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# Solution of Fractional Order Differential Equation Problems by Triangular Functions for Biomedical Applications

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*Abstract*— **Fractional Order Differential equations are used for modelling of a wide variety of biological systems but the solution process of such equations are quite complex. In this paper Orthogonal Triangular functions and their operational matrices have been used for finding an approximate solution of Fractional Order Differential Equations. This technique has been found to be more powerful in solving Fractional Order Differential Equations owing to the fact that the differential equations are reduced to systems of algebraic equations which are easy to solve numerically and the percentage error is lower compared to other methods of solutions (like: Laplace Transform Method). Also due to the recursive nature of this method, it can also be concluded that this method is less complex and more efficient in solving varieties of the Fractional Order Differential Equations.**



Keywords-Triangular function, Riemann–Liouville fractional integral, Operational matrix, Laplace Transform, Fitzhugh Nagumo equations, Viscoelasticity of tissues.


## I. INTRODUCTION

Fractional or non-integer order derivation and integration originated at the same time as integral order calculus. However, it is in the past century the most fascinating leaps have been made in terms of both engineering and scientific fields. Due to the fact that fractional order differential (FOD) equations can be closely related to both biological and memory-based systems there has a wide spread application of fractional order differential equations. Also, the results derived from the fractional system are of a more general nature. FOD can be solved using various methods and numerical solutions: the iteration method, the series method, the Fourier transform technique, the Laplace transform technique, and the operational calculus method[1].Apart from the above mentioned techniques lately several mathematical methods including the Adomian decomposition method, variational iteration method and homotopy perturbation method have been developed[] to obtain the exact and approximate analytic solutions. The use of fractional differentiation for the mathematical modeling of real-world physical and bio-medical problems has been widespread in recent years for example, the modeling of earthquake, the fluid dynamic traffic model with fractional derivatives, viscosity properties etc. In this paper we discuss the solution of fractional order differential equations using orthogonal Triangular Functions (TF) [2, 3,4].

In this paper we have applied the developed technique to solve fractional differential equations used in bio-medical applications like modelling viscoelasticity of tissues [5] and Fitzhugh Nagumo equation [7] which models the electrical activity of the heart. The solutions show the efficacy of the developed method with respect to the popular algorithms for solving FDEs.

## 2. THE TF ESTIMATE OF FRACTIONAL ORDER INTEGRAL OF FUNCTION $f(t)$ THROUGH LAPLACE TRANSFORM METHOD

The Riemann–Liouville fractional order integral **[3-4]** of the function $f(t)$ is given as

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau = \frac{1}{\Gamma(\alpha)} t^{\alpha-1} * f(t) \quad (1)$$

By TF estimation we have

$$f(t) \cong C^T T1_m(t) + D^T T2_m(t) \quad (2)$$

So, the integral in equation (1) can be written as

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} t^{\alpha-1} * [C^T T1_m(t) + D^T T2_m(t)]$$
$$= \frac{1}{\Gamma(\alpha)} \left( C^T \sum_{i=0}^{m-1} t^{\alpha-1} * T1_i(t) + D^T \sum_{i=0}^{m-1} t^{\alpha-1} * T2_i(t) \right)$$

The function of $T1_i(t)$ versus $t$ and $T2_i(t)$ versus $t$ can be written as:

$$T1_i(t) = u(t-ih) - \frac{t-ih}{h} u(t-ih) + \frac{t-(i+1)h}{h} u(t-(i+1)h) \tag{3}$$

$$T2_i(t) = \frac{t-ih}{h} u(t-ih) - \frac{t-(i+1)h}{h} u(t-(i+1)h) - u(t-(i+1)h) \tag{4}$$

Taking Laplace transform of the above,

$$T1_i(s) = \frac{e^{-ihs}}{s} - \frac{e^{-ihs}}{hs^2} + \frac{e^{-(i+1)hs}}{hs^2} \tag{5}$$

$$T2_i(s) = \frac{e^{-ihs}}{hs^2} - \frac{e^{-(i+1)hs}}{hs^2} - \frac{e^{-(i+1)hs}}{s} \tag{6}$$

