FLUID DYNAMICS

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INDIAN ASSOCIATION FOR THE CULTIVATION OF SCI
JADAVPUR, CALCUTTA-32.

Ripon Professorship Lecture for 1947 delivered at the Indian Association for the Cultivation of Science, in August-September 1952

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PRINTED BY KALIPADA MUKHERJEE
EKA PRESS, 204/1 B. T. ROAD, CALCUTTA-35

A. THEORY

CHAPTER I

FUNDAMENTAL PRINCIPLES OF FLOW

I. 1. FUNDAMENTALS

Two most common elemants in Nature with which man is familiar are "Air" and "Water". Besides the uses to which he puts these elements for his personal physical needs the multifarious duties to which these two elements are now being increasingly applied make a systematic study of their physical and dynamical characteristics of the utmost importance to the progress of human civilization. The ruins of Roman Aequducts near Rome and of the canals on the hill-side near Martanda in Kashmir still bear testimony to the hydraulic engineering skill of the ancients. The use of air as a medium of transport was very limited before the Wright Brothers in the early years of the present century showed the possibilities of aerial flight. The rapid developments of air and water as a medium of transport and for generation of energy and power have been phenomenal since then. This increasing everyday use to which these elements are now being put has made a systematic study of these two mediums indispensable.

Both these two elements are classed as fluids. Both satisfy the fundamental properties of "Ideal Fluid" to different extent. According to definition "ideal fluid" should satisfy the following characteristics :—

1. It should be "Homogeneous".
2. It should be "Incompressible".
3. It should be "Frictionless".

Under ordinary conditions of temperature and pressure "Water" satisfies characteristics 1 and 2 of Ideal Fluid completely. Friction effects in the two mediums "Water" and "Air" are usually negligible; only under special circumstances do friction effects become operative. In general, the agree- ment between the theoretical and experimental results therefore become better as the viscosity becomes smaller.

This statement, however, is true with one important exception. Only in the cases when the "Boundary Layer" formed under the influence of the viscosity remains in contact with the body can an approximation of the actual fluid motion by means of a theory in terms of "Ideal Fluid" be attempted, whereas in all cases where the boundary layer leaves the body, a theoretical treatment leads to results which do not agree at all with experiments.

A classical example is the problem of the resistance of a body (for instance, a sphere) moving through a liquid with uniform velocity. The theory on the basis of a "frictionless fluid" leads to the curious result that the resistance or drag of such a sphere is zero. The reason for the discrepancy is that in the actual case the boundary layer leaves the sphere, so that the picture for the flow is entirely different from the one examined in the theoretical calculations.

Since the hydrodynamics of frictionless fluid leads to results regarding the resistance problem that do not agree with nature and a consideration of viscosity in the equation of motion has offered insurmountable difficulties, there remains only the experimental procedure for determining the laws of drag. For this purpose, extensive series of tests have been carried through, especially for air and water. Model experimentations were greatly accelerated since the beginning of this century by the enormous development in Aeronautics, by the multipurpose river development projects, by the insistent demand for reclamation of low lying areas subject to tidal action for food production and by the rapid development of irrigation practice.

Before starting the discussion of the laws of similarity for model experimentations let us consider the elements of the theory of flow and the principles of internal friction of an ideal fluid.

I. 2. Equations of Motion of an Ideal Fluid

The equation of motion of an Ideal Fluid, otherwise known as Equation of Euler is a vector equation. This can be derived from the fundamental equation of the dynamics of a fluid particle.

$$\text{Force} = \text{mass} \times \text{acceleration}.$$

The forces affecting a particle in a fluid medium are :—

1. The weight per unit vol. $= g\,\rho = \sigma$
2. The pressure drop per unit vol. $= -\text{ grad } p$
3. The friction or viscosity force, μ.

The motion of a fluid is governed essentially by inertia and friction. In most cases we can ignore the effect of gravity, which in compressible homogenous fluid makes itself noticeable only by a change of density. Experience shows that when we are dealing with large volumes of fluid or gases the action of friction in the interior of the fluid is much less important than that of inertia. The differences of pressure that occur here are in almost all cases due to the forces of inertia. The state of affair is entirely reversed while dealing with small masses of fluid in which generally the accelerations are very small; here the differences of pressure that occur are balanced almost entirely by frictional forces and the effect of inertia can be neglected.

Uptil now it has been found impossible to make a mathematical approach to those types of motion in which the frictional and inertia forces are of the same order of magnitude. From practical point of view the question then becomes of importance to know the difference between the actual motion of a fluid with small viscosity (*e.g.*, water) and the theoretical motion of the ideal frictionless fluid of hydrodynamics. From our knowledge of the motion of a fluid with small viscosity it can be stated that the action of viscosity in such liquid is generally confined to a very thin layer next to the boundaries of the rigid body. In this layer—better known as Prandtl's Boundary Layer—there is a steep increase in velocity to which the frictional forces are proportional. Although ideal frictionless fluids are supposed to slide over boundary surface, the particles of fluids of even small viscosity adhere to the surface so that their velocity with respect to the body is zero. When however we are dealing with a liquid of small internal friction this change of velocity takes place in a very thin layer, since considerable velocities—such as would exist in frictionless fluid—appear at a very short distance from the body. The large velocity gradient thus set up is connected with the existence of viscous forces in the boundary layer which are of the same order as the pressure forces.

As long as this thin layer, in which the action of viscosity takes place, remains in contact with the body, the actual stream pattern does not differ greatly from that derived theoretically on the basis of ideal frictionless fluid. When, however, as occurs most frequently, the streaming motion breaks away from the body, the general character of the streamline pattern is immediately changed. Part of the boundary layer leaves the body as a vortex which alters the character of the motion entirely. Under such circumstances the assumption of an ideal frictionless fluid leads to conclusions entirely different from phenomenon met with in nature.

Let us consider the motion of a streamlined body such as a slender air-ship or an airfoil. Here also the frictional forces are at work in the boundary layer because the velocity is reduced to nil on the surface of the body, but since the layer remains clinging to the body and the streaming motion does not separate it does not introduce any fundamental change in the form of the streamlines. In the case of motion round a sphere or a flat plate placed at right angles to the stream, the boundary layer, however, does not remain attached to the body but detaches itself in definite places and then proceeds along the stream and diffuses as vortices. It, therefore, becomes of fundamental importance to know the circumstances under which the boundary layer separates from the body. In those cases when the boundary layer does not separate from the surface a useful approximation to the actual flow can be calculated by considering the fluid as frictionless. If, however, the boundary layer is allowed sufficient time to develop, separate and form vortices the picture of frictionless motion does no longer persist. But as in many cases useful information is obtained from an analysis in which the viscosity of the fluid is entirely neglected, the theory of ideal fluid has been of great help to practice. The results, however, should be accepted with great caution.

I. 3. Euler's Equation

By applying Newton's law of motion to a fluid particle we get the equation derived first by Euler in the form

$$\frac{\delta \omega}{\delta t} + \omega \operatorname{grad} \omega = g - \rho \operatorname{grad} p \tag{I}$$

Using with this the equation of continuity

$$\frac{\delta \rho}{\delta t} + \omega \operatorname{grad} \rho + \rho \operatorname{div} \omega = 0 \tag{I.2}$$

completes the Equations of Motion of an Incompressible Fluid.

An integral of great practical importance can be derived by integrating this equation for steady motion along a streamline C. This for the special case of incompressible fluids (ρ=constant) and with gravity as the body force can be written as

$$\frac{\omega^2}{2g} + \frac{p}{\rho} + z = \text{constant} \tag{I.3}$$

This equation is known as Bernoulli's equation and is used extensively by engineers. This equation in the form (1.3) states that the sum of the velocity head, the pressure head and z, "Static Head", is constant at every point of some definite streamline, if we are dealing with steady motion of an incompressible fluid under the influence of a force function. This constant is generally different for different streamlines and it is only in the case of no rotation that the constants of all streamlines are identical.

As stated before this equation of Bernoulli's has great application in engineering practice.

