View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by HAL Clermont Université

Interference Effects, Time Reversal Violation and Search for New Physics in Hadronic Weak Decays

Ziad Ajaltouni, E Di Salvo

▶ To cite this version:

Ziad Ajaltouni, E Di Salvo. Interference Effects, Time Reversal Violation and Search for New Physics in Hadronic Weak Decays. International Journal of Modern Physics A, World Scientific Publishing, 2012, 27, 1250086 (26 p.). <10.1142/S0217751X12500868>. <in2p3-00698478>

HAL Id: in2p3-00698478

http://hal.in2p3.fr/in2p3-00698478

Submitted on 16 May 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Interference Effects, Time Reversal Violation and Search for New Physics in Hadronic Weak Decays

Z. J. Ajaltouni[1*], E. Di Salvo[1,2†]

[1] Laboratoire de Physique Corpusculaire de Clermont-Ferrand,
IN2P3/CNRS Université Blaise Pascal, F-63177 Aubière Cedex, France
[2] I.N.F.N. - Sez. Genova,
Via Dodecaneso, 33, 16146 Genova, Italy

Abstract

We propose some methods for studying hadronic sequential two-body decays involving more spinning particles. It relies on the analysis of T-odd and T-even asymmetries, which are related to interference terms. The latter asymmetries turn out to be as useful as the former ones in inferring time reversal violating observables; these in turn may be sensitive, under some particular conditions, to possible contributions beyond the standard model. Our main result is that one can extract such observables even after integrating the differential decay width over almost all of the available angles. Moreover we find that the correlations based exclusively on momenta are quite general, since they provide as much information as those involving one or more spins. We generalize some methods already proposed in the literature for particular decay channels, but we also pick out a new kind of time reversal violating observables. Our analysis could be applied, for example, to data of LHCb experiment.

PACS numbers: PACS Nos.: 11.30.Er, 11.80.Et, 13.25.-k, 13.30.Eg

*ziad@clermont.in2p3.fr

†Elvio.Disalvo@ge.infn.it

1 Introduction

Interference is a typical quantum mechanical effect and can be exploited experimentally to detect important phenomena. For example, parity violation was discovered in the fifties by hypothesizing[1], and successively exhibiting[2], an interference term between two decay amplitudes which behave differently under parity inversion; these were identified later[3] as matrix elements of the vector and axial current respectively. As is well-known, such a term is proportional to a scalar product of the type $\mathbf{J} \cdot \mathbf{p}$[1], where $\mathbf{J}$ and $\mathbf{p}$ are respectively an angular momentum and a momentum of particles involved in the process under study. It is worth recalling that this dependence was picked out by a suitable asymmetry[2]. Similarly, the asymmetries connected to direct CP violation − revealed, it is worth mentioning, only many years after the discovery of indirect CP violation in the $K_L \to \pi\pi$ decays[4, 5] − can be detected if two different amplitudes, with different weak and strong phases, contribute to the decay[6, 7].

It could be tempting to propose for time reversal violation (TRV) something similar to parity violation. That is, we could study interference terms between amplitudes which behave differently under time reversal (TR). These terms correspond to correlations of the type[8]

$$C = \mathbf{v}_1 \cdot \mathbf{v}_2 \times \mathbf{v}_3, \tag{1}$$

where $\mathbf{v}_i$ (i = 1,2,3) are either momenta or angular momenta of particles involved in the process. Such correlations, named T-odd[7, 9, 10, 11, 12], can be defined in processes where more than two particles are involved. Correlation (1) may be revealed by the asymmetry[8]

$$\mathcal{A} = \frac{(C > 0) - (C < 0)}{(C > 0) + (C < 0)}. \tag{2}$$

However in this case we are not so lucky as with parity violation. Indeed, the genuine TRV effects are mimicked by fictitious T-odd interference terms, caused for example by strong or electromagnetic spin-orbit interactions[13, 14]. Such effects, present in scattering processes[10, 11, 12, 13, 14] as well as in decays (see for example refs. [7, 8, 9, 15, 16, 17, 18, 19, 20, 21, 22, 23]), produce the same T-odd correlations (1) as real TRV. In particular, they mask real TRV in a weak decay, owing to final state interactions (FSI). It is therefore quite a serious problem to separate the two contributions[7, 8, 24]. But combining the asymmetry of a given triple product with the one of its CP-conjugated gives rise to a real TRV[8, 24, 25].

CP violations (CPV), as well as TRV, which can be seen as the counterpart of CPV owing to the CPT symmetry, are often used for investigating possible clues of new physics (NP). Indeed, baryon-antibaryon asymmetry in the Universe cannot be explained by the sole CKM mechanism of CPV[26]. A common strategy to find hints beyond the standard model (SM) consists of investigating experimentally those processes where the CPV (or TRV) are tiny in the SM predictions. Indeed some of such violations may indicate deviations from the SM. Especially, we recall $B \to \pi K$ decays[27, 28, 29], $B_s - \overline{B}_s$ mixing[30, 31] and the like-sign dimuon asymmetry[32]. However till now experimental and theoretical uncertainties do not allow any conclusive results. Confirmations to the first few discrepancies are demanded. In this sense, experiments on sequential two-body decays, mainly of the type

$$B_{(s)} \to V_1 V_2 \tag{3}$$

(with $V_{1,2}$ vector mesons), have been performed[17, 28, 30, 31] and suggested[21, 33, 34, 35, 36, 37, 38, 39]. Such types of decays, where more spinning particles are involved, offer the advantage of the angular correlations[7, 8, 24, 25, 40, 41, 42, 43], which may provide the moduli and the relative phases of the helicity or transversity amplitudes[44, 45]. It is worth noting, in the context of such decays, that the above mentioned T-odd correlations are more and more frequently proposed[18, 19, 20, 21, 22] and also used[15, 16, 17] for discovering possible clues of NP. This is because these correlations may be preferred to direct CP-asymmetries in the cases when the relative strong phases are negligibly small[25, 46]. Moreover, as regards the helicity amplitudes, they are a further, suitable tool for testing the SM[38, 39, 47], possibly by examining fake[38, 48] T-odd observables.

The aim of the present paper is to illustrate some methods for analyzing two-body hadronic weak decays. In particular we study the properties of asymmetries connected to T-odd products relative to a sequential two-body decay; moreover, we consider some T-even asymmetries, seldom used in the literature but equally useful.

First of all, we define suitable T-odd and T-even correlations, of the type (1) or slightly more complicated, and the corresponding asymmetries. Then we show how to use such asymmetries for extracting observables sensitive to TRV and, possibly, to NP. In particular, we illustrate some tests for probing clues to physics beyond the SM. This work implements in some way a previous one[49],where we proposed tests based on the knowledge of moduli and relative phases of the amplitudes of decays of the type considered here. Moreover it generalizes some of the methods adopted in the literature in decays of the type (3).

The paper is organized as follows. Sects. 2 to 4 are devoted to the definitions and to the analytical expressions of single and double T-odd and T-even asymmetries, written as functions of helicity amplitudes. Several TRV observables are defined in sect. 5. In sect. 6 we carry out an explicit example. Sect. 7 is dedicated to relations with previous papers. Various tests of the SM are presented in sect. 8. Lastly some conclusions are drawn in sect. 9.

2 Single T-odd and T-even Asymmetries

Here and in the following section we illustrate some T-odd and T-even asymmetries, which can be inferred from a sequential decay of the type

$$J \rightarrow a \quad b, \qquad a \rightarrow a_1 \quad a_2, \qquad b \rightarrow b_1 \quad b_2. \tag{4}$$

Such observables depend on correlations of the type (1) or similar. The T-odd asymmetries have been proposed by several authors, in different contexts. The most recent contributions regard hadronic[21, 23, 24, 25, 34, 38, 40, 41, 50, 51, 52] and semi-leptonic[53, 54, 55, 56, 57] decays, searches for top decays[9, 58, 59] and for new particles[18, 20, 22, 60]. Also some experiments[15, 16, 17] adopt this technique. On the contrary, T-even asymmetries are not so frequently suggested in the literature[9, 51, 61]. In order to introduce these observables, we define preliminarily suitable reference frames.

2.1 Reference Frames

First of all, we define a canonical frame, at rest with respect to the parent resonance J:

$$\hat{\mathbf{y}} = \frac{\mathbf{p}_{in}}{|\mathbf{p}_{in}|}, \qquad \hat{\mathbf{z}} = \frac{\mathbf{p}_{in} \times \mathbf{p}_J}{|\mathbf{p}_{in} \times \mathbf{p}_J|}, \qquad \hat{\mathbf{x}} = \hat{\mathbf{y}} \times \hat{\mathbf{z}}, \tag{5}$$

where $\mathbf{p}_{in}$ and $\mathbf{p}_J$ are, respectively, the momenta of the initial beam and of the resonance J in the laboratory frame.

On the other hand, the successive decays of the particles a and b are more conveniently described in the helicity frames. For example, as regards particle a, it is given by the following three mutually orthogonal unit vectors:

$$\hat{\mathbf{e}}_L = \frac{\mathbf{p}_a}{|\mathbf{p}_a|}, \qquad \hat{\mathbf{e}}_T = \frac{\hat{\mathbf{z}} \times \hat{\mathbf{e}}_L}{|\hat{\mathbf{z}} \times \hat{\mathbf{e}}_L|}, \qquad \hat{\mathbf{e}}_N = \hat{\mathbf{e}}_T \times \hat{\mathbf{e}}_L, \tag{6}$$

$\mathbf{p}_a$ being the momentum of a in the canonical frame. Analogous definitions hold true for particle b.

2.2 Definition of Single Asymmetries

2.2.1 Single T-odd Asymmetries

Consider the scalar product

$$T_a^N = \mathbf{p}_{a_1} \cdot \hat{\mathbf{e}}_N, \tag{7}$$

where $\mathbf{p}_{a_1}$ is the momentum of the particle a_1 in the rest frame of a. This is a T-odd quantity. Correspondingly, we define the T-odd asymmetry

$$A_a^N = \frac{N(T_a^N > 0) - N(T_a^N < 0)}{N(T_a^N > 0) + N(T_a^N < 0)}, \tag{8}$$

where $N(T_a^N > 0)$ $[N(T_a^N < 0)]$ is the number of decays such that, for a given $\mathbf{p}_{a_1}$, the scalar product above is positive (negative). Another, independent, T-odd product can be defined as

$$T_b^N = \mathbf{p}_{b_1} \cdot \hat{\mathbf{e}}_N, \tag{9}$$

where $\mathbf{p}_{b_1}$ is the momentum of particle b_1 in the rest frame of b. Obviously, this product induces an asymmetry analogous to (8).

If different than zero, such asymmetries do not imply TRV, since, as told, they may derive contributions also from strong or/and electromagnetic interactions[7, 8, 13, 14]. However, as we shall see in sect. 5, it is possible to define quantities sensitive to such a violation[8, 24, 25].

