# UNCERTAINTY, MONEY AND THE "FAIR" RATE OF INTEREST 

Zdenek Chytil, doc., Ing., CSc. Lukas Maslo, Ing.<br>University of Economics, Prague, Czech Republic


#### Abstract

This paper presents a redefinition of notion of the inter-temporal distributional neutrality underlying the "fair" rate of interest of Lavoie (1999). The authors re-define the notion by replacement of the labor-time constant purchasing power method by a more feasible method of a discounted value of consumption. Next, a general proof is provided by the authors of Lavoie's postulate of the equality of the real "fair" rate of interest to the productivity growth rate. This proof is provided separately for a nonproductive and a productive economy. Subsequently, the authors present their own $45^{\circ}$ "fair" rate model which inter-relates both the real and nominal "fair" rates with the productivity growth rate in a single graphical scheme. Finally, the authors amend their own center-equilibrium underemployment model by this $45^{\circ}$ "fair" rate model to produce a complex fair-rate-amended center-equilibrium underemployment model. This model incorporates Lavoie's "fair" rate of interest into a fundamental-uncertainty-based model of underemployment with an endogenous money supply.


Keywords: New Consensus, underemployment equilibrium, "fair" rate of interest, natural rate of interest, inter-temporal distribution

## Introduction

Current discussions about both the theoretical and practical consequences of a negative rate of interest, referred to as breaking through the zero lower bound ${ }^{68}$, have called into question the monetary policy of New Consensus (NC) based on the Wicksellian two-rate hypothesis with a new intensity. Keynes's income-balancing mechanism (Keynes, 1936, p.

[^0]$64)^{69}$ which effectively disposed the natural rate of interest begged a positive question of a determination of the interest rate. Since Keynes's answer to this question was that the interest rate is positively determined by mere conventions (Keynes, 1936, p. 152), then, it was legitimate to ask a normative question of the interest rate determination: "What should the interest rate be?" A feasible answer to this question seems to be provided by a post-keynesian concept of the "fair" rate of interest which "preserves the intertemporal distribution of income between borrowers and lenders." Lavoie (1999, p. 4) ${ }^{70}$. In other words, it is such a rate of interest at which neither the borrowers enrich themselves at the expense of lenders, nor the lenders enrich themselves at cost of borrowers. Replacement of the current natural-interestrate framework by a new one based on the normatively determined "fair" rate of interest would necessary lead to a fundamental reshape of the monetary policy as we know it.

We are going to re-define the "fair" rate of interest - and the intertemporal distributional neutrality concept which underlies the "fair" rate of interest - by means of replacing the labor-time constant purchasing power method of Lavoie (1999, p. 4) by discounting the future value, which we regard more feasible. Next, we are going to make an effort to provide a general proof of Lavoie's (1999, p. 4) postulate of an equality of the real "fair" rate of interest to the productivity growth rate, for both a nonproductive and a productive economy. Our ambition will be to provide a simple model which would capture the inter-relatedness of the real "fair" rate of interest, nominal "fair" rate of interest and the productivity growth rate in a single graphical scheme. Finally, we will built this simple model into the center-equilibrium underemployment model that we presented in our preceding paper to produce a complex fair-rate-amended center-equilibrium underemployment model that will incorporate the "fair" rate of interest concept of Lavoie into a fundamental-uncertainty-based model of underemployment with an endogenous money supply.

[^1]
## Monetary Theory and Policy of New Consensus

In the framework of neoclassical economics - where we count all schools of economic thought which accept Say's law ${ }^{71}$ - the money is neutral in the long run. In the short run, though, money is not neutral. Increase in the rate of money supply growth causes money illusions which shifts a short-run equilibrium in the labor market to the right, pushes down the rate of unemployment below its natural level and increases the real product. In the long run, though, the money illusions disappear, the short-run equilibrium in the labor market shifts left to its long-run position, the rate of unemployment increases to its natural level and the real product returns to its potential level. However, higher money supply and higher price level are not the only two things which have been permanently changed. The expected inflation rate has increased permanently, too. The growth rate of real product has not been affected by the increase in the inflation expectations, though. The LR Phillipse curve is vertical.

From the viewpoint of the New Consensus (NC) - and also the monetarists but unlike the RBC school and the post-keynesians - it is the change in the rate of money supply growth which is a source of the short-run fluctuations in the real product. The reason for this was described by Knut Wicksell (1898, pp. 102-121) who distinguished a money interest rate ${ }^{72}$ $\left(\mathrm{IR}_{\mathrm{M}}\right)$ from a natural interest rate $\left(\mathrm{IR}_{\mathrm{n}}\right)$. Changes in the former bring the money market into its equilibrium, while changes in the latter restore the equilibrium of the capital market. As soon as these two interest rates are not equal, the price level starts increasing or decreasing. As long as the money interest rate is lower than the natural interest rate, the price level increases. As long as the money interest rate is higher than the natural interest rate, the price level decreases. If the money were neutral both in the long run and the short run - which is a situation compatible with a vertical AS curve - then these price level changes would not affect the real product either in the long run or the short run which is a position of proponents of the rational expectations hypothesis (Lucas, Sargent, Wallace). Since this is not the case in the short run - according to either monetarists or the NC adherents (Blinder, Taylor) - the changes in the price level bring about fluctuations in the real product in the short run, which is a situation compatible with an upward-sloping short-run AS curve. So, short-run fluctuations in the real product can be traced back to the inequality of the money interest rate and the natural interest rate, according to the NC (Fontana, 2006, p. 14). The first question to be answered is, then, whether the equality $\mathrm{IR}_{\mathrm{M}}=\mathrm{IR}_{\mathrm{n}}$ can be better provided by a one-level system of free banking, where individual

