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Thermomagnetic convection in stratified ferrofluids permeated with dusty particles through a porous medium

Kapil Kumar[1*], Rajan Singh[2] , Anand Kumar[3]

[1]Department of Applied Mathematics, Government Polytechnic, Pilibhit, India.
[2]Department of Mathematics, I.F.T.M. University, Moradabad, India.
[3]Department of Mathematics, Ramabai Govt. Girls P.G. College Akbarpur, Ambedkarnagar, India.

*corresponding author: kkchaudhary000@gmail.com

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Abstract: In the present note, the stability problem of an incompressible dusty stratified ferromagnetic fluid is investigated through a porous medium when the fluid layer is subjected to vertical magnetic field intensity. The governing nonlinear equations are linearized using perturbation technique and the cases of exponentially varying stratifications for various physical parameters are discussed. The system is found to be stable for stable stratification in both the absence and presence of magnetic field. For unstable stratification, the system has both stabilizing and destabilizing effects in the presence of magnetic field under certain conditions, whereas in the absence of magnetic field, the system has only destabilizing effect. The variations in the growth rate with respect to kinematic viscosity, medium porosity, medium permeability, square of Alfvén velocity and suspended particle parameter are also shown analytically.

1. INTRODUCTION

The basic concepts to understand various fascinating and diverse applications of fluid mechanics have been given in Bansal (2004) and Gupta and Gupta (2013). A rigorous and elegant overview about hydrodynamic stability problem of an incompressible Newtonian fluid has been well addressed in Chandrasekhar (1981) and Drazin and Reid (1981). The behaviour and flow characteristic of non-Newtonian fluids are significantly different with those of Newtonian fluids. Ferrofluids are electrically non-conducting colloidal suspensions of solid ferromagnetic particles in a carrier fluid such as kerosene. Ferrofluids manifest simultaneously both liquid and paramagnetic properties.

The study of stratified fluids has produced great interest in recent years due to its numerous industrial and technological applications such as thermal stratification of reservoirs and oceans, density, temperature and gravitational stratification of the atmosphere, salinity stratification in rivers, oceans and estuaries, layer stratification in the earth’s interior and several heterogeneous mixtures in food processing industry. Stratified fluids are abundant and the understanding of the behaviour and dynamics of these fluids are mandatory for scientific and industrial purposes. During the last few decades, the study on non-Newtonian fluids has attracted several researchers and investigators to study such industrially important fluids.

Rosensweig (1985) and Odenbach (2002) discussed about the fundamental concepts behind the use of ferrofluids and also provided a comprehensive and detailed account of ferrohydrodynamics and its applications in various commercial usages such as novel zero-leakage rotary shaft seals used in computer disk drives (Bailey, 1983); semiconductor manufacturing (Moskowitz, 1975); pressure seals for compressor and blowers (Rosensweig, 1985) and more.

Other applications include the uses and interesting effects of ferrofluids in a wide range of technological and bio-medical purposes such as vacuum technology, instrumentation, lubrication mechanism, acoustics theory, recovery of metals, detection of tumours, drug delivery to a target site, magnetic fluid bearings, non-destructive testing, sensors and actuators, sorting of industrial scrap metals such as titanium, aluminium and zinc, tracer of blood flow in non-invasive circulatory measurements (Newbower, 1972) and in loudspeakers to conduct heat away from speakers coil (Hathaway, 1979). The stability of ferrofluids intended for medical use is a current topic of frontier research and also attractive from a theoretical point of view. Thus, the overall field of ferrofluid research has a highly interdisciplinary character, bringing physicists, engineers, chemists and mathematicians together. Finlayson (1970) discussed the convective instability of ferromagnetic fluid layer heated from below under the effect of a uniform vertical magnetic field and analyzed the instability with or without considering the effect of body force (gravity force). He also concluded that convection can be induced in a ferromagnetic fluid due to variation in magnetization which depends upon the strength of magnetic field, temperature gradient and density of fluid and is known as ferroconvection, which is very similar to Bénard convection (Chandrasekhar, 1981).