I. 4. We shall discuss a few of these cases :—

1. *Motion of a Fluid over a weir.*

The weir as a measuring device is well known. Since here the surface of the liquid is a surface of equal pressure we have for the Bernoulli's equation

$$\frac{\omega^2}{2g} + z = \text{constant} \tag{I.4}$$

If the shape of the surface is known say by photography, the velocity at any point of the surface can be calculated from this formula (1.4). In this connection it may be pointed out that if any geometrical relationship can be obtained for any fluid motion it is possible to calculate further results by means of Bernoulli's Equation.

2. *Jet impinging at right angles on a flat plate.*

For symmetry the middle streamline must strike the surface of the plate at right angles with zero velocity. The Bernoulli's Equation for two points on the middle streamline are

$$\frac{\omega^2}{2} + \frac{p_0}{\rho} = C$$

$$0 + \frac{p_1}{\rho} = C$$

$$\therefore \quad \frac{\omega^2}{2} + \frac{p_0}{\rho} = \frac{p_1}{\rho}$$

$$or, \qquad p_1 = p_0 + \frac{\rho\omega^2}{2g}. \tag{I.5}$$

The point at which the middle stream-line comes to rest on the plate is called the "Stagnation Point" and the pressure p_1 there is known as "Stagnation Pressure". The pressure p_0 is known as "Static Pressure".

3. *Cavitation.*

The phenomenon of "Cavitation" in fluid flow is well known to hydraulic engineers. The Equation of Bernoulli can explain this phenomenon in the following way. If the flow is steady the following relationship will hold good between pressure and velocity at two points of the same streamline.

$$\rho\,\frac{\omega^2}{2} + p_1 = \rho\,\frac{\omega^2}{2} + p_2 = C \tag{I.6}$$

In the case of steady irrotational flow if p_0 and v_0 refer to a convenient reference zone (preferably where the flow is uniform) and p, v to any point in the fluid medium then the above equation may be re-written in the following form :—

$$\frac{p-p_0}{\rho v_0^2/2} = 1-\left(\frac{v}{v_0}\right)^2. \tag{I.7}$$

Here we have on the left a term for relative pressure distribution which evidently depends entirely on the relative velocity distribution as given on the right. Since both the terms are in non-dimensional form, they should be independent of the actual linear scale, velocity, fluid density and pressure load on the system so long as the flow continues to be steady and free from rotation.

Equation (1.7) shows that as $\frac{v}{v_0}$ increases and becomes greater than unity the pressure p reduces in value and becomes less than p_0. In the case of gases this change in pressure will result in density changes which can not be ignored. In the case of liquid, however, this drop in pressure will produce no change in density until its vapour pressure is approached when bubbles of water vapour along with such gases as are carried in solution will form about minute particles of foreign matter that happen to be in suspension.

This is how the phenomenon is described by Prof. Straub of Minneapolis :—"When a flowing liquid vaporises as the result of velocity changes which reduce the pressure to its vapour limit, the phenomenon is known as "cavitation". As cavitation just begins tiny vapour bubbles form in rapid succession at the point of lowest pressure and are carried downstream by the flow into a zone of higher pressure where they immediately collapse as the vapour within them condenses. The process of formation and collapse is so nearly instantaneous that with the naked eye only a continuous opaque blur can be distinguished. However, as each of the countless individual bubble collapses, the resulting impact of opposing masses of liquid produces an extremely great local pressure which is transmitted radially outward with the speed of sound, followed by a negative pressure wave which may lead to one or more repetitions of the vaporization-condensation cycle. Boundary materials in the immediate vicinity are, therefore, subject to rapidly repeated stress reversals and may eventually fail through fatigue the first sign of which is surface pitting."

As the velocity increases beyond that required for incipient cavitation no further reduction in pressure can take place, only the zone over which the vapor limit prevails will extend and the size of air bubbles will increase till at an advanced stage a more or less stable vapour pocket is formed which is very much similar to the zone of separation next to a protuberance on a smooth surface. With the formation of such a pocket inside the fluid the flow pattern in the surrounding medium will also change with attendant pressure change. The expression

$$\frac{p_0 - p}{\rho v_0^2/2} \qquad (I.8)$$

is known as Cavitation Number σ.

Plotting a dimensionless pressure distribution curve $[(p_0-p)/\frac{1}{2}\rho v_0^2, x/D]$, where x is a distance measured along the surface and D a representative length of a body of given form, the occurrence of cavitation is not to be expected at any point on the boundary so long as σ has an appreciably greater numerical value than the minimum ordinate on this curve. When σ approaches this minimum value cavitation is to be expected. At values appreciably below this limit a marked effect on the pressure distribution will be observed.

In cases of bodies with forms that will lead to separation of the boundary layer the cavitation is generally noticed to begin within the eddies formed at the surface of separation long before the boundary pressure attains its vapour limit. Under these circumstances it is not possible to predict the value of x either by analytical means or by actual measurement of pressure distribution in flow without cavitation.

Boundary Layer. We have talked of "Boundary Layer" before. This conception of "Boundary Layer" in fluid movement had been introduced by Prof. Prandtl of Göttingen and has been very useful in the analysis of the movement of different bodies in fluid mediums. We have seen before that when a body moves in an incompressible homogeneous fluid of small internal friction, a layer forms round the body in which the velocity with reference to the body changes from a zero value to that at infinity within a very short distance. The layer through which this rapid change of velocity takes place is known as "Boundary Layer". As the velocity change $\frac{\delta \omega}{\delta n}$ is very large though the velocity ω itself may be very small, the shear stress $T = \mu . \frac{\delta \omega}{\delta n}$ assumes values which cannot be ignored.

In the region outside this layer, where the velocity gradient does not become so large the influence of viscosity is negligible. Here the streamline picture is entirely determined by the action of pressure, that is, it is a potential flow. In general it can be stated that the layer in which the velocity is reduced to zero owing to the action of viscosity is thin for small viscosities or to be more general, is thinner the greater the Reynold's Number, $R_e = \frac{lv}{\gamma}$, where γ is the kinematic viscosity.

A consideration of the orders of magnitude of the various terms in the two-dimensional form of Navier-Stokes equation for the flow along a flat plate leads one to the following conclusions :—

(1) Inside the boundary layer the effects of the friction forces are of the same order as those of the Inertia Forces.

(2) In a thin boundary layer, the pressure is approximately equal to that in the outside flow so that in a sense the outside flow forces its pressure upon it.

(3) The velocity in the boundary layer perpendicular to the boundary is negligible.

(4) The velocity in the boundary layer parallel to the boundary changes from zero at the boundary to the velocity of the outside fluid.

Therefore the total flow phenomenon of a fluid of small viscosity round a solid body can be split up into a flow in a very thin layer where the internal friction has a definite influence, and an outside flow where the viscosity has practically no effect. The pressure inside the boundary layer is determined by the flow outside it.

The most important characteristic of the boundary layer is that under certain conditions a back flow takes place in it which leads to creation of vortices and to a complete change in the flow pattern. This phenomenon takes place on the downstream side of a flow through a channel that flares out from a narrow section to a wider section, as in the case of the downstream end of a fall in a canal. Irrigation Engineers know it too well that if the flaring is too rapid the main flow will leave the downstream walls and severe erosion will take place in the earthen channel bank where the rigid structure of the fall ends. To avoid this undesirable action gentle flaring is generally adopted, the degree of flare depending on the velocities of flow.

Another method of preventing back flow or eddy formation at rapidly flaring boundary surfaces was suggested by Prandtl. It consists of sucking away into the interior of the boundary surface those particles of the boundary layer that are just on the point of standing still before flowing back. This method was utilised in avoiding the undesirable effect of a guide bund which could not be flared sufficiently gently upstream of the course of the river Bokaro immediately above a barrage that was put up for the supply of circulating water to the Thermal Power Station. Incidentally this helped in preventing the formation of a sand deposit in the pocket of the barrage.

A phenomenon well known to Hydraulic Engineers and that is being increasingly used in dissipating the energy of high velocity flow over spillways through tunnels and draft tubes is what is known as "Air Entrainment". As the nappe goes down the glacis its smooth glossy surface appears to be suddenly pierced through by air boils making the whole flow frothy. The impact of the solid jet of water coming down the bucket and impinging on the river bed down below is thereby considerably reduced. Investigations carried out have now proved that this phenomenon of "Air-Entrainment" is caused by the development of a boundary layer starting from the rough surface of the glacis and enlarging till it attains the thickness of the jet when suddenly air appears to be sucked in and make the whole jet frothy. With the help of this device the scour downstream of the Fontana Dam T.V.A. (480 ft. high) has been reduced to a negligible extent.