[^2]commercial banks issue their own respective currencies, or by a two-level system with a central bank, where credits provided by individual commercial banks are denominated in the same currency and where the central bank is a lender of the last resort and a clearing center. However interesting this discussion is ${ }^{73}$, we cannot pay attention to it in this paper because it is both beyond its scope and beyond the main line of the post-keynesian neoclassical controversy about monetary policy. The standpoint of both the mainstream economics and the post-keynesian economics is the same in this matter (although for different reasons), i. e. the system with a central bank is more feasible.

The second question to be answered is, then, how the central bank should achieve its goal to make the monetary IR and the natural IR equal and, through this, to eliminate or reduce the short-run fluctuations in the real product. The problem is the "capital market" is a highly abstract phenomenon which cannot be observed directly in practice and, therefore, neither the value of the equilibrium interest rate of the capital market $I R_{n}$ * can be known by the central bank (or anyone). Nonetheless, it must hold true that if the central bank sets the money interest rate at the right level - that is at the level which is equal to the equilibrium natural interest rate - then the inflation rate will be constant and the real product will not fluctuate. In other words, if we observe a constant inflation rate and no real fluctuations, then, it is an evidence that the central bank set the money interest rate at the right level, i. e. $\mathrm{IR}_{\mathrm{M}}=\mathrm{IR}_{\mathrm{M}}{ }^{*}=\mathrm{IR}_{\mathrm{n}}{ }^{*}$. The monetary policy rule, known as the Taylor rule (see Taylor, 1993, p. 202), consists in adjustment of the money interest rate whenever the inflation rate deviates from the target. An absence of real product fluctuations can be presented indirectly as an absence of fluctuations of the unemployment rate u above or below its natural level $\mathrm{u}^{*} .{ }^{74}$ Underlying the 3 -equation conceptualization of the NC model ${ }^{75}$ is the simplifying assumption of only 3 macro-markets: 1) the money market whose equilibrium can be captured as an equality of the quantity of money balances demanded to the money balances supplied at the equilibrium money interest rate $\mathrm{IR}_{\mathrm{M}}$; 2) the capital market - whose equilibrium can be captured as an equality between the savings supplied and the savings demanded

[^3](investment) at the equilibrium natural interest rate $\mathrm{IR}_{\mathrm{n}}{ }^{*}$; 3) the labor market - whose equilibrium can be expressed as an equality of the current unemployment rate $u$ to the natural unemployment rate $u^{*}$. These three markets are inter-related, so if one of them is out of equilibrium, the system is out of its general equilibrium (whether such exists, at all). The general equilibrium is a following configuration
$\left(\mathrm{IR}_{\mathrm{M}}{ }^{*}=\mathrm{IR}_{\mathrm{n}}{ }^{*}\right)$ AND $\left(\mathrm{u}=\mathrm{u}^{*}\right)$
which is compatible with a situation
$$
(\Delta \Pi=0) \text { AND }\left(\Pi=\Pi^{\mathrm{T}}\right)
$$
i. e. the inflation rate is constant AND equal to its target value. Whether this inflation target $\Pi^{\mathrm{T}}$ can be chosen arbitrarily or not is a matter of disputes. ${ }^{76}$

## Interest Rate Policy Revised

A need for a natural rate of interest as a balancing mechanism of the capital market was denied by Keynes. ${ }^{77}$ Since he supplanted the interest-rate mechanism by his income mechanism, he did not "need" the interest rate to explain either the equilibrium-restoration process of the capital market, or the restoration of full employment, and the problem he was facing was now that the determination of the interest rate via the capital market was so excluded. ${ }^{78}$ Explanation of the interest rate which Keynes set forth is his liquidity preference theory according to which the interest rate is a purely monetary phenomenon, having nothing to do with savings, investment and capital market as such. In a world ruled by fundamental uncertainty, possession of liquid assets becomes "a safe haven for not committing one's monetary claims on resources when the threat of uncertainty becomes great"

[^4](Davidson, 1991, p. 139). Liquidity of an asset is a valuable characteristic. Banknotes and current accounts do not yield an interest, though. If anyone decided to issue new interest-free bonds, no one would buy them unless the liquidity of these bonds is equal to the liquidity of banknotes and current accounts. As long as this is not the case, the bond issuer must compensate the potential buyer for lower liquidity thereof by a non-zero rate of return. The interest rate is so a compensation of an asset holder for the benefits of liquidity (liquidity-premium) ${ }^{79}$ of the alternative more liquid asset he gives up. Or, vice versa, if someone holds cash balances, he bears the (implicit) costs of holding the cash balances which are equal to the liquidity-premium of the cash balances which are equal to the sacrificed rate of return of the alternative asset (e. g. bond).

