Baryon structure in chiral effective field theory on the light front

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We propose a new approach to treat the nucleon structure in terms of an effective chiral Lagrangian. We formulate the state vector, $\phi(p)$, of the nucleon in light-front dynamics, and solve the eigenvalue equation $\hat{P}^2\phi(p) = M^2\phi(p)$ in a truncated Fock space. The effective Lagrangian is decomposed in terms of pion fields in order to match the Fock expansion of the state vector. We use a general renormalization scheme consistent with Fock state truncation. We present our first results in a two-body Fock space truncation.

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1. Introduction

For the last 40 years pion-nucleon systems and their theoretical investigation within the general framework of chiral perturbation theory have been of great interest. Since the nucleon mass is not zero in the chiral limit, all momentum scales are involved in the calculation of baryon properties (like masses or electro-weak observables) beyond tree level. This is at variance with the meson sector for which a meaningful power expansion of any physical amplitude can be done. While there is not much freedom, thanks to chiral symmetry, for the construction of the effective Lagrangian in Chiral Perturbation Theory (CPT), $\mathscr{L}_{CPT}$, in terms of the pion field - or more precisely in terms of the U field defined by $U = e^{i\tau.\pi/f_\pi}$ where f_π is the pion decay constant - one should settle an appropriate approximation scheme in order to calculate baryon properties. Up to now, two main strategies have been adopted. The first one is to force the bare (and hence the physical) nucleon mass to be infinite, in Heavy Baryon Chiral Perturbation Theory [1]. In this case, by construction, an expansion in characteristic momenta can be developed. The second one is to use a specific regularization scheme [2] in order to separate contributions which exhibit a meaningful power expansion, and hide the other parts in appropriate counterterms. In both cases however, the explicit calculation of baryon properties relies on an extra approximation in the sense that physical amplitudes are further calculated by expanding $\mathscr{L}_{CPT}$ in a finite number of pion fields.

Following [3], we propose to calculate nucleon properties using the general eigenvalue equation for the state vector projected on the light front. This non-perturbative equation is then solved in a truncated Fock space. This enables to consider irreducible contributions arising from $\pi\pi NN$ contact interactions in a systematic way, as shown in Fig.(1). It was calculated in perturbation theory in [4]. This decomposition of the state vector in a finite number of Fock components implies

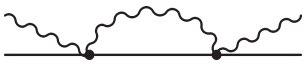

Figure 1: Two-body irreducible contribution to the nucleon state vector.

to consider an effective Lagrangian which enables all possible elementary couplings between the pion and the nucleon fields to the same order. This is indeed easy to achieve in chiral perturbation theory since each derivative of the U field involves one derivative of the pion field. In the chiral limit, the chiral effective Lagrangian of order p involves p derivatives and at least p pion fields. In order to calculate the state vector in the N-body truncation, with one fermion and $(N-1)$ pions, one has to include contributions up to $2(N-1)$ pion fields in the effective Lagrangian, as shown in Fig. (2). We thus should calculate the state vector in the N-body truncation with an effective Lagrangian, denoted by $\mathscr{L}^N_{eff}$, and given by

$$\mathscr{L}^N_{eff} = \mathscr{L}^{p=2(N-1)}_{CPT} \,. \tag{1.1}$$

2. Description of bound states in light-front dynamics

One of the main advantages of Light-Front Dynamics (LFD) is that the vacuum state of a

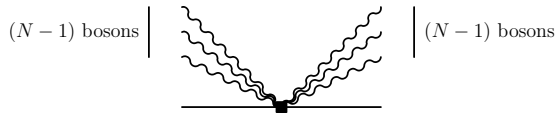

Figure 2: General vertex including a maximum of $(N-1)$ pion fields in the initial and final states.

physical system coincides with the free vacuum. All intermediate states result from fluctuations of the physical system. So it is very natural to decompose the state vector $\phi(p)$ in a series of Fock sectors:

$$\phi(p) = |1\rangle + |2\rangle + ... + |N\rangle + ...$$

Each term of this expansion denotes a state with a fixed number, n, of particles from which the physical system can be constructed. For obvious practical reasons, this expansion should be truncated. We shall call N the maximal number of Fock sectors under consideration. Each Fock sector is then described by a non-perturbative many-body component, called vertex function. Graphically, the vertex function of order n, i.e. including n particles, in a truncation to order N, is represented by the diagram of Fig. (3). It is denoted by $\Gamma_n^{(N)}$. This state vector is a solution of the general eigenvalue equation

$$\hat{P}^2\phi(p) = M^2\phi(p)\ , \tag{2.1}$$

where $\hat{P}$ is the full momentum operator and M is the physical bound state mass.

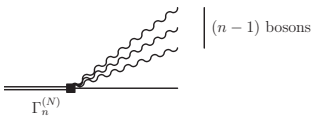

Figure 3: Vertex function of order n for the N-body Fock space truncation.