Consider the convolution integral $\frac{1}{\Gamma(\alpha)} t^{\alpha-1} * T1_i(t)$

Taking Laplace transform of the above convolution, we have

$$\frac{1}{\Gamma(\alpha)} t^{\alpha-1} * T1_i(t) = \frac{1}{\Gamma(\alpha)} \frac{(\alpha-1)!}{s^{\alpha}} \left[ \frac{e^{-ihs}}{s} - \frac{e^{-ihs}}{hs^2} + \frac{e^{-(i+1)hs}}{hs^2} \right]$$
$$= \frac{1}{\Gamma(\alpha)} \left[ \frac{(\alpha-1)! e^{-ihs}}{s^{\alpha+1}} - \frac{(\alpha-1)! e^{-ihs}}{hs^{\alpha+2}} + \frac{(\alpha-1)! e^{-(i+1)hs}}{hs^{\alpha+2}} \right]$$

Taking inverse Laplace, the expression in time domain obtained is

$$= \frac{1}{\Gamma(\alpha+2)} \left[ \left( (\alpha+1)(t-ih)^{\alpha} - \frac{(t-ih)^{\alpha+1}}{h} \right) u(t-ih) + \frac{(t-(i+1)h)^{\alpha+1}}{h} u(t-(i+1)h) \right]$$

$$= \frac{1}{\Gamma(\alpha+2)} \left[ (\alpha+1)(t-ih)^{\alpha} - \frac{(t-ih)^{\alpha+1}}{h} \right] \sum_{j=i}^{m-1} [T1_j(t) + T2_j(t)] + \frac{1}{\Gamma(\alpha+2)} \frac{(t-(i+1)h)^{\alpha+1}}{h} \sum_{j=i+1}^{m-1} [T1_j(t) + T2_j(t)]$$

Using the definition of TF and rearranging the terms with respect to $T1_j(t)$ and $T2_j(t)$ the above equation becomes:

$$= \frac{h^{\alpha}}{\Gamma(\alpha+2)} \left( \begin{array}{l} \sum_{j=i+1}^{m-1} [(\alpha+1)(j-i)^{\alpha} - (j-i)^{\alpha+1} + (j-i-1)^{\alpha+1}] T1_j(t) + \sum_{j=i}^{m-1} [(\alpha+1)(j-i+1)^{\alpha} \\ -(j-i+1)^{\alpha+1} + (j-i)^{\alpha+1}] T2_j(t) \end{array} \right)$$

Replacing $(j-i)$ with $k$ the above equation of LHTF can be written in form of matrices as:

$$\frac{1}{\Gamma(\alpha)} t^{\alpha-1} * T1_m(t) = P_1^{\alpha} T1_m(t) + P_2^{\alpha} T2_m(t) \tag{7}$$

$$P_1^{\alpha} = \frac{h^{\alpha}}{\Gamma(\alpha+2)} \begin{bmatrix} 0 & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \dots & \varepsilon_{m-1} \\ 0 & 0 & \varepsilon_1 & \varepsilon_2 & \dots & \varepsilon_{m-2} \\ 0 & 0 & 0 & \varepsilon_1 & \dots & \varepsilon_{m-3} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \varepsilon_1 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad P_2^{\alpha} = \frac{h^{\alpha}}{\Gamma(\alpha+2)} \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 & \dots & \varepsilon_m \\ 0 & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \dots & \varepsilon_{m-1} \\ 0 & 0 & \varepsilon_1 & \varepsilon_2 & \dots & \varepsilon_{m-2} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \varepsilon_2 \\ 0 & 0 & 0 & 0 & \dots & \varepsilon_1 \end{bmatrix}$$

where $\varepsilon_k = \left[ [(\alpha+1)-k]k^{\alpha} + (k-1)^{\alpha+1} \right]; \quad k = 1, 2, 3, \dots, m$