2.2.2 Single T-even Asymmetries

The scalar products

$$T_a^{T(L)} = \mathbf{p}_{a_1} \cdot \hat{\mathbf{e}}_{T(L)} \tag{10}$$

are T-even quantities. Analogously to the T-odd asymmetries of subsect. 2.2.1, we define the T-even asymmetries

$$A_a^{T(L)} = \frac{N(T_a^{T(L)} > 0) - N(T_a^{T(L)} < 0)}{N(T_a^{T(L)} > 0) + N(T_a^{T(L)} < 0)}. \tag{11}$$

Also in this case it is possible to define two more (T-even) asymmetries by substituting $\mathbf{p}_{b_1}$ to $\mathbf{p}_{a_1}$ in the definitions (10).

2.3 Differential Two-Body Decay Width

We deduce, here and in the following subsections, the expressions of the differential width of a two-body decay and of the asymmetries just defined before. The calculations are based on the formalism of the density matrix[62, 63, 64, 65]. The starting point is the differential decay width for sequential two-body decays [66, 67]. It reads as

$$\begin{aligned} & \frac{1}{\Gamma_{ab}\Gamma_{a_1a_2}\Gamma_{b_1b_2}} \frac{d^9\Gamma_{a_1a_2b_1b_2}}{d^2\Omega d^2\Omega_a d^2\Omega_b dp_J^2 dp_a^2 dp_b^2} \\ = \ & \mathcal{N}_{Jab} W \sum_{\mu_a,\mu_b} a_{\mu_a}^{s_a} a_{\mu_b}^{s_b} \mathcal{T}_{\mu_a\mu_b}(\Omega, \Omega_a, \Omega_b). \end{aligned} \tag{12}$$

Here the Γ's are the partial decay widths of the decays (4); moreover

$$\mathcal{N}_{Jab} = \frac{2s_a+1}{4\pi}\frac{2s_b+1}{4\pi}\frac{2J+1}{4\pi}, \tag{13}$$

$$W = |B(p_J^2)|^2 |B(p_a^2)|^2 |B(p_b^2)|^2, \tag{14}$$

$$\mathcal{T}_{\mu_a\mu_b}(\Omega,\Omega_a,\Omega_b) = \sum_{\lambda_a,\lambda_b}\sum_{\lambda'_a,\lambda'_b} \alpha^J_{\lambda_a\lambda_b}\alpha^{J*}_{\lambda'_a\lambda'_b}\mathcal{D}^{s_a*}_{\lambda_a\mu_a}(\Omega_a)$$
$$\times\mathcal{D}^{s_a}_{\lambda'_a\mu_a}(\Omega_a)\mathcal{D}^{s_b*}_{\lambda_b\mu_b}(\Omega_b)\mathcal{D}^{s_b}_{\lambda'_b\mu_b}(\Omega_b)\sum_{M,M'}\mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda'}(\Omega)\rho^{(0)}_{MM'}, \tag{15}$$

$$\alpha^J_{\lambda_a\lambda_b} = A^J_{\lambda_a\lambda_b}(\sum_{\lambda_a,\lambda_b}|A^J_{\lambda_a\lambda_b}|^2)^{-1/2}, \tag{16}$$
$$a^{s_a}_{\mu_a} = \sum_{\mu_{a_1}-\mu_{a_2}=\mu_a}|A^{s_a}_{\mu_{a_1}\mu_{a_2}}|^2(\sum_{\mu_{a_1},\mu_{a_2}}|A^{s_a}_{\mu_{a_1}\mu_{a_2}}|^2)^{-1}, \tag{17}$$
$$a^{s_b}_{\mu_b} = \sum_{\mu_{b_1}-\mu_{b_2}=\mu_b}|A^{s_b}_{\mu_{b_1}\mu_{b_2}}|^2(\sum_{\mu_{b_1},\mu_{b_2}}|A^{s_b}_{\mu_{b_1}\mu_{b_2}}|^2)^{-1}. \tag{18}$$

J, s_a and s_b are the spins, respectively, of the parent resonance and of particles a and b. $A^J_{\lambda_a\lambda_b}$, $A^{s_a}_{\mu_{a_1}\mu_{a_2}}$ and $A^{s_b}_{\mu_{b_1}\mu_{b_2}}$ are the rotationally invariant two-body decay amplitudes relative to the three decays (4). They depend, respectively, on the helicities $\lambda_a(\lambda'_a)$ and $\lambda_b(\lambda'_b)$, μ_{a_1} and μ_{a_2}, and μ_{b_1} and μ_{b_2}, such that

$$\lambda_a-\lambda_b=\Lambda, \quad \lambda'_a-\lambda'_b=\Lambda', \quad \mu_{a_1}-\mu_{a_2}=\mu_a, \quad \mu_{b_1}-\mu_{b_2}=\mu_b. \tag{19}$$

$M(M')$ is the third component of the spin of J in the canonical frame. Ω, Ω_a, Ω_b denote the directions of, respectively, particles a, a_1 and b_1 in the rest frames of their parent resonances: as usual, we have set $\Omega \equiv (\theta,\phi)$, where θ and ϕ are, respectively, the polar and azimuthal angle; similar definitions hold for Ω_a and Ω_b. $\rho^{(0)}_{MM'}$ and the $B(p^2)$'s are, respectively, the spin density matrix of the parent resonance and the relativistic Breit-Wigner functions of the resonances, normalized as

$$\sum_M \rho^{(0)}_{MM}(p^2,p^2)=1, \qquad \int_0^\infty dp^2|B(p^2)|^2=1. \tag{20}$$

Lastly, $\mathcal{D}^J_{M\Lambda}(\Omega)$ is a Wigner $\mathcal{D}$-matrix element, defined as[68]

$$\mathcal{D}^J_{M\Lambda}(\Omega) = e^{-iM\phi}d^J_{M\Lambda}(\theta), \tag{21}$$
$$d^J_{M\Lambda}(\theta) = \langle JM|e^{-iJ_y\theta}|J\Lambda\rangle, \tag{22}$$

where J_y is the y-component of the angular momentum.

Problems arise if one takes into account the energy dependence of the decay amplitudes and of the density matrix; however, if the resonances are sufficiently narrow, as it usually happens, such a dependence can be neglected. Therefore, in an analysis, it would be convenient to integrate over the energies of the resonances. But we do not perform such integrations, in order to point out an important theoretical subtlety, to be clarified in sect. 5.

Now we integrate eq. (12) over $d^2\Omega_a d^2\Omega_b$. To this end, we have to take into account the normalization

$$\int d^2\Omega \mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda}(\Omega) = \frac{4\pi}{2J+1}\delta_{MM'} \tag{23}$$

for Wigner's $\mathcal{D}$-matrices. As a result we get the expression of the differential two-body decay width, *i. e.*,

$$\Gamma(\Omega) = \mathcal{N}_J W \sum_{\lambda_a,\lambda_b} |\alpha^J_{\lambda_a\lambda_b}|^2 \sum_{M,M'} \mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda}(\Omega)\rho^{(0)}_{MM'}. \tag{24}$$

Here

$$\mathcal{N}_J = \frac{2J+1}{4\pi}. \tag{25}$$

Rigorously speaking, the distribution Γ depends also on p_J^2, p_a^2 and p_b^2, through the factor W. However, in order to simplify the notation, we have omitted such a dependence in the argument of the distribution. We shall adopt this convention for all of the asymmetries that we shall define in the present paper.

More important, we remark that the differential decay width (12), and therefore all of the expressions derived from it, do not depend strictly on the decay amplitudes, but rather on the dimensionless quantities (16); as we shall see, these play an important role in the definitions of the asymmetries and, from now on, will be named "reduced" amplitudes.

2.4 Analytical Expression of the T-odd Asymmetry

The expression of the observable $\Gamma(\Omega)A_a^N(\Omega)$ reads

$$\begin{aligned}\Gamma(\Omega)A_a^N(\Omega) &= \int_0^\pi d\theta_a sin\theta_a \Big[\int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2}\Big] d\phi_a \int d^2\Omega_b \Delta\Gamma(\Omega,\Omega_a\Omega_b) \\ &= \mathcal{N}_{Ja} W \sum_{\mu_a} a^{s_a}_{\mu_a} \sum_{\lambda_a,\lambda_b} \sum_{\lambda'_a}{}' \Theta^{s_a}_{\lambda_a\lambda'_a\mu_a} \frac{4(-)^{D_a}}{\lambda_a - \lambda'_a} \\ &\times \ \alpha^J_{\lambda_a\lambda_b}\alpha^{J*}_{\lambda'_a\lambda_b} \sum_{M,M'} \mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda'}(\Omega)\rho^{(0)}_{MM'}.\end{aligned} \tag{26}$$

Here $\Delta\Gamma(\Omega, \Omega_a\Omega_b)$ is a short notation for the differential decay width (12). Moreover we have set

$$
\begin{aligned}
\mathcal{N}_{Ja} &= \frac{2J+1}{4\pi}\frac{2s_a+1}{4\pi}, \qquad D_a = (\lambda_a - \lambda'_a - 1)/2, && (27)\\
\Theta^{s_a}_{\lambda_a\lambda'_a\mu_a} &= \int_0^\pi d\theta_a sin\theta_a d^{s_a}_{\lambda_a\mu_a}(\theta_a) d^{s_a}_{\lambda'_a\mu_a}(\theta_a), && (28)
\end{aligned}
$$

the d-matrices being defined by eq. (22). Lastly, we have denoted by $\sum'$ the sum over those λ'_a for which $\lambda_a - \lambda'_a$ is odd.

2.5 Expressions of the T-even Asymmetries

The expression of the asymmetry A^T_a is given by

$$
\begin{aligned}
\Gamma(\Omega)A^T_a(\Omega) &= \int_0^\pi sin\theta_a d\theta_a \Big[\int_0^\pi - \int_\pi^{2\pi}\Big] d\phi_a \int d^2\Omega_b \Delta\Gamma(\Omega, \Omega_a\Omega_b)\\
&= \mathcal{N}_{Ja} W \sum_{\mu_a} a^{s_a}_{\mu_a} \sum_{\lambda_a,\lambda_b} \sum_{\lambda'_a}{}' \Theta^{s_a}_{\lambda_a\lambda'_a\mu_a} \frac{4i}{\lambda_a - \lambda'_a}\\
&\times \alpha^J_{\lambda_a\lambda_b}\alpha^{J*}_{\lambda'_a\lambda_b} \sum_{M,M'} \mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^J_{M'\Lambda'}(\Omega)\rho^{(0)}_{MM'}. && (29)
\end{aligned}
$$

On the other hand, the asymmetry A^L_a reads

$$
\begin{aligned}
\Gamma(\Omega)A^L_a(\Omega) &= \Big[\int_0^{\pi/2} - \int_{\pi/2}^\pi\Big] sin\theta_a d\theta_a \int_0^{2\pi} d\phi_a \int d^2\Omega_b \Delta\Gamma(\Omega, \Omega_a\Omega_b)\\
&= 8\pi\mathcal{N}_{Ja} W \sum_{\mu_a>0} \Delta a^{s_a}_{\mu_a}\\
&\times \sum_\Lambda \Delta a^J_{\Lambda\mu_a} \sum_{M,M'} \mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^J_{M'\Lambda}(\Omega)\rho^{(0)}_{MM'}. && (30)
\end{aligned}
$$

Here

$$
\begin{aligned}
\Delta a^{s_a}_{\mu_a} &= \frac{1}{2}(a^{s_a}_{\mu_a} - a^{s_a}_{-\mu_a}), && (31)\\
\Delta a^J_{\Lambda\mu_a} &= \frac{1}{2}\sum_{\lambda_a>0} \delta^{s_a}_{\lambda_a\mu_a}(|\alpha^J_{\lambda_a\lambda_b}|^2 - |\alpha^J_{-\lambda_a\lambda'_b}|^2), && (32)
\end{aligned}
$$

$\lambda_b = \lambda_a - \Lambda$, $\lambda'_b = -\lambda_a - \Lambda$ and

$$
\delta^{s_a}_{\lambda_a\mu_a} = \Big[\int_0^{\pi/2} - \int_{\pi/2}^\pi\Big][d^J_{\lambda_a\mu_a}(\theta)]^2 sin\theta d\theta. \qquad (33)
$$

2.6 Remarks

Two short remarks are in order. Firstly, the expressions of the single asymmetries calculated in this section for particle a can be extended in a straightforward way to particle b. Secondly, we observe that the procedure just described, concerning single asymmetries, is not applicable to a decay of the type (3), but only to cases where both the parent resonance and at least one of the decay products are spinning $(J, s_a(s_b) \geq 1/2)$; for example, it may be applied to

$$\Lambda_b \to \Lambda(\Lambda_c) \;\; P(V), \qquad\qquad \Lambda_c \to \Lambda \;\; P(V), \tag{34}$$

P and V denoting, respectively, a pseudoscalar and a vector particle.