In the standard version of LFD the state vector is defined on the plane $t + \frac{z}{c} = 0$. This plane is not invariant under spatial rotations. This may lead to many unpleasant consequences for any approximate calculation. We use in our work the so-called explicitly Covariant formulation of LFD (CLFD) [5]. Within this formalism, the state vector is defined on the plane characterized by the equation $\omega \cdot x = 0$, where ω is an arbitrary light-like 4-vector. The standard LFD plane corresponds to the particular choice $\omega = (1,0,0,-1)$. Of course, physical observables should coincide in both approaches in any exact calculation. However, in approximate calculations, the use of CLFD is of particular interest. In that case, this framework allows to separate physical observables from ω-dependent unphysical ones in a very transparent way, while exact calculations should not depend on the arbitrary position of the light front.

In order to make definite predictions for physical observables we should define a renormalization scheme. It should be done with care since any truncation of the Fock space may induce uncancelled divergences. Let us look for instance to the calculation of the physical fermion propagator in second order perturbation theory as represented in Fig. (4). The propagator has three contributions: free propagator, the self-energy contribution and the mass counterterm. The last

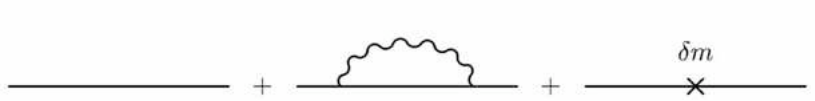

Figure 4: Renormalization of the fermion propagator in second order of perturbation theory

contribution corresponds to the one-body Fock sector (single fermion). It should however be opposite to the two-body Fock sector contribution (one fermion plus one boson) at $p^2 = M^2$, in order to recover the free physical propagator. It is thus clear that if the Fock state is truncated, such cancellation, to all orders, may be broken. A general renormalization scheme to deal with Fock state truncation in Light-Front dynamics has been developed in [6].

3. Eigenvalue equation

In order to show how one should proceed, we start from the following typical pion-nucleon interaction Lagrangian

$$\mathscr{L}_{int} = -\frac{1}{2}\frac{g_A}{F_0}\bar{\Psi}\gamma^\mu\gamma_5\tau^b\partial_\mu\phi^b\Psi - \frac{1}{4F_0^2}\bar{\Psi}\gamma^\mu\vec{\tau}\cdot\vec{\phi}\times\partial_\mu\vec{\phi}\,\Psi. \tag{3.1}$$

The first term is the standard pseudo-vector pion-nucleon coupling, and the second one is the leading contact $\pi\pi NN$ interaction. Other contributions involving two pion fields arise from the second order πN chiral perturbation theory Lagrangian [7]. They can be included in a very similar way to the $\pi\pi NN$ contact interaction.

Solving the eigenvalue equation (2.1) [6], we can represent the system of coupled equations for the vertex functions in the two-body truncated Fock space by the diagrams of Fig. (5). It can be written as

$$\bar{u}(p_1)\Gamma_1 u(p) = \bar{u}(p_1)(V_1+V_2)u(p), \tag{3.2}$$
$$\bar{u}(k_1)\Gamma_2 u(p) = \bar{u}(k_1)(V_3+V_4)u(p). \tag{3.3}$$

Here p is the momentum of the physical fermion, p_1 and k_1 are the momenta of the constituent fermion in the one-body and two-body Fock space truncation respectively. The graph denoted by V_4 corresponds to the second term in the interaction Lagrangian (3.1).

The vertex functions Γ_1 and Γ_2 should be decomposed in independent spin structures. Using the explicit covariance of our approach, we can write

$$\bar{u}(k_1)\Gamma_1 u(p) = (m^2-M^2)a_1\bar{u}(k_1)u(p) \tag{3.4}$$
$$\bar{u}(k_1)\Gamma_2 u(p) = i\bar{u}(k_1)\left((\not{k}_2 - \not{\omega}\tau)\,b_1(\mathbf{R}_\perp,x) + \frac{m\,\not{\omega}}{\omega\cdot p}\,b_2(\mathbf{R}_\perp,x)\right)\gamma_5 u(p)\,, \tag{3.5}$$

where

$$\tau = \frac{s-M^2}{2\,\omega\cdot p} \tag{3.6}$$

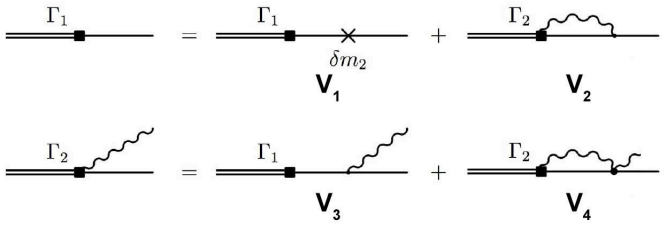

Figure 5: System of equations for the pion-nucleon vertex functions in the two-body Fock space truncation.