Magneto-hydrodynamics theory of electrically conducting fluids has several scientific and practical applications in astrophysics, geophysics, space sciences etc. Magnetic field is also used in several clinical areas such as neurology and orthopaedics for probing and curing the internal organs of the body in several diseases like tumours detection, heart and brain diseases, stroke damage etc. Sunil et al. (2004, 2005) considered, theoretically, the thermal and thermosolutal convection problems for ferromagnetic fluid to include the effects of magnetic field and dust particles saturating a porous medium. Sharma et al. (2006) pointed out the combined effects of magnetic field and rotation on the stability of stratified visco-elastic Walters' (model B') fluid through a porous medium and concluded that the system is found to be unstable at stable stratification, whereas for unstable stratification, magnetic field is found to stabilize the small wavelength perturbations. It is also shown that the growth rate increases with the increase in kinematic viscosity and permeability, whereas it decreases with the increase in kinematic viscoelasticity. Kumar et al. (2013, 2015a, b) discussed thermal convection problem for Oldroydian, couple-stress and ferrofluid under the effects of magnetic field, rotation, compressibility, variable gravity, suspended particles and heat source strength through a Darcy as well as Brinkman porous medium.

The flow through a porous medium is of fundamental importance in solidification, chemical processing industry, geophysical fluid dynamics, petroleum industry, filtering equipment, recovery of crude oil from earth's interior etc. A detailed study of convection through porous medium has been given by Nield and Bejan (2006) in his famous monograph. In many branches of sanitary work, notably in the study of factory conditions, the enumeration of the actual number of dust particles present is quite as important as the determination of the total weight of dust.Dust comes from a wide variety of sources, including soil, vegetation (pollens and fungi), sea salt, fossil fuel combustion, burning of biomass, and industrial activities.

In geophysical context, the fluid is often not pure but may instead be permeated with dust particles. The effects of suspended particles on the stability of superposed fluids have industrial and scientific importance in geophysics, chemical engineering and astrophysics. Scanlon and Segel (1973) have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by that of the particles. The governing hydrodynamic equations of motion are solved using a regular perturbation technique. The intention is to investigate theoretically the stability of a stratified ferrofluid in the presence of magnetic field and suspended particles through a porous medium using linear stability theory. The use of Boussinesq approximation has been made throughout in the equations of motion which states that the density variations occurs only in the external force term or gravitational force term. The purpose and practical relevance of the present investigation is in determining the influence of the impurities and magnetic strength in a stratified ferrofluid in thermal convection phenomena.

2. MATHEMATICAL FORMULATION AND GOVERNING EQUATIONS

The physical configuration considered here consists of a ferromagnetic fluid of variable density ρ, kinematic viscosity υ, medium porosity $\in$, medium permeability k_1, particle number density N and thermometric conductivity κ, arranged in a rectangular channel bounded by two infinite horizontal stratum separated by an altitude *d* apart in a porous medium. A uniform vertical magnetic field $\mathbf{H}(0,0,H)$ pervades the system with gravity acting vertically downward. The Cartesian axes are chosen with the *z*-axis vertically upward and the *x*-axis in the direction of applied horizontal temperature gradient. It is also assumed that the flow in the porous medium is governed by the Darcy's law in the equation of motion.

