, root locus, and frequency response sections of this problem, it will be interested only in the control of the pendulum's position. This is because the techniques used in these sections are best-suited for single-input, single-output (SISO) systems. Therefore, none of the design criteria deal with the cart's position. It will, however, be investigated the controller's effect on the cart's position after the controller has been designed. For these sections, the design of a controller to restore the pendulum to a vertically upward position after it has experienced an impulsive "bump" to the cart. Specifically, the design criteria are that the pendulum returns to its upright position within 5 seconds and that the pendulum never moves more than 0.05 radians away from vertical after being disturbed by an impulse of magnitude 1 Nsec. The pendulum will initially begin in the vertically upward equilibrium, $\theta = \pi$. In summary, the design requirements for this system are:

- Settling time for θ of less than 5 seconds
- Pendulum angle θ never more than 0.05 radians from the vertical

Pole Zero Map of Uncompensated Open Loop System: The poles position of the linearized model of Inverted Pendulum (in open loop configuration) shows that system is unstable, as one of the poles of the transfer function lies on the Right Half Side of the s-plane. Thus the system is absolutely unstable.

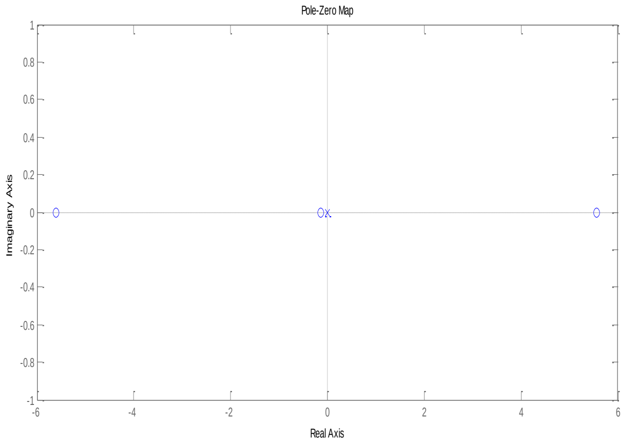


Figure-03: zeros and poles of pendulum position.

From figure,
Zeros =0
Poles =5.5651, -5.6041, -0.1428

Likewise, the zeros and poles of the system where the cart position is the output are found as follows:

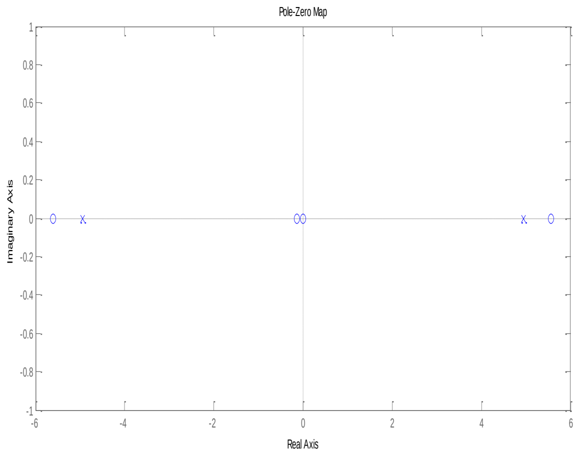


Figure-04: zeros and poles of cart position

The clear results are:
Zeros = 4.9497,   -4.9497
Poles =0, 5.5651, -5.6041, -0.1428
As predicted, the poles for both transfer functions are identical. The pole at 5.5651 indicates that the system is unstable since the pole has positive real part (V., 1991). In other words, the pole is in the right half of the complex s-plane. This agrees with what we observed above.

Step Response of Uncompensated Open Loop System:

Since the system has a pole with positive real part its response to a step input will also grow unbounded. The verification of this using the "lsim" command which can be employed to simulate the response of LTI models to arbitrary inputs. In this case, a 1-Newton step input will be used. Adding the MATLAB code to "m-file" and running it in the MATLAB command window generates the plot given below:

