A Simplified Analytical Method to Calculate the Lifting Condensation Level from a Skew-T Log-P Chart

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Abstract

In this work, a methodological framework is presented to automate the Lifting Condensation Level (LCL) height estimate process in meteorological applications, by assimilating dependencies on the saturation mixing ratio from a Skew-T log-P chart. Results of the methodology implementation show a high correlation with the measurements made by atmospheric soundings. In addition, LCL height maps were built for the Caribbean region, these were generated by the proposed automatized process using daily reanalysis data from 1948 to 2010.

Keywords. Lifting Condensation Level, Cloud Base Altitude, Cloud Physics, Espy's Equation.

Un Método Analítico Simplificado para Calcular el Nivel de Condensación por Ascenso a partir de un Diagrama oblicuo-T log-P

Resumen

En este trabajo, se propone un marco metodológico para automatizar el proceso de estimación de la altura del nivel de condensación por ascenso (LCL, por sus siglas en inglés) desde un enfoque de meteorología operativa, considerando la dependencia de una parcela de aire convectiva, respecto a la razón de mezcla de saturación, usando para esto, un diagrama oblicuo-T log-P. Los resultados de la evaluación de la metodología muestran una alta correlación cuando son comparados respecto a mediciones realizadas con radiosondas. Adicionalmente, se muestra el resultado de la generación automatizada de mapas promedios mensuales y anual de LCL sobre el Caribe, para una serie temporal de datos reanálisis que comprende datos diarios desde 1948 a 2010.

Palabras Clave. Nivel de Condensación por Ascenso, Altitud de base de Nube, Física de Nubes, Ecuación de Espy.

Introduction

The LCL computation has important applications from a meteorological point of view; as it is a technique long used to estimate boundary layer cloud heights has been the lifting condensation level (LCL) calculation [4]. The LCL is defined as the pressure-temperature point at which a dry-adiabatically rising air parcel of initially specified pressure, temperature, and moisture content reaches saturation [18]. On a Skew-T log-P chart, this point is the interception of the line of constant saturation mixing ratio w_s, drawn from the initial dew point temperature T_d, and the dry-adiabatic Γ_d line drawn from the initial surface temperature T_o. Thus, if T_o and T_d are known, the LCL height z_{LCL} can be estimated by a Skew-T log-P chart.

In mathematical terms, when T_o and T_d are known the z_{LCL} can be calculated by the equation 1 [6], in which the dry-adiabatic lapse rate Γ_d is approximately 9.8 *K/km*, and the dew point lapse rate Γ_{dew} is 1.8 *K/km*. Therefore, generally the diference between Γ_d and Γ_{dew} is defined as 8.0 *K/km*, and then equation 1 is simplified to 2, where T_o and T_d are expressed in Kelvin and z_{LCL} in meters.

$$z_{LCL} = \frac{T_0 - T_d}{\Gamma_d - \Gamma_{dew}} \tag{1}$$

Muñoz et al.

Av. Cienc. Ing. (Quito), **2015**, *Vol. 7, No. 2, Pags. C124-C129*

$$z_{LCL} = 125(T_0 - T_d) \qquad (2)$$

The approach taken in the equation 2 is based upon the assumption that the dew point temperature lapse rate and the dry-adiabatic lapse rate are both constants in the troposphere. Moreover, gross errors in the z_{LCL} calculations could result if the surface temperature and/or the dewpoint do not represent the thermodynamic profile in the boundary layer [4]. Considering that Γ_{dew} varies from 1.6 to 1.8 *K/km*, and that it is assumed to be a constant when applying the equation 2, uncertainties higher than 200 meters could be expected. Regarding the accuracy level provided by the calculation of z_{LCL} from a Skew-T log-P chart, where the dependence of Γ_{dew} on the saturation mixing ratio is considered, the availability of an analytical equation to estimate z_{LCL} is useful for the automation of the LCL estimate process.

Even when several analytical equations have been proposed for z_{LCL} calculation [1, 10, 17], neither of them allow the replication, in an automatic way, of the methodology of a thermodynamical diagram, such as the Skew-T log-P chart. In this paper, an altenative analytical equation is proposed, in order to automate the methodology involved in the Skew-T log-P chart to calculate z_{LCL}. The main feature of this model is the assimilation of the dependence of Γ_{dew} on the saturation mixing ratio from a Skew-T log-P chart. Taking into account the dependence of Γ_{dew} on w_s, an enhancement of the estimation accuracy of z_{LCL} is expected.