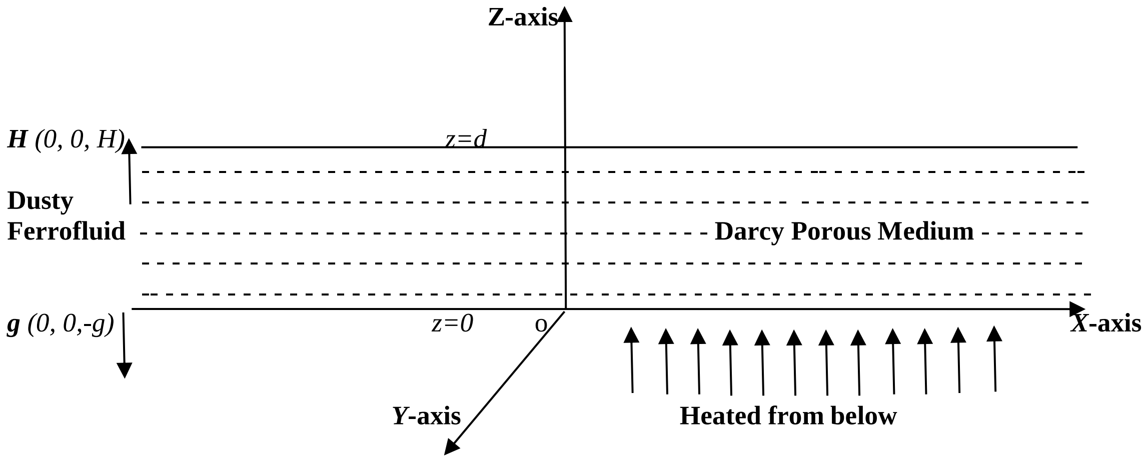

Fig1. Geometrical sketch of the physical problem

The governing equations of conservation of mass and momentum balance for an incompressible magnetized ferrofluid in a porous medium are as follows

$$\frac{\rho}{\in}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\in}(\mathbf{q}.\nabla)\mathbf{q}\right]=-\nabla p+\rho \mathbf{X_i}+\mu_0{}'(\mathbf{M}.\nabla)\mathbf{H}-\frac{\mu}{k_1}\mathbf{q}$$
$$+\frac{K'N(\mathbf{q_d}-\mathbf{q})}{\in}+\frac{\mu_e}{4\pi}\left[(\nabla\times\mathbf{H})\times\mathbf{H}\right] \qquad (2.1)$$

$$\nabla.\mathbf{q}=0 \qquad (2.2)$$

$$\in\frac{\partial \rho}{\partial t}+(\mathbf{q}.\nabla)\rho=0 \qquad (2.3)$$

where, in the above equations, the symbols $\rho, t, \mu, \mathbf{q}, \mathbf{q_d}, \nabla p, N, \mu_e, \mu_0{}', \mathbf{H}, \mathbf{B}$ *and* $\mathbf{X_i}=-\mathbf{g}\lambda_i$ denote, respectively, the density of ferromagnetic fluid, the time, co-efficient of viscosity, velocity of fluid particles, velocity of suspended particles, pressure gradient for ferromagnetic fluid, suspended particles number density, magnetic permeability of medium, magnetic permeability of vacuum, magnetic field strength, magnetic induction and the external force due to gravity. The term $K'=6\pi\rho\delta\upsilon$, where δ being the particle radius, represent the Stokes drag co-efficient.

The equation for energy balance which obeys Fourier's law of heat conduction is

$$\left[\in\rho c_v+\rho_s c_s(1-\in)\right]\frac{\partial T}{\partial t}+\rho c_v(\mathbf{q}.\nabla)T+mNc_{pt}\left(\in\frac{\partial}{\partial t}+\mathbf{q_d}.\nabla\right)T=k_T\nabla^2 T \qquad (2.4)$$

where, ρ_s, c_s, c_v, T *and* k_T denote, respectively, the density of solid material, heat capacity of solid material, specific heat at constant volume, the temperature and the thermal conductivity of fluid particles.

The Maxwell's equations of electromagnetism are

$$\in\frac{\partial \mathbf{H}}{\partial t}=\nabla\times(\mathbf{q}\times\mathbf{H}) \qquad (2.5)$$

$$\nabla.\mathbf{H}=0 \qquad (2.6)$$

where, the electrical resistivity η is taken as zero.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the dust particles are

$$mN\left[\frac{\partial \mathbf{q_d}}{\partial t}+\frac{1}{\in}\left(\mathbf{q_d}.\nabla\right)\mathbf{q_d}\right]=K'N\left(\mathbf{q}-\mathbf{q_d}\right) \tag{2.7}$$

$$\in\frac{\partial N}{\partial t}+\nabla.\left(N\mathbf{q_d}\right)=0 \tag{2.8}$$

The density equation of state is

$$\rho=\rho_0\left[1+\alpha\left(T_0-T\right)\right] \tag{2.9}$$

where, $\alpha,\ \rho_0\ and\ T_0$ denote, respectively, the coefficient of thermal expansion, reference density and the temperature at the lower boundary.