3 Double T-odd and T-even Asymmetries

Here we define some double asymmetries, analogous to the single asymmetries of the previous section. As an example, we set

$$A_{ab}^{NT} = \frac{\left[N(T_a^N \cdot T_b^T > 0) - N(T_a^N \cdot T_b^T < 0)\right]}{\left[N(T_a^N \cdot T_b^T > 0) + N(T_a^N \cdot T_b^T < 0)\right]}. \tag{35}$$

It is instructive to notice that the numerator of this asymmetry can be written as

$$\begin{aligned} \text{Num} \;\; &= \;\; \left[N(T_a^N > 0, T_b^T > 0) - N(T_a^N > 0, T_b^T < 0)\right] \\ &- \;\; \left[N(T_a^N < 0, T_b^T > 0) - N(T_a^N < 0, T_b^T < 0)\right], \end{aligned} \tag{36}$$

which justifies the name of "double" asymmetry. This is a T-odd asymmetry, as appears from eq. (35). Another asymmetry of this type can be defined as

$$A_{ab}^{LN} = \frac{\left[N(T_a^L \cdot T_b^N > 0) - N(T_a^L \cdot T_b^N < 0)\right]}{\left[N(T_a^L \cdot T_b^N > 0) + N(T_a^L \cdot T_b^N < 0)\right]}. \tag{37}$$

Two more T-odd asymmetries can be obtained, respectively, from eqs. (35) and (37) by interchanging a with b.

Viceversa, one can define five T-even asymmetries, in a quite analogous way: A_{ab}^{NN}, A_{ab}^{TT}, A_{ab}^{LL}, A_{ab}^{TL} and A_{ab}^{LT}. For example,

$$A_{ab}^{NN} = \frac{\left[N(T_a^N \cdot T_b^N > 0) - N(T_a^N \cdot T_b^N < 0)\right]}{\left[N(T_a^N \cdot T_b^N > 0) + N(T_a^N \cdot T_b^N < 0)\right]}. \tag{38}$$

Analytically, the first double, T-odd asymmetry reads as

$$
\begin{aligned}
& \Gamma(\Omega) A_{ab}^{NT}(\Omega) \\
= & \Big[\int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2}\Big] d\phi_a \Big[\int_0^{\pi} - \int_{\pi}^{2\pi}\Big] d\phi_b \int_0^{\pi} sin\theta_a d\theta_a \int_0^{\pi} sin\theta_b d\theta_b \Delta\Gamma(\Omega, \Omega_a \Omega_b) \\
= & \mathcal{N}_{Jab} W \sum_{\mu_a,\mu_b} a_{\mu_a}^{s_a} a_{\mu_b}^{s_b} \sum_{\lambda_a,\lambda_b} {\sum_{\lambda'_a}}' {\sum_{\lambda'_b}}' \Theta_{\lambda_a \lambda'_a \mu_a}^{s_a} \Theta_{\lambda_b \lambda'_b \mu_b}^{s_b} \frac{16i(-)^{D_a}}{(\lambda_a - \lambda'_a)(\lambda_b - \lambda'_b)} \\
\times & \alpha_{\lambda_a \lambda_b}^{J} \alpha_{\lambda'_a \lambda'_b}^{J*} \sum_{M,M'} \mathcal{D}_{M\Lambda}^{J*}(\Omega) \mathcal{D}_{M'\Lambda'}^{J}(\Omega) \rho_{MM'}^{(0)}. \qquad (39)
\end{aligned}
$$

The other double, T-odd asymmetry is given by

$$
\begin{aligned}
\Gamma(\Omega) A_{ab}^{LN}(\Omega) = & \; 2\pi \mathcal{N}_{Jab} W \sum_{\mu_a,\mu_b} a_{\mu_a}^{s_a} a_{\mu_b}^{s_b} \sum_{\lambda_a,\lambda_b} {\sum_{\lambda'_b}}' \delta_{\lambda_a \mu_a}^{s_a} \Theta_{\lambda_b \lambda'_b \mu_b}^{s_b} \frac{4(-)^{D_b}}{\lambda_b - \lambda'_b} \\
\times & \; \alpha_{\lambda_a \lambda_b}^{J} \alpha_{\lambda_a \lambda'_b}^{J*} \sum_{M,M'} \mathcal{D}_{M\Lambda}^{J*}(\Omega) \mathcal{D}_{M'\Lambda'}^{J}(\Omega) \rho_{MM'}^{(0)}, \qquad (40)
\end{aligned}
$$

with $D_b = (\lambda_b - \lambda'_b - 1)/2$. Concerning the double, T-even asymmetries, we have

$$
\begin{aligned}
& \Gamma(\Omega) A_{ab}^{NN}(\Omega) \\
= & \mathcal{N}_{Jab} W \sum_{\mu_a,\mu_b} a_{\mu_a}^{s_a} a_{\mu_b}^{s_b} \sum_{\lambda_a,\lambda_b} {\sum_{\lambda'_a}}' {\sum_{\lambda'_b}}' \Theta_{\lambda_a \lambda'_a \mu_a}^{s_a} \Theta_{\lambda_b \lambda'_b \mu_b}^{s_b} \frac{16(-)^{D_a}(-)^{D_b}}{(\lambda_a - \lambda'_a)(\lambda_b - \lambda'_b)} \\
\times & \alpha_{\lambda_a \lambda_b}^{J} \alpha_{\lambda'_a \lambda'_b}^{J*} \sum_{M,M'} \mathcal{D}_{M\Lambda}^{J*}(\Omega) \mathcal{D}_{M'\Lambda'}^{J}(\Omega) \rho_{MM'}^{(0)}; \qquad (41) \\
& \Gamma(\Omega) A_{ab}^{TT}(\Omega) \\
= & \mathcal{N}_{Jab} W \sum_{\mu_a,\mu_b} a_{\mu_a}^{s_a} a_{\mu_b}^{s_b} \sum_{\lambda_a,\lambda_b} {\sum_{\lambda'_a}}' {\sum_{\lambda'_b}}' \Theta_{\lambda_a \lambda'_a \mu_a}^{s_a} \Theta_{\lambda_b \lambda'_b \mu_b}^{s_b} \\
\times & \frac{(-16)}{(\lambda_a - \lambda'_a)(\lambda_b - \lambda'_b)} \alpha_{\lambda_a \lambda_b}^{J} \alpha_{\lambda'_a \lambda'_b}^{J*} \sum_{M,M'} \mathcal{D}_{M\Lambda}^{J*}(\Omega) \mathcal{D}_{M'\Lambda'}^{J}(\Omega) \rho_{MM'}^{(0)}; \qquad (42)
\end{aligned}
$$

$$
\begin{aligned}
\Gamma(\Omega) A_{ab}^{LL}(\Omega) = & \; 16\pi^2 \mathcal{N}_{Jab} W \sum_{\mu_a>0} \sum_{\mu_b>0} \Delta a_{\mu_a}^{s_a} \Delta a_{\mu_b}^{s_b} \sum_{\Lambda} \Delta_{\Lambda \mu_a \mu_b}^{(2)} \\
\times & \sum_{M,M'} \mathcal{D}_{M\Lambda}^{J*}(\Omega) \mathcal{D}_{M'\Lambda}^{J}(\Omega) \rho_{MM'}^{(0)}. \qquad (43)
\end{aligned}
$$

Here

$$
\Delta_{\lambda_b \mu_a \mu_b}^{(2)} = \sum_{\lambda_a} |\alpha_{\lambda_a \lambda_b}^{J}|^2 \delta_{\lambda_a \mu_a}^{s_a} \delta_{\lambda_a \mu_b}^{s_b}, \qquad (44)
$$

with $\lambda_b = \lambda_a - \Lambda$. Lastly,

$$\begin{aligned} \Gamma(\Omega) A^{LT}_{ab}(\Omega) &= 2\pi \mathcal{N}_{Jab} W \sum_{\mu_a,\mu_b} a^{s_a}_{\mu_a} a^{s_b}_{\mu_b} \sum_{\lambda_a,\lambda_b} \delta^{s_a}_{\lambda_a\mu_a} \sum_{\lambda'_b}{}' \Theta^{s_b}_{\lambda_b\lambda'_b\mu_b} \frac{4i}{\lambda_b - \lambda'_b} \\ &\times \; \alpha^J_{\lambda_a\lambda_b} \alpha^{J*}_{\lambda_a\lambda'_b} \sum_{M,M'} \mathcal{D}^{J*}_{M\Lambda}(\Omega) \mathcal{D}^J_{M'\Lambda'}(\Omega) \rho^{(0)}_{MM'}. \end{aligned} \tag{45}$$

The expressions of the asymmetries A^{TN}_{ab}, A^{NL}_{ab} and A^{TL}_{ab} are obtained from, respectively, A^{NT}_{ab}, A^{LN}_{ab} and A^{LT}_{ab} by interchanging particle a with b.

The asymmetries defined and elaborated in this section may be applied to a wider class of decays, including those of the type (3).