Similarly, matrices for RHTF can be written as:

$$\frac{1}{\Gamma(\alpha)} t^{\alpha-1} * T2_m(t) = P_3^{\alpha} T1_m(t) + P_4^{\alpha} T2_m(t) \tag{8}$$

$$P_3^{\alpha} = \frac{h^{\alpha}}{\Gamma(\alpha+2)} \begin{bmatrix} 0 & \chi_1 & \chi_2 & \chi_3 & \dots & \chi_{m-1} \\ 0 & 0 & \chi_1 & \chi_2 & \dots & \chi_{m-2} \\ 0 & 0 & 0 & \chi_1 & \dots & \chi_{m-3} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \chi_1 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad P_4^{\alpha} = \frac{h^{\alpha}}{\Gamma(\alpha+2)} \begin{bmatrix} \chi_1 & \chi_2 & \chi_3 & \chi_4 & \dots & \chi_m \\ 0 & \chi_1 & \chi_2 & \chi_3 & \dots & \chi_{m-1} \\ 0 & 0 & \chi_1 & \chi_2 & \dots & \chi_{m-2} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \chi_2 \\ 0 & 0 & 0 & 0 & \dots & \chi_1 \end{bmatrix}$$

where $\chi_k = \left[ k^{\alpha+1} - (\alpha+k)(k-1)^{\alpha} \right]; \quad k = 1, 2, 3, \dots, m$

where, $P_1^{\alpha}, P_2^{\alpha}, P_3^{\alpha}, P_4^{\alpha}$ are square matrices of order $m$

From equations (7) and (8)

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau = (C^T P_1^{\alpha} + D^T P_3^{\alpha}) T1_m(t) + (C^T P_2^{\alpha} + D^T P_4^{\alpha}) T2_m(t) \tag{9}$$

Similarly, considering initial conditions gives

$$J^{\alpha-\beta_k} y(t) = (C^T P_1^{\alpha-\beta_k} + D^T P_3^{\alpha-\beta_k}) T1_m(t) + (C^T P_2^{\alpha-\beta_k} + D^T P_4^{\alpha-\beta_k}) T2_m(t) + \sum_{j=1}^{m_k} y^{m_k-j}(0) \frac{t^{m_k-j}}{(m-j)!} \tag{10}$$

such that $m_k < \alpha - \beta \leq m_k + 1$ with initial conditions $y^j(0) = a_j \;\; j = 1, 2, \dots\dots m_k - 1 \;\; \alpha, \beta \in R \;\; m_k \in N$

Using the definition of TF, $f(t) = \sum_{j=1}^{m_k} y^{m_k-j}(0) \frac{t^{m_k-j}}{(m-j)!}$ becomes

$$f(t) = \sum_{j=1}^{m_k} y^{m_k-j}(0) \frac{t^{m_k-j}}{(m-j)!} = C_{m_k}^T T1_m(t) + D_{m_k}^T T2_m(t) \tag{11}$$

Where $C_{m_k}^T$ and $D_{m_k}^T$ are the initial value of coefficient matrices.

Using Equation (10) in Equation (11) gives

$$J^{\alpha-\beta_k} y(t) = (C^T P_1^{\alpha-\beta_k} + D^T P_3^{\alpha-\beta_k} + C_{m_k}^T) T1_m(t) + (C^T P_2^{\alpha-\beta_k} + D^T P_4^{\alpha-\beta_k} + D_{m_k}^T) T2_m(t) \tag{12}$$

## 3.APPROXIMATE SOLUTION OF MULTI-ORDER FDEs

In this section, we are trying to formulate an approximate method to solve fractional order differential equations using orthogonal TF and the proposed matrices obtained from the TF estimate of Riemann–Liouville fractional integral proposed in section-I

### a. Linear multi-order FDE

We consider the following general form of linear multi-order FDE.

$$D^{\alpha} y(t) = \sum_{k=1}^{r} b_k D^{\beta_k} y(t) + c y(t) + f(t) \quad n-1 < \alpha \le n, \quad t \in [0,T] \quad \text{where } \alpha, \beta_k, b_k, c \in R \tag{13}$$

with initial conditions $y^{(s)}(0) = a_s, s = 0,1,2,\ldots,n-1$

Here $D^{\alpha} y(t)$ is the Caputo fractional derivative of order $\alpha$ satisfying the relation $\alpha > \beta_1 > \beta_2 \cdots \beta_r, b_k, c$ are real constants and $f(t)$ is a known function.