CHAPTER II

TIDES AND TIDAL CHANNELS

II. 1. INTRODUCTION

Tides and tidal channels are well known to the residents of lower Bengal. The whole of this part of the country has been built up by the enormous amount of silt brought down by the rivers Ganga, Brahmaputra and Meghna with their numerous tributaries that drain almost the entire countryside north of Deccan plateau. These millions and millions of tons of silt carried in suspension and rolling on the bed of these three rivers are met by tides from the Bay of Bengal and drop on the bed to form the land that is known as the Gangetic delta. This is one of the biggest delta in the world. It will thus be seen that the great range and intensity of the tides of the Bay of Bengal are also partly responsible for the rapid rise of the delta. We will now examine the phenomenon of tides more carefully.

It is well known that tides are produced by the attraction of the Sun and the Moon on the volume of water in the ocean. But besides the accurate knowledge of the external forces giving rise to tides, it will be necessary to have exact informations about the part played by the configuration of the land masses of the earth surface, by the depths of the ocean, by the rotation of the earth, by the internal forces set up in disturbing the water surface from its normal level and also by the frictional forces expressed by moving objects. It will thus be seen that besides the knowledge of tides which by itself is a fairly complicated subject a knowledge of the effect of land configuration on the characteristics of flow, through tidal estuaries and channels will be equally important. We will however briefly derive the various laws of tidal flow in oceans and straight channels.

The tide-producing forces exerted by the moon and sun are similar in their action, and mathematical expressions obtained for one may therefore by proper substitutions be adapted to the other. Because of the greater importance of the moon in its tide-producing effects, the following development will apply primarily to that body, the necessary changes to represent the solar tides being afterwards indicated.

The tide-producing force of the moon is that portion of its gravitational attraction which is effective in changing the water level on the earth's surface. This effective force is the difference between the attraction for the earth as a whole and the attraction for the different particles which constitute the yielding part of the earth's surface; or, if the entire earth were considered to be a plastic mass, the tide-producing force at any point within the mass would be the force that tended to change the position of a particle at that point relative to a particle at the centre of the earth. The part of the earth's surface which is directly under the moon is nearer to that body than is the center of the earth and is therefore more strongly attracted since the force of gravity varies inversely as the square of the distance. For the same reason the centre of the earth is more strongly attracted by the moon than is that part of the earth's surface which is turned away from the moon.

The tide-producing force, being the difference between the attraction for particles situated relatively near together, is small compared with the attraction itself. It may be interesting to note that, although the sun's attraction on the earth is nearly 200 times as great as that of the moon, its tide-producing force is less than one-half that of the moon. If the forces acting upon each particle of the earth were equal and parallel, no matter how great those forces might be, there would be no tendency to change the relative positions of those particles, and consequently there would be no tide-producing force.

Let us denote the forces acting at any point P on the earth's surface by

F_v = vertical components of tide producing force and
F_a = horizontal ,, ,, ,, ,,

$$\frac{F_v}{g} = \frac{M}{E}\left(\frac{a}{d}\right)^2\left[\frac{\cos z - r/d}{\left\{1 - 2\left(\frac{r}{d}\right)\cos z + \left(\frac{r}{d}\right)^2\right\}^{3/2}} - \cos z\right] \tag{II.1}$$

$$\frac{F_a}{g} = \frac{M}{E}\left(\frac{a}{d}\right)^2\left[\frac{\sin z}{\left\{1 - 2\left(\frac{r}{d}\right)\cos z + \left(\frac{r}{a}\right)^2\right\}^{3/2}} - \sin z\right] \tag{II.2}$$

where M = Mass of moon
E = Mass of earth
a = Mean radius of earth
d = Distance from centre of earth to centre of moon
r = Distance of the point P from the centre of the earth
z = Moon's Azimuth distance

Assuming $r = a$ and neglecting terms containing powers of $\frac{a}{d}$ higher than the cube we obtain

$$F_v = g\frac{M}{E}\left(\frac{a}{d}\right)^3\left[2\cos^2 z - \sin^2 z\right] \tag{II.3}$$

$$F_a = \frac{3}{2}g\frac{M}{E}\left(\frac{a}{d}\right)^3 \sin 2z \tag{II.4}$$

We will not go into more detail analysis of these tide generating forces of the Sun and the Moon. They can be found in any book on "Tides".

II. 2. Meteorological and Shallow-Water Tides

In addition to the elementary constituents obtained from the development of the tide-producing forces of the moon and the Sun, there are a number of harmonic terms that have their origin in meteorological changes or in shallow-water conditions. Variations in temperature, barometric pressure, and in the direction and force of the wind may be expected to cause fluctuations in the water level. Although in general such fluctuations are very irregular, there are some seasonal and daily variations which occur with a rough periodicity that admit of being expressed by harmonic terms. The meteorological constituents usually taken into account in the tidal analysis are the three terms involving periods corresponding respectively to the tropical year, the half tropical year, and the solar day. These constituents are represented also by terms in the development of the tide-producing force of the sun but these are considered of greater importance than meteorological tides. One of the terms occurs in the development of the principal solar force while other ones would appear in a development involving the 4th power of the solar parallax. In the analysis of tide observations both these terms are usually found to have an appreciable effect on the water level.

The shallow-water constituents result from the fact that when a wave runs into shallow water its trough is retarded more than its crest and the wave loses its simple harmonic form. The shallow-water constituents are classified as overtides and compound tides, the over-tide having a speed that is an exact multiple of one of the elementary constituents and the compound tide a speed that equals the sum or difference of the speeds of two or more elementary constituents.

The importance of a knowledge of the tide-generating forces was stressed at the beginning. Hitherto no assumptions have been made as to the response of the water to these forces, so that the knowledge we have acquired as to the variations of the forces is strictly accurate. Now without going into more detailed analysis of the effect of these forces on the volume of water in the ocean, estuaries and river channels, it may be broadly stated from commonsense that the variation of these forces will naturally lead to variation in actual tides.

Hence it is possible to make the following very important deductions :—

1. Tides caused by the moon will recur at intervals of a lunar day.
2. There will generally be two well-marked types of oscillations, one having a period of a lunar day and the other having a period of half a lunar day.

3. Both types of oscillations will be affected in amplitude by the changing distance of the moon, and they will probably change approximately according to the cube of the lunar parallax, this effect being shown from the occurrence of the factor $(a/d)^3$ in the expression for the forces.
4. The declination of the moon will play a very important part, the range of the semidiurnal tides being diminished at times of high declination, and the diurnal tide approximately being proportional to the declination of the moon.
5. Tidal streams will be set in motion by the tractive force, and the characteristics of the streams will be similar to those of the forces. It will be necessary, for instance, to resolve the streams into their components and these again into the diurnal and semi-diurnal parts in order to consider their variations.
6. The solar forces will bring about variations in the tides and in the tidal streams in much the same way as the lunar forces, but because the solar forces are generally less than half the lunar forces in absolute magnitudes the solar tides will be somewhat less than half the lunar tides, on the average.

II. 3. The Distortion of a Wave

The mean rate of propagation of a wave given by

$$C = \sqrt{gh} \qquad \text{(II.5)}$$

where C = rate of travel of a point in the wave profile,

g = gravity force,

h = the mean depth of water in the channel.

is independent of the elevation of the water surface and has been obtained on the assumption that the elevation is small compared with the depth. In very deep water all parts of the wave will travel at the same rate and distortion will not take place.

The more accurate mathematical analysis shew that the rate of propagation is partly dependent upon the elevation and is given by

$$C = \sqrt{g\,(h+3y)} \qquad \text{(II.6)}$$

where y = the elevation of the point above mean level.

It may be noted that it is the flux across the section which is strictly proportional to the elevation and that the flux is proportional to the product of the velocity of the streams and the actual depth from the surface to the bottom. If the elevation varies according to a simple cosine law, so also will the flux but the law of variation of the rate of the stream will not be quite so simple.