## The Concept of the "Fair" Rate of Interest

With the concept of the natural interest rate gone, two crucial questions emerged. The first question is a positive one: if the supply-demand scheme in the capital market does not determine the interest rate, what does then? The liquidity-preference theory explains a negative dependence of a demand for cash balances on the interest rate but it does not explain the precise value of the interest rate for any given quantity demanded. That is why Keynes resorts to conventions as the ultimate determinant. ${ }^{80}$ Both the discount rate of the central bank and normal rate of profit are given by conventions and from these rates others are derived, depending on the maturity, expected risk and liquidity premium of the particular asset. The second question following a disposal of natural-interest-rate concept is a normative one: what should the interest rate be? A normative question like this is irrelevant with respect to a process of market clearing. Asking a question what a price of apples should be like does not make any sense as long as a price of apples is co-determined by a supply and a demand. There is nothing like a "fair" price here. There is just a market price at which there

[^5]is neither an excess demand, nor an excess supply. At the moment when this balancing mechanism is eliminated and the price is determined by conventions, it is legitimate to ask whether these conventions are good or not. It is only in such a framework that a term like "fair" rate of interest can make any sense. However, a macroeconomic environment ruled by the fundamental uncertainty is exactly that kind of a framework. An environment where the prevalence of fundamental uncertainty makes the existence of money necessary even in the long run. An environment where the existence of money helps reducing the risk of bankruptcy caused by an asset-liability mismatch. An environment where the liquidity premium of money for any given amount of cash balances is determined by mere conventions.

A concept of the "fair" rate of interest is an answer to the normative question "what should be the interest rate?". According to Lavoie (1999, p. 4), the "fair" rate of interest is such a rate which "preserves the intertemporal distribution of income between borrowers and lenders." Lavoie (1999, p. 4) postulates an equality of the real "fair" rate of interest - we will denote this $\mathrm{r}_{\mathrm{f}}^{\mathrm{R}}$ - with the productivity growth rate ${ }^{81}$
$r_{f}^{R}=\eta$.(1)
Making use of the Samuelson-Solow formula for price inflation
$\Pi=g_{w}-\eta$, (2)
where $\Pi$ is a rate of the price inflation, $g_{w}$ is a rate of the nominal wage inflation, $\eta$ is a productivity growth rate, the real "fair" rate of interest can also be expressed
$\mathrm{r}_{\mathrm{f}}^{\mathrm{R}}=\mathrm{g}_{\mathrm{w}}-\Pi$,(3)
i. e. $r_{f}^{R}$ is equal to the real wage growth rate. ${ }^{82}$ Since the nominal "fair" rate of interest is
$\mathrm{r}_{\mathrm{f}}^{\mathrm{N}}=\mathrm{r}_{\mathrm{f}}^{\mathrm{R}}+\Pi,(4)$
we can write
$r_{f}^{N}=g_{w}$, (5)
and since
$\mathrm{g}_{\mathrm{w}}=\eta+\Pi$,(6)
the nominal "fair" rate of interest can also be expressed as ${ }^{83}$
$\mathrm{rf}^{\mathrm{N}}=\eta+$ П. (7)
Postulating an equality of the rate of interest to the productivity growth rate as a condition for inter-temporal distributional neutrality does

[^6]not lack plausibility. Lavoie (1999, p. 4) explains the concept of the "fair" rate of interest by means of a "constant purchasing power in terms of labor time" on a following example.
\[

$$
\begin{aligned}
& \Pi=5 \% \\
& W=10 \$ / h
\end{aligned}
$$
\]

$$
\text { a loan }=10000 \$
$$

$$
\eta=2 \%
$$

From the formulas above we can compute both the nominal and real "fair" rates of interest:
$\mathrm{r}_{\mathrm{f}}^{\mathrm{N}}=7 \%$
$r_{f}^{R}=2 \%$.
The borrower takes a loan which corresponds to a labor-time equivalent of 1000 hours. The lender provides the borrower with a loan of 1000 hours. Now, the borrower has to pay back to the lender a sum of $10000 \$ .1,07=10700 \$$ which - since the nominal wage has increased by 7 $\%$ to $10,7 \$$ - corresponds to 1000 hours of labor. Let us assume the real rate of interest
$r^{R}=1 \%<r_{f}{ }^{R}$
and the nominal rate of interest