Maxwell's equations for an electrically non-conducting fluid with no displacement currents, become

$$\nabla.\mathbf{B}=0,\qquad \nabla\times\mathbf{H}=0 \tag{2.10}$$

According to Penfield and Haus (1967), the magnetic induction *(B)*, magnetization *(M)* and the intensity of magnetic field *(H)* are coupled by the relation

$$\mathbf{B}=\mu_0'(\mathbf{H}+\mathbf{M}) \tag{2.11}$$

Now, for the problem under consideration, it is assumed that the magnetization (M) does not depend on the magnetic field (H) and is a function of temperature (T) only. So, as an initial approximation, we consider the form

$$\mathbf{M}=M_0[1+\chi\left(T_0-T\right)] \tag{2.12}$$

where, $T_0\ and\ M_0$ are the reference temperature and reference magnetization, respectively and $\chi=-\frac{1}{M_0}\left(\frac{\partial M}{\partial T}\right)_{H_0}$ stands for the pyromagnetic co-efficient with $H=|\mathbf{H}|,\ M=|\mathbf{M}|\ and\ M_0=M\left(T_0\right).$

Now, the stability of the basic state defined below is analyzed using the regular perturbation technique.

$$\begin{gathered}\mathbf{q_b}=\left(0,0,0\right),\ p=p_b\left(z\right)=p_0-g\rho_0\left[1+\alpha\beta'z^2/2\right],\ T_b\left(z\right)=T_0-\beta'z,\\ \mathbf{H}=\mathbf{H_b}\left(0,0,\mathbf{H_z}\right),\ \mathbf{M}=\mathbf{M_b}\left(z\right),\ N=N_b,\ \rho=\rho_b\left(z\right)=\rho_0\left(1+\alpha\beta'z\right).\end{gathered} \tag{2.13}$$

Let the conduction state described by Eq. (2.13) be slightly perturbed by assuming perturbations of the form

$$\begin{gathered}\mathbf{q}=\mathbf{q_b}+\mathbf{q},\ p=p_b\left(z\right)+\delta p,\ T=T_b\left(z\right)+\theta,\mathbf{H}=\mathbf{H_b}+\mathbf{h}\left(h_x,h_y,h_z\right),\\ \rho=\rho_b\left(z\right)+\delta\rho,\ N=N_b+N',\ \ \mathbf{M}=\mathbf{M_b}\left(z\right)+\mathbf{m}\left(m_x,m_y,m_z\right).\end{gathered} \tag{2.14}$$

The changes in density $\delta\rho$ and magnetization m caused by perturbations θ and χ in temperature and concentration, respectively, are given by

$$\delta\rho=-\alpha\rho_m\theta,\quad m=-\chi M_0\theta \tag{2.15}$$

Assuming these perturbation quantities to be very small, the relevant linearized perturbation equations for the magnetized ferrofluid become

$$\begin{aligned}\frac{\rho}{\in}\frac{\partial\mathbf{q}}{\partial t}=&-\nabla\left(\delta p\right)-g\left(\delta\rho\right)\lambda_i-\mu_0'\chi M_0\nabla\mathbf{H}\theta+\mu_0'\left(\mathbf{M}.\nabla\right)\mathbf{h}-\frac{\mu}{k_1}\mathbf{q}\\ &+\frac{K'N'\left(\mathbf{q_d}-\mathbf{q}\right)}{\in}+\frac{\mu_e}{4\pi}\left[\left(\nabla\times\mathbf{h}\right)\times\mathbf{H}\right]\end{aligned} \tag{2.16}$$

$$\nabla.\mathbf{q}=0 \tag{2.17}$$

$$\in\frac{\partial}{\partial t}\left(\delta\rho\right)=-\mathbf{w}\left(D\rho\right) \tag{2.18}$$

$$\left(E+b\in\right)\frac{\partial\theta}{\partial t}=\beta'\left(\mathbf{w}+b\mathbf{s}\right)+\kappa\nabla^2\theta \tag{2.19}$$

$$\in\frac{\partial\mathbf{h}}{\partial t}=\left(\nabla.\mathbf{H}\right)\mathbf{q} \tag{2.20}$$

$$\nabla.\mathbf{h}=0 \tag{2.21}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\mathbf{q_d}=\mathbf{q} \tag{2.22}$$

$$\in\frac{\partial N'}{\partial t}+\nabla.\left(N'\mathbf{q_d}\right)=0 \tag{2.23}$$