The value of c shows that the crests of the waves travel faster than the troughs, so that any particular point in the channel high water appears to be accelerated and low water to be retarded with the consequence that the tide rises more than it falls. This is generally characteristic of tides in estuaries and tidal channels.

II. 4. General Description of Bores

In most parts of the world the tides change slowly and continuously throughout the cycle of the phenomena, but in certain regions there are discontinuities in the motion which produce very striking effects. The tide may rise suddenly and very rapidly, so that a wall of water, so to speak, rushes up a channel and menaces with death or destruction all that lies in its path.

Such a sudden inrush of water is generally known as a bore, and the principal characteristic of a bore is the relatively quick rise in level. It is only very rarely that a bore has a vertical front like a step, and in all cases the vertical front is only a small fraction of the total elevation of the bore. The main part of the profile of the bore, as seen from the banks of the channel, consists of a steep slope and this again is furrowed by large waves, more or less permanent. In many cases where the vertical front does not occur the rapid rise of water level is prominently associated with the passage of these characteristic waves. A bore can travel along a water surface as in a river or along the sands of an estuary, the principal characteristic, as has been said before, being that of the abnormal rapidity of change of water level. In the Hooghly Estuary the water rushes forward as a huge wave 3 to 6 feet high, which is resolved into a great mass of water in violent perturbation. In the Severn the bore rushes up the river at the rate of about 10 to 20 miles per hour, with a height of 3 to 7 feet.

There is evidence to show that at one time this bore in the Hooghly exist, and did not experience has shown that artificial changes in a river have destroyed bores that once existed. Before the Seine was improved (commencing about 1780) it was subject to a very dangerous bore which affected the whole river to Rouen. The bore now only occurs at the greatest spring tides and affects only a short length of the river. All the evidence points to a bore being due to a nice balance between various conflicting forces; natural changes, by silting or otherwise, in an estuary will make all the difference between the existence and non-existence of a bore.

The shape of the surface of the water in the bore has been determined for several places. The profiles of the bores in the Petitcodiac river and in the Trent have been examined very carefully by various works. H. N. Chambers had carried out a systematic survey of the profile and other characteristics of the bore in the river Trent. These data have been examined by R. H. Corkan of the Liverpool Observatory and Tidal Institute. The analysis of these observations disclosed very significant phenomenon. The Trent flows into the Humber and at the point where the river enters the Humber, the rise in the river bed is very rapid and there is no bore. From this point to some distance upstream the gradient of the bed of the river Trent is much less steep and the bore is maximum. Above this point the gradient again diminishes and the bore also diminishes. It was also shown that the rate of travel of the bore diminshes in the last reach.

Though no such elaborate analysis of the bores in the Hooghly river has been carried out by the authorities of the Port of Calcutta, yet careful observations of bore velocities in different stretches of the river have been made. The bore in the Hooghly travels between Budge Budge and Cossipore with a velocity of 16 nautical miles per hour and the height of the bore varies between 3 to 6 ft.

II. 5.

Analysis of data in connection with bores in different parts of the world indicate that one of the essential features to be considered is that of the steepness of the gradient of the river bed, and probably another factor arises from restrictions in the channel.

II. 6. Generation of Bores

We will now consider briefly the causes of formation of bores. There are various theories :—

1. *Restrictions in the channels.*

Very great caution is required before making assertions as to the effects of restrictions in a channel. Take, for instance, a channel in which a steady current is flowing. If at some point the channel is narrowed it is obvious that the velocity of the current must be increased in order to convey the same amount of water in the same interval of time as in the wider part of the channel. A point that is by no means easy to settle is whether a mere change of velocity is or is not likely to give equal rates of transport of volume as well as equal rates of transport of energy. The channel is supposed to be of unlimited length and the bed of the channel to be a level plane. Then the mathematical analysis yields the result :

$$\frac{\text{change in elevation}}{\text{change in breadth}} = \frac{u^2/gb}{1-u^2/gh} \qquad \text{(II.8)}$$

where b is the mean breadth of the broad and narrow parts of the channel

h is the mean depth of the fluid in the two portions

and u is the mean velocity of current.

This result is of great theoretical and practical importance, for we see that whether the elevation of the surface increases or decreases as the breadth decreases depends on whether u^2 is greater or less than gh. It will be remembered that the mean rate of propagation of a free progressive wave is equal to $\sqrt{gh}$.

If therefore, the current has a mean velocity which is greater than the rate of propagation of a progressive wave a constriction in the channel will cause the elevation to be increased. Otherwise the elevation will fall.

It is a simple deduction that if the channel, instead of having parallel plane walls, has one side plane and the other corrugated, then the elevation will have a shape similar to that of the corrugated wall but on a different scale and having peaks where the passage is narrow and troughs where the passage is broad, provided that the rate of current exceeds the rate of propagation of a free progressive wave in water of the same depth. If however, the current has a smaller velocity than this critical value then the profile will be inverted, and the peaks will correspond with the wide portions of the channel.

2. *Flow up an inclined plane.*

If the flow of water is extremely slow then the gradient of the surface is zero; that is, the water finds a normal horizontal level irrespective of the inclined bed. In fact, if u is very small, we have the conditions almost of a lake where the inequalities of the bed are not apparent on the surface.

If the inclination of the bed of the stream to the horizontal is denoted by i then the gradient of the water surface relative to the bed of the stream is given by

$$\left.\begin{array}{l}\text{gradient of surface}\\ \text{relative to bed of channel}\end{array}\right\} = i \ (u = 0) \qquad \text{(II.9)}$$

When, however, the current is not zero, then the mathematical analysis shows that

$$\left.\begin{array}{l}\text{gradient of surface}\\ \text{relative to bed of channel}\end{array}\right\} = \frac{i}{1-u^2/gh} \qquad \text{(II.10)}$$

We see, as in the case of restrictions in a channel that the relation between the velocity of the current (u) and the rate of propagation of a free progressive wave ($\sqrt{gh}$) is a critical one. The gradient of the surface is always greater than i, but when u^2 approaches the value gh then the gradient may become very great, according to the simple theory just utilised.

Here, then, we have something which may be very local in its effects, as distinct from the theory referred to at the beginning. Suppose as in the case of the Hooghly, that the tide rises up a steep slope. Then as the depth rapidly diminishes a slope may be reached in which we have the critical relation.

$$u^2 = gh \tag{II.11}$$

as indicated in the formula. The result of this is that for a very small change in distance the change in elevation would become very great, and would tend to give such a steep rise of front to the water surface as to approximate to a wall of water. It should be pointed out here that other forces would come into operation and modify the flow. We note that the conditions of Eq. II.10 would hold as at first u^2 would be less than gh as in normal tidal motion and would be forced upto a critical value. The steepness of the slope would not greatly affect the tendency to such discontinuity in the profile of water surface, but according to Eq. II.10 the greater the inclination of the bed of the channel the greater the change in elevation. Hence a steep rise definitely favours the formation of a bore in two ways, firstly because of the general magnification of the slope of the water surface, and secondly by accelerating the rate at which the velocity of the water tends to approach the critical value $\sqrt{gh}$.

Again, we see that a restriction in the channel, whether in the walls or in the bed, would tend to yield the same result. We saw that when u is less than the critical value the effect of restriction in the channel is to diminish the elevation inside the narrow channel, and if u tends to the critical value $\sqrt{gh}$ this effect is greatly exaggerated. Hence a steep rise and a narrow entrance to a channel tend to give the same result, and if the conditions are favourable as regards depth so that the critical relations can be quickly obtained, a discontinuity in profile will result, and a bore will be formed.

It must be realised, however, that as such critical conditions are approached the conditions on which the formulae have been obtained are largely violated so that the formulae cannot be used with great precision. They can only indicate the possibilities. As the critical value is approached, instability sets in, and the wave motion associated with the bore follows as a necessity.