$$
\mathrm{r}^{\mathrm{N}}=6 \%<\mathrm{r}_{\mathrm{f}}^{\mathrm{N}}
$$

Now, the borrower would take a loan of a value corresponding to 1000 hours. Next year, he would pay back $10000 \$ .1,06=10600 \$$ which since the nominal wages increased by $\Pi+\eta=7 \%$ to $10,7 \$$, again - would only correspond to $10600 \$ / 10,7 \$=990,65$ hours of labor. Thanks to a rate of interest lower than the "fair" rate of interest, the borrower would enrich himself at the expense of the lender. Let us suppose an opposite case:
$r^{R}=3 \%>r_{f}^{R}$
$\mathrm{r}^{\mathrm{N}}=8 \%>\mathrm{r}_{\mathrm{f}}^{\mathrm{N}}$.
In this case, the borrower takes a loan of 1000 hours of labor and pay back $10800 \$ / 10,7 \$=1009,35$ hours of labor. As a consequence of an interest rate bigger than the "fair" rate of interest, the lender enriches himself at cost of the borrower. Of course, this is just a numerical example, not a proof. To make a proof of the statement $r_{f}^{R}=\eta$, we need to show that this statement holds true irrespective of the exact value of the borrower's present income, savings (loan) and future income and the lender's present income and future income. First of all, let us re-define the constant purchasing power. Instead of recalculating the nominal inflated money prices and wages to a constant indicator of labor hours, we will use a simple discounting of the future value. E. g. the future value (FV) of the investment is computed as
$\mathrm{FV}_{\text {investment }}=$ loan. $(1+\eta) \cdot(1+\Pi)(8)$
$\mathrm{FV}_{\text {investment }}=10000 \$ .1,02.1,05$
$\mathrm{FV}_{\text {investment }}=10700 \$$.
$10700 \$$ will be discounted by the alternative nominal rate of return, which can be said to be the same for all market participants in equilibrium since the rates of return of investments tend to be equal in the long run and, as a result, the rate of return of the second best alternative is equal to the rate of return of the current investment. So, we get
$\mathrm{r}^{\mathrm{N}}=\mathrm{r}^{\mathrm{R}}+\Pi(9)$
$\mathrm{r}^{\mathrm{N}}=3 \%+5 \%$
$\mathrm{r}^{\mathrm{N}}=8 \%$.
Then, the present value ( PV ) of the future cash-flow from the investment is
$P V_{\text {investment }}=F V_{\text {investment }} /\left(1+r^{N}\right)(10)$
$\mathrm{PV}_{\text {investment }}=10700 \$ / 1,08$
$\mathrm{PV}_{\text {investment }}=9907,4$.
The future value of the repayment will consist of the principal (loan)
plus interest:
$F V_{\text {repayment }}=$ loan. $\left(1+r^{\mathrm{R}}\right) \cdot(1+\Pi)(11)$

$$
\mathrm{FV}_{\text {repayment }}=10000 \$ \cdot 1,03 \cdot 1,05
$$

$\mathrm{FV}_{\text {repayment }}=10800 \$$.
The present value of the repayment is then
$P V_{\text {repayment }}=F V_{\text {repayment }} /\left[\left(1+r^{R}\right) .(1+\Pi)\right],(12)$
which yields us
$\mathrm{PV}_{\text {repayment }}=$ loan $\cdot\left(1+\mathrm{r}^{\mathrm{R}}\right) \cdot(1+\Pi) /\left[\left(1+\mathrm{r}^{\mathrm{R}}\right) \cdot(1+\Pi)\right],(13)$
i. e.
$\mathrm{PV}_{\text {repayment }}=$ loan.(14)
Since
$\mathrm{PV}_{\text {investment }}-\mathrm{PV}_{\text {repayment }}<0(15)$
in this particular case, the lender enriches himself at the cost of the borrower. As a result, an application of the discounting method leads to the same conclusion which Lavoie draw by an application of the labor-time method which means that we can use the discounting method to derive a proof of Lavoie's postulate without running a risk of inconsistency.

## The "Fair" Rate of Interest in a Non-Productive Economy: A Proof

At this point, we need to make a general proof of the statement $r_{f}{ }^{R}=$ $\eta$. In our preceding paper, we started with a simple model of non-productive economy. Let us, therefore, start with a derivation of the "fair" rate of interest for the case of a non-productive economy. In a non-productive economy, we can write
$\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today } *}\right]+\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today } *}\right]=$
 $/(1+\mathrm{r})$ ]
or
$\Delta \mathrm{S}^{\mathrm{A}}+\Delta \mathrm{S}^{\mathrm{B}}=\Delta \mathrm{C}^{\mathrm{A}} /(1+\mathrm{r})+\Delta \mathrm{C}^{\mathrm{B}} /(1+\mathrm{r})(17)$
or
$\Delta \mathrm{S}=\Delta \mathrm{C} /(1+\mathrm{r}) .(18)$
In other words, a sum of savings of $A$ and $B$ today needs to be equal to a sum of consumption increase of $A$ and $B$ tomorrow. For example, if A saves 50 today and lends it to B , so that B can increase his today consumption by 50 , then, A will increase his tomorrow consumption by $(1+r) .50$ and $B$ will have to decrease his tomorrow consumption by the same amount to be able to repay his debt to A. In case of a "fair" inter-temporal distribution, it must hold true
(1) $\Delta \mathrm{S}^{\mathrm{A}}-\Delta \mathrm{C}^{\mathrm{A}} /(1+\mathrm{r})=0$

AND
(2) $\Delta \mathrm{C}^{\mathrm{B}} /(1+\mathrm{r})-\Delta \mathrm{S}^{\mathrm{B}}=0 .(20)$

The first condition means that if, for example, A saves 50 today, his tomorrow consumption needs to increase by $(1+r) .50$ the present value of which is 50 , again. The second condition means that if, for example, B saves 0 and takes a loan 50 from A, then, B's consumption tomorrow needs to decrease by $(1+r) .50$ the present value of which is 50 , again.