where, ***w*** and ***s***denote, respectively, the vertical fluid velocity and suspended particle velocity and $E=\in+\left(1-\in\right)\left(\frac{\rho_s C_s}{\rho C_v}\right)$, $b=\frac{mN'C_{pt}}{\rho C_v}$, $\upsilon=\frac{\mu}{\rho}$, $\kappa=\frac{k_T}{\rho C_v}$, $\beta'=\left|dT/dz\right|$, $\lambda_i=\left(0,0,1\right)$, $D=\frac{d}{dz}$.
Now, considering an exponential solution with a dependence on *x, y* and *t* of the form

$$\exp\left(ik_x x+ik_y y+nt\right) \tag{2.24}$$

where, k_x and k_y are the wave numbers along *x* and *y* directions, respectively and $k^2=\left(k_x{}^2+k_y{}^2\right)$ is the overall horizontal wave number and *n* is the growth rate of harmonic disturbance, which is, in general, a complex quantity.
Equations (2.16) - (2.23) yield

$$\frac{\rho}{\in}n'\mathbf{u}=-ik_x\left(\delta p\right)+\mu_0'\mathbf{M}\left(D\mathbf{h_x}\right)-\frac{\mu}{k_1}\mathbf{u}+\frac{\mu_e\mathbf{H}}{4\pi}\left[\left(D\mathbf{h_x}-ik_x\mathbf{h_z}\right)\right] \tag{2.25}$$

$$\frac{\rho}{\in}n'\mathbf{v}=-ik_y\left(\delta p\right)+\mu_0'\mathbf{M}\left(D\mathbf{h_y}\right)-\frac{\mu}{k_1}\mathbf{v}+\frac{\mu_e\mathbf{H}}{4\pi}\left[\left(D\mathbf{h_y}-ik_y\mathbf{h_z}\right)\right] \tag{2.26}$$

$$\frac{\rho}{\in}n'\mathbf{w}=-D\left(\delta p\right)-g\left(\delta\rho\right)\lambda_i+\mu_0'\mathbf{M}\left(D\mathbf{h_z}\right)-\mu_0'\chi M_0\left(D\mathbf{H}\right)\theta-\frac{\mu}{k_1}\mathbf{w} \tag{2.27}$$

$$ik_x\mathbf{u}+ik_y\mathbf{v}+D\mathbf{w}=0 \tag{2.28}$$

$$n\in\left(\delta\rho\right)=-\mathbf{w}\left(D\rho\right) \tag{2.29}$$

$$\left(nE'-\kappa D^2\right)\theta=\beta'\left[1+\frac{b}{\left(1+n\gamma\right)}\right] \tag{2.30}$$

$$n\in\mathbf{h_x}=\mathbf{H}\left(D\mathbf{u}\right) \tag{2.31}$$

$$n\in\mathbf{h_y}=\mathbf{H}\left(D\mathbf{v}\right) \tag{2.32}$$

$$n\in\mathbf{h_z}=\mathbf{H}\left(D\mathbf{w}\right) \tag{2.33}$$

$$ik_x\mathbf{h_x}+ik_y\mathbf{h_y}+D\mathbf{h_z}=0 \tag{2.34}$$

$$\mathbf{s}=\left\{\mathbf{w}/\left(1+n\gamma\right)\right\} \tag{2.35}$$

$$n\in N'+\nabla.\left(N'\mathbf{q_d}\right)=0 \tag{2.36}$$

where, $\gamma=\left(\frac{m}{K'}\right)$, $E'=E+b\in$, $n'=n\left[1+\left\{mN'/\rho\left(1+\gamma n\right)\right\}\right]$ (2.37)

Multiplying Eq. (2.25) by ik_x and Eq. (2.26) by ik_y and adding, we obtain

$$\frac{\rho}{\in}n'D\mathbf{w}=-k^2\left(\delta p\right)+\mu_0'\mathbf{M}\left(D^2\mathbf{h_z}\right)-\frac{\mu}{k_1}D\mathbf{w}+\frac{\mu_e\mathbf{H}}{4\pi}\left[\left(D^2-k^2\right)\mathbf{h_z}\right] \tag{2.38}$$