II. 7. The Propagation of a Bore

The question now arises as to how a bore is maintained and the rate at which it moves. If the tide in an estuary continues to rise, then a bore is forced onward and it will tend to travel as a free wave. Mathematical analysis shows that the rate of propagation depends on the height of the bore (B) and the mean depth (h) before and after the bore, so we get

$$C = \left(1 + \tfrac{1}{2}\frac{B}{h}\right)\sqrt{gh} \qquad \text{(II.12)}$$

If however we have some upland discharge coming down the river with a current velocity u then the rate of travel of the bore becomes

$$C = \left(1 + \tfrac{1}{2}\frac{B}{h}\right)\sqrt{gh} - u \qquad \text{(II.13)}$$

We see, therefore, that the bore travels forward at a rate considerably greater than that of a progressive wave ($\sqrt{gh}$), and that the rate of travel is diminished if the velocity of the river current increases.

Precise data are not available to illustrate these results for an actual bore, but Wheeler, in his "Tides and Waves" gives some very rough observations of the bore in the Trent. He states the rate of travel of the bore as 15 statute miles per hour (*i.e.*, 22 ft. per second) and the velocity of current immediately after the passage of the bore as 5 miles per hour (*i.e.*, 7 ft. per second). He estimates the depth of the river before and after the passage of the bore as 6 ft. and 12 ft. respectively. Since the mean depth is 9 ft. and $g = 32$ ft. per second, then $\sqrt{gh} = \sqrt{288}$ ft. per second, whence Eq. II.12 gives

$$C = \text{rate of travel of bore } \frac{12}{9}\sqrt{288} \text{ ft. per second} = 23 \text{ ft. per second}$$

and Eq. II.13 gives

$$C - u = \sqrt{288} \text{ ft. per second} = 17 \text{ ft. per second}$$

The value of u is thus 6 ft. per second. The value for C is quite close to Wheeler's value of 22 ft. per second, and the mean current as given by Wheeler is 3.5 ft. per second if the bore invades still water. If we take Wheeler's value for C, then the value of u from the last formulae becomes equal to 5 ft. per second.

The agreement between the observed and theoretical values is remarkable.

Owing to the instability of motion in a bore the formulae given above can only be regarded as of use in indicating conditions favourable to the formation and rapid propagation of bores. It has been found, as was indicated earlier that artificial changes in a river may suppress a bore. The reason for this is clearly revealed in the preceding investigations, for the object of training walls or embankments is to deepen the river, and since u^2 is normally less than gh it follows that if the depth of a river is increased then the chances of formation of a bore are lessened.

There are certain facts regarding bores which can be easily related to the theories. It has been shown that a bore is generated when certain restrictions or conformations of a channel bring about a critical stage of velocity of water. It has also been remarked that the velocities of water normally experienced are below the critical values. It is evident therefore that there is a greater possibility of a bore being caused when the depth of water is low, so that we should expect the greatest bores to occur after low water on a spring tide. Further, the greater the range of tide the greater is the probability of very small depths of water occurring at low water so that large bores are usually associated with large tides.

II. 8. Shoaling Process in Tidal Channels

We have now derived from Mathematical Hydrodynamics as much as is possible to know about tidal cycles and their rate of travel in open sea, estuaries and channels. The problem becomes immediately complicated when we introduce tortuous channels with erodable banks and bed. Not only do the streamlines assume unpredictable configuration but the presence of flexible bank and bed composed of movable material such as sand, silt and clay make the problem of shoaling and scouring of channels only amenable to model experiments. I will not go into the details of these experiments here to-day. You perhaps appreciate how difficult it is to predict not to say to calculate any of the streamlines, and the shoaling of the navigable channel depends mainly on the configuration of these streamlines.

II. 9. Basic Principles of Shoaling

There are two types of shoaling. One involves the following basic processes : (1) pickup of material in one area; (2) its transportation to another area and deposits in that area; while the other takes place in slack waters only and is confined to very fine material mainly in suspension.

Shoaling material may be brought to the site of a tidewater shoal by :

i) Fresh-water streams emptying into a tidal waterway.

ii) Flood-tide currents carrying littoral drift or other material from the open sea through an entrance into a tidal waterway, and so depositing it that it is not again carried out by ebb currents.

iii) Ebb-tide currents, usually reinforced by fresh-water discharge, carrying shoaling material seaward through a tidal entrance and depositing it outside the entrance to form an outside bar.

iv) Littoral drift moving material along the shore to an outside bar across a tidal entrance.

v) Wind-blown material from the land, especially from beaches, into the waterway.

vi) Eroded materials on both banks formed before and carrying them down the stream during freshets.

In the regimen of a tidal waterway, currents are constantly changing both in direction and in velocity; materials are deposited at slack water, and are picked up again and moved elsewhere during the flow tide. The formation of a shoal is therefore usually not a simple process of a single deposit of material in a given location, but is the end product of a complicated series of pickups and deposits, of movements to and fro. In this complex process, our knowledge of the behaviour of currents and the formation of shoals of comparatively heavy material in relatively simple one-way stream flow, while applicable to some degree and under some conditions, must be intelligently modified to accord with the more complex regimen of the tidal waterway.

In the upper reaches of a tidal section of a river or estuary, in which the fluvial characteristics predominate over the tidal and in which the shoaling material is altogether or largely sand or silt, the knowledge of shoaling processes in nontidal streams may be found largely applicable. At or near the mouth of such a tidal waterway, however, the tidal influences usually control, and the shoaling then does not follow the pattern for non-tidal streams; this is especially true of tidal waterways in which the tidal range is large and the shoaling material is predominantly silt.

II. 10.

In tidal waterways, current velocities and directions at a given point and time are determined by the following factors besides those that generally govern non-tidal flow :—

The phase and range of the tide.
The fresh-water discharge from tributary streams.
The interaction of fresh and salt water.
Interaction between tidal currents coming up different channels in the delta.

Very little knowledge is available of the operation and effects of most of these factors individually. When only one is involved in a given tidal waterway, a fairly dependable analysis of the shoaling situation may be possible; but when many or all are involved, sometimes reinforcing and sometimes opposing one another, an assessment of the weight to be given to each in the production of shoaling must be a very uncertain matter.

II. 11.

One complication frequently encountered in tidal waterways is that the ebb current will largely follow one channel and the flood another, with a middle shoal between the two. This tends to encourage shoaling in the ebb channel during flood flows and in the flood channel during ebb flows.

II. 12.

The scouring and carrying power of a current depends to a great extent on the magnitude of its flow and also on the degree of turbulence. Flow in tidal waterways is almost always more or less turbulent, especially immediately after the reversal of the currents. Changes in turbulence, however produced, are in many cases a major factor in the shoaling processes of tidal waterways. Some study has been given to the subject, and some knowledge of the action of turbulence in producing scouring and shoaling is available, but evaluation of its effect under the complex and ever-changing conditions in most tidal waterways remains largely a matter for individual judgment.

Materials carried as bed load or in suspension, and not subject to chemical or electrochemical reactions, will settle out when the transporting power of the current becomes insufficient to carry them. Coarse and heavy particles will settle out first, the finer and lighter materials later, in general accord with Stokes' law. If the loss of transporting power is gradual, the materials will be sorted and distributed over a considerable reach or area; if the change is abrupt, the materials will be deposited in a short distance.

Materials carried in very fine suspension or in colloidal form settle out slowly as the transporting power of the stream diminishes. In cases, they are carried downstream and may be carried out to sea, spread out over extensive areas of shoal water, or flocculated upon contact with salt water, after which they settle out much more rapidly. The last process is of particular interest in the problem of shoaling in some navigable tidal waters.

All colloidal particles bear a charge of electricity which may be either positive or negative according to the natural of the colloid. When two colloidal gels of opposite electric charges are mixed, they precipitate each other. Colloids are also precipitated by electrolytes of opposite sign in solution. Therefore many materials and reagents may undergo such a change in the formation of shoals; further study of the entire subject is needed. It should be remembered that some silts are not subject to this reaction.

In tidal waterways into which considerable quantities of colloidal clays are carried, this reaction gives rise to shoaling processes considerably different from those in freshwater streams. Since salt water is necessary for the reaction, its presence not only causes the flocculation of the materials, but to a large extent determines its subsequent movement and its ultimate deposit in the waterway to cause shoaling.

We thus see that the contributory causes of shoaling in tidal channels are turbulence, configuration of streamlines and behaviour of salt-water wedge.