The above said equation (16) does not imply these conditions, though. For example, we could assume a case
$\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today } *}=0(21)$
$\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow } *} /(1+\mathrm{r})=\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow }+} /(1+\mathrm{r}),(22)$
in other words, A saves all of his today's income but, at the same time, he does not increase his tomorrow's consumption by any amount.This would imply
$\Delta \mathrm{S}^{\mathrm{A}}=\Delta \mathrm{C}^{\mathrm{B}} /(1+\mathrm{r})-\Delta \mathrm{S}^{\mathrm{B}}$,(23)
that is, if A saves 50 today and lends it to B and B saves 0 today, then, either $B$ increases his consumption by 50 today or tomorrow but he will not repay this loan to A and, as a result, he will not have to decrease his tomorrow consumption. The above said equation (16) still holds true but neither the "fair" inter-temporal distribution condition (1) nor the condition (2) is fulfilled, as obvious.

Let us derive a "fair" rate of interest for a non-productive economy. From the first condition it can be inferred that
$\mathrm{r}_{(1)}=\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow } *}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow }+}\right] /\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today } *}\right]-1$. (24)
From the second condition it can be inferred that
$\mathrm{r}_{(2)}=\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow } *}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow }+}\right] /\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today } *}\right]-1 .(25)$
Since obviously
$r_{(1)}=r_{(2)}=r_{f},(26)$
because the "fair" rate we are looking for must be the same for A and B, by definition,
then, the "fair" rate of interest can simply be computed by plugging into either $\mathrm{r}_{(1)}$ or $\mathrm{r}_{(2)}$. ${ }^{84,85}$

## The "Fair" Rate of Interest in a Productive Economy: A Proof

Now, let us examine the "fair" rate of interest for a case of productive economy with a productivity growth rate $\eta$ :
$\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today* }}\right]+\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today } *}\right]<$
 $/(1+\mathrm{r})]$
(27)

Or
$\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }}\right]-\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow } *} /(1+\mathrm{r})-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow }+} /(1+\mathrm{r})\right]<$ $<\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow } *} /(1+\mathrm{r})-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow }+} /(1+\mathrm{r})\right]-\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }}\right]$. (28)

In other words, reduction in today consumption by e. g. 50 will increase tomorrow consumption by $\eta .50$ as a result of the investment. How this additional consumption will be distributed between $A$ and $B$ is not determined by this inequality. Substituting for
$\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow } * /(1+\mathrm{r})=\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow }+} /(1+\mathrm{r})+\mathrm{I} .(1+\mathrm{\eta}) /(1+\mathrm{r})-\mathrm{I},(29), ~(2)}$
i. e. B can increase his tomorrow consumption by the return from the investment after subtraction of the repayment of the loan plus interest to A . At the same time, it holds true that
$\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }}=\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }},(30)$
in other words, the loan I does not increase B's current consumption because B invests this I.

The loan I is A's saving, i. e. the amount by which A reduces his today consumption:
$\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today* }}\right]=\mathrm{I} .(31)$

[^7]After substitution, we can write
$-\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow } *} /(1+\mathrm{r})-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow }+} /(1+\mathrm{r})\right]<\mathrm{I} .(1+\eta) /(1+\mathrm{r})-2 . \mathrm{I}$.
(32)

So, for a "fair" inter-temporal distribution, it must hold true
$-\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow } * *} /\left(1+\mathrm{r}_{\mathrm{f}}\right)-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow }} /\left(1+\mathrm{r}_{\mathrm{f}}\right)\right]=\mathrm{I} .(1+\mathrm{\eta}) /\left(1+\mathrm{r}_{\mathrm{f}}\right)-2 . \mathrm{I}$,
(33)
i. e. the additional production needs to be distributed between A and B in such a way that the present value of the additional consumption of A is equal to present value of the additional consumption o B , which implies
$-\mathrm{I}=\mathrm{I} .(1+\eta) /\left(1+\mathrm{r}_{\mathrm{f}}\right)-2 . \mathrm{I}(34)$
from which it can be derived that
$(1+\eta)=\left(1+\mathrm{r}_{\mathrm{f}}\right)(35)$
which yields us
$\mathrm{r}_{\mathrm{f}}=\eta$.(36)
By this, we have made a general proof of Lavoie's postulate of the equality of the "fair" rate of interest to the productivity growth rate.

## A Fair-Rate-Amended Center-Equilibrium Underemployment Model Without the Natural Rate of Interest

Now, we can built the "fair" rate concept into our center-equilibrium underemployment model that we presented in our preceding paper. This model captures the characteristic feature of post-keynesian theoretical approach, the feature of fundamental uncertainty. This fundamental uncertainty is reflected by the absence of the natural-interest-rate balancing mechanism of the capital market and by the presence of Keynes's incomebalancing mechanism. This is our fair-rate-amended underemployment equilibrium model:


Fig. 5. Fair-Rate-Amended Center-Equilibrium Underemployment Model Without the Natural Rate of Interest: An Economy With Fundamental Uncertainty (own source)