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To determine z_{LCL} in automatic mode, and following the procedures established in the Skew-T log-P chart methodology, we analyzed mathematic functions that could explain how the dew point temperature lapse rate depends on the saturation mixing ratio. In a Skew-T log-P chart, this dependence determines the location of the interception point between the dry adiabatic line for the state defined by the initial surface temperature and the dew point temperature for a convective air parcel.

When a convective parcel of air ascends adiabatically in the atmospheres until reaching saturation, the temperature at the LCL height (T_{LCL}), also known as the condensation temperature, can be calculated by using the empirical equation of Barnes [1]. Equation 3 shows the model of Barnes, in which the difference between T_{LCL} and T_d, divided by the difference between T_o and T_d is a constant as a consecuence of the relation between T_{LCL} and T_d. Then the equation 4 leads to establish a functional dependence of k on T_d.

$$k = \frac{T_d - T_{LCL}}{T_o - T_d} \qquad (3)$$

$$k = 0.001296T_d + 0.1963 \qquad (4)$$

Where T_d is measured in Celsius. Combining the equations (3) and (4), and isolating T_{LCL} we have:

$$T_{LCL} = T_d - (0.001296T_d + 0.1963)(T_o - T_d) \quad (5)$$

The equation 5 is useful, because if T_o and T_d are known, T_{LCL} can be estimated directly, and with an uncertainty near to $0, 5C$ [1]. This is a starting point that establishes a conceptual framework to build an analytical expression to calculate z_{LCL}, on the basis of fundamental atmospheric physics.

It is also known that the temperature of the convective parcel decreases linearly according to the pseudoadiabatic lapse rate Γ_s [5, 11, 12, 16], until it reaches z_{LCL}, then:

$$T_{LCL} = T_o - \Gamma_s(z_{LCL} - z_o) \qquad (6)$$

Isolating z_{LCL}, and considering that z_o is the initial height (in meters) above sea level of the convective parcel, we obtain:

$$z_{LCL} = z_o + \frac{T_o - T_{LCL}}{\Gamma_s} \qquad (7)$$

As Γ_s is a constant in equation 7 ($\Gamma_s = 6.5K.km^{-1}$, approximately), it could be analogous to equation 1, in the sense that both lead to z_{LCL} estimates when T_o and T_d are known. However this scenario is only feasible in a free atmosphere. In contrast, variations in Γ_s are frequent in a standard atmosphere; hence, there is a posibility to include such changes into the analytical formulation by using equation 8 [8].

$$\Gamma_s = \Gamma_d \frac{[1 + \frac{Lw_s}{R'T_o}]}{[1 + \frac{L^2 \varepsilon w_s}{R' c_p T_o^2}]} \qquad (8)$$

where L is latent heat, R' is the universal gas constant, and c_p is the heat capacity at constant pressure [2], the set of variables presented in equation 8 can be considered constants except the saturation mixing ratio w_s. This last variable could be considered like a function of T_d ($w_s(T_d)$) [2]. To define the $w_s(T_d)$ function from a regression model, an extensive choice of ordered pairs was achieved from a digitized Skew-T log-P chart, obtaining:

$$w_s = 3,8166e^{0,0665*T_d} \qquad (9)$$

The purpose of deducing equation (9) is to emulate the Skew-T log-P chart methodology, since the fact that for each T_d, a value for w_s was assigned. The set conformed by the equations 5, 7, 8 and 9 represents the conceptual framework to estimate z_{LCL} considering dependences on the saturation mixing ratio. Additionally,

from equations (6) and (7) we deduce that the pressure experimented by the air parcel at the LCL height, also known as the condensation pressure, is given as:

$$p_{LCL} = p_o e^{[-\frac{g}{RT_{LCL}}(z_{LCL}-z_o)]} \qquad (10)$$

where g is the gravity and R is the gas constant for dry air. Then, equation 10 defines the pressure exerted over the convective parcel at the LCL point.
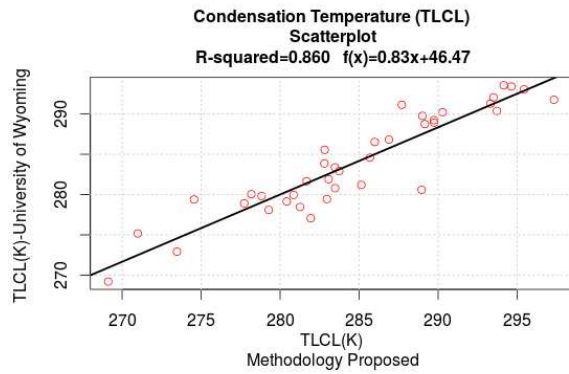