CHAPTER III

SEDIMENT TRANSPORTATION

One of the most important problems which Hydraulic Engineers have to handle is the regime of canals and rivers, both tidal and non-tidal. A number of empirical formulae have been developed and used successfully for the flow in canals both rigid and erodable, and in non-tidal rivers only. Similar formulae even empirical have not been obtained as yet for tidal channels. Besides the usual mathematical difficulties met with while working out theoretical formulae for fluid flow in natural boundaries, the presence of a movable incoherent and coherent material on the boundary and in suspension has made the problem almost insoluble. The variable constants included in Manning's, Kutter's and Kennedy's formulae do not explicitly contain any term involving the bed or the suspended load. The first attempt along this line was made by G. Lacey of U.P. Irrigation who explicitly introduced a factor called "Silt Factor" in otherwise empirical formulae. Later authors like Bose and Malhotra, Lane and Kalinske have developed formulae containing certain characteristic of silt rolling on the bed. Einstein has also introduced silt load in some of his theoretical formulae. The present position so far as the flow in canals and non-tidal rivers is concerned is not very satisfactory, yet the engineers have a set of formulae to work out the dimensions of his canals and river training works more or less accurately. But the position so far as the tidal rivers and channels are concerned is completely chaotic; no formulae even empirical are available to work out the designs of channels and training works in tidal flow. The work in the River Research Institute, Bengal had been directed to this aspect of the problem for the last few years. Some of the results so far obtained will be discussed here to-day.

As in the case of flow in non-tidal channels mathematical treatment of the subject of tidal flow has been successful in a limited number of simplified cases. With complicated boundaries and with sediment in movement no solution has been uptil now possible either by theoretical or "electric analog" method. In tidal estuaries the transportation of sediment is further complicated by the salinity of the sea water that is known to affect the rate of settlement of the suspended colloidal material. A series of hydraulic observations were set up in a tidal channel called "Kultigong" that flows into a tidal estuary Rai Mangal that opens into the Bay of Bengal. These observations consisted in collecting information of two kinds :—

1. Purely hydraulic data such as gauge readings, cross-sections, velocities and direction of currents;
2. Data other than hydraulic, such as total solid content, salinity and sewage content of the river.

As the object of these investigations was to examine the regime condition of the river with a view to devise methods for stopping deterioration of the river if any, a stretch of the river was chosen for examination between Nowi-Sunti junction at the uppermost point where the tidal effect was inappreciable and the flow was governed entirely by the upland discharge and Bermajur, a point in the lower reach of the river upto which it was desired to study the condition. Within this reach the following points were selected for observation.

i. Nowi-Sunti juction

ii. Haroahat

iii. Malancha

iv. Bermajur

Besides these routine observations, a special series of velocity observations with current meter were carried out at a cross section 850 ft. downstream of Kulti outfall. These observations were carried out for 27 hours at a stretch starting from 9-30 hr. on 1-3-51 and were taken at three verticals across a section for 5 different depths (0, .2D, .4D, .6D, .8D). Three current meters were used from boats at the three verticals and velocity observations were repeated at every half an hour interval, observation at each point taking on an average 2 to 3 minutes. The water surface level at the cross section were taken at every 10 minutes interval. From the velocity-time curves plotted from these observations at different depths for the three verticals, instantaneous vertical velocity distribution curves have been worked out. Simultaneous gauge readings were taken at the cross section and at a point 850 ft. upstream. These have been plotted. A close examination of these curves shew the following :—

1. A reversal of the current direction does not take place as soon as the High Water Level or Low Water Level is attained. For the Kultigong at this cross section it generally takes 60 minutes for ebb-flow-transition and 40 minutes for flow-ebb-transition.
2. The reversal does not take place simultaneously over the whole cross section. It generally takes 15 to 20 minutes for both the reversals.

3. Both during ebb and flow tides, the average velocity point in a vertical is at about 0.6D, the same as in one way channel.
4. During flow tide the rate of rise in the velocity at 0.8D which attains positive value at one-third flow is very high in the beginning (80×10^{-5} ft./sc^2) and falls off gradually to zero value.
5. During ebb tide however the rate of rise of the velocity at 0.8D which attains negative value at quarter ebb is lower (60×10^{-5} ft./sc^2). As the scouring capacity of a stream depends on the rate of change of velocity of the flow as well as on the absolute magnitude of the velocity, the scouring effect during ebb is generally less than during flow.
6. Samples of bed sand was collected at a number of points of the river and were analysed. The average diameter works upto 0.17 mm.

The informations so far collected in the regime of tidal channels have not been sufficient as yet to devise any sort of relationship between the various hydraulic constants such as, discharge Q, slope S, hydraulic mean radius R, wetted perimeter P and silt characteristic m the average diameter of bed silt sample.

It has long been recognised that in the economy of a river system silt plays a very important part. Both in tidal and non-tidal channels it is silt both suspended and rolling on the bed more than the discharge that determines the regime of the channel. Silt has been broadly divided into two components,—one is known as the rolling silt, a silt that hops and rolls and moves slowly as sand dunes on the bed of the channel. This silt is mainly confined to the bottom few inches or few feet depending on the magnitude and character of the flow. The other component is known as the suspended silt—a silt that is once in suspense remains almost always in suspension even when the flow is extremely slow. The rolling silt really forms the so-called boundary layer of Prandtl between the solid bed of the channel and the flowing fluid. It will be recognised that there must be a fundamental difference between the character of the flow on a rigid rough boundary and that on an erodable bed consisting of incoherent materials that form a boundary adjusting itself to the intensity and character of the flow. The writer has been of opinion that the distribution of silt size in the layer really determines the energy distribution in the general body of the fluid and hence determine the channel section and slope. Accordingly silt samples were collected from the bottom layer of water that contains the silt that moves in dunes. To see if in a regime channel this silt does not change in character appreciably from day to day and from stretch to stretch a number of bed samples were collected from a regime channel and analysed. The result of the analysis is given here, giving the average diameter of the sample collected from different points and at different times :

No.	Samples from three tin	Taken within the same hour	Taken within the same day	Taken during the same week	Effect of tempera-ture	Taken from diff. point simultaneously
1	2	3	4	5	6	7
1	.227 mm	.193 mm	.242 mm	.239 mm	.222 (17°C)	.218 mm
2	.223 ,,	.203 ,,	.236 ,,	.241 ,,	.226 (20°C)	.194 ,, (10′u/s)
3	.226 ,,	.212 ,,	.230 ,,	.235 ,,	.232 (23°C)	.217 ,, (20′u/s)
4	.223 ,,	.225 ,,	.228 ,,	.221 ,,	.239 (26°C)	.204 ,, (10′d/s)
5	.223 ,,	.211 ,,	.230 ,,	.237 ,,	.249 (29°C)	.232 ,, (20′d/s)
6	.226 ,,	.226 ,,	.220 ,,	.223 ,,	.252 (32°C)	.217 ,, (30′d/s)
Mean	.225 ,,	.216 ,,	.231 ,,	.233 ,,	—	.214 ,,
Standard deviation	.002 ,,	.018 ,,	.002 ,,	.003 ,,	—	—

The size distribution curve provides a good idea of the proportion in which particles of various sizes are present in the samples analysed. But as the main object of investigation was to obtain an idea of the inter-relation of bed silt with the discharge and other hydraulic elements of a canal that are all capable of numerical single-valued expression, it was necessary to devise one or more indices which could give a numerical idea of the various properties of bed-silt sample. The simplest concept appeared to be that of a mean size, which could describe the sample adequately and allow it to be distinguished from other samples. The correspondence of the percentage size curve to the frequency distribution curve so often used in statistics led to the calculation of weighted mean size '*m*' and of the "standard deviation" *S*. The mean by itself, has a limitation that it cannot distinguish between samples which have a large number of coarse and fine particles and a small number of medium particles, and samples that have a large proportion of medium and small porportion of coarse and fine particles. In other words it does not provide a measure of the heterogenity of the samples. The standard deviation is generally considered a suitable index for describing the dispersion of sizes about the mean size. If it is large the sample can be taken to be composed of a large number of sizes present roughly in comparable proportion. If it is small, the particles congregate in a narrow band on either side of the mean and the samples are comparatively homogenous. It has been shown that these distribution curves reproduce to a remarkable extent the life history of a channel. For the purpose of mathematical treatment of the subject of regime of channels it is necessary to introduce certain constants that will define these distribution curves uniquely. The mean and standard deviation are obvious statistical constants. It became clear as the analysis proceeded that the average diameter size alone could not define completely all the hydraulic characteristics of silt. A study of the distribution curves of the bed silt sample of a regime channel from day to day made it apparent that bed silt samples with almost the same average diameter can have very different distributions. These curves for the same channel show a swing of the peak from the left to the right and then from the right to the left. For smaller channels this swing is not so pronounced as for big channels. It appears that this swing is a characteristic of the turbulence in the channel so that studying these curves from this point of view we shall get some idea about probable stability of the channel.