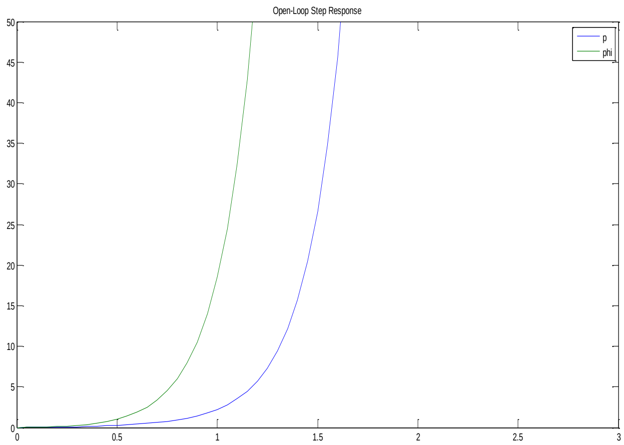


Figure-05: Step Response of Uncompensated Open Loop System

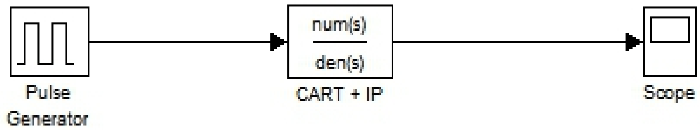
The above results confirm the expectation that the system's response to a step input is unstable.

It is apparent from the analysis above that some sort of control is needed to be designed to improve the response of the system. PID, root locus, frequency response, and state space are the controllers can be used but here PID controller is designed.

Simulink Model for the Open Loop Impulse Response of the Inverted Pendulum System

SIMULATION PARAMETERS:
Impulse is applied for 0.5 s
Start Time: 0

Stop Time: 1.5
Solver Algorithm: Variable-step ODE45 (Dormand-Prince), Maximum Step Size: 0.03



The impulse response of open-loop uncompensated system is given below:

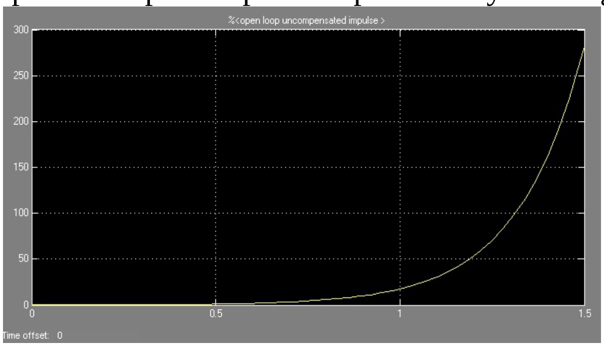


Figure-06: open loop impulse response (Scope view)

This model shows that impulse response of inverted pendulum. This model is highly unstable as theta diverges rapidly with time. Applying step for 1s reveals that the pendulum remains upright, but becomes highly unstable as step comes.

Simulink Modelling And Pid Controller

Nonlinear Simscape Model: SimMechanics software is a block diagram modeling environment for the engineering design and simulation of rigid body machines and their motions, using the standard Newtonian dynamics of forces and torques, instead of representing a mathematical model of the system (Said,L., Latifa, B.,, 2012). The inverted pendulum model using the physical modeling blocks of the Simscape extension to Simulink has been built. The blocks in the Simscape library represent actual physical components; therefore, complex multi-body dynamic models can be built without the need of mathematical equations from physical principles by applying Newton's laws. Establishing and saving SimMechanics model of the inverted pendulum and cart, the animated view of the physical system is created which is given below:

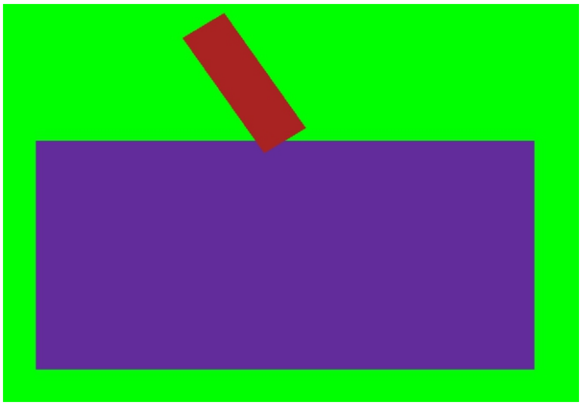


Figure-07: System animation without controller (20 degree displace with vertical).

In the Scope, clicking the **Autoscale** button, the following output for the pendulum angle and the cart position has been found which is nonlinear in practice.

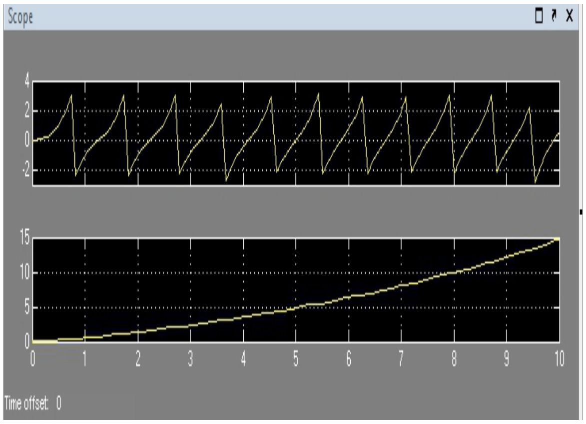


Figure-08: Scope condition of pendulum angle and cart position.