Figure 1: Scatterplot for the condensation temperature between soundings measurements and computations using the Barnes equation
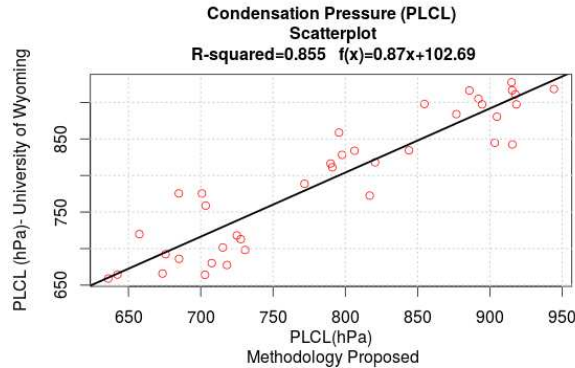

Figure 2: Scatterplot for the condensation pressure between soundings measurements and the method proposed

Results and Discussions

The main advantage of using the methodology proposed, is the automatization of the LCL calculations following the methodology of the Skew-T log-P chart. In this context, it is important to remark that the complete procedure to calculate the z_{LCL} includes i) computation of w_s by using equation 9, ii) computation of Γ_s using equation 8, iii) computation of T_{LCL} by using equation 5, iv) the z_{LCL} computation using equation 7, and v) an option to calculate the p_{LCL} from equation 10. In order to assess the accuracy of the implementation of the method hereby presented, we have compared the
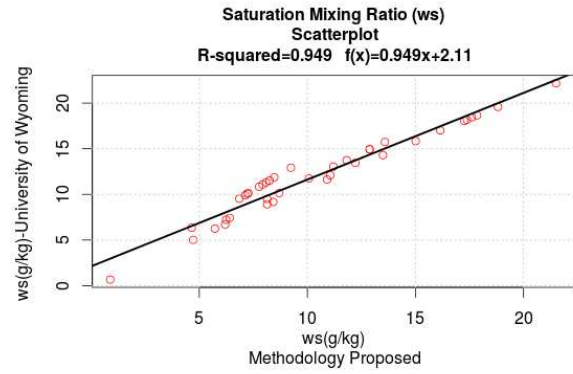

Figure 3: Scatterplot for the saturation mixing ratio between soundings measurements and the method proposed
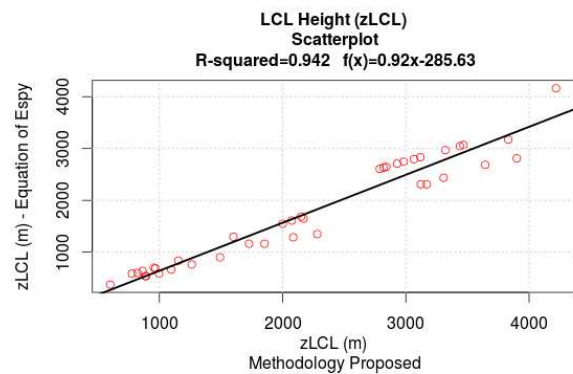

Figure 4: Scatterplot for LCL height estimates between the Espy's equation and the method proposed

output for 39 soundings provided for the University of Wyoming. The variables T_{LCL}, p_{LCL} and w_s have been compared in a scatterplot with the soundings measurements, and the results can be observed in figures 1, 2, and 3 respectively. The University of Wyoming's soundings do not report the z_{LCL} directly, for this reason the z_{LCL} calculation were evaluated by considering the Espy's equation (2), and the result is shown in the figure 4.