Carrying out fractional integration of order $\alpha$ on both sides of Equation (13) gives

$$y(t) = \sum_{k=1}^{r} b_k J^{\alpha-\beta_k} y(t) + c J^{\alpha} y(t) + J^{\alpha} f(t) \tag{14}$$

Expanding the unknown function $y(t)$ and known function $f(t)$ using TF with their respective coefficient matrices gives

$$y(t) \cong C^T T1_m(t) + D^T T2_m(t) \tag{15}$$

$$f(t) \cong C_0^T T1_m(t) + D_0^T T2_m(t) \tag{16}$$

From Equation (13) and Equations (14) – (16) the Linear order FDE can be rearranged to LHTF and RHTF to find unknown matrices $C^T$ and $D^T$

$$C^T = \sum_{k=1}^{r} b_k \left( C^T P_1^{\alpha-\beta_k} + D^T P_3^{\alpha-\beta_k} + C_{m_k}^T \right) + c \, (C^T P_1^{\alpha} + D^T P_3^{\alpha} + C_{n-1}^T) + (C_0^T P_1^{\alpha} + D_0^T P_3^{\alpha}) \tag{17}$$

$$D^T = \sum_{k=1}^{r} b_k \left( C^T P_2^{\alpha-\beta_k} + D^T P_4^{\alpha-\beta_k} + D_{m_k}^T \right) + c \, (C^T P_2^{\alpha} + D^T P_4^{\alpha} + D_{n-1}^T) + (C_0^T P_2^{\alpha} + D_0^T P_4^{\alpha}) \tag{18}$$

Upon solving the above Equations (17) – (18) the function $y(t)$ can be obtained

$$y(t) \cong C^T T1_m(t) + D^T T2_m(t) \tag{19}$$

### b. Nonlinear multi-order FDE

We consider the following general form of nonlinear multi-order FDE.

$$D^{\alpha} y(t) = \sum_{k=1}^{r} b_k D^{\beta_k} y(t) + c(y(t))^p + f(t), \quad n-1 < \alpha \le n, \quad t \in [0,T],$$

where $\alpha, \beta_k, b_k, c \in R; \ p \in N$

(20)

With initial conditions $y^{(s)}(0) = a_s, s = 0,1,2,\ldots,n-1$ Here $D^{\alpha} y(t)$ is the Caputo fractional derivative of order $\alpha$ satisfying the relation $\alpha > \beta_1 > \beta_2 \cdots \beta_r$ are and $f(t)$ is known function.

Carrying out fractional integration of order $\alpha$ on both sides of Equation (20) gives

$$y(t) = \sum_{k=1}^{r} b_k J^{\alpha-\beta_k} y(t) + c J^{\alpha} \left( y(t) \right)^p + J^{\alpha} f(t) \tag{21}$$

The TF estimate of nonlinear term $(y(t))^p$ using TF estimate

$$(y(t))^p \cong \left( C^T \right)_p T1_m(t) + \left( D^T \right)_p T2_m(t) \tag{22}$$

Expansion in the TF domain for the unknown function *y(t)* and known function *f(t)*

$$y(t) \cong C^T T1_m(t) + D^T T2_m(t) \tag{23}$$

$$f(t) \cong C_0^T T1_m(t) + D_0^T T2_m(t) \tag{24}$$

From Equation (12) and Equations (21) – (24) the Non-Linear order FDE can be rearranged to LHTF and RHTF to find the unknown matrices $C^T$ and $D^T$

$$C^T = \sum_{k=1}^{r} b_k \left( C^T P_1^{\alpha-\beta_k} + D^T P_3^{\alpha-\beta_k} + C_{m_k}^T \right) + c \, \left( \left( C^T \right)_p P_1^{\alpha} + \left( D^T \right)_p P_3^{\alpha} \right) + (C_0^T P_1^{\alpha} + D_0^T P_3^{\alpha}) \tag{25}$$

$$D^T = \sum_{k=1}^{r} b_k \left( C^T P_2^{\alpha-\beta_k} + D^T P_4^{\alpha-\beta_k} + D_{m_k}^T \right) + c \, \left( \left( C^T \right)_p P_2^{\alpha} + \left( D^T \right)_p P_4^{\alpha} \right) + (C_0^T P_2^{\alpha} + D_0^T P_4^{\alpha}) \tag{26}$$

