PROCESS OF DEVELOPMENT OF MODEL BASED ON LINEAR PROGRAMMING TO SOLVE RESOURCE ALLOCATION TASKS WITH EMPHASIS ON FINANCIAL ASPECTS

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Abstract

Article deals with the topic of Linear Programming application on tasks of allocating limited resources in production companies. These restrictions come from both inside of company and market conditions. There is usually limited amount of resource available on market. Moreover the purchase price is also an important decisive factor when estimating the optimal amount of resource needed. The objective of article is to present the process of development of two models based on linear programming which describe situations production companies have to face in terms of resource allocation. We chose to look into this problem from financial point of view. Therefore most important parameters in models are of financial nature. We look into resource allocation in terms of possible costs and revenues. These two models describe the two possible optimization problems – the minimizing tasks and the maximizing tasks. Examples of such tasks from managerial practice are also included.

Keywords: Linear Programming, resource allocation, costs, revenues

Introduction:

Nowadays the global economic crisis deteriorates the business environment and makes it more difficult for companies to manage. Therefore they must learn to adapt and make an effort to secure an effective and promising development in these constantly changing conditions. Managers should pay more attention to methods, which would help their company not only to survive but also to best fulfill the goals.

This article focuses on the Linear Programming methods. The aim of this paper is to explore the process of development of a model based on linear programming. As an example we chose a task of resource allocation. We chose to look into this problem from financial point of view. Therefore most important parameters in models are of financial nature. We look into resource allocation in terms of possible costs and revenues. These two models describe the two possible optimization problems – the minimizing tasks and the maximizing tasks.

The structure of Linear Programming tasks:

One of the characteristics of optimizing tasks is the large amount of solutions matching the basic task conditions. The selection of a particular solution as the best to a problem depends on the overall objective that is implied in the statement of the problem. A solution which satisfies both the conditions of the problem and the give objective is considered to be the optimal one (Zemánková, Komorníková, 2008; Gass, 2010).

The analysis of any given optimizing problem involves the transformation of necessary data into the set of equations. Then this set of these equations is solved and the optimal solution is determinate. Similarly according to Gass (2010) the complete mathematical statement of a Linear Programming task includes the set of linear equations which represent the conditions of the problem and a linear function which expresses the objective of the problem. In managerial practice, Linear Programming deals with nonnegative solutions to determinate system of linear equations. Due to this limitation, negative solutions are eliminated from the decision-making process. The other possibility is to incorporate this necessary limitation into the set of conditions.

Consequently we can summarize the following three components of the linear model:

- the objective function (the function which represents given problem and whose extreme has to be found);
- the conditions (the limitations of the task);
- the nonnegative condition for the resulting values (Sákal, Jerz, 2003; Borrelli, Bemporad, Morari, 2003).

According to Ivaničová, Brezina and Pekár (2002) and Liu et al. (2007) the mathematical statement of the Linear Programming task can be formulated as following:

objective function $\min f(X) = f(x_1, x_2, \dots x_n)$

conditions

$$g_1(x_1, x_2, \dots x_n) \geq c_1$$
$$g_2(x_1, x_2, \dots x_n) \geq c_2$$
$$\dots$$
$$g_n(x_1, x_2, \dots x_n) \geq c_n$$

The process of the first model development – maximizing tasks

Development of any type of model is a challenging process. We present the process of development of two models. These models use Linear Programming as a foundation and therefore are rather simple in their nature. This aspect is one of the main advantages of any Linear Programming model. Such tasks are easily solved and do not require any expensive application programs (software). To solve simple tasks it is even sufficient to use the Solver application in Microsoft Excel.

The process of development of models we built consists of three steps for each model. Firstly we look into task of maximizing the amount of one factor as its main objective. In practice it could be a task of maximizing amount of substance or material used in product; maximizing the level of active ingredient in final substance and so on. In our case the objective of Linear Programming task is to maximize the revenues company gains from selling its products. We omit the fact that it may not be possible for company to sell all of the products it produces. Therefore the only restriction comes from the maximal amount of material it has available. These factor and processes can be displayed in table (Table 1).

Table 1 Parameters of the first model

factor / process	A_1	A_2	A_3	…	A_m	maximal amount available
S_1	S_{1A1}	S_{1A2}	S_{1A3}	…	S_{1Am}	S_{1max}
S_2	S_{2A1}	S_{2A2}	S_{2A3}	.	S_{2Am}	S_{2max}
S_3	S_{3A1}	S_{3A2}	S_{3A3}	.	S_{3Am}	S_{3max}
. . .	. . .	. . .	. . .	.	. . .	. . .
S_n	S_{nA1}	S_{nA2}	S_{nA3}		S_{nAm}	$S_{n\,max}$
criterion	P_{A1}	P_{A2}	P_{A3}	…	P_{Am}	max.

Source: Own elaboration, 2013.