4 Summary

We have shown in the two previous sections that the expressions of the differential decay widths and of the T-odd and T-even asymmetries depend on interference terms or on the moduli squared of the "reduced" decay amplitudes. Linear combinations of such parameters can be obtained from experimental data, as we show in Appendix for several cases of interest. Indeed, we apply there the method of the moments[66, 67] to the distributions defined in sects. 2 and 3. As is well-known, each moment is factorized into a term which depends solely on the production density matrix, times another one which contains information only on decay amplitudes. Moreover, for each moment, the former factor is independent of the distribution considered; therefore the ratio of, say, a given moment of some asymmetry, to the corresponding moment of the differential decay width (24), provides linear relations among the moduli squared or among the interference terms of the "reduced" amplitudes. Therefore we obtain linear systems with respect to these quantities, and constraints to be imposed on the parameters which can be extracted from data. The results of the analysis that we have proposed can be used for determining TRV observables, as we shall show in the next section. In particular, the linear system obtained may be over-determined, provided all asymmetries defined in sects. 2 and 3 are nonzero. Unfortunately, if particle a or b (or both) in (4) have a strong or electromagnetic decay, some asymmetries may vanish because of parity conservation, as we shall see in a specific example (sect. 6).

However we shall establish that, in the decay considered, the nonzero distributions are sufficient to determining all moduli and relative phases of the amplitudes. In cases when this is possible, these quantities can be used as inputs for the tests of NP suggested in our preceding paper[49].

5 Determining TRV Observables

Some TRV observables are deduced directly from experimental distributions, while others require some elaboration. In this section we shall consider both kinds of observables, starting from the former ones.

5.1 Some TRV Observables from Distributions

Consider the differential width of the sequential two-body decay CP-conjugated to (4), *i. e.*,

$$\overline{J} \to \overline{a} \;\; \overline{b}, \qquad \overline{a} \to \overline{a}_1 \;\; \overline{a}_2, \qquad \overline{b} \to \overline{b}_1 \;\; \overline{b}_2. \tag{46}$$

The direction corresponding to Ω is

$$\overline{\Omega} \equiv (\pi - \theta, \pi + \phi). \tag{47}$$

Consider the T-odd asymmetries for $\overline{J}$. As an example, first of all, we elaborate the expression of the single T-odd asymmetry $\overline{A}_a^N(\overline{\Omega})$. It is defined through

$$\begin{aligned} \overline{\Gamma}(\overline{\Omega})\overline{A}_a^N(\overline{\Omega}) \;&=\; \mathcal{N}_{Ja}\overline{W}\sum_{\mu_a}\overline{a}^{s_a}_{-\mu_a}\sum_{\lambda_a,\lambda_b}\sum_{\lambda'_a}{}' \,\Theta^{s_a}_{-\lambda_a-\lambda'_a-\mu_a}\frac{4(-)^{D_a}}{\lambda'_a-\lambda_a} \\ &\times\; \overline{\alpha}^J_{-\lambda_a-\lambda_b}\overline{\alpha}^{J*}_{-\lambda'_a-\lambda_b}\sum_{M,M'}\mathcal{D}^{J*}_{M-\Lambda}(\overline{\Omega})\mathcal{D}^J_{M'-\Lambda'}(\overline{\Omega})\overline{\rho}^{(0)}_{MM'}. \end{aligned} \tag{48}$$

Here $\overline{W}$ is analogous to the factor (14) and $\overline{\rho}^{(0)}_{MM'}$ is the spin density matrix of the resonance $\overline{J}$. If the spin of the parent resonances is 0, it is natural to define a TRV observable as[8, 21, 25, 34, 38]

$$\Delta_a^N = \Gamma(\Omega)A_a^N(\Omega) + \overline{\Gamma}(\overline{\Omega})\overline{A}_a^N(\overline{\Omega}), \tag{49}$$

where the + sign in eq. (49) is a consequence of the change of sign under CP reflection in the scalar product (7). For a nonzero spin of J, Δ_a^N is a TRV observable only if the production process of $\overline{J}$ is CP-conjugated to the one of J; this implies a relation between the density matrices of the two resonances, practically impossible to realize. However, as we shall see, we can equally derive useful TRV quantities from the experimental data. To this end, we consider an ideal experiment, in which we assume that the production process of the parent resonance J and the decays of particles a and b are invariant under CP reflection. Moreover we suppose the production processes of J and $\overline{J}$ to be described by the same density matrix. Lastly, we take account of the CPT theorem, which yields $\overline{W} = W$, and of the following properties of the $\mathcal{D}$- and d-matrices:

$$\mathcal{D}^{J*}_{M-\Lambda}(\overline{\Omega})\mathcal{D}^{J}_{M'-\Lambda'}(\overline{\Omega}) = \mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda'}(\Omega), \tag{50}$$

$$\Theta^{s_a}_{-\lambda_a-\lambda'_a-\mu_a} = -\Theta^{s_a}_{\lambda_a\lambda'_a\mu_a} \quad \text{for} \quad \lambda_a - \lambda'_a \quad \text{odd.} \tag{51}$$

As a result, we get

$$\begin{aligned}\overline{\Gamma}(\overline{\Omega})\overline{A}^N_a(\overline{\Omega}) \;&=\; -\mathcal{N}_{Ja}W\sum_{\mu_a} a^{s_a}_{\mu_a}\sum_{\lambda_a,\lambda_b}\sum_{\lambda'_a}{}' \Theta^{s_a}_{\lambda_a\lambda'_a\mu_a}\frac{4(-)^{D_a}}{\lambda_a-\lambda'_a} \\ &\times\; \overline{\alpha}^J_{-\lambda_a-\lambda_b}\overline{\alpha}^{J*}_{-\lambda'_a-\lambda_b}\sum_{M,M'}\mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda'}(\Omega)\rho^{(0)}_{MM'}.\end{aligned} \tag{52}$$

Substituting eqs. (26) and (52) into (49), we get

$$\Delta^N_a = 4\mathcal{N}_{Ja}W\sum_{\lambda_a,\lambda_b}\sum_{\lambda'_a}{}'\sum_{M,M'}\mathcal{C}^{M,M'}_{\lambda_a,\lambda_b,\lambda'_a}\mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda'}(\Omega). \tag{53}$$

Here

$$\mathcal{C}^{M,M'}_{\lambda_a,\lambda_b,\lambda'_a} = \frac{(-)^{D_a}}{\lambda_a-\lambda'_a}\rho^{(0)}_{MM'}T_{\lambda_a,\lambda'_a}\cdot\epsilon_{\lambda_a,\lambda_b,\lambda'_a}, \qquad T_{\lambda_a,\lambda'_a} = \sum_{\mu_a} a^{s_a}_{\mu_a}\Theta^{s_a}_{\lambda_a\lambda'_a\mu_a} \tag{54}$$

and

$$\epsilon_{\lambda_a,\lambda_b,\lambda'_a} = \alpha^J_{\lambda_a\lambda_b}\alpha^{J*}_{\lambda'_a\lambda_b} - \overline{\alpha}^J_{-\lambda_a-\lambda_b}\overline{\alpha}^{J*}_{-\lambda'_a-\lambda_b}, \tag{55}$$

with $\lambda'_a - \lambda_a$ odd. If Δ^N_a is different from zero, one has TRV. But this implies that at least one of the $\epsilon_{\lambda_a,\lambda_b,\lambda'_a}$'s is non-vanishing. Although deduced under particular assumptions on the production process and on the decay of particle a, this condition depends solely on the decays of J and of its anti-particle, which are independent of the production process and of the successive decays of a and b. Therefore the $\epsilon_{\lambda_a,\lambda_b,\lambda'_a}$'s are TRV observables for odd $\lambda'_a - \lambda_a$.

One can define quite similarly the quantities Δ_b^N, Δ_{ab}^{NL}, Δ_{ab}^{TN} and those obtained by interchanging, respectively, N with L and T with N. Obviously, analogous procedures can be applied to these observables: Δ_{ab}^{NL} gives again rise to the difference (55), while Δ_{ab}^{TN} yields

$$\epsilon_{\lambda_a,\lambda_b,\lambda'_a\lambda'_b} = \alpha^J_{\lambda_a\lambda_b}\alpha^{J*}_{\lambda'_a\lambda'_b} - \overline{\alpha}^J_{-\lambda_a-\lambda_b}\overline{\alpha}^{J*}_{-\lambda'_a-\lambda'_b}, \tag{56}$$

for odd $\lambda'_a - \lambda_a$ and $\lambda'_b - \lambda_b$.

It is worth noting that eqs. (55) and (56) are both TRV and CP-odd, independent of the CPT symmetry.

5.2 More TRV Observables

More TRV observables can be obtained from the moduli and relative phases of the decay amplitudes. To this end it is convenient to pass from the helicity representation to the $l-s$ one, where l is the orbital angular momentum and s the overall spin of the two-particle state. In particular, interesting TRV quantities may be defined as

$$\varepsilon^J_{lss'} = \Im(\alpha^J_{ls}\alpha^{J*}_{l+1s'} + \overline{\alpha}^J_{ls}\overline{\alpha}^{J*}_{l+1s'}). \tag{57}$$

Here

$$\alpha^J_{ls} = \sum_{\lambda_a\lambda_b} C^{s\ \ l\ J}_{\Lambda\ 0\ \Lambda}\ \ C^{s_a\ \ s_b\ \ s}_{\lambda_a\ -\lambda_b\ \Lambda}\ \ \alpha^J_{\lambda_a\lambda_b} \tag{58}$$

and the C's are the usual Clebsch-Gordan coefficients. The $\varepsilon^J_{lss'}$, which can be inferred from information on moduli and relative phases, generalize the TRV observable proposed, *e. g.*, in refs. [8, 25].

5.3 Remarks

We conclude this section with some remarks, in part connected to the CPT symmetry.

First of all, the observables defined in this section are expressed as functions of the "reduced" amplitudes (16). This is not the same as obtained, *e. g.*, in refs. [8, 24, 25], where the very amplitudes are involved. Therefore our analysis has picked out new T-odd and TRV observables, different from the analogous observables defined in those references.

Secondly, the CPT symmetry implies that the TRV observables just defined are of the type

$$\sum_k \phi_k R_k, \tag{59}$$

where the R_k's are finite, T-even quantities and the ϕ_k's are phases causing time reversal violation.

Thirdly, it sometimes happens that the weak phase of a decay amplitude is negligibly small in comparison with the strong one[48]. It is more convenient in these cases to consider T-odd quantities analogous to eqs. (55) or (56), but with the + sign in place of the - sign, or of the type (57), but with the - sign between the two terms. These observables are invariant under TR and, according to the CPT symmetry, also CP-even; they are called fake T-odd. We shall see in sect. 8 that they may be employed in some tests, as already proposed[38, 48].

Fourthly, as regards the T-even asymmetries, consider differences of the type

$$\Delta_a^T = \Gamma(\Omega)A_a^T(\Omega) - \overline{\Gamma}(\overline{\Omega})\overline{A}_a^T(\overline{\Omega}), \tag{60}$$

where the same notations and assumptions as in eqs. (47) to (49) and (52) have been adopted. If significantly different than zero, these differences may consist, either of possible CPT-odd terms and products, or of a fake T-odd amplitude times a real TRV one. If CPT symmetry is assumed, these observables may be used to set constraints on decay models.

Lastly, it is worth spending some words on studies of CPT violation, a very hot topic at present[69, 70]. Indeed, the possibility of such a violation − connected to Lorentz invariance violation − was considered more than ten years ago by Coleman and Glashow[71, 72], who suggested experiments of neutrino oscillations. Among the most recent contributions, we mention the MiniBooNE[73] and Minos[74, 75] experiment, theoretical speculations[76, 77] and the numerous references cited therein. Incidentally, neutrino oscillations offer also the possibility of studying CPV and TRV in the leptonic sector[69, 70]. Moreover, intrinsic CPT violations, if any, have to be disentangled from fake violations, induced by neutrino-matter interaction[78], which is known as the MSW effect[79, 80].