Upon solving the simultaneous nonlinear equations (25) and (26), $y(t)$ is obtained

$$y(t) \cong C^T T1_m(t) + D^T T2_m(t) \tag{27}$$

## 4. FRACTIONAL CALCULUS IN BIOMEDICAL SYSTEM MODELLING

Fractional differential equations have been extensively used in modelling of signal processing, fractional order control systems, viscoelasticity, electrochemistry, electromagnetics, network analysis, heat transfer, and diffusion. This enables the application of Fractional Differential equations in analyzing the dynamics of biological systems. Some popular application areas include vestibular ocular models (analyzed as fractional order biological control systems governed by FDEs), fractional VOR dynamics, cardiac cellular dynamics, fractional viscoelastic models of cells and tissues, fractional order models of respiratory mechanics etc. Among these, we have illustrated a model for analyzing viscoelasticity of tissues **[5]** and Fitzhugh Nagumo equation (for analyzing cardiac cell dynamics) **[7]** in the following section to show the efficacy of the developed model.

Numerical examples

**Example 1.** Fractional viscoelastic properties of tissues **[5]**

$$\sigma(t)+\tau^{\alpha}\frac{d^{\alpha}\sigma(t)}{dt^{\alpha}}=E_R\left[\varepsilon(t)+\rho^{\alpha}\frac{d^{\alpha}\varepsilon(t)}{dt^{\alpha}}\right],\ \text{where } \varepsilon(t)=\text{u}(t) \qquad \text{For unit step input}$$

$$\varepsilon(t)=t.\text{u}(t) \qquad \text{For unit ramp input}$$

**Solution:** Solving using Laplace Transform (LT):

- For unit step input

$$\sigma(t)=E_R\left[1+\left(\frac{\rho^{\alpha}-\tau^{\alpha}}{\tau^{\alpha}}\right)E_{\alpha,1}\left(-\left(\frac{t}{\tau}\right)^{\alpha}\right)\right] \tag{28}$$

With initial condition $y(0)=0$

- For unit ramp input:

$$\sigma(t)=E_R\left[t+\left(\frac{\rho^{\alpha}-\tau^{\alpha}}{\tau^{\alpha}}\right)tE_{\alpha,2}\left(-\left(\frac{t}{\tau}\right)^{\alpha}\right)\right] \tag{29}$$

With initial condition $y(0)=0$

Where $E_{\alpha,\beta}(t)$ is the Mittag Leffler function with two parameters given by

$$E_{\alpha,\beta}(t)=\sum_{k=0}^{\infty}\frac{t^k}{\Gamma(\alpha k+\beta)}$$

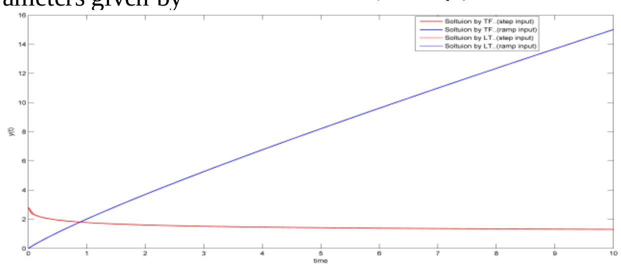


FIG 1: TF and LT solution of example 2 with $E_R$ =1, $\rho=1$ , $\tau$=0.9 for unit step input and unit ramp input