At the level of product $\mathrm{Y}_{1}$, both the money market and capital market are in their respective equilibria but the labor market displays involuntary unemployment $\left(\mathrm{L}_{3}-\mathrm{L}_{1}\right)$ at the real wage $(\mathrm{W} / \mathrm{P})_{1}$. The first diagram on the left captures our $45^{\circ}$ "fair" rate model. The "fair" rate of interest is equal to the productivity growth rate in real terms. The nominal "fair" rate of interest $r_{f}{ }^{N}$ is the productivity growth rate $\eta$ plus the price inflation rate $\Pi$. The real "fair" rate of interest is equal to the productivity growth rate along the $45^{\circ}$ line intersecting the start of the coordinate system (the $\Pi=0$ line). The nominal "fair" rate of interest is equal to ( $\eta+\Pi$ ) along the $45^{\circ}$ line intersecting the ( $0 ; 1 \%$ ) point in this case (the $\Pi=1 \%$ line). As a result, $r_{f}{ }^{N}$ is exogenous to the money market, and so the $\mathrm{M}_{\mathrm{S}}$ curve is horizontal at the level of $r_{f}{ }^{N}$. Amount of savings supplied is determined by the level of product $\mathrm{Y}_{1}$ at the level of $\mathrm{S}\left(\mathrm{Y}_{1}\right)$ which is equal to the amount of investments that the firms are willing to make. The independence of the capital market equilibrium of the interest rate is projected into the vertical IS curve, while the horizontal money supply curve - which implies the endogeneity of the
money supply - determines the horizontal shape of the LM curve. Since both the money market and the capital market find themselves in their respective equilibria at the level of product $\mathrm{Y}_{1}$, the economy finds itself in the intersection of the IS and LM curves. However, the labor market is out of its equilibrium at this product.

If the nominal and real wages decreased to the level $(\mathrm{W} / \mathrm{P})_{2}$, the labor market would restore its equilibrium at the level of employment $\mathrm{L}_{2}$. Higher employment would increase the product to the level $\mathrm{Y}_{2}$ which would lead to a higher supply of savings $\mathrm{S}\left(\mathrm{Y}_{2}\right)$ and, effectively, bring the capital market out of its equilibrium. Higher product would result in a higher money demand $\left(\mathrm{M}_{\mathrm{D} 2}\right)$ which, because of the horizontal $\mathrm{M}_{\mathrm{S}}$ curve, would not affect the interest rate, though, and, therefore, the economy would shift along the LM curve to the right, away off the $\mathrm{IS}_{1}$ curve. Since this fair-rate-amended model is also a center-equilibrium system, it possesses a unique equilibrium but it cannot reach this equilibrium as long as left to its own endogenous dynamics. Therefore, an exogenous force must overcome the long-run (!) and endogenously insurmountable barriers which protect the system from getting to its equilibrium. When the government increases the aggregate demand by the governmental investments $G$, the labor demand will shift up to $\mathrm{L}_{\mathrm{D} 2}$, and so the labor market gets to a new equilibrium at the level of employment $\mathrm{L}_{3}$ and the real wage $(\mathrm{W} / \mathrm{P})_{1}$. At the same time, higher aggregate demand shifts the equilibrium product from the level $\mathrm{Y}_{1}$ to the level $\mathrm{Y}_{3}$, at which the supply of savings increases to the level $\mathrm{S}\left(\mathrm{Y}_{3}\right)$ and, simultaneously, the private investment I get increased by the governmental investment G, and so the new level of willingness to invest corresponds exactly to the willingness to save. The capital market restores its equilibrium at a new level $S\left(Y_{3}\right)=I+G$ which shifts the $\mathrm{IS}_{1}$ curve to a position $\mathrm{IS}_{2}$. At the same time, a higher product results in a higher money demand $\mathrm{M}_{\mathrm{D} 3}$, without any change in the interest rate, so, the economy shifts along the LM curve to the right and the IS curve shifts as well, resulting in a new intersection of the IS and LM. All three markets are in equilibrium now.

## Conclusion

We presented the concept of the "fair" rate of interest (Lavoie, 1999) as an alternative foundation of the monetary policy in case that the natural-interest-rate hypothesis is rejected. We pointed out that in absence of natural interest rate, two questions must follow. Since Keynes (1936, p. 152) had answered the positive question (what determines the interest rate?) by determination of the interest rate by conventions, the normative question is now mostly legitimate (what should the interest rate be?). After explaining the concept, we suggested to re-define the notion of the inter-temporal distributional neutrality underlying the "fair" rate of interest by means of
replacing the labor-time constant purchasing power method by discounting the future value. We re-defined the "fair" rate of interest as such a rate of interest which guarantees that the additional production is distributed between the borrower and the lender in such a way that the present value of the additional consumption of the former is equal to present value of the additional consumption of the latter. Since we showed that this redefinition does not affect Lavoie's conclusion, then, we could maintain that we do not run a risk of inconsistency when we derive a proof of Lavoie's postulate for this re-defined "fair" rate.

Following this adjustment, we provided a general proof of Lavoie's (1999, p. 4) postulate of the equality of the real "fair" rate of interest to the production growth rate. We provided a general derivation of the nominal and real "fair" rates of interest, separately for a non-productive and a productive economy. Next, we designed a simple $45^{\circ}$ "fair" rate model which interrelates the nominal "fair" rate of interest, real "fair" rate of interest and the productivity growth rate in a single graphical scheme. Then, we built this $45^{\circ}$ "fair" rate model of ours into a center-equilibrium underemployment model presented by us in our preceding paper. Like this, we got our fair-rateamended center-equilibrium underemployment model which incorporates Lavoie's concept of "fair" rate of interest into a fundamental-uncertaintybased model of underemployment with an endogenous money supply.