Now, subtracting Eq. (2.27) (after multiplying by k^2) from Eq. (2.38) and also using Eqs. (30), (33) and (37), an expression is obtained as

$$\frac{n}{\in}\left[D\left(\rho D\mathbf{w}\right)-\rho k^2\mathbf{w}\right]+\frac{n}{\in\left(1+\gamma n\right)}\left[D\left\{mN'\left(D\mathbf{w}\right)\right\}-mN'\left(k^2\mathbf{w}\right)\right]$$

$$+\frac{gk^2\left(D\rho\right)\mathbf{w}}{\in n}+\frac{\mu}{k_1}\left(D^2-k^2\right)\mathbf{w}-\frac{\mu_e\mathbf{H}^2}{4\pi\in n}\left[\left(D^2-k^2\right)D^2\mathbf{w}\right] \tag{2.39}$$

$$+\frac{\mu_0{}'\chi\mathbf{M}_0k^2\left(D\mathbf{H}\right)\beta'\left\{1+b/\left(1+n\gamma\right)\right\}\mathbf{w}}{\left\{nE'-\kappa D^2\right\}}-\frac{\mu_0{}'\mathbf{MH}}{\in n}\left[\left(D^2-k^2\right)D^2\mathbf{w}\right]=0$$

where, $\rho = mN'$ *i.e.* mass of particles per unit volume

Equation (2.39) represents the general dispersion relation for stratified ferromagnetic fluid in terms of magnetic field and dust particle parameters in a porous medium.

3. EXPONENTIALLY VARYING STRATIFICATIONS

Considering the stratifications in various physical parameters of the forms

$$\left[\rho,\mu,\in,k_1,N',\mathbf{H}^2,E',\kappa\right]=\left[\rho_0,\mu_0,\in_0,k_{10},N_0{}',H_0{}^2,E_0',\kappa_0\right]e^{\beta z} \tag{3.1}$$

where, $\rho_0,\mu_0,\in_0,k_{10},N_0,H_0,E_0',\kappa_0$ *and* β all are constants.

On using stratification expression (3.1) in relation (2.39) and obtain

$$\left[\frac{n}{\in_0}+\frac{mnN_0{}'}{\rho_0\in_0\left(1+\gamma n\right)}+\frac{\upsilon_0}{k_{10}}\right]\left\{nE_0'-\kappa_0D^2\right\}\left(1+\gamma n\right)\left(D^2-k^2\right)\mathbf{w}$$

$$+\frac{g\beta k^2}{\in_0 n}\left\{nE_0'-\kappa_0D^2\right\}\left(1+\gamma n\right)\mathbf{w}-\left[\frac{V_A{}^2}{\in_0 n}+\frac{\mu_0{}'\mathbf{M}H_0}{\rho_0\in_0 n}\right]\left\{nE_0'-\kappa_0D^2\right\} \tag{3.2}$$

$$\left(1+\gamma n\right)\left\{\left(D^2-k^2\right)D^2\mathbf{w}\right\}+\frac{\mu_0{}'\chi\mathbf{M}_0k^2\beta\beta'H_0}{\rho_0}\left(B+\gamma n\right)\mathbf{w}=0$$

where, $V_A{}^2=\dfrac{\mu_eH_0{}^2}{4\pi\rho_0}$ is the square of Alfvén velocity named after Hannes Alfvén with $B=\left(1+b\right)$.

The boundary conditions (for the case of free boundaries) are defined as

$$\mathbf{w}=D^2\mathbf{w}=0 \;\; at \;\; z=0 \;\; and \;\; d. \tag{3.3}$$

Now, a proper solution for w satisfying the boundary condition (3.3) can be proposed as

$$\mathbf{w}=w_0\sin l\pi z \tag{3.4}$$

where, w_0 *and* l are constants and $l=1$ for the lowest mode.