is the off-shell energy, with $s = (k_1 + k_2)^2$. The usual longitudinal momentum fraction, with respect to the position ω of the light front, is given by

$$x = \frac{\omega \cdot k_2}{\omega \cdot p} \ . \tag{3.7}$$

The transverse momentum $R_\perp$ with respect to ω is defined from

$$R_2 = k_2 - xp \quad \text{with} \quad R_2 = (R_2^0, \mathbf{R}_\perp, R_2^0) \ , \tag{3.8}$$

since by construction $\omega \cdot R_2 = 0$. The last component in (3.8) corresponds to the longitudinal momentum R_2 with respect to ω. The mass of the physical bound state is denoted by M, while m is the mass of the constituent fermion. In the final result, one should take the limit $m \to M$. Generally speaking, $b_1(\mathbf{R}_\perp, x)$ and $b_2(\mathbf{R}_\perp, x)$ are scalar functions depending on the dynamical variables (momenta). In the two-body truncation, with the Lagrangian (3.1), they are just constants. The spin decomposition of the two-body component in terms of independent spin structures is of course not unique. We choose here the most convenient one.

On the energy shell, i.e. for $s = M^2$, the two-body vertex function Γ_2 should be independent of the arbitrary position of the light front, so that, since b_2 is a constant, one should have the condition

$$b_2 = 0. \tag{3.9}$$

Due to the Fock state truncation, this may be however not the case in practice. In order to inforce this condition, one may consider an ω-dependent vertex counterterm in the effective Lagrangian, of the form

$$\delta \mathscr{L}_{int} = i Z_3 \bar{\Psi} \, \not{\omega} \gamma_5 \tau^b \phi^b \Psi. \tag{3.10}$$

This amounts to extend the pion-nucleon vertex in Fig. (5) by a new contribution proportional to $Z_3 \not{\omega}$.

The loop contributions of Fig. (5) are a-priori divergent, and one should use an appropriate regularization scheme in order to give them a mathematical sens. In our preliminary study, we shall use the Pauli-Villars (PV) regularization scheme as applied to CLFD [6]. We thus should extend all physical components to incorporate PV particles. For simplicity, we have left over all indices relative to physical and PV particles in the vertex functions in the eigenvalue equations (3.2,3.3) and in Fig. (5).

The general strategy to solve the coupled eigenvalue equations shown in Fig. (5) is detailed in [6]. Its solution for the simplest case, namely, without the $\pi\pi NN$ contact interaction, i.e. without the graph V_4 from the system presented in Fig. (5) can be found analytically as a function of PV masses. The necessary condition (3.9) is satisfied automatically. We are analyzing the ω-independent component. It should correspond exactly to the perturbative calculation.

We also have found a solution in the more general case given by the Lagrangian (3.1), i.e. with $\pi\pi NN$ contact interaction. The solution is also analytic but much more complicated and in that case the physical component $b_2(s = M^2)$ is a constant different from 0. It should be killed by an appropriate counterterm (3.10).

4. Perspectives

We have outlined in this preliminary study the main steps in the calculation of nucleon properties within light front chiral effective field theory. Our formalism is based on the Fock expansion of the nucleon state vector, projected on the light front. Using the properties of the explicitly covariant formulation of light-front dynamics, and an adequate renormalization scheme when the Fock expansion is truncated, we have calculated explicitly the spin components of the state vector in the two-body truncation.

We are now analyzing our results in terms of an expansion of the nucleon mass as a function of the pion mass, and calculate the scalar form factor.

References

[1] E. Jenkins and A.V. Manohar, *Phys. Lett.* **B255**, 558 (1991).

[2] R.J. Ellis and H.B. Tang, *Phys. Rev.* **C57** (1998) 3356;
T. Becher and H. Leutwyler, *Eur. Phys. J.* **C9** (1999) 643.

[3] J.-F. Mathiot, *Chiral effective field theory on the light front*, PoS(EFT09)014

[4] D. Djukanovic, J. Gegelia and S. Scherer, *Eur. Phys. J.***A29** (2006) 337.

[5] V.A. Karmanov, *Zh. Eksp. Teor. Fiz.* **71** (1976) 399. [transl.:*Sov. Phys. JETP* **44** (1976) 210] ;
J. Carbonell, B. Desplanques, V.A. Karmanov and J.-F. Mathiot, *Phys. Rep.* **300** (1998) 215.

[6] V.A. Karmanov, J.-F. Mathiot and A.V. Smirnov, *Phys.Rev.D* **77** (2008) 085028.

[7] S. Scherer, *Adv.Nucl.Phys.* **27** (2003) 277 and references therein