6 An Explicit Example

In this section we specialize the main formulae of the previous sections to the sequential decay

$$\Lambda_b \to (p\pi^-)_\Lambda(\mu^+\mu^-)_{J/\psi}, \tag{61}$$

a particular case of decays (34). In this situation, the differential decay width and the asymmetries can be expressed as functions of the components along the unit vectors (6) of the polarization vector $\vec{\mathcal{P}}$ of the Λ_b resonance. Indeed, in the canonical frame, the Λ_b density matrix $\rho^{(0)}$ reads as

$$\rho^{(0)}_{\pm\pm} = \frac{1}{2} \pm \mathcal{P}_z, \qquad \rho^{(0)}_{\mp\pm} = \mathcal{P}_x \pm i\mathcal{P}_y. \tag{62}$$

On the other hand, the unit vectors (6) can be expressed as functions of the angles θ and ϕ in the canonical frame:

$$\begin{aligned} \hat{\mathbf{e}}_L &\equiv (sin\theta cos\phi, sin\theta sin\phi, cos\theta), & (63)\\ \hat{\mathbf{e}}_T &\equiv (-sin\phi, cos\phi, 0), & (64)\\ \hat{\mathbf{e}}_N &\equiv (-cos\theta cos\phi, -cos\theta sin\phi, sin\theta). & (65) \end{aligned}$$

The decay is characterized by four amplitudes, which we denote by $A_{1/2,1}$, $A_{-1/2,-1}$, $A_{1/2,0}$ and $A_{-1/2,0}$, dropping - from now on - the superscript J introduced in eq. (16). The differential decay width reads

$$\begin{aligned} \Gamma(\Omega) &= \frac{1}{4\pi}W(1 + 2\mathcal{P}_L\Delta G_L), & (66)\\ \Delta G_L &= |\alpha_{1/2,0}|^2 - |\alpha_{-1/2,0}|^2 - |\alpha_{1/2,1}|^2 + |\alpha_{-1/2,-1}|^2, & (67)\\ \mathcal{P}_L &= \vec{\mathcal{P}} \cdot \hat{\mathbf{e}}_L. & (68) \end{aligned}$$

Now we give the expressions of some of the asymmetries:

$$\begin{aligned} \Gamma(\Omega)A^L_\Lambda(\Omega) &= \frac{1}{2\pi}W\Delta a^\Lambda(B^+_L + 2\mathcal{P}_L B^-_L); & (69)\\ \Gamma(\Omega)A^N_\Lambda(\Omega) &= \frac{1}{\pi}W\Delta a^\Lambda[\Re(\alpha_{1/2,0}\alpha^*_{-1/2,0})\mathcal{P}_N + \Im(\alpha_{1/2,0}\alpha^*_{-1/2,0})\mathcal{P}_T]; & (70)\\ \Gamma(\Omega)A^T_\Lambda(\Omega) &= \frac{1}{\pi}W\Delta a^\Lambda[\Im(\alpha_{1/2,0}\alpha^*_{-1/2,0})\mathcal{P}_N - \Re(\alpha_{1/2,0}\alpha^*_{-1/2,0})\mathcal{P}_T]; & (71) \end{aligned}$$

$$\Gamma(\Omega)A_\Lambda^{LN}(\Omega) = \frac{3\sqrt{2}}{8\pi}W\Delta a^\Lambda[\Re(C)\mathcal{P}_N - \Im(C)\mathcal{P}_T]; \quad (72)$$

$$\Gamma(\Omega)A_\Lambda^{LT}(\Omega) = -\frac{3\sqrt{2}}{8\pi}W\Delta a^\Lambda[\Im(C)\mathcal{P}_N + \Re(C)\mathcal{P}_T]. \quad (73)$$

Here we have set

$$\Delta a^\Lambda = \frac{1}{2}(a_+^\Lambda - a_-^\Lambda), \quad (74)$$

$$B_L^\pm = |\alpha_{1/2,1}|^2 \pm |\alpha_{1/2,0}|^2 \mp |\alpha_{-1/2,-1}|^2 - |\alpha_{-1/2,0}|^2, \quad (75)$$

$$\mathcal{P}_N = \vec{\mathcal{P}} \cdot \hat{\mathbf{e}}_N, \qquad \mathcal{P}_T = \vec{\mathcal{P}} \cdot \hat{\mathbf{e}}_T, \quad (76)$$

$$C = \alpha_{1/2,1}\alpha^*_{1/2,0} - \alpha_{-1/2,0}\alpha^*_{-1/2,-1}, \quad (77)$$

$a_\pm^\Lambda$ being the quantities (17) referred to the decay of the Λ-resonance, with positive (+) or negative (-) helicity.

The remaining asymmetries defined in the preceding sections vanish, because they are proportional to the quantity

$$\Delta a^V = \frac{1}{2}(a_1^V - a_{-1}^V), \quad (78)$$

where V indicates the vector meson: this observable is zero, owing to parity conservation in strong and electromagnetic decays. However, information that we can extract from the non-vanishing asymmetries is sufficient to determining the moduli and relative phases of the reduced amplitudes. Indeed, the moduli can be inferred from $\Gamma(\Omega)$ and $A_\Lambda^L(\Omega)$; moreover the relative phase between $\alpha_{1/2,0}$ and $\alpha^*_{-1/2,0}$ can be extracted from $A_\Lambda^N(\Omega)$ or from $A_\Lambda^T(\Omega)$; lastly, the sine and cosine of the other two relative phases are related, respectively, to $\Im(C)$ and to $\Re(C)$, whence the two phases can be deduced.

7 Relations to Previous Works

In this section we compare our results with those of other authors. In particular, as we shall see, our choice of considering correlations constructed exclusively from momenta results to be rather general. Indeed, such correlations turn out to be spin dependent, yielding results equivalent to those which involve one or more spins. A strong indication in this sense comes from the observation that the asymmetries defined in sects. 2 and 3 vanish if both the parent resonance and the decay products are spinless.

7.1 Hadronic Λ_b Decays

Let us focus on decays of the type (4), such that J and s_a or/and s_b are nonzero. For example, consider decays (34), studied by several authors[24, 40, 50, 51, 55] and also detected experimentally[81, 82, 83, 84]. We define the following products:

$$\mathbf{s}_{a(b)} \cdot \mathbf{e}_N, \qquad \mathbf{s}_{a(b)} \cdot \mathbf{e}_T, \qquad \mathbf{s}_{a(b)} \cdot \mathbf{e}_L, \tag{79}$$

whose mean values are the components of the polarization vector of one of the two decay products, a or b; we denote such components, respectively, as P_N, P_T and P_L. P_N is T-odd, while P_T and P_L are T-even. If both a and b are spinning, one can also define polarization correlations, like, *e. g.*,

$$P_{TN} = \langle \mathbf{s}_a \cdot \mathbf{e}_T \; \mathbf{s}_b \cdot \mathbf{e}_N \rangle. \tag{80}$$

This is the case of the decays (34), if a vector meson V is involved. Here the normal component P_N of the polarization of the final fermion reads as[61]

$$\Gamma(\Omega) P_N^\Lambda(\Omega) \; = \; \frac{1}{4\pi}(G_N^\Lambda \; + \; \Delta G_N^\Lambda \mathcal{P}_L), \tag{81}$$

where

$$G_N^\Lambda \;\; = \;\; \Re\Big(A_{1/2,1} A^\star_{-1/2,0} + A_{-1/2,-1} A^\star_{1/2,0}\Big), \tag{82}$$

$$\Delta G_N^\Lambda \;\; = \;\; -2\Re\Big(A_{1/2,1} A^\star_{-1/2,0} - A_{-1/2,-1} A^\star_{1/2,0}\Big). \tag{83}$$

As regards the polarization correlation P_{TN}, we have[61]

$$\Gamma(\Omega) P_{TN}(\Omega) = \frac{1}{4\pi\sqrt{2}}(\Delta G_{TN} \; + \; G_{TN} \mathcal{P}_L), \tag{84}$$

with

$$G_{TN} \;\; = \;\; 2\Im(A_{-1/2,-1} A^\star_{1/2,0} + A_{1/2,1} A^\star_{-1/2,0}), \tag{85}$$

$$\Delta G_{TN} \;\; = \;\; \Im(A_{-1/2,-1} A^\star_{1/2,0} - A_{1/2,1} A^\star_{-1/2,0}). \tag{86}$$

As can be seen, such polarization formulae consist of linear combinations of the parameters which appear in the single and double asymmetries defined in sects. 2 and 3. Also the other components of the polarization vector and the polarization correlations are related to our present results in a similar way[61]: see also sect. 6. It is reasonable to think that this result is not peculiar to the decay considered; we conjecture that the asymmetries defined in the present paper provide, in principle, information on the components of the polarization vector and polarization correlations. Even, the results of our analysis may be quite suitable for determining the components of the polarization vector of a decay product which carries a spin greater than 1/2; indeed, it is not so easy to measure directly this polarization.

7.2 $B_{(s)} \to V_1 V_2$ Decays

As we have seen in sect. 3, double asymmetries include decays of the type (3), for which our analysis yields the following T-odd observables:

$$\Im(\alpha_{11}\alpha^*_{00}) \qquad \text{and} \qquad \Im(\alpha_{-1-1}\alpha^*_{00}). \tag{87}$$

A linear combination of these terms corresponds to the usual T-odd quantity[21, 25, 34, 38, 48]

$$\Im(A_\perp A^*_0), \tag{88}$$

where the A's are the decay amplitudes in the transversity representation. In fact, we have the following relations:

$$A_0 = F\alpha_{00}, \quad A_\parallel = \frac{1}{\sqrt{2}}F(\alpha_{11} + \alpha_{-1-1}) \quad A_\perp = \frac{1}{\sqrt{2}}F(\alpha_{11} - \alpha_{-1-1}), \tag{89}$$

with

$$F^2 = \sum_\lambda |A_\lambda|^2 = |A_0|^2 + |A_\parallel|^2 + |A_\perp|^2. \tag{90}$$

Moreover, for the decays considered, eq. (56) yields the following TRV observables:

$$\Im(\alpha_{11}\alpha^*_{00} - \overline{\alpha}_{-1-1}\overline{\alpha}^*_{00}) \qquad \text{and} \qquad \Im(\alpha_{-1-1}\alpha^*_{00} - \overline{\alpha}_{11}\overline{\alpha}^*_{00}), \tag{91}$$

which are linearly related to $\Im(A_\perp A^*_0)$, to $\Im(A_\parallel A^*_0)$ and to their CP-conjugated quantities.