The stations used for the analysis are listed in table (1). For each station, and for each variable, a selection of monthy measurements data was taken for the period from january to november of 2015. Henceforth, the results reported on this paper, involve estimates for a representative set of interannual temporal and spatial conditions. In addition, in figure 5 are shown the average monthly LCL height maps for a portion of the Caribbean region. These maps were generated considering daily reanalysis data from 1948 to 2010. In general, it could be observed that the LCL height patterns match well with the annual convective activity in the region [13, 14], and also with the dynamic behavior of the Intertropical Convergence Zone (ITCZ) [7, 15, 19, 20].

Some importants features about the feasibility of the implementation of the method proposed can be derived
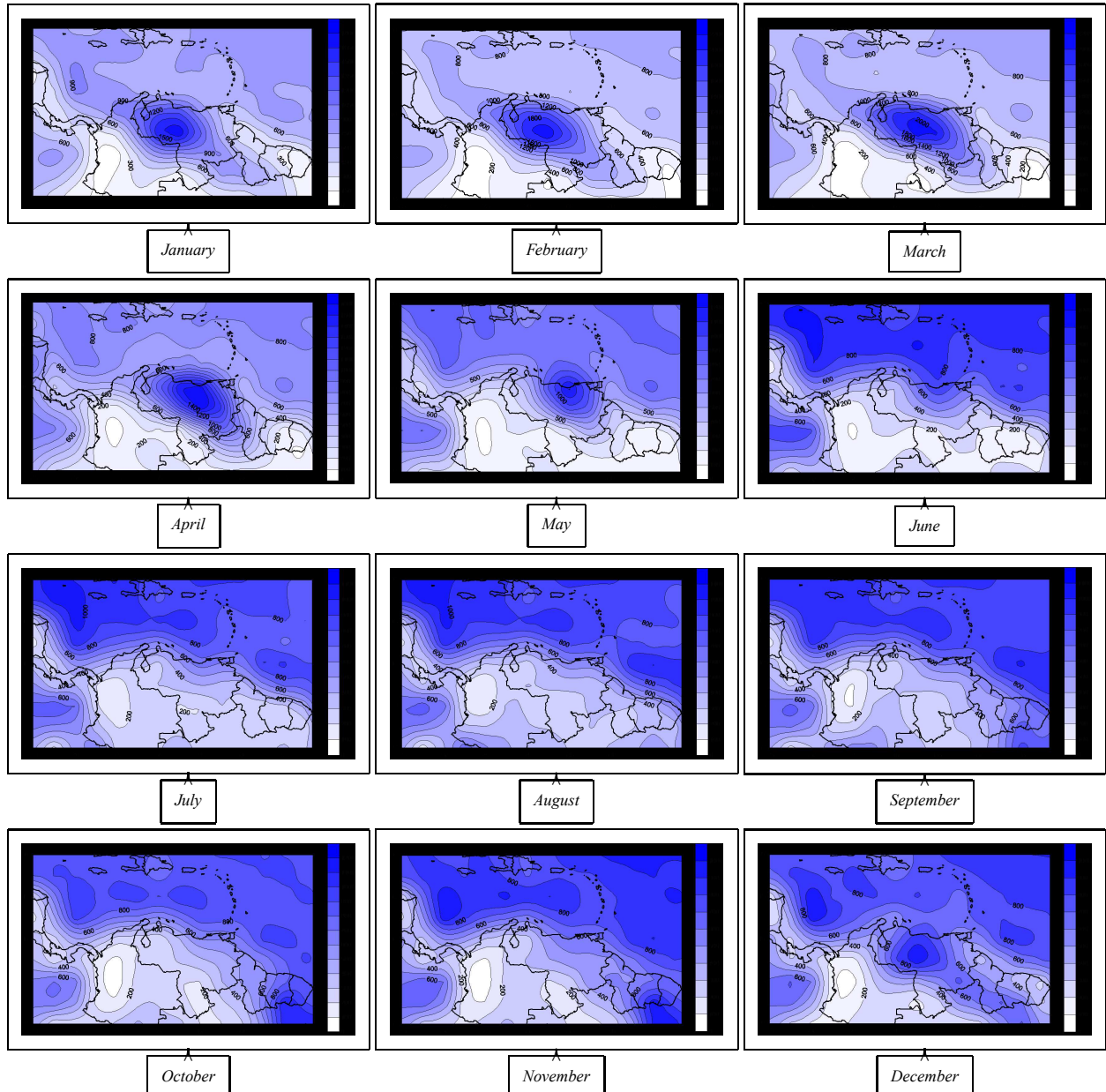
Figure 5: Monthly calculation of z_{LCL} for the Caribbean region following the methodology proposed. The data used as input was assimilated from the reanalysis database, taking both the daily surface temperature and the dew point temperature as input variable for the time series from 1948 until 2010 for a spatial grid conformed by 135 points between the longitude range from -85 to -55, and the latitude 0 to 20. These data was processed using the Statistical Package for the Social Sciences (SPSS) in order to filter and write the data in un specific format, and the set of equations (5, 7, 8 and 9) were programmed into the Grid Analysis and Display System (GrADS) to calculate the LCL. It is remarkable the fact that all these LCL maps have been generated in an automatic mode following the conceptual methodology of the Skew-T log-P chart in a very short period of time.