On using solution (3.4), Eq. (3.2) yields

$$\left[\frac{n}{\in_0}+\frac{mnN_0'}{\rho_0\in_0\left(1+\gamma n\right)}+\frac{\upsilon_0}{k_{10}}\right]\left\{nE_0'+\kappa_0\pi^2l^2\right\}\left(\pi^2l^2+k^2\right)\left(1+\gamma n\right)$$

$$-\frac{g\beta k^2}{\in_0 n}\left\{nE_0'+\kappa_0\pi^2l^2\right\}\left(1+\gamma n\right)+\left[\frac{V_A{}^2}{\in_0 n}+\frac{\mu_0{}'\mathbf{M}H_0}{\rho_0\in_0 n}\right]\left\{nE_0'+\kappa_0\pi^2l^2\right\} \tag{3.5}$$

$$\left(1+\gamma n\right)\left(\pi^2l^2+k^2\right)\pi^2l^2-\frac{\mu_0{}'\chi\mathbf{M}_0k^2\beta\beta'H_0}{\rho_0}\left(B+\gamma n\right)=0$$

Simplifying Eq. (3.5), a polynomial of degree four is obtained as

$$a_0 n^4 + a_1 n^3 + a_2 n^2 + a_3 n + a_4 = 0 \tag{3.6}$$

where, the coefficients a_0, a_1, a_2, a_3 *and* a_4 all are constants and defined as

$$a_0 = \rho_0 \gamma E_0{}' S_1$$

$$a_1 = \left[\left(\rho_0 + mN_0{}' \right) E_0{}' S_1 + \frac{\rho_0 \upsilon_0 \in_0 \gamma E_0{}' S_1}{k_{10}} + \rho_0 \gamma \kappa_0 \pi^2 l^2 S_1 \right]$$

$$a_2 = \begin{bmatrix} \left(E_0{}' + \gamma \kappa_0 \pi^2 l^2 \right) \dfrac{\rho_0 \upsilon_0 \in_0 S_1}{k_{10}} + \left(\rho_0 + mN_0{}' \right) \kappa_0 \pi^2 l^2 S_1 + \left(V_A{}^2 \rho_0 + \mu_0{}' M H_0 \right) \\ \gamma E_0{}' \pi^2 l^2 S_1 + \beta k^2 \gamma \left(g \rho_0 E_0{}' - \mu_0{}' \chi M_0 \beta' H_0 \in_0 \right) \end{bmatrix}$$

$$a_3 = \begin{bmatrix} \dfrac{\rho_0 \upsilon_0 \in_0 \kappa_0 \pi^2 l^2 S_1}{k_{10}} + \left[\left(V_A{}^2 \rho_0 + \mu_0{}' M H_0 \right) \pi^2 l^2 S_1 - g \beta k^2 \rho_0 \right] \\ \left(E_0{}' + \gamma \kappa_0 \pi^2 l^2 \right) - \mu_0{}' \chi M_0 k^2 \beta \beta' H_0 \in_0 B \end{bmatrix}$$

$$a_4 = \left[\left(V_A{}^2 \rho_0 + \mu_0{}' M H_0 \right) \kappa_0 \pi^4 l^4 S_1 - g \beta k^2 \rho_0 \kappa_0 \pi^2 l^2 \right]$$

where, in the above coefficients, it is assumed $S_1 = \left(\pi^2 l^2 + k^2 \right)$. Equation (3.6) is bi-quadratic in the growth rate n and therefore it must give four roots.

4. DISCUSSION AND RESULTS

The effect of various embedded parameters responsible for the stability/instability of the system is discussed and some more important results are also obtained.

Stable stratification cases:

Case1: Let $\alpha_1, \alpha_2, \alpha_3$ *and* α_4 be the four roots of Eq. (3.6) then for the conditions $\beta < 0,\ \beta' > 0,\ \mu_0{}' > 0,\ B > 0$ *and* $g \rho_0 E_0{}' < \mu_0{}' \chi M_0 \beta' H_0 \in_0$, all the coefficients a_0, a_1, a_2, a_3 *and* a_4 will be positive and therefore Eq. (3.6) does not admit any positive root of *n*.So, the product of roots $\alpha_1 \alpha_2 \alpha_3 \alpha_4 = \left(a_4 \big/ a_1 \right) > 0.$

This indicates that the system is stable for disturbances of all wave numbers.

Case2: In the absence of magnetic field $\left(i.e.\ H_0 = 0,\ V_A{}^2 = 0 \right)$, the system may have stabilizing effect under the conditions $\beta < 0,\ B > 0$ *and* $\mu_0{}' > 0.$

Unstable stratification cases:

Case1. If $\beta > 0,\ \beta' > 0,\ \mu_0{}' > 0,\ B < 0,\ g \rho_0 E_0{}' > \mu_0{}' \chi M_0 \beta' H_0 \in_0$ and $\left(V_A^2 \rho_0 + \mu_0{}' M H_0 \right) \pi^2 l^2 S_1 > g \beta k^2 \rho_0$ then all the coefficients a_0, a_1, a_2, a_3 *and* a_4 will be positive and therefore Eq. (3.6) does not admit any positive root. So, the system has stabilizing effect for disturbances of all wave numbers.