The pendulum repeatedly swings through full revolutions where the angle rolls over 360 degrees. Furthermore, the cart's position grows unbounded, but oscillates under the influence of the swinging pendulum.

PID control design: In the design process, it has been assumed a single-input, single-output plant as described by the transfer function (Kumar R., Singh B., Das J., 2013). Closed loop impulse with PID controller is given below:

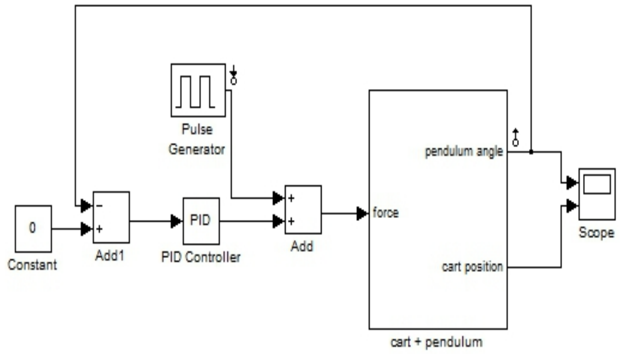


Figure-09: Feedback control system for the Iverted Pendulum

To design a compensator using the automated PID Ziegler-Nichols open-loop tuning algorithm, this tuning method computes the proportional, integral, and derivative gains using the Chien-Hrones-Resnick (CHR) setting with a 20% overshoot. The response of the closed-loop system to an impulse disturbance for this initial set of control gains: Kp = 1; Ki = 1;Kd = 1;

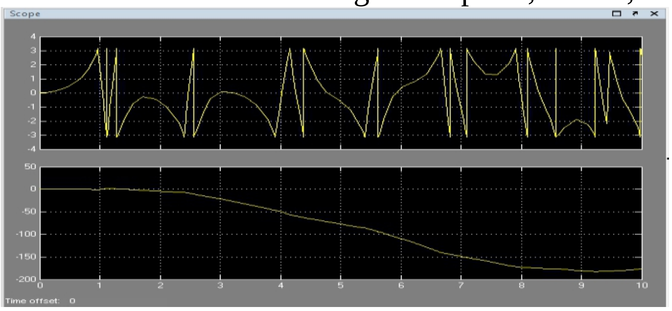


Figure-10: Zigzag movement of pendulum K_p=1;K_i=1;K_d =1

This response is still not stable. To modify the response, an iteration process is followed by manipulating proportional, integral and derivative gain.

Results

The Inverted Pendulum was given an initial angle inclination, as indicated by an initial 20 magnitude of pendulum's angular displacement. the system is completely controlled under operating condition. The design criteria of the PID controller are: K_p=100; K_i=1; K_d=20; Increasing amplitude of impulse, it is seen error increases further. As is shown in the plot, the settling time of the system is less than 5 seconds. Impulse response, scope view and final animation are given below:

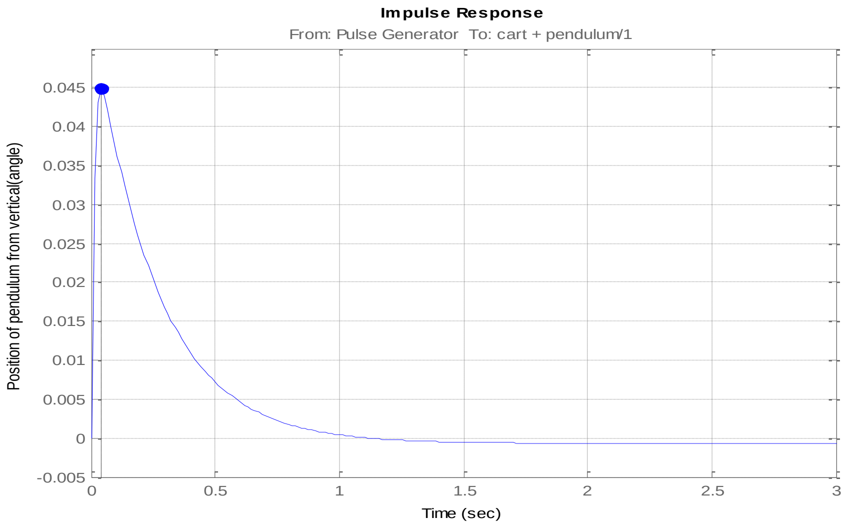


Figure-11: Final simulation result (LTI view).