Some remarks are in order.

a) It is worth stressing that also in this case our method yields results which turn out to be very similar to those found by the authors just mentioned, although they adopt the T-odd product

$$\mathbf{p}_a \cdot \mathbf{s}_a \times \mathbf{s}_b. \tag{92}$$

b) The usual observables f_L and f_T[21, 23, 35, 38, 39, 48], concerning respectively longitudinal and transverse polarization of the vector mesons in a decay (3), and used for testing the SM[38, 39, 47], are simply related to our "reduced" amplitudes:

$$f_L = |\alpha_{00}|^2, \qquad f_T = |\alpha_{11}|^2 + |\alpha_{-1-1}|^2. \tag{93}$$

c) The T-odd observable[25] $\Im(A_\perp A_\parallel^*)$ does not appear among the parameters of our analysis. This is due to the fact that we have integrated over some angular variables in the sequential decay, whereas one usually adopts a full angular analysis[16, 17, 28, 29, 85], in terms of two polar angles and an azimuthal one[42, 44, 86]. But if, as discussed in sects. 4 and 6, our method allows to extract the moduli and relative phases of *all* of the decay amplitudes, the above mentioned term can be deduced indirectly, as well as those which do not appear as parameters of our analysis. Our procedure may be suitably followed in cases where the statistics is not so rich as in Babar[28] and Belle[29] experiments.

d) The TRV observables proposed by previous authors, *e. g.*,

$$\Im(A_\perp A_0^* + \overline{A}_\perp \overline{A}_0^*), \tag{94}$$

involve just the decay amplitudes. But we observe that the proportionality constant $\overline{F}$ relative to the CP-conjugated decay is different than F. This implies that the observable (94) is substantially different than (91). More generally, it confirms once more that the TRV observables defined in subsects. 5.1 and 5.2 are really different than those proposed in the literature.

7.3 Detection of Top Decays and New Particle Decays

Also some of the top decays[9, 58, 59] and of the possible new particle sequential decays[18, 20, 22, 60] are studied by means of triple products. In particular, it is pointed out that correlations from three or more momenta[18] appear to be more appropriate for experimental detection and that the maximal asymmetry is obtained by operating in the rest frame of the decaying particle[18].

8 Tests of Self-Consistency and of SM

Here we propose some tests, based on the observables illustrated in sections 2, 3 and 5. Moreover, as shown in sect. 4 and in Appendix, the analysis that we have illustrated allows to determine quantities of the type

$$|A_1|^2, \qquad |A_2|^2, \qquad \Re(A_1 A_2^*), \qquad \Im(A_1 A_2^*), \tag{95}$$

where A_1 and A_2 are two different helicity amplitudes of a given decay mode. Now we propose various kinds of tests, based on the knowledge of such parameters and of others, related to them.

8.1 Self-consistency Tests

A first kind of tests is a self-consistency one. We suggest to exploit the identities of the type

$$[\Re(A_1 A_2^*)]^2 + [\Im(A_1 A_2^*)]^2 = |A_1|^2 |A_2|^2, \tag{96}$$

as constraints on the parameters to be extracted from data.

8.2 Tests of Moduli and Phases

As told in the introduction, it is always convenient to look for observables whose values are small according to the SM, since they may give a clear indication of NP. In particular, it appears suitable to study decays such that the SM predicts negligibly small TRV or/and direct CPV.

Some tests in this sense were already proposed in a preceding paper[49]. For instance, we suggested to consider the asymmetries

$$\mathcal{A}_{CP} = \frac{\Phi_{\lambda_a\lambda_b} - \overline{\Phi}_{-\lambda_a-\lambda_b}}{\Phi_{\lambda_a\lambda_b} + \overline{\Phi}_{-\lambda_a-\lambda_b}} \quad \text{and} \quad \mathcal{A}_M = \frac{|A_{\lambda_a\lambda_b}|^2 - |\overline{A}_{-\lambda_a-\lambda_b}|^2}{|A_{\lambda_a\lambda_b}|^2 + |\overline{A}_{-\lambda_a-\lambda_b}|^2}. \tag{97}$$

Here $\Phi_{\lambda_a\lambda_b}$ is the relative phase of the amplitude $A_{\lambda_a\lambda_b}$ to a fixed one, taken as a reference.

Aside from that, as already observed by various authors (see, *e. g.*, ref. [8, 21, 24, 25, 34]), a hierarchy among the CP-odd or T-odd observables has to be established, according to their sensitivity to NP. For example, consider the interference term $A_1A_2^*$. In order to evaluate qualitatively the behavior of this term, we assume a quite simplified model, such that each amplitude is of the form

$$A = Te^{i\omega_T} + Pe^{i(\omega_P+\delta_P)}. \tag{98}$$

Here we have omitted, for the sake of simplicity, the index 1 or 2; moreover the former term corresponds to the tree contribution and the latter to the penguin graph; in particular, the ω's (assumed to be helicity independent) and the δ's are, respectively, the weak and the strong phases. As usual, we assume δ_P to be attributed exclusively the absorptive part of the penguin diagram[7]. Now we substitute eq. (98) into the interference term and compare it with its CP-conjugated quantity. As a result we get

$$\Re(A_1A_2^* - \overline{A}_1\overline{A}_2^*) = -2sin\Delta\omega(|P_1||T_2|sin\delta_{P1} + |P_2||T_1|sin\delta_{P2}), \tag{99}$$

$$\Im(A_1A_2^* - \overline{A}_1\overline{A}_2^*) = 2sin\Delta\omega(|P_1||T_2|cos\delta_{P1} - |P_2||T_1|cos\delta_{P2}), \tag{100}$$

where $\Delta\omega = \omega_P - \omega_T$. Then we conclude that, if the strong phase is small, the real part of the interference term is surely negligible, while the imaginary part may be sizeable, provided the difference $||P_1||T_2| - |P_2||T_1||$ is sufficiently large[7]. Therefore the imaginary parts of the complex quantities (55) and (56), as well as the observable (57), appear favourite in the search for possible clues of NP.

8.3 Helicity tests

Here we suggest an interesting test of the SM, as a generalization of the one proposed in ref.[48]. This test is not connected to interference terms, but it is an important byproduct of the helicity representation, used in our treatment. Indeed, the SM predicts positive helicity amplitudes to be strongly suppressed with respect to negative helicity ones, if heavy quarks are involved in a decay. This is verified at an empirical level, although theoretical arguments are not so sound[39]. Then violation of this inequality may be an indication of NP. Incidentally, if the decay products are spinning, and at least one of them is a vector meson, it is interesting to compare the 0-helicity amplitudes with the corresponding ones with ± 1-helicity amplitudes. Factorization, generally satisfied by the tree contribution, predicts that the 0-helicity amplitudes are much greater[39]. On the contrary, the penguin contribution, for which factorization fails, yields comparable 0- and ± 1-helicity amplitudes[39]. Therefore the ratios between such amplitudes indicate the relative weight of penguin to tree term. It is worth noting that such kinds of tests can be suitably performed by means of the fake T-odd observables introduced in sect. 5, especially if the weak phases are negligibly small[48].

8.4 Λ_b and Λ_c Decays

It is useful to consider the application of the previous tests to decays of heavy baryons, for example to those described by eq. (34), in view of the forthcoming LHCb data. Contributions in this sense, especially if one of the decay products is a vector meson V, have been already given[24, 40, 49, 50, 51, 55, 61, 87]. According to the helicity tests, we expect $|A_{1/2,1}| << |A_{-1/2,-1}|$ and $|A_{1/2,0}| << |A_{-1/2,0}|$, the A's being the decay amplitudes introduced in sect. 6.

Moreover the ratios $|A_{-1/2,-1}/A_{-1/2,0}|$ and $|A_{1/2,1}/A_{1/2,0}|$ may give indications on the proportions with which penguin and tree diagram contribute to the decay amplitudes. Let us consider the case of $V = J/\psi$[61]. Assuming amplitudes of the type (98), the SM predicts

$$Te^{i\omega_T} = V_{bc}V^*_{sc}T', \qquad\qquad Pe^{i(\omega_P+\delta_P)} = \sum_q V_{bq}V^*_{sq}P'_q. \tag{101}$$

Here the V's are elements of the CKM matrix, T, P and T' are real positive numbers and $q = u, c, t$. Taking into account the orthogonality condition

$$\sum_q V_{bq}V^*_{sq} = 0, \tag{102}$$

and the fact that $V_{bu}V^*_{su}$ is negligible in comparison with the other two terms, we conclude that the phase difference $\Delta\omega$ is quite small according to the SM. However, recent experimental results, concerning both $B_s - \overline{B}_s$ mixing[30, 31] and direct CP violation asymmetry in $B \to K\pi$ decays[27, 28, 29], indicate that this phase could be considerably larger. Therefore it is worth applying our analyses to these decay modes. Incidentally, we signal that a commonly used model[88, 89, 51] predicts $P/T \sim 0.134$[51].

Even more intriguing is the case of $V = \rho^0, \omega$[87], or the decay $\Lambda_b \to \Lambda\pi^0$. Indeed, here the penguin contribution is greater than the tree; therefore the interference between the two terms, which includes the CP-violating phase, is quite relevant. Moreover, the diagrams involved are quite similar to those which occur in the decays $B \to K\pi$[27, 28, 29]. Therefore it is worth doing efforts for revealing such decay modes of Λ_b and for applying to them the tests suggested in this section.

As a further example of decay, consider[90]

$$\Lambda_c \to \Lambda\pi^+. \tag{103}$$

In this case no CP violation has been observed[90]; however an analysis of the triple product asymmetry is suggested, since one has to do with a decay where the relative strong phase of the two amplitudes is quite small. Obviously, our method appears to be appropriate also in this case.

9 Conclusions

We have elaborated some methods for analyzing hadronic sequential two-body decays involving more spinning particles. In particular, we have suggested to investigate several distributions, based on T-odd and T-even, simple or double correlations. Unlike other authors[43], who try to disentangle different CP eigenstates, we exploit just the interference between such eigenstates. The decays considered offer a richer range of observables sensitive to CPV and TRV. They may also help to find hints to NP, provided we focus preferably on observables for which the SM predicts quite small values. Now we exhibit the highlights of our analysis.

a) Our main result is that, given a set of data concerning the above mentioned decays, one can always infer a set of TRV observables, even after integrating over all variables, except for the direction of one of the two decay products in the initial decay. We stress that this would not be possible if less than two of the particles involved in this decay are spinning. Some of these observables are especially sensitive to TRV and to possible clues beyond the SM. Among them we signal the imaginary parts of the interference terms like (55) and (56), which are rather small according to the SM predictions. In particular, the study of the interference terms in the decay modes $\Lambda_b \to \Lambda\rho^0, \omega$ appears very interesting in this sense.

b) We have shown that the T-odd correlations based just on momenta (three or more), which may be more easily adopted in an experiment, provide the same results as those involving one or more spins.

c) Thirdly, T-even observables - quite helpful, although a bit neglected in the current literature - may be combined with their CP-conjugated ones, so as to include either possible CPT-odd terms or products of a fake T-odd amplitude, times a real TRV one. These observables, as well as fake T-odd ones, may be used to set constraints on the T-odd terms caused by strong interactions, typically spin-orbit ones.

d) Our treatment recovers, as particular cases, some methods suggested by other authors. For example, the analysis of the helicity amplitudes, rather efficient as a test of the SM, is an important byproduct of our choice of the helicity representation. Furthermore, new TRV observables have been picked out, the "reduced" amplitudes, which generalize the polarization parameters f_L and f_T, used for vector mesons.

e) Lastly, our method may allow, at least in some cases, to determine the moduli and the relative phases of the decay amplitudes. These quantities allow, in turn, to infer further, especially sensitive TRV observables and the polarization vectors of the decay products, otherwise difficult to determine for, say, a vector meson.