Case2: If $\beta > 0$ *and* $\left(V_A{}^2 \rho_0 + \mu_0{}' M H_0 \right) \pi^2 l^2 S_1 < g \beta k^2 \rho_0$ then the constant term a_4 in Eq. (3.6) will be negative and therefore Eq. (3.6) has at least one positive root thereby implying the instability of the system.

Case3: In the absence of magnetic field $\left(i.e.\ H_0 = 0,\ V_A{}^2 = 0 \right)$, the term a_4 will be negative for $\beta > 0$. Hence the system is unstable for all wave numbers.

The variation in the growth rate parameter n for unstable stratification with respect to various parameters such as kinematic viscosity υ_0, medium porosity $\in_0$, medium permeability k_{10}, square of Alfven velocity $V_A^{\ 2}$ and suspended particle parameter B has been examined analytically by evaluating various derivatives i.e. $\frac{dn}{d\upsilon_0}, \frac{dn}{d\in_0}, \frac{dn}{dk_{10}}, \frac{dn}{dV_A^{\ 2}}$ *and* $\frac{dn}{dB}$ respectively.

$$\frac{dn}{d\upsilon_0} = -\frac{\frac{\rho_0 \in_0 S_1 n}{k_{10}}\left\{\gamma E_0' n^2 + \left(E_0' + \gamma\kappa_0\pi^2 l^2\right)n + \kappa_0\pi^2 l^2\right\}}{4a_0n^3 + 3a_1n^2 + 2a_2n + a_3} \tag{4.1}$$

$$\frac{dn}{d\in_0} = -\frac{\left[\begin{array}{l}\frac{\rho_0\upsilon_0\gamma E_0' S_1 n^3}{k_{10}} + \left\{\left(E_0' + \gamma\kappa_0\pi^2 l^2\right)\frac{\rho_0\upsilon_0 S_1}{k_{10}} - \beta k^2\gamma\mu_0'\chi M_0\beta' H_0\right\}n^2 \\ + \left\{\frac{\rho_0\upsilon_0\kappa_0\pi^2 l^2 S_1}{k_{10}} - \beta k^2\mu_0'\chi M_0\beta' H_0 B\right\}n\end{array}\right]}{4a_0n^3 + 3a_1n^2 + 2a_2n + a_3} \tag{4.2}$$

$$\frac{dn}{dk_{10}} = \frac{\frac{\rho_0\upsilon_0 \in_0 S_1 n}{k_{10}^{\ 2}}\left\{\gamma E_0' n^2 + \left(E_0' + \gamma\kappa_0\pi^2 l^2\right)n + \kappa_0\pi^2 l^2\right\}}{4a_0n^3 + 3a_1n^2 + 2a_2n + a_3} \tag{4.3}$$

$$\frac{dn}{dV_A^{\ 2}} = -\frac{\rho_0\pi^2 l^2 S_1\left\{\gamma E_0' n^2 + \left(E_0' + \gamma\kappa_0\pi^2 l^2\right)n + \kappa_0\pi^2 l^2\right\}}{4a_0n^3 + 3a_1n^2 + 2a_2n + a_3} \tag{4.4}$$

$$\frac{dn}{dB} = \frac{\mu_0'\chi M_0 k^2\beta\beta' H_0 \in_0 n}{4a_0n^3 + 3a_1n^2 + 2a_2n + a_3} \tag{4.5}$$

From the aforementioned derivatives (4.1) – (4.5), it is clear that the growth rate increases with an increase in medium permeability and suspended particles implying thereby the destabilizing effects of medium permeability and suspended particle parameter. The parameters kinematic viscosity and square of Alfvén velocity both have stabilizing effects as the growth rate decreases under these parameters. The medium porosity has a dual character as the growth rate both increases and decreases according as the numerator in Eq. (4.2) is negative or positive respectively.

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