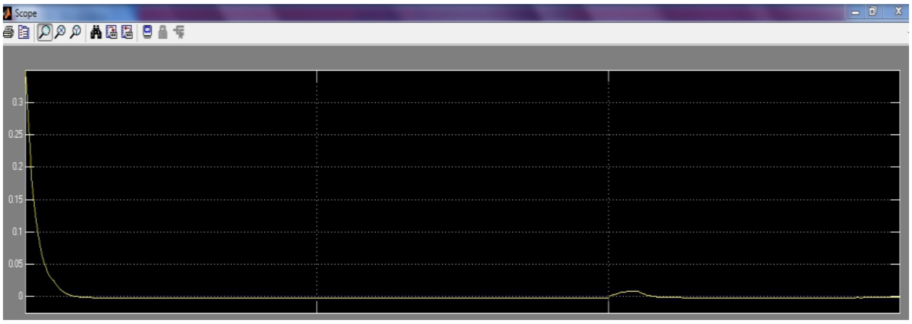


Figure-12: Actual movement of pendulum (scope)

A video of system simulation is extracted using recording options. the cart moves in the negative direction with approximately constant velocity.

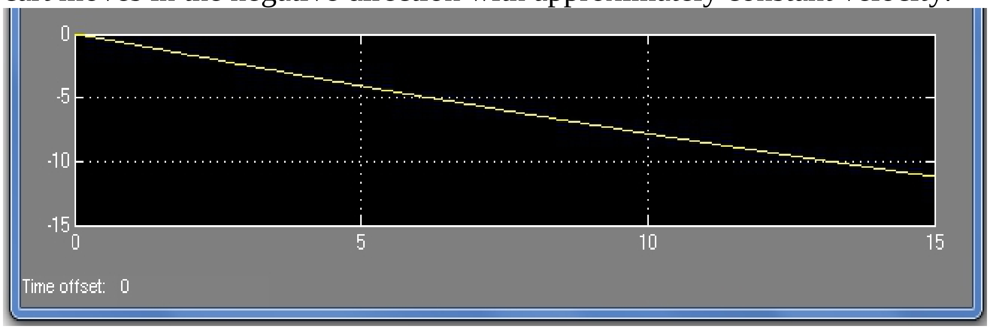


Figure-13: cart position with time.

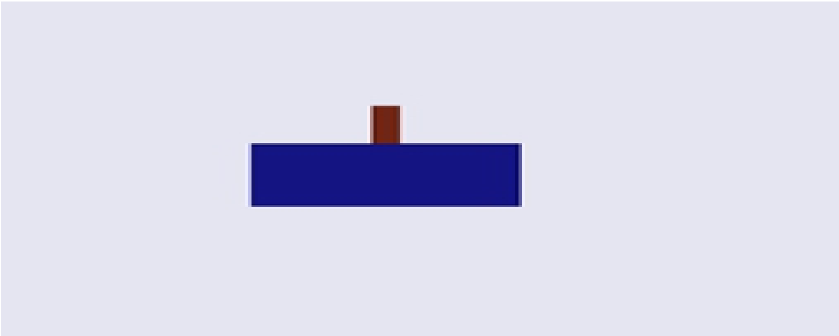


Figure-14: Final animation of model of inverted pendulum and cart.

Conclusion

From the analysis, it creates an ability of the control of a nonlinear model by any linear feedback control system. PID controller designed here is followed an iteration process. In the project only friction force is assumed as external impedance but in reality there would have air impedance. The cart velocity decreases in the negative direction. The actuator needs a very small effort and power to stable the pendulum as it quickly stabilizes without too much fluctuations. In the project a unit feedback gain is considered as it becomes simple and avoiding so much complexity of calculation. System properties are taken reasonable as the simulation is completed in SimMechanics model by breaking mechanical elements into building blocks. Control System Toolbox provides an app and functions for analyzing linear models. Impulse response plot is used and settling time, peak amplitude or maximum overshot are defined using linearized tools. Applying controller to cart position, the system would be implemented. Root locus method has been used to define that the system is unstable. ODE45 is based on an explicit Runge-Kutta (4, 5) formula, the Dormand-Prince pair. It is a one-step solver; that is, in computing $y(t_n)$, it needs only the solution at the immediately preceding time point, $y(t_{n-1})$. In general, ode45 is the best solver to apply as a first try for most problems. For this reason, ode45 is the default solver used for models with continuous states and been used for this problem.

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