The analysis proposed here could be usefully applied to forthcoming data, especially at LHCb, but also in experiments where the available statistics is not so rich and abundant.

Acknowledgments

The authors are very grateful to their friend J. Orloff for very useful and illumi-

nating discussions.

Appendix

We describe a method for extracting from data the moduli squared of the helicity decay amplitudes and some interference terms between them. The method is based on the moment expansion of the various distributions defined in the text, that is, $\Gamma(\Omega)$, $A_a^N(\Omega)\Gamma(\Omega)$, etc., which we denote generically by $F(\Omega)$. Each such distribution contains the product $\mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda'}(\Omega)$ of Wigner $\mathcal{D}$-matrices. Therefore our starting point is the well-known relation

$$\mathcal{D}^{J*}_{M\Lambda}(\Omega)\mathcal{D}^{J}_{M'\Lambda'}(\Omega) = \sum_L C^{J\ L\ J}_{M'NM} C^{JLJ}_{\Lambda'\nu\Lambda} \mathcal{D}^{L*}_{N\nu}(\Omega). \tag{A. 1}$$

Here the C's are the usual Clebsch-Gordan coefficients. Inserting this into $F(\Omega)$, we get an expansion of the type

$$F(\Omega) = \sum_{LN\nu} H^J_{LN\nu} \mathcal{D}^{L*}_{N\nu}(\Omega). \tag{A. 2}$$

Here

$$H^J_{LN\nu} = t^{J*}_{LN} f^J_{L\nu}, \qquad t^{J*}_{LN} = \sum_{MM'} \rho^{(0)}_{MM'} C^{J\ L\ J}_{M'NM} \tag{A. 3}$$

and the $f^J_{L\nu}$'s vary from distribution to distribution, as we shall specify below. They depend only on the decay amplitudes, while the coefficients t^{J*}_{LN} depend only on the production reaction of the parent resonance.

A.1 - Extracting Moduli of Decay Amplitudes

Now we give the expressions of the coefficients $f^J_{L\nu}$ for three distributions, which depend solely on the moduli of the decay amplitudes, that is, $\Gamma(\Omega)$, $\Gamma(\Omega)A^L_{a(b)}(\Omega)$ and $\Gamma(\Omega)A^{LL}(\Omega)$, whose expressions are given, respectively, by eqs. (24), (30) and (43). We have

$$f^{J(\Gamma)}_{L\nu} = \mathcal{N}_J W \delta_{\nu 0} \sum_{\lambda_a,\lambda_b} |\alpha^J_{\lambda_a\lambda_b}|^2 C^{JLJ}_{\Lambda 0\Lambda}, \tag{A. 4}$$

$$f^{J(A^\ell_a)}_{L\nu} = 8\pi \mathcal{N}_{Ja} W \delta_{\nu 0} \sum_{\mu_a>0} \Delta a^{s_a}_{\mu_a} \sum_\Lambda \Delta a^J_{\Lambda\mu_a} C^{JLJ}_{\Lambda 0\Lambda}, \tag{A. 5}$$

$$f_{L\nu}^{J(A^{\ell\ell})} = 16\pi^2 \mathcal{N}_{Jab} W \delta_{\nu 0} \sum_{\mu_a>0} \sum_{\mu_b>0} \Delta a_{\mu_a}^{s_a} \Delta a_{\mu_b}^{s_b} \sum_{\Lambda} \Delta^{(2)}_{\Lambda\mu_a\mu_b} C^{JLJ}_{\Lambda 0 \Lambda}. \quad \text{(A. 6)}$$

Here the various symbols introduced are defined in the text. We just reproduce those expressions which contain the "reduced" amplitudes $\alpha^J_{\lambda_a\lambda_b}$:

$$\Delta a^J_{\Lambda\mu_a} = \frac{1}{2} \sum_{\lambda_a>0} \delta^{s_a}_{\lambda_a\mu_a} (|\alpha^J_{\lambda_a\lambda_b}|^2 - |\alpha^J_{-\lambda_a\lambda_b'}|^2), \quad \text{(A. 7)}$$

$$\Delta^{(2)}_{\Lambda\mu_a\mu_b} = \sum_{\lambda_a} |\alpha^J_{\lambda_a\lambda_b}|^2 \delta^{s_a}_{\lambda_a\mu_a} \delta^{s_b}_{\lambda_a\mu_b}. \quad \text{(A. 8)}$$

Moreover the upper indices of the coefficients $f^J_{L\nu}$ refer to the specific distributions considered; in particular, the index ℓ refers to "longitudinal asymmetry", not to be confused with the order L of the moment. Lastly, the coefficient $f_{L\nu}^{J(A_b^\ell)}$ can be defined analogously to eq. (A. 5), by interchanging index a with index b. Now we consider the ratios

$$r_L^{a\ell} = H_{LN0}^{J(A_a^\ell)} / H_{LN0}^{J(\Gamma)}, \quad \text{(A. 9)}$$

$$r_L^{b\ell} = H_{LN0}^{J(A_b^\ell)} / H_{LN0}^{J(\Gamma)}, \quad \text{(A. 10)}$$

$$r_L^{\ell\ell} = H_{LN0}^{J(A^{\ell\ell})} / H_{LN0}^{J(\Gamma)}. \quad \text{(A. 11)}$$

Here, again, the upper indices of the H^J's are typical of the distribution considered. First of all, it is important to note that the ratios (A. 9) to (A. 11) are independent of N. This constitutes a check for the moments of the distributions. Furthermore, by recalling the relations above, we obtain the following linear system in the moduli squared of the various $\alpha^J_{\lambda_a\lambda_b}$:

$$r_L^{a\ell} \sum_{\lambda_a,\lambda_b} |\alpha^J_{\lambda_a\lambda_b}|^2 C^{JLJ}_{\Lambda 0\Lambda} = 8\pi \frac{\mathcal{N}_{Ja}}{\mathcal{N}_J} \sum_{\mu_a>0} \Delta a^{s_a}_{\mu_a} \sum_{\Lambda} \Delta a^J_{\Lambda\mu_a} C^{JLJ}_{\Lambda 0\Lambda}, \quad \text{(A. 12)}$$

$$r_L^{b\ell} \sum_{\lambda_a,\lambda_b} |\alpha^J_{\lambda_a\lambda_b}|^2 C^{JLJ}_{\Lambda 0\Lambda} = 8\pi \frac{\mathcal{N}_{Ja}}{\mathcal{N}_J} \sum_{\mu_b>0} \Delta a^{s_b}_{\mu_b} \sum_{\Lambda} \Delta a^J_{\Lambda\mu_b} C^{JLJ}_{\Lambda 0\Lambda}, \quad \text{(A. 13)}$$

$$r_L^{\ell\ell} \sum_{\lambda_a,\lambda_b} |\alpha^J_{\lambda_a\lambda_b}|^2 C^{JLJ}_{\Lambda 0\Lambda} = 16\pi^2 \frac{\mathcal{N}_{Jab}}{\mathcal{N}_J} \sum_{\mu_a>0} \sum_{\mu_b>0} \Delta a^{s_a}_{\mu_a} \Delta a^{s_b}_{\mu_b} \sum_{\Lambda} \Delta^{(2)}_{\Lambda\mu_a\mu_b} C^{JLJ}_{\Lambda 0\Lambda}. \quad \text{(A. 14)}$$

The parameters $\Delta a^{s_{a(b)}}_{\mu_{a(b)}}$, relative to the secondary decays, are generally known. The linear system (A. 12)-(A. 14) is *not* homogeneous, owing to the constraint

$$\sum_{\lambda_a,\lambda_b} |\alpha^J_{\lambda_a\lambda_b}|^2 = 1. \quad \text{(A. 15)}$$

It is generally over-determined, as we shall see in some examples of interest, provided all of the asymmetries are nontrivial.

A.2 - Inferring Relative Phases of Decay Amplitudes

The relative phases of the decay amplitudes may be determined starting from the other distributions considered in the text, where interference terms between amplitudes with different helicities are involved. Here we limit ourselves to two of the T-odd distributions, but the method we suggest can be extended to any distribution of sects. 2 and 3.

According to our formalism, the moments related to the distribution $\Gamma(\Omega)A_{ab}^{NT}(\Omega)$ (see the first eq. (A. 3)) yield the following decay coefficients $f^J_{L\nu}$:

$$\begin{aligned} f^{J(NT)}_{L\nu} &= \mathcal{N}_{Jab} W \sum_{\mu_a,\mu_b} C^{JLJ}_{\Lambda'\nu\Lambda} a^{s_a}_{\mu_a} a^{s_b}_{\mu_b} \\ &\times \sum_{\lambda_a}\sum_{\lambda_b}\sum_{\lambda'_a}{}'\sum_{\lambda'_b}{}' \Theta^{s_a}_{\lambda_a\lambda'_a\mu_a}\Theta^{s_b}_{\lambda_b\lambda'_b\mu_b}\frac{16i(-)^{D_a}}{(\lambda_a-\lambda'_a)(\lambda_b-\lambda'_b)} \\ &\times \alpha^J_{\lambda_a\lambda_b}\alpha^{J*}_{\lambda'_a\lambda'_b}, \end{aligned} \tag{A. 16}$$

$$\Lambda = \lambda_a-\lambda_b, \qquad \Lambda' = \lambda'_a-\lambda'_b, \qquad D_a = (\lambda_a-\lambda'_a-1)/2. \tag{A. 17}$$

Here, as in the text, the symbol $\sum'$ means that $\lambda_a-\lambda'_a$ or $\lambda_b-\lambda'_b$ is an odd quantity. Analogously, the distribution $\Gamma(\Omega)A_{ab}^{LN}(\Omega)$ yields

$$\begin{aligned} f^{J(LN)}_{L\nu} &= 2\pi\mathcal{N}_{Jab} W \sum_{\mu_a,\mu_b} C^{JLJ}_{\Lambda'\nu\Lambda} a^{s_a}_{\mu_a} a^{s_b}_{\mu_b} \\ &\times \sum_{\lambda_a}\sum_{\lambda_b}\sum_{\lambda'_b}{}' \delta^{s_a}_{\lambda_a\mu_a}\Theta^{s_b}_{\lambda_b\lambda'_b\mu_b}\frac{4(-)^{D_b}}{\lambda_b-\lambda'_b} \\ &\times \alpha^J_{\lambda_a\lambda_b}\alpha^{J*}_{\lambda_a\lambda'_b}. \end{aligned} \tag{A. 18}$$

Moreover $D_b = (\lambda_b-\lambda'_b-1)/2$ and eqs. (A. 17) hold. Other equations can be obtained from the other distributions considered in sects. 2 and 3. Once the moduli of the decay amplitudes are known, the ratios $H^{J(NT)}_{LN\nu}/H^{J(\Gamma)}_{LN0}$, $H^{J(LN)}_{LN\nu}/H^{J(\Gamma)}_{LN0}$ and the other ratios obtained from the various distributions depend only on the relative phases of such amplitudes. It can be checked that, in the cases of interest (see the next subsection), a complete set of $n-1$ *independent* relative phases - n being the number of helicity amplitudes - is present in the equations that we obtain from these ratios.