The goal is to find optimal values of x_i – amount of A_1, A_2, A_3, … A_m, where $x_i \in N$ and $i \in 1, 2, 3, \dots m$, whereas each x_i consists of various combinations of substances S_r, where $r \in 1, 2, 3, \dots n$. Therefore the first step is to define the objective function and conditions of the task which represent the restrictions due to limited availability of each substance. The objective function is to maximize revenues from selling the products. For demonstration we use an example of a situation when all of produced products are sold. Parameters of the model go as following:

$\max z_x = P_{A1} . x_1 + P_{A2} . x_2 + P_{A3} . x_3 + \ldots + P_{Am} . x_m$

$S_{1A1} . x_1 + S_{1A2} . x_2 + S_{1A3} . x_3 + \ldots + S_{1Am} . x_m \leq S_{1max}$

$S_{2A1} . x_1 + S_{2A2} . x_2 + S_{2A3} . x_3 + \ldots + S_{2Am} . x_m \leq S_{2max}$

$S_{3A1} . x_1 + S_{3A2} . x_2 + S_{3A3} . x_3 + \ldots + S_{3Am} . x_m \leq S_{3max}$

…

$S_{nA1} . x_1 + S_{nA2} . x_2 + S_{nA3} . x_3 + \ldots + S_{nAm} . x_m \leq S_{n\,max}$

The second step is to edit given parameters as following:

$\max z_x = \sum P_i . x_i$, where $i \in 1, 2, 3, \ldots m$

$\sum S_{1i} . x_i \leq S_{1max}$

$\sum S_{2i} . x_i \leq S_{2max}$

$\sum S_{3i} . x_i \leq S_{1max}$

…

$\sum S_{ni} . x_i \leq S_{n\,max}$

The third and final step of model development goes as following:

$\max z_x = \sum P_i . x_i$, where $i \in 1, 2, 3, \ldots m$

$\sum \sum S_{ri} . x_i \leq S_{r\,max}$, where $r \in 1, 2, 3, \ldots n$

This way a simple model of Linear Programming task can be built. It can be used to describe any given Linear Programming task with objective function that needs to be maximized.

The process of the second model development – minimizing tasks

The second Linear Programming model we develop is focused on solving tasks whose ultimate goal is to minimize the target value. From financial point of view it would be the best to think of achieving the minimal value of costs during production as the goal of solving the task. We can find many examples of such tasks in managerial practice. Production companies always struggle to minimize their production cost. One of the other examples involves creating an optimal production program of company with emphasis on using the best combination of production factors based on their purchase prices.

Table 2 shows the parameters of one type of minimizing Linear Programming tasks. The ultimate goal is to minimize the sum of purchase prices of consumed production factors. However company needs to use a certain minimal amount of each material in order to produce the final product.

Table 2 Parameters of the second model

factor / process	I_1	I_2	I_3	…	I_m	criterion
F_1	a_{F1I1}	a_{F1I2}	a_{F1I3}	…	a_{F1Im}	P_{F1}
F_2	a_{F2I1}	a_{F2I2}	a_{F2I3}	.	a_{F2Im}	P_{F2}
F_3	a_{F3I1}	a_{F3I2}	a_{F3I3}	.	a_{F3Im}	P_{F3}
. . .	. . .	. . .	. . .	.	. . .	. . .
F_n	a_{FnI1}	a_{FnI2}	a_{FnI3}		a_{FnIm}	P_{Fn}
minimal amount necessary	I_{1min}	I_{2min}	I_{3min}	…	I_{mV}	min

Source: Own elaboration, 2013.

The goal is to find optimal values of y_j – amount of F_1, F_2, F_3,…F_n, where $y_j \in N$ and $j \in 1, 2, 3, \ldots n$. The first step is to define the objective function and conditions of model. In this particular case they are formulated as following:

$\min z_y = P_{F1} . y_1 + P_{F2} . y_2 + P_{F3} . y_3 + \ldots + P_{Fn} . y_n$

$a_{F1I1} . y_1 + a_{F2I1} . y_2 + a_{F3I1} . y_3 + \ldots + a_{FnI1} . y_n \geq I_{1min}$
$a_{F1I2} . y_1 + a_{F2I2} . y_2 + a_{F3I2} . y_3 + \ldots + a_{FnI2} . y_n \geq I_{2min}$
$a_{F1I3} . y_1 + a_{F2I3} . y_2 + a_{F3I3} . y_3 + \ldots + a_{FnI3} . y_n \geq I_{3min}$
…
$a_{F1Im} . y_1 + a_{F2Im} . y_2 + a_{F3Im} . y_3 + \ldots + a_{FnIm} . y_n \geq I_{m\,min}$

The second step is to edit given parameters as following:

$\min z_y = \sum P_{Fj} . y_j$, where $j \in 1, 2, 3, \ldots n$
$\sum a_{FjI1} . y_j \geq I_{1min}$
$\sum a_{FjI2} . y_j \geq I_{2min}$
$\sum a_{FjI3} . y_j \geq I_{3min}$
…
$\sum a_{FjIm} . y_j \geq I_{m\,min}$

The third and final step of development of this model goes as following:

$\min z_y = \sum P_{Fj} . y_j$, where $j \in 1, 2, 3, \ldots n$
$\sum \sum a_{FjIt} . y_j \geq I_{t\,max}$, where $t \in 1, 2, 3, \ldots m$.

Conclusion:

The aim of the article was to describe the process of Linear Programming model development using two examples. We focused on resource allocation tasks with emphasis on the financial aspects of these tasks. Therefore the objective function representing the ultimate goal was either maximization of revenues or minimization of costs.

We tried to set the models as general as possible and that way make them applicable to any given Linear Programming task. In practice it is possible to describe the majority of problems as Linear Programming tasks and resolve them as such. However the models we prepared do not serve to solve these tasks. They only help to better understand all facets of the problem and enable looking at it from different angles using descriptive mathematic methods. Therefore we can state that these models can serve as a foundation for solving Linear Programming tasks using any of the methods or thorough available application program (software).

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