The distribution curves of bed silt can be broadly divided into three types : the left Hand Type, the Middle Type, and the Right Hand Type. The two types, Left Hand and Right Hand types can be further subdivided into Extreme Left Hand and Extreme Right Hand Type and Left Hand and Right Hand Types. Of course these types cannot be put in absolutely cut and dried compartments and very often they merge one into another. Just as the spectrum of a source of light runs from the ultra violet through the visible part to the infra red end of the spectrum, silt curves of a regime canal run through the above types if the samples are analysed all throughout the year. Each of these types indicates a certain condition of equilibrium of a canal. So the annual frequency distribution of these types of curves for any canal is a very reliable indication of the stage of equilibrium of that channel. This problem can be attacked from two points of view; either by conducting controlled experiments on channels that are known to be in a certain stage of development and comparing the silt curves or by studying the annual distribution of silt curves of different canals and comparing them with deduction from their hydraulic data. In the absence of the former, recourse to the latter method has been taken. The former method would have been preferable as it would have cleared our fundamental ideas and would have been free from assumptions that are inevitable in the latter method.

A different line of attack on this silt problem suggests itself when it is considered that the characteristic of the flow of water in channels that keeps silt in motion is turbulence. In what follows it will be shown how the distribution of silt varies with the state of turbulence in the canal, so that if the turbulence could be detected and analysed it would be easier to get a single constant to represent the full dynamical effect of silt movement in canals. This line of investigation has not been explored—the main difficulty being how to measure turbulence.

By the help of the Prandtl theory of turbulence the distribution of velocity along a vertical situated at a sufficient distance from the sides of an open channel can be investigated in the following way. Taking Prandtl's notation and assumption we have

$$dv = \frac{1}{K}\sqrt{\frac{T}{P}}\,\frac{dy}{y} \qquad \text{(III.1)}$$

where T is the shear stress, is equal to $gPRS$ so that $\frac{T}{P} = gRS$,

$$dv = \frac{1}{K}\sqrt{gRS}\,\frac{dy}{y}, \quad v = \frac{1}{K}\sqrt{gRS}\log y + A$$

Assuming the value of K to be the same as for a pipe ($K = .4$) we have

$$v = 32.5\sqrt{RS}\log y + A \qquad \text{(III.2)}$$

Of the three types of distribution curves mentioned above let us examine the middle type. Before investigating its significance, it is interesting to compare the velocity distribution curves of flow of fluids in pipes under different conditions. The velocity distribution curves in a stream line flow in pipe is a parabola, whereas when turbulence sets in this parabola flattens into a curve which shows a very steep gradient at the boundaries and is almost flat over the remaining portion of the section. This can be easily explained by the fact the turbulence churns up the whole of the fluid and equalises the velocities of the water particles from one boundary to the other. Reynold's famous experiments on the onset of turbulence shew also these churning effects. The middle type of silt curve is very much similar to these curves of turbulent flow and can therefore be associated with a stage in the canal flow when the churning action is going on briskly and silts of all grades are made to move on the bed of the canal. This effect will only be apparent when there is some scouring action taking place on the bed of the canal. So that the presence of this Middle Type indicates that no heavy silting or scouring is taking place in the canal. This type has been found associated with the most stable of the channels examined so far and as soon as a channel starts going off regime these curves become rarer and rarer till with certain channels they are altogether absent.

The Left Hand Types indicate that the predominating components of the bed samples consist mostly of silt that are comparatively fine, and either the coarser components have already settled down in the upper reaches or are absent in the head. In most of the canals the first assumption is correct. In the head reach generally the Right Hand Type curves predominate and as we go down the canal, curves of middle type begin to appear more frequently so that towards the tail end of a canal the right hand type of curves altogether disappear and left hand one predominate.

It therefore appears that canals in which all the three types occur frequently with no bias for any particular type, will be the most stable. In canals with big discharges and consequently with a large range of fluctuation in the discharge these three types will appear periodically. Type I will indicate that the channel has some fine silt that has settled down from suspension. Type II will indicate that scouring action has started and has churned up silt of all sizes. Type III will indicate that the maximum velocity has been attained and all fine components have gone into suspension. Soon after there is an abatement of turbulence, fine silt that had gone into suspension will settle down and the curves will revert back to Type I, sometimes through an intermediate stage. In smaller channels with lesser turbulence all the variations might not occur. The peak of Type II curve will swing about its mean value but will not go beyond Type II.

From a study of the silt distribution curves of different types it has been apparent that these curves though having the same value for the average diameter may differ widely in their shape. As a matter of fact it has been showed just now that these changes in the shape of the distribution curves are associated with certain changes in the condition of regime of the channels and by studying these changes it is possible to find the life history of these channels from day to day. In order to bring this idea in the statistical analysis of the hydraulic constants of the channels, in addition to the average diameter of the bed silt size, which as has been shown is a real characteristic of the bed silt, a new constant defining the shape of these curves has been introduced. This has been called the 'mix of the samples, which has been defined by the "Skewness" and 'Flatness' of the curves. Hence we have the three following characteristic defining completely a bed silt sample

$$\text{Average diameter} = m = \frac{\int xf(x)\,dx}{\int f(x)\,dx} \tag{III.3}$$

$$S_1 = \frac{\mu_3^2}{\mu_2^2} \tag{III.4}$$

$$S_2 = \frac{\mu_4^2}{\mu_1^2} \tag{III.5}$$

where $y-\int(x)$ is the equation of the distribution curve, y giving the percentage of particles of diameter size x and

$$\mu_2 = \frac{\int x^2 f(x)\,dx}{\int f(x)\,dx} \tag{III.6}$$

$$\mu_3 = \frac{\int x^3 f(x)\, dx}{\int f(x)\, dx} \tag{III.7}$$

$$\mu_4 = \frac{\int x^4 f(x)\, dx}{\int f(x)\, dz} \tag{III.8}$$

The limits of integration being the whole range of the diameter size of the sample.

Having now analysed size distribution curves of bed samples completely, hydraulic data and bed samples were collected from sites in canals known to be in regime and kept under observation for a considerable length of time with the following points in view :—

1. First and foremost, to find, if possible, a representative characteristic of silt, a characteristic that will define silt, both from its physical and hydraulic standpoints. It has been shown before that *m*, the average diameter of the silt defines silt completely and for sites in regime the "mix" of the samples does not vary from place to place nor from time to time. This quantity *m* has, therefore, been used to define silt completely.
2. Certain relevant hydraulic constants were chosen and their variations studied from day to day—in some cases over a period of 24 months. Some of these variations were found to be increasing with time—in some channels these increases were noticeable after 10 or 12 months had elapsed, in others they started at the beginning of the observations. In the first case, data have been averaged and others, where the channels are slowly going off regime, but their rate of drift is small though perceptible after the lapse of considerable time, average values for these channels have also been worked out. Only a few channels have been rejected.

The collection and selection of data having been completed, the help of correlation analysis was taken at the stage. It will be pointed out here that correlation analysis is generally used for two purpose—

1. To detect any evidence of causation.
2. To detect any evidence of association between the statistics so collected. In its first role this analysis is rather a dangerous weapon to handle. When investigating causation it is usually better to decide first on other grounds that a casual relationship is likely and then to conduct a close analysis of other factors before using the correlation co-efficient as evidence.

If a linear regression formulae is used to predict one characteristic from another, the correlation co-efficient may safely be used to define the accuracy of the prediction. It was decided therefore to use this analysis for the latter purpose only.

This analysis first started on the assumption that the relationship that was to be deduced from these sets of data should be of immediate practical application. An engineer's first concern when he is going to design a channel is to decide what slope to give to the channel if it is to carry a certain discharge and a certain silt.