from these results. For instance, the equation of Barnes showed a high R squared when is compared with soundings measurements, it is a remarkable evidence about the quality of this semiempirical expression to determine the condensation temperature of a convective mass of air, when T_o and T_d are known. A key element considered in the method was to model the dependencie of LCL on the saturation mixing ratio from equation 9. The performance of this equation has been evaluated in the figure 3 against the saturation mixing ratio measured by sounding measurements finding a R squared of 0.949 for 39 experiments. Fitting performance is maximized for high values of w_s, while for low values an alternative fitting strategy could increase the R squared. The relevance for ensuring a high accuracy for equation 9, remains in the fact that it will determines the value of the psudoadiabatic lapse rate from equation 8 for a given dew point temperature. This feature is an advantage over the equation of Espy aimed to improve the accuracy of the LCL estimates.

Identifier	Latitude (deg)	Longitud (deg)	Elevation (m)
SBAT	-9.86	-56.10	288
SACO	-31.32	-64.22	474
SBBR	-15.86	-47.93	1061
SKBO	4.70	-74.13	2548

Table 1: List of sounding stations used for the analysis. The criteria for the selection is based on the geographycal location, and the elevation of the station, in order to evaluate the results in different scenarios.

Regarding the LCL height estimates evaluation, the soundings carried out by the University of Wyoming do not report results for z_{LCL}; for this reason, the evaluation was performed by the scatterplot between the Espy's equation and the method output (see figure 4). The R squared is 0.942, which mean that there is a deep correlation between the two approaches, however the difference in most of the puntual samples is higher than 200m. Depending on the objectives guiding the LCL height computation, the magnitude of this difference can affect substantially the results, for example, when the LCL height is used to seed clouds through bombing silver iodide into the atmosphere [9], the theoretical methodology demands that the bombing should be done close to the LCL height for reaching an optimum degree of condensation during the convection process [3], hence, in this case 200m could be a big source of uncertainty when a satisfactory quality about results is expected. In this context, a further research could be oriented to estimate the level of uncertainty of this method.

In figure 2, is exposed the scatterplot for the condensation pressure measured by sounding, against the calculation made using the equation 10. The fitting performance obtained is quite satisfactory, because it is usuful to validate the output generates by the equations 5, 7, 8 and 9, as an integrated methodology, to estimate z_{LCL}. The approach proposed implicates that p_{LCL} depends on the calculated values for T_{LCL}, w_s, and z_{LCL}, then the good fitting exhibited against the sounding data provides a remarkable framework of robustness for the method proposed.

Conclusions

The main task of this research was to provide a methodological framework to automate the LCL height estimates process in meteorological operational applications by assimilating dependencies on the saturation mixing ratio from a Skew-T log-P chart. We achieved an integration of a) the equation of Barnes to estimate the condensation temperature (T_{LCL}), b) the Skew-T log-P chart approach -used to find a mathematical relationship between the dew point temperature and the saturation mixing ratio in order to take variations on the pseudoadiabatic lapse rate for each value of the dew point temperature-, and c) the thermodynamical models (equation 7, 8) for convective dynamics. These three elements combined allowed the calculation of the LCL height (z_{LCL}) and the condensation pressure (p_{LCL}).

The results shown on this research, lead us to conclude

that the method turned out to be useful to calculate LCL height. However, we sugest to conduct further analyses in order to determine whether the method could be applied to other regions and even at a global scale, considering that its effective implementation could be associated to similar levels of uncertainties as the Skew-T log-P chart approach.

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