## References:

Arestis, P. 2009. New Consensus Macroeconomics: A Critical Appraisal. The Levy Economics Institute Working Paper No. 564
Arestis, P.; Sawyer, M. 2008. A Critical Reconsideration of the Foundations of Monetary Policy in the New Consensus Macroeconomics Framework. Cambridge Journal of Economics 2008, 32, pp. 761-779
Davidson, P. 1991. Is Probability Theory Relevant for Uncertainty? A Post Keynesian Perspective. The Journal of Economic Perspectives, Vol. 5, No. 1. (Winter, 1991), pp. 129-143
Fontana, Giuseppe. 2006. The "New Consensus" View of Monetary Policy: A New Wicksellian Connection? The Levy Economics Institute working paper No. 476. WWW: http://www.levyinstitute.org/pubs/wp 476.pdf
Hayek, Friedrich A. 1976. Denationalisation of Money. The Institute of Economic Affairs, 1990, $3^{\text {rd }}$ ed. ISBN 0-255 36239-0.
Holman, Robert. 2005. Dějiny ekonomického myšlení. Praha: C. H. Beck, 2005, ISBN 80-7179-380-9
Ilgmann, Cordelius; Menner, Martin. 2011. Negative Nominal Interest Rates: History and Current Proposals. CAWM Discussion Paper No. 43 - January 2011. WWW:
https://www.wiwi.uni-
muenster.de/cawm/forschen/Download/Diskbeitraege/DP43 IlgmannMenne r.pdf

Keynes, John M. 1936. The General Theory of Employment, Interest and Money. Collected Writings of John Maynard Keynes, Volume VII. Royal Economic Society, Cambridge University Press, 1978. ISBN 0333107292. Keynes, John Maynard. 1937. Alternative Theories of the Rate of Interest. The Economic Journal, Vol. 47, No. 186. (Jun., 1937), pp. 241-252.
Kriesler, P.; Lavoie, M. 2007. The New Consensus on Monetary Policy and Its Post-Keynesian Critique. Review of Political Economy, Vol. 19, No. 3, pp. 387-404
Lavoie, Marc. 2006. A Post-Keynesian Amendment to the New Consensus on Monetary Policy. Metroeconomica, 57 (2), pp. 165-192
Lavoie, Marc. 2008. Towards Taming the New Consensus: Hystereris and Some Other Post-Keynesian Amendments. ROBINSON Working Paper No. 08-08
Lavoie, Marc. 1999. Fair Rates of Interest and Post-Keynesian Economics: The Canadian Case. Working Paper, WWW: http://aix1.uottawa.ca/~robinson/english/wp/fairratecla.pdf
Rochon, Louis-P. 2004. Wicksell After the Taylor Rule. A Post-Keynesian Critique of the New Consensus. ROBINSON working paper, October 29, 2004. WWW:
http://aix1.uottawa.ca/~robinson/english/workshop paper/2004/New Conse nsus1.pdf
Sutch, Richard. 2014. The Liquidity Trap, the Great Depression, and Unconventional Policy:
Reading Keynes at the Zero Lower Bound. BEHL Working Paper WP201405, October $2014 . \quad$ WWW: http://behl.berkeley.edu/files/2014/11/BEHLWP2014-05_Sutch_10-3014.pdf

Setterfield, M. 2005b. Is Inflation Targeting Compatible With Post Keynesian Economics? WWW: http://tmypfunam.files.wordpress.com/2012/09/is-inflation-targeting-compatible-with-post-keynesian-economics.pdf
Seccareccia, M.; Lavoie, M. 2015. Income Distribution, Rentiers and their Role in a Capitalist Economy: A Keynes-Pasinetti Perspective. Presented at the INET Conference Liberté, Égalité, Fragilité in Paris, April 8-11, 2015. WWW: http://beta.ineteconomics.org/uploads/papers/Seccareccia-Lavoie-Paris-INET-27-03-15.pdf
Summers, Lawrence H. 2014. U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound. Business Economics, Vol. 49, No. 2. pp. 65-83.

Taylor, John B. 1993. Discretion Versus Policy Rules in Practice. CarnegieRochester Conference Series on Public Policy 39 (1), pp. 195-214
Wicksell, K. G. 1898. Interest and Prices. London: Macmillan and Co., 1936


[^0]:    ${ }^{68}$ E. g. Seccareccia, M.; Lavoie, M. 2015; Summers, Lawrence H. 2014; Sutch, Richard. 2014; Ilgmann, Cordelius; Menner, Martin. 2011.

[^1]:    ${ }^{69}$ "Saving, in fact, is a mere residual. The decisions to consume and the decisions to invest between them determine incomes. Assuming that the decisions to invest become effective, they must in doing so either curtail
    consumption or expand income. Thus the act of investment in itself cannot help causing the residual or margin, which we call saving, to increase by a corresponding amount." (Keynes, 1936, p. 64).
    ${ }^{70}$ The concept of "fair" rate of interest originates from Luigi Pasinetti's "The Rate of Interest and the Distribution of Income in a Pure Labor Economy", yet, the term itself comes from Lavoie and Seccareccia: "Fair Interest Rates", Encyclopedia of Political Economy, Vol. 1, ed. by P.A. O’Hara, London/New York, 1999, Routledge.