Again, if the asymmetries are all nontrivial, this system is over-determined in the cases of interest.

A.3 - Some Examples

We present some cases of interest of two-body decays in Table 1, where we give the number of unknowns and of available equations. As regards relative phases, we have taken into account only the conditions deriving from the knowledge of the moments defined in subsect. A.2.

Table 1: Moduli and relative phases in different channels

Decay	N_m	N_m^e	N_p	N_p^e
$1 \to 0 \quad 0$	1	1	0	0
$0 \to 1 \quad 1$	3	4	2	2
$0 \to 1/2 \quad 1/2$	2	4	1	2
$1/2 \to 1/2 \quad 0$	2	7	1	4
$1/2 \to 1/2 \quad 1$	4	7	3	4
$3/2 \to 1/2 \quad 1$	6	13	5	8

Here $N_{m(p)}$ is the number of moduli (relative phases) of of the decay amplitudes in the various channels and $N^e_{m(p)}$ the respective numbers of equations.

References

[1] T.D. Lee and C.N. Yang: *Phys. Rev.* **104** (1956) 254

[2] C.S. Wu *et al.*: *Phys. Rev.* **105** (1957) 1413

[3] R.P. Feynman and M. Gell-Mann: *Phys. Rev.* **109** (1958) 193

[4] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay: *Phys. Rev. Lett.* **13** (1964) 138

[5] J.W. Cronin, P.F. Kunz, W.S. Risk and P.C. Wheeler: *Phys. Rev. Lett.* **18** (1967) 25

[6] T. Brown, S.F. Tuan and S. Pakvasa: *Phys. Rev. Lett.* **51** (1983) 1823

[7] L. Wolfenstein: *Phys. Rev. D* **43** (1991) 151

[8] G. Valencia: *Phys. Rev. D* **39** (1989) 3339

[9] D. Atwood *et al.*: *Phys. Rept.* **347** (2001) 1

[10] A. DeRujula *et al.*: *Nucl. Phys. B* **35** (1971) 365

[11] A. Bilal *et al.*: *Nucl. Phys. B* **355** (1991) 549

[12] D. Sivers: *Phys. Rev. D* **74** (2006) 094008

[13] S.J. Brodsky, D.S. Hwang and I. Schmidt: *Phys Lett. B* **530** (2002) 99

[14] S.J. Brodsky, D.S. Hwang and I. Schmidt: *Nucl. Phys. B* **642** (2002) 344

[15] V.M. Abazov *et al.*, D0 Coll.: *Phys. Lett. B* **668** (2008) 357

[16] B. Aubert *et al.*, BABAR Coll.: *Phys. Rev. Lett.* **93** (2004) 231804

[17] K.F. Chen *et al.*, BELLE Coll.: *Phys. Rev. Lett.* **94** (2005) 221804

[18] G. Moortgart-Pick *et al.*: *JHEP* **1001** (2010) 004

[19] K. Rolbiecki *et al.*: *Fortsch. Phys.* **58** (2010) 699

[20] J. Ellis *et al.*: *Eur. Phys. J. C* **60** (2009) 633

[21] A. Datta *et al.*: *Phys. Rev. D* **76** (2007) 034015

[22] P. Langacker *et al.*: *JHEP* **07** (2007) 055

[23] S. Baek *et al.*: *Phys. Rev. D* **72** (2005) 094008

[24] W. Bensalem, A. Datta and D. London: *Phys. Rev. D* **66** (2002) 094004

[25] A. Datta and D. London: *Int. Jou. Mod. Phys. A* **19** (2004) 2505

[26] M.B. Gavela *et al.*: *Nucl. Phys. B* **430** (1994) 382

[27] S. Chen *et al.*, CLEO Coll.: *Phys. Rev. Lett.* **85** (2000) 525

[28] B. Aubert *et al.*, BABAR Coll.: *Phys. Rev. Lett.* **98** (2007) 051801

[29] S.W. Lin *et al.*, BELLE Coll.: *Nature* **452** (2008) 332

[30] T. Aaltonen *et al.*, CDF Coll.: *Phys. Rev. Lett.* **100** (2008) 161802

[31] V.M. Abazov *et al.*, D0 Coll.: *Phys. Rev. Lett.* **101** (2008) 241801

[32] V.M. Abazov *et al.*, D0 Coll.: *Phys. Rev. D* **82** (2010) 032001

[33] L. Hofer, D. Scherer and L. Vernazza: *JHEP* **1102** (2011) 080

[34] X.-W. Kang and H.-B. Li: *Phys. Lett. B* **684** (2010) 137

[35] C.W. Chiang *et al.*: *JHEP* **1004** (2010) 31

[36] S. Faller, R. Fleischer and T. Mannel: *Phys. Rev. D* **79** (2009) 014005

[37] S. Stone and L. Zhang: *Phys. Rev. D* **79** (2009) 074024

[38] A. Datta, M. Imbeault and D. London: *Phys Lett. B* **671** (2009) 256

[39] M. Beneke, J. Rohrer and D. Yang: *Nucl. Phys. B* **774** (2007) 64

[40] W. Bensalem, A. Datta and D. London: *Phys. Lett. B* **538** (2002) 309

[41] W. Bensalem and D. London: *Phys. Rev. D* **64** (2001) 116003

[42] A.S. Dighe, I. Dunietz, H.J. Lipkin and J.L. Rosner: *Phys. Lett. B* **369** (1996) 144

[43] I. Dunietz *et al.*: *Phys. Rev. D* **43** (1991) 2193

[44] C.W. Chiang and L. Wolfenstein: *Phys. Rev. D* **61** (2000) 074031

[45] C.W. Chiang: *Phys. Rev. D* **62** (2000) 014017

[46] C.-H. Chen, C.-Q. Geng and L. Li: *Phys Lett. B* **670** (2009) 374

[47] A.L. Kagan: *Phys. Lett. B* **601** (2004) 151

[48] A. Datta, M. Duraisamy and D. London: *Phys Lett. B* **701** (2011) 357

[49] Z. J. Ajaltouni and E. Di Salvo: *Jou. Phys. G: Nucl. Part. Phys.* **37** (2010) 125001

[50] S. Arunagiri and C.Q. Geng: *Phys. Rev. D* **69** (2004) 0307307

[51] Z.J. Ajaltouni *et al.*: *Phys. Lett. B* **614** (2005) 165

[52] N. Nagashima, A. Szynkman and D. London: *Mod. Phys. Lett. A* **19** (2008) 1175

[53] W. Bensalam, D. London, N. Sinha and R. Sinha: *Phys. Rev. D* **63** (2003) 034007

[54] C.-H. Chen and C.-Q. Geng: *Phys. Rev. D* **66** (2002) 094018

[55] C.-H. Chen, C.-Q. Geng and J.N. Ng: *Phys. Rev. D* **65** (2002) 091502

[56] T.M. Aliev *et al.*: *Phys. Lett. B* **542** (2002) 229

[57] C.-H. Chen and C.-Q. Geng: *Phys. Rev. D* **64** (2001) 074001

[58] O. Antipin and G. Valencia: *Phys. Rev. D* **79** (2009) 013013

[59] K. Hagiwara, K. Mawatari and H. Yokoya: *JHEP* **0712** (2007) 41

[60] A. Bartl *et al.*: *Phys. Rev. D* **70** (2004) 095007

[61] E. Di Salvo and Z. J. Ajaltouni: *Mod. Phys. Lett. A* **24** (2009) 109

[62] M. Jacob and G.C. Wick: *Ann. Phys.* **7** (1959) 404

[63] M. Ademollo, R. Gatto and G. Preparata: *Phys. Rev.* **140 B** (1965) 192

[64] P. Minnaert: *Phys. Rev.* **151** (1966) 1306

[65] N. Byers and S. Fenster: *Phys. Rev. Lett.* **11** (1963) 52

[66] M. Ademollo and R. Gatto: *Phys. Rev.* **133 B** (1964) 531

[67] M. Ademollo, R. Gatto and G. Preparata: *Phys. Rev.* **139 B** (1965) 1608

[68] A.R. Edmonds: "Angular momentum in quantum mechanics", Princeton University Press, Princeton, N. J., 1957

[69] L.S. Kisslinger, E.M. Henley and M.B. Johnson: arXiv:1105.2741/hep-ph

[70] E.M. Henley, M.B. Johnson and L.S. Kisslinger: *Int. Jou. Mod. Phys. E* **20** (2011) 2463

[71] S. Coleman and S.L. Glashow: *Phys Lett. B* **405** (1997) 249

[72] S. Coleman and S.L. Glashow: *Phys. Rev. D* **59** (1999) 116008

[73] MiniBooNE Coll.: *Phys. Rev. Lett.* **105**(2010) 181801

[74] Minos Coll.: *Phys. Rev. Lett.* **103**(2009) 261802

[75] Minos Coll.: *Phys. Rev. D* **81** (2010) 052004

[76] H. Davoudiasl, H.-S. Lee and W.J. Marciano: *Phys. Rev. D* **84** (2011) 013009

[77] M.C. Gonzalez-Garcia, M. Maltoni and J. Salvado: *JHEP* **1105** (2011) 075

[78] M. Jacobson and T. Ohlsson: *Phys. Rev. D* **69** (2004) 013003

[79] L. Wolfenstein: *Phys. Rev. D* **17** (1978) 2369

[80] S.P. Mikheev and A.Yu. Smirnov: *Sov. J. Nucl. Phys.* **42** (1985) 913

[81] T. Aaltonen *et al.*, CDF Coll.: *Phys. Rev. Lett.* **104** (2010) 102002

[82] V.M. Abazov *et al.*, D0 Coll.: *Phys. Rev. Lett.* **99** (2007) 142001

[83] T. Abulencia *et al.*, CDF Coll.: *Phys. Rev. Lett.* **98** (2007) 122001

[84] V.M. Abazov *et al.*, D0 Coll.: arXiv:1105.0690

[85] K.F. Chen *et al.*, BELLE Coll.: *Phys. Rev. Lett.* **91**(2003) 201801

[86] C. Sharma and R. Sinha: *Phys. Rev. D* **73** (2006) 014016

[87] Z. J. Ajaltouni *et al.*: *Eur. Phys. J. C* **29** (2003) 215

[88] T. Mannel, W. Roberts and Z. Ryzak: *Nucl. Phys. B* **355** (1991) 38

[89] C.S. Huang and H.G. Yan: *Phys. Rev. D* **59** (1999) 114022 [Erratum: ibid. D **61**(2000) 039901]

[90] X.-W. Kang, H.-B. Li, G.-R. Lu and A. Datta: *Int. Jou. Mod. Phys. A* **26** (2011) 2523