Statistical analysis of these data, led to the following relationship between S, Q and m :—

$$S \times 10^3 = 2.09 \frac{m^{\cdot 86}}{Q^{\cdot 21}} \qquad \text{(III.9)}$$

where S is the slope of the water surface measured in feet per thousand feet, Q the full supply discharge in cusecs, and m the average diameter in mm. of the bed silt that the channel can carry.

This relationship has a very high correlation—about 0.90 which is a very close approach to a law of nature in spite of the fact that silt can and does vary considerably from month to month.

Having determined the slope, the next problem is to determine the shape of the channel, that is whether the channel is to be broad and shallow or narrow and deep. This conception can be mathematically expressed by the ratio between the depth and the width of a channel, or, what is better from the hydraulic standpoint, by the ratio between the hydraulic mean radius R and the wetted perimeter P, so that shape can be expressed by $\lambda = \frac{R}{P}$.

It has been found from field observations as well as from laboratory experiments that the shape of a channel is very closely related to its slope and silt. Gibson notes in his Report on the Model Experiments on the Severn River; "It may be of interest to note that a distortion of scale is usual in nature, small streams flowing through alluvial ground having much steeper side slopes and gradient than larger rivers of similar regime in similar ground. In a very large river such as the Mississippi, the Ganges, or the Irawadi, the maximum depth will rarely exceed 1 : 100 of the maximum width while in a small stream in similar ground the ratio will seldom be less than 1 : 5".

This indicated the method to be adopted for the investigation of the relationship between X, S and m, and it was found that the relationship obtained was as significant as that between Q, S and m in equation (III.9). This relationship is—

$$\lambda = \frac{R}{P} = \frac{1}{6.25} \frac{S^{\frac{1}{4}}}{m} \qquad \text{(III.10)}$$

The shape of a channel is admittedly an indication as to whether it is in good or bad regime. But this by itself is not enough to solve the whole problem of design. This relation together with the only relation of Lacey not involving "*f*" silt factor, *viz.* :—

$$P = 2.8\,Q^{\frac{1}{2}} \qquad (6.3)$$

will solve the whole problem of design.

These three relations, which are reproduced below furnish all the information that is required for the design of a channel if the discharge and the silt that it is going to carry are known.

$$S \times 10^{3} = 2.09\,\frac{m^{\cdot 86}}{Q^{\cdot 21}} \qquad (\text{III.11})$$

$$\lambda = \frac{R}{P} = \frac{1}{6.25}\,\frac{S^{\frac{1}{4}}}{m} \qquad (\text{III.12})$$

$$P = 2.8\,Q^{\frac{1}{2}} \qquad (\text{III.13})$$

These relationships are empirical and have not been derived theoretically. However, they have been derived from a collection a regime data and the high correlations that are found to exist between them justify their adoption. These relationships are now put before engineers so that they may be tested and modified if necessary. The only quantity which an engineer will find difficult to fix is *m*. It has been found that there is an annual variation in the values of *m* of any particular channel. It will now be shown how this can be explained by the variation of temperature of the water of the channel.

It is well-known that there is a considerable variation in the temperature of canal water. In the Punjab canals it has usually a range from 8° or 9°C. to 27° to 28°C. This big variation in the temperature of water will have a corresponding variation in the viscosity of water. The variation in the Kinematic Viscosity $\gamma = \dfrac{\text{Viscosity}}{\text{Density}}$ of water in this range of temperature is from 0.0136 to 0.0084—C.G.S. units.

This change in temperature though considerable, will not have an appreciable effect on the average movement in the general body of the fluid, since the magnitude of the Reynolds' number being very high the effect of viscosity will be almost negligible. The only zone where this change in the kinematic viscosity of water will have any effect will be in the boundary layer in which there is the rolling silt. It is well-known that in the flow of water through pipes turbulence starts at the wall of the pipe and proceeds inwards so that it is apparent that a rational solution of the movement of water in stable channels in alluvium must begin at this boundary layer through which the turbulence is transferred from the solid ground to the general body of the fluid. In the absence of any definite knowledge as to how this transference takes place through the cushion of silt and water in this layer, the following simplifying assumption is made about the probable effect of temperature variation on the movement of silt in this boundary layer.

It has been shown above that the effect of a temperature variation from 9°C to 28°C is to diminish the value of the kinematic viscosity from 0.0136 to 0.0084. This change of viscosity in the boundary layer will make silt particles of the same effective size with reference to diameter fall faster in summer than in winter. So that assuming that the magnitude of the upward lift of the turbulent eddies that move along the boundary layer remains more or less steady throughout the year, (as this is one of the necessary conditions of regime), then the silt that will be found in motion in the boundary layer will be of smaller size in summer than in winter. This deduction has been verified by the analysis of silt samples collected during the different parts of a year from regime site. Of course this variation in the diameter size of the bed silt will be marked only in regime channels, because if the channel be not in regime, there will be a continuous increase in the diameter size of the particles or a continuous diminuation, or the change in 'm' will be irregular so that the effect of tenperature will be masked.

To take account of these temperature variations all the values of 'm' have been reduced to one definite temperature, the average temperature for the whole year. This is about 20°C. So that in making use of the regime relationship in which 'm' occurs, the value of m for the site under observation must be reduced to that for 20°C. This is done in the following way, which has been worked out from Zahm's extension of Stokes' Equation for the time of fall of spheres in a column of water. Suppose it is required to find the regime slope for a channel carrying Q c.ft./sc. and bed silt collected at 9°C of average diameter size $m = .310$ mm. In order to find the slope from equation (III.9) this diameter must be reduced to 20°C which is $m_{20^\circ} = .276$ mm. and used in the equation.

B. PRACTICE

CHAPTER IV

MODEL EXPERIMENTS FOR FLUID DYNAMICS PROBLEM

IV. 1. DIFFICULTIES IN PRACTICE

We have seen by now that in fluid dynamic problems, rigorous theoretical solution is in very few cases practicable. This may be due to either of the following factors :—

1. Limitation of mathematical technique.
2. Imperfect knowledge of the various physical features involved in the fluid flow phenomenon under consideration.
3. Introduction of such foreign substance as sand, silt and clay in the fluid medium with consequent change in the properties of the liquid and the characteristics of the boundaries.

IV. 2. LIMITATION OF MATHEMATICAL TECHNIQUE.

We have seen before how the failure to solve the Navier Stokes Equations for viscous fluid movement led to various simplifying assumptions such as are required for Oseen's Solution, Prandtl's Boundary Layer Equation, Karman's Vortex Sheet Movement and Aerofoil Theory (Trag flugel Theorie). It is not necessary for me to go into more detail about these various simplifying assumptions. These are well known to the audience. These are only been mentioned here to indicate the limitation of our present day mathematical technique.

IV. 3. IMPERFECT KNOWLEDGE OF THE VARIOUS PHYSICAL FACTORS INVOLVED.

This point is very well illustrated by the theory of Drag as developed by Prandtl for the movement of an object, a sphre or cylinder in a viscous medium. It will be helpful if we recapitulate briefly the essentials of this theory as these will be useful for us while developing the theory of model experiment.

IV. 4. DRAG OF BODIES MOVING THROUGH FLUIDS.

When a body is moved with a uniform velocity along a straight line through a fluid at rest, it experiences a force in the direction opposite to that of the motion. This force is called the drag or resistance. The first resistance law was proposed by Newton. With slight modifications this law still holds good to-day for motions where the drag is due to inertia which often is the case for fluids of very small viscosity, like water or air. Newton's law can be written as

$$D = f. \quad A\rho\omega^2 \tag{IV.1}$$

where D is the drag, w the velocity of the body ρ the density of the fluid, A the projected area of the body in the direction of flow and f the factor of proportionality which will depend on the geometry of the body and the condition of flows.

Newton's assumption leads to a very simple formula for the proportionality factor 'f'. Experiments however have later on proved that these factors did not agree with the experimental ones. For example, Newton's calculations for the motion of a square plate perpendicular to its direction of motion gave the value of the factor f as unity while experiments indicated a value 0.55. The difference between theoretical and experimental values were still greater for skew plates or rounded bodies like spheres, while for streamline bodies like airships