[^2]:    ${ }^{71}$ Keynes referred to them as classics. (Holman, p. 359).
    ${ }^{72}$ In Wicksell's terminology a rate of interest on loans.

[^3]:    ${ }^{73}$ For example Hayek (1990), in opposition to the anarcho-capitalist stream inside the Austrian school (Rothbard, de Soto), does not subscribe to the full-reserves-banking approach and - drawing upon Wicksell's theory of two interest rates - suggests a partialreserves one-level free-banking system which displays some common features with the inflation targeting approach of the New Consensus.
    ${ }^{74}$ For a discussion about an optimal definition of the output gap see Kriesler, Lavoie, 2007, pp. 393-395.
    ${ }^{75}$ Compare various modifications in Rochon, 2004, pp. 7-8; Fontana, 2006, p. 12; Lavoie, 2006, pp. 169-170; Lavoie, 2008, pp. 3-4; Setterfield, 2005b, pp. 5-6; Arestis, Sawyer, 2008, p. 762 .

[^4]:    ${ }^{76}$ E. g. Lavoie (2006, pp. 176-177) points out an inconsistency of New Consensus in this regard. According to the proclaimed long-run money neutrality, the output growth should be compatible with any inflation target. However, the inflation target is usually suggested relatively low which implies that the inflation target matters in the long run. Lavoie calls this the hidden equation of the New Consensus.
    77 "But the notion that the rate of interest is the balancing factor which brings the demand for saving in the shape of new investment forthcoming at a given rate of interest into equality with the supply of saving which results at that rate of interest from the community's psychological propensity to save, breaks down as soon as we perceive that it is impossible to deduce the rate of interest merely from a knowledge of these two factors." (Keynes, 1936, p. 165).

    78 "As I have said above, the initial novelty lies in my maintaining that it is not the rate of interest, but the level of incomes which ensures equality between saving and investment. The arguments which lead up to this initial conclusion are independent of my subsequent theory of the rate of interest, and in fact I reached it before I had reached the latter theory. But the result of it was to leave the rate of interest in the air. If the rate of interest is not determined by saving and investment in the same way in which price is determined by supply and demand, how is it determined?" (Keynes, 1937, p. 250). - We want to thank Robin Kraffer for pointing out this paper of Keynes and, especially, this passage of it to us.

[^5]:    79 "(...) it is something for which people are ready to pay something. The amount (measured in terms of itself)
    which they are willing to pay for the potential convenience or security given by this power of disposal (exclusive of yield or carrying cost attaching to the asset), we shall call its liquidity-premium 1." (Keynes, 1936, p. 226).
    80 "The essence of this convention - though it does not, of course, work out quite so simply lies in assuming that the existing state of affairs will continue indefinitely, except in so far as we have specific reasons to expect
    a change. This does not mean that we really believe that the existing state of affairs will continue indefinitely. We know from extensive experience that this is most unlikely. [...] Nevertheless the above conventional method of calculation will be compatible with a considerable measure of continuity and stability in our affairs, so long as we can rely on the maintenance of the convention." (Keynes, 1936, p. 152).

[^6]:    ${ }^{81}$ More precisely "rate of increase in the productivity of the total amount of labour that is required, directly or indirectly, to produce consumption goods and to increase productive capacity" (Lavoie, 1999, p. 4).
    ${ }^{82}$ Of course, provided that productivity (and, as a result, real remuneration growth rate) of other factors of production has not changed.
    ${ }^{83}$ Lavoie, 1999, p. 4.

[^7]:    ${ }^{84}$ More precisely, we can find the "fair" rate of interest by plugging into any of the following 4 equations: 1$) \mathrm{r}_{\mathrm{f}}=\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow } *}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow }}\right] /\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today } *}\right]-1$ or 2$) \mathrm{r}_{\mathrm{f}}$ $=\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow } *}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow }+}\right] /\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today } *}\right]-1$ or 3$) \mathrm{r}_{\mathrm{f}}=\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow } *-}\right.$ $\left.\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {tomorrow }+}\right] /\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today*} *}\right]-1$ or 4$) \mathrm{r}_{\mathrm{f}}=\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow } *}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow }+}\right] /\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }+}-\right.$ $\left.\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today } *}\right]-1$.
    ${ }^{85}$ That the real "fair" rate of interest in a non-productive economy is not equal to zero, as the formula $r_{f}^{R}=\eta$ would indicate, is not a mistake. Zero value of the productivity growth rate is not the only difference between the model of non-productive economy and the productive economy. The other difference that we assume here is the use of the savings. While the savings lent by A to B is consumed by B in the first period in the non-productive economy, i. e. $\left[\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }}-\left(\mathrm{Y}^{\mathrm{A}}\right)^{\text {today }}\right]=\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }+}-\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today } *}\right]$, these savings are not consumed by B in the first period in the productive economy but invested, i. e. $\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }}=\left[\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {today }}\right.$ and $\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow }} /(1+\mathrm{r})=\left(\mathrm{Y}^{\mathrm{B}}\right)^{\text {tomorrow }+} /(1+\mathrm{r})+\mathrm{I} .(1+\mathrm{\eta}) /(1+\mathrm{r})-\mathrm{I}$.