TABLE 1: Comparison between LT and TF solution of example 1 with unit step input with $m=100$ , $\alpha$=0.5

| Sl. No. | **Solution using LT Equation DisplayText ca** $[x_{LT}]$ | **Solution using TF Equation (19)** $[x_{TF}]$ | **% Error** $\varepsilon=\frac{x_{TF}-x_{LT}}{x_{LT}}\times100\%$ |
|---|---|---|---|
| 1 | 1.4092 | 1.4093 | 0.007096 |
| 2 | 1.5287 | 1.5289 | 0.013081 |
| 3 | 1.4693 | 1.4694 | 0.006805 |
| 4 | 1.3281 | 1.3281 | 0 |
| 5 | 1.3677 | 1.3677 | 0 |
| 6 | 1.3801 | 1.3801 | 0 |
| 7 | 1.5688 | 1.569 | 0.012747 |
| 8 | 1.4464 | 1.4465 | 0.006913 |
| 9 | 1.3462 | 1.3462 | 0 |
| 10 | 1.7939 | 1.7946 | 0.039006 |

**Example 2.** The Fitzhugh–Nagumo model (FHN): **[7]**

$$\frac{dv(t)}{dt}=c_1\big(v(t)\big(v(t)-a\big)\big(1-v(t)\big)\big)-c_2w(t)+I_{ext}(t) \qquad \text{where,} \quad I_{ext}(t)=\begin{cases}0.08, & 10\le t\le 20\\ 0, & \text{elsewhere}\end{cases}$$

$$\frac{dw(t)}{dt}=b\big(v(t)-c_3w(t)\big)$$

**Solution:** The equations are solved by TF method and compared with the solution by Euler's Method **[8]**with initial conditions $v(0)=0$ and $w(0)=0$ using Equation (30) and Equation (31)

The $(i+1)^{th}$ iteration in Euler's methood for Example 5 gives

$$v((i+1)h)=v(ih)+hv'(ih) \tag{30}$$

$$w((i+1)h)=w(ih)+hw'(ih) \tag{31}$$

Where $i=0,1,2,\ldots,m-1$

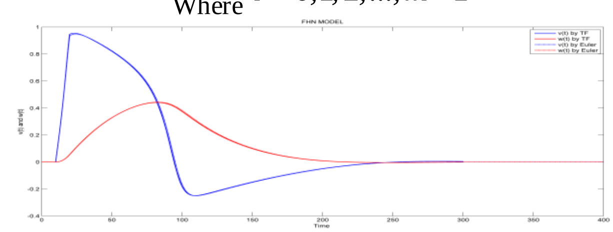


FIG 2: Solution using TF and EULER method of example 5 with a=0.13, b=0.013, c1=0.26, c2=0.1, c3=1

TABLE 2: Comparison between Euler's method and TF solution for values of $v(t)$ in example 2 with $m=800$.

| Sl. No. | **Solution using LT Equation DisplayText ca** | **Solution using TF Equation (19)** $[x_{TF}]$ | **% Error** $\varepsilon=\frac{x_{TF}-x_{LT}}{x_{LT}}\times100\%$ |
|---|---|---|---|

| | $\left[x_{LT}\right]$ | | |
|---|---|---|---|
| 1 | -0.1562 | -0.15633 | 0.08322 |
| 2 | -0.19065 | -0.19009 | 0.29373 |
| 3 | -0.19637 | -0.19685 | 0.24443 |
| 4 | -0.24201 | -0.24217 | 0.06611 |
| 5 | -0.2152 | 0.21339 | -0.8689 |
| 6 | 0.42315 | 0.4279 | 1.02638 |
| 7 | 0.67905 | 0.6779 | 0.18555 |
| 8 | 0.7778 | 0.7834 | 0.72892 |
| 9 | 0.8269 | 0.83433 | 0.54691 |

## 5. CONCLUSION

In this paper we have applied Triangular orthogonal functions [2] to solve the Fractional Differential equations. From the examples illustrated above it can be concluded that a wide range of fractional differential equations (specifically used in bio-medical applications) can be solved with high accuracy using Triangular functions.

The maximum error recorded was less than 1%. In future works we will try to minimize this error and further establish the efficacy of the developed algorithm by proving its significance in reducing the computational cost of solving FDEs compared to the widely used solvers.

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