The "Twin Paradox" Resolved

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Abstract

The so-called "Twin Paradox", wherein a relativistic effect is hypothesized to produce verifiably different clock rates between bodies, has not been resolved to the satisfaction of many theorists. There has been an abiding difficulty with determining how arbitrary periods of uniform motion, when both twins will observe the other's clock to move more slowly, can be resolved upon their reunion. Spacetime diagrams are used here to demonstrate visually and mathematically that there is a non-paradoxical explanation for the supposed discrepancy that has not been previously proposed.

Keywords:

Introduction

The "paradox" of the twins dates from Einstein's first paper on relativity (Einstein, 1905), although he only alluded there to a "peculiar consequence" of his theory which didn't involve a re-uniting of twins, just distant clocks becoming un-synchronized by one's acceleration. The consequence doesn't properly belong in a discussion of Special Relativity, which is "special" because it excludes the consideration of accelerations, and Einstein had yet to fully explore relativistic implications of acceleration. But in subsequent investigations of inertial (non-gravitatioal) acceleration and gravitation he and others succeeded in attributing and confirming the "peculiarity." The problem was taken up and coined a "twin paradox" by Paul Langevin (1911) and explicitly tied to the effect of inertial acceleration, and Einstein later (1918) discussed it in terms of gravitation.

Numerous experiments have confirmed that both inertial accelerations and gravitational effects produce a *dilation*, or *slowing* of a body's clock, which unlike the relationship between bodies in relative uniform motion, is absolute – agreed upon from any frame of reference (Rossi, Hall 1941, Pound, Rebka Jr. 1959, Hafele, Keating 1972, Bailey, et al 1977, Botermann, et al 2014). The plausibility of a "twin effect", whereby people who undergo prolonged and intense accelerations can age significantly less than those who don't, has thus acquired a solid experimental foundation.

The alleged paradox derives from this: If a twin accelerates toward a distant star-system, decelerates at the destination, accelerates back toward earth, then decelerates for a reunion with the stay-at-home twin, it is believed that the traveling twin will have aged less than the stay-at-home, and it would be consistent with relevant experiments. (Some theorists have tried to attribute the effect not to the accelerations but to the traveling twin's change of coordinate systems in the return (Laue 1913), but I'll demonstrate below that this is not the case.)

A thought-experiment with twins that attributes the "peculiarity" of time dilation to acceleration works fine when the traveling twin is constantly accelerating or decelerating, and is not really a paradox, as it can fully account for the differences in age at the reunion. But if the traveling twin spends any time during the journey moving uniformly, the principle of Special Relativity applies, and each twin will regard the other's clock as being only *relatively* dilated during such periods. A period of time in relative motion between the accelerations introduces this conundrum: If each twin has been observing the other's clock moving *relatively* slower during any part of their separation, how can their clocks agree on that part of the journey, how can the mutual observation of the other's relative dilations be resolved when they are reunited?

A Visualization

The spacetime diagram used below to visualize the Twin problem differs from the conventional Minkowski diagram (1908) in that it plots a *world-line* (the motion of a body in spacetime) of an observed body according to its own clock rather than the clock of the observer. (Characteristically, the Minkowski diagram portrays a ray of light as moving in spacetime at 1 light-second per second (hence the "light-cones"), but according to Relativity light moves at a relative *zero* seconds per light-second, while the observer records the motion in 1 second.) This relativistic flaw in the Minkowski diagram has been discussed elsewhere (Arnold 2015), but the alternative will be used below without comparison and argumentation, proceeding simply on its evident utility.

A diagram (*figure 1*) conforming to Special Relativity and the Lorentz transformations, and treating both space and time as relative, provides a heuristic representation by means of which the relativistic relationship can be visualized as Minkowski originally intended, so that "physical laws might find their most perfect expression" (Minkowski 2008, p.76).

The x-axis in *figure 1* represents space calibrated in light-years (*ly*), while its perpendicular, the y-axis, represents time calibrated in years (*yr*) – both according to observer ***A***, who is considered to be at rest in space and moving in time along the y-axis. Vector ***B*** represents a body in motion relative to ***A***. Body ***B*** moves from the vicinity of ***A*** at a velocity which will take it 4 *ly* in 5 *yr* according to **A** with an elapsed time of 3 *yr* on **B**'s clock. The x' and y' axes represent **B**'s coordinate system.

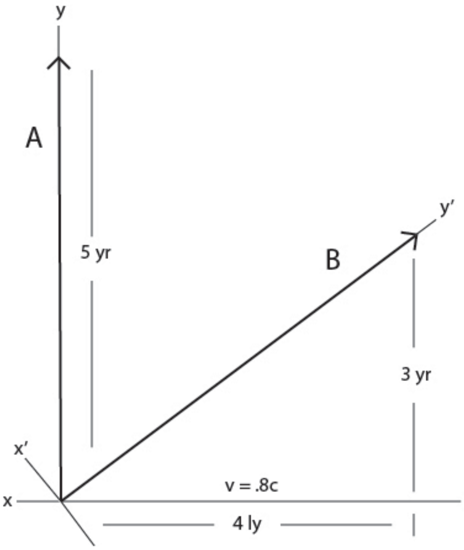

figure 1

An alternative to Minkowski's spacetime diagram. The world-line of the observed body **B** is projected as moving in space as measured by the observer **A** and in time both according to **B**'s own clock and the clock of **A**. According to Special Relativity, a body moving at 80% the speed of light will go 4 light-years (*ly*) by A's reckoning and in 5 years (*yr*) on **A**'s clock, but **B**'s clock will be observed by **A** to have elapsed only 3 *yr*.

The relativistic relationship can be expressed by $t' = t\ \sqrt{(1-v^2)}$ per a Lorentz transformation (with t being the observer's time, t' the observed time, and v the relative velocity proportional to c), which in the above example yields $5*\sqrt{(1-.8^2)} = 3$. Strictly speaking therefore, body **B** travels a relative 4 *ly* in 5 *yr* according to **A**, with an elapse of 3 *yr* according to **B**'s proper time (its clock is observed by **A** to elapse 3 years).

(It will be relevant in later discussion to note that un-accelerated world-lines as in *figure 1* are necessarily equal in length. This is because the world-line of an observer relates to the observed body's time and velocity as the hypotenuse to the sides of a right triangle ($t = \sqrt{(t'^2 + x^2)}$), with t' as the observed clock and x as the distance traveled by the observed body according to the observer), and the world-line of the observed body *is* the hypotenuse of the triangle.)

The spacetime diagram works to represent the relationship determined by the Lorentz transformations only if a body moving uniformly (or “at rest”) in space is actually moving, time-wise, perpendicular to space in its own coordinate system, thus providing a basal frame of reference in the diagram. Given that a body observed to be in relative motion is also moving along its time-axis perpendicular to space in its own coordinate system, its space axis must be different than that of a body taken to be the observer at rest. Accordingly, *figure 2* shows two reference frames at once, with ***A*** and ***B*** each moving in time perpendicular to space according to their own coordinate system. It depicts, as the Minkowski diagram cannot, the strange phenomenon wherein each observer measures the other’s clock as moving more slowly than her own. By rotating the diagram, the mirror image of **A**’s perspective can be seen from that of **B**.

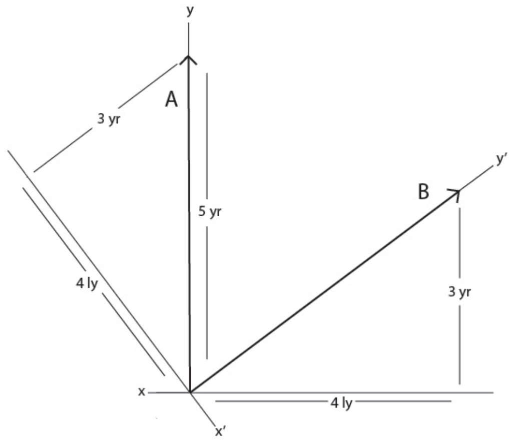

figure 2

Two bodies in two different coordinate systems, *x*-*y* and *x’*-*y’*, are shown to mirror their mutual relativistic effects. By rotating the diagram either system can be represented as at rest in space and the other in relative motion with a slower clock, and each by the same measures.

Figure 2 is a fully accurate depiction of the relativistic relationship. It expresses the duality that students of relativity often have difficulty comprehending: It is because each body has its own orientation in the spacetime continuum that each mirrors the relativistic effects of the other.

Analysis

Granting the absolute effects of acceleration and gravitation on clocks, as has been satisfactorily confirmed by experiments, and given the entirely relative effects of un-accelerated (uniform) motion, any uniform motion that might be included in a test of the so-called Twin Paradox can be considered in abstraction from the accelerations. By doing so the problem of mutual time dilation during periods of uniform relative motion and its reconciliation at the twins' reunion can be isolated and more easily resolved.

Figure 3 takes the perspective of a stay-at-home Twin A's coordinate system. World-lines $\mathbf{A_1}$ and $\mathbf{B_1}$ represent the spacetime paths of the twins in the uniform part of Twin B's journey to a distant star-system according to Twin A; vectors $\mathbf{A_2}$ and $\mathbf{B_2}$ are the world-lines of the twins in the uniform part of B's return trip, also according to Twin A. As in *figure 1*, the Pythagorean 3-4-5 relationship that obtains from a relative velocity of .8c is used for the sake of simplicity and clarity.

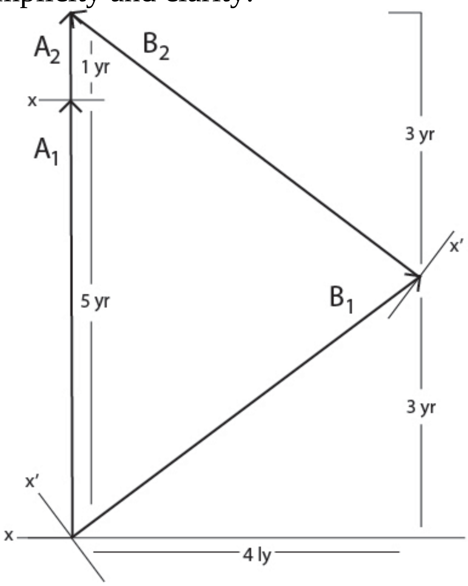

figure 3

Uniform portions of the journey of Twin B to-and-from a distant star-system are shown from the perspective of the stay-at-home Twin A. Vectors $\mathbf{A_1}$ and $\mathbf{B_1}$ represent the un-accelerated *away* segment of B's journey, and vectors $\mathbf{A_2}$ and $\mathbf{B_2}$ represent the period of B's un-accelerated *return* segment. At the 5 year mark on Twin A's clock corresponding to her observation of the uniform part of Twin B's journey away, the clock of the latter indicates that she has begun her deceleration near the destination after 3 years of moving uniformly. Following her deceleration and acceleration for the return home, she coasts for 3 years before beginning her deceleration to earth, where both agree that 6 years of uniform motion have elapsed.

Figure 3 illustrates how Twin A can agree that Twin B's clock ends up with an identical recording of 6 years moving uniformly: Because Twin B had already began her return, according to A's clock, two years prior to A's corresponding time of 5 years, they both agree that Twin B has spent 6 years moving uniformly while A was waiting at home for 6 years.

(Note that Twin B's vectors are drawn to intersect the spacetime points of destination and reunion for the sake of clarity, but it would be the decelerations subsequent to the uniform segments that would actually mark those arrivals.)

There are several indications that *figure 3* is an imperfect representation of two-way periods of uniform motion in spacetime: The diagram isn't rotate-able, as was *figure 2*. To accurately treat two reference frames that separate and re-converge it would be necessary to somehow balance their perspectives, otherwise the time dilation observed by one is treated as more "real" than the other. But Twin A is portrayed as having been absolutely at rest and Twin B as in absolute motion in the un-accelerated segments spanned by the diagram, with no way to reverse or balance their roles, because one twin reverses directions while the other maintains her continuous direction in spacetime – one world-line thus forms an angle, and the other does not. Another problem is that the sum of the lengths of the twins' world-lines are not equal like they are in *figures 1* and *2*, as Twin B's vectors add to 10 units ($\sqrt{(3^2+4^2)}$ in each direction) compared to 6 units for A's. This is because Twin B is moving in A's coordinate system according to A's measure of space, which requires the representation of B to conform as-if to an absolute space metric. The Lorentz transformation for the relative measure of space corresponding to the transformation related to time is $x' = x \sqrt{(1-v^2)}$, which in the example would be $4*\sqrt{(1-.8^2)} = 2.4$ *ly*, which is Twin B's measure of the space traversed in each direction per 3 *yr* period. Unlike Twin B's proper time (3 *yr* each way in the example), her own measure of the distance involved is not directly observed by Twin A, and isn't therefore represented (or representable) in a spacetime graph depicting A's observations, and this is what accounts for the over-extension of Twin B's world-lines in *figure 3*.

The unavoidable spatial imperfections of *figure 3* aside, what is important for understanding the relationship between the twins' clocks is that their relative time-frames *are* accurately represented, consistent with the temporal expression of the Lorentz transformations. The twins' clocks are not simultaneous except at the beginning and end of their relative motion, but they are *correlative* at every moment in between, in alignment along Twin A's space axis (x), as for example when the distance from A's x-axis to

Twin B's position in spacetime is at the 3 *yr* mark, A's clock is likewise 3 *yr* away from its original point in time on its space-axis. Therefore, regardless of the periods and velocities in uniform relative motion during the twins' separation, their clocks for such periods will necessarily remain correlative, and will be synchronous at their reunion.

Discussion

Given the inherent spatial distortion in *figure 3*, it may be helpful to consider a rotate-able, although physically impossible perspective on the twins' relative motion, one that can transcend the limitation of a realistic single reference frame as in *figure 3*.

Figure 4 takes an unnatural but more comprehensible perspective as of a *demiurge*, or if it is preferred, a *God's-Eye view* of the spatial relationship between the twins, illustrating that except when Twin B is accelerating, one twin isn't at rest while the other is in motion; both are at once relatively at-rest and relatively in-motion.

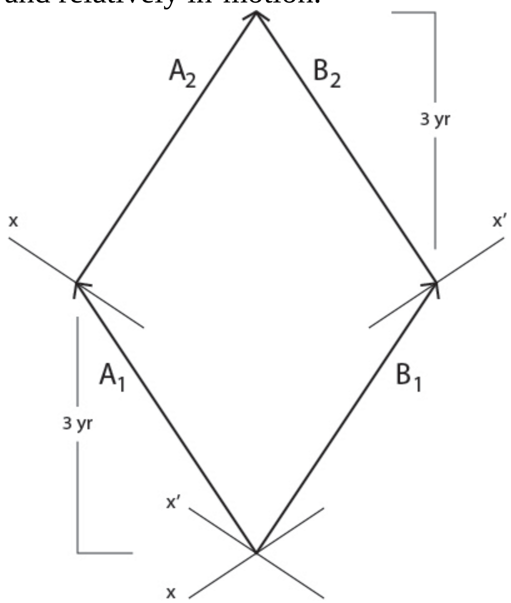

figure 4

A composite and transcendent perspective on the twins' adventure provides a comprehensible but unrealistic representation of the un-accelerated portion of their mutual separation and reconvergence. The lengths of their world-lines are equal, as are the durations of their clocks.

The illustration of uniform motion in *figure 4* balances the to-and-from segments for the sake of clarity, but the periods of uniform motion to-and-from needn't be equal for the final reckoning of clocks to be in agreement. Given the correlative relationship between clocks discussed above, the convergent vectors can have a different length than the divergent vectors. And there needn't be any uniform motion at all in one direction for concurrency to be maintained; such a situation can be envisioned as involving only one period of uniform motion, using one pair of the vectors in *figure 4* – vectors $\mathbf{A_1}$ and $\mathbf{B_1}$ or vectors $\mathbf{A_2}$ and $\mathbf{B_2}$. In each case, no matter how long or how relatively fast their uniform motion in either direction, there is always a correlation between clocks.

Conclusion

The "Twin Effect" has been shown to be entirely explicable in terms of Relativity Theory, and supported by the principle of *correlative clocks* entailed by relative motion. The experimentally confirmed effects of acceleration are neither complicated nor in any way affected by intermittent periods of uniform motion in space due to the correlation of relative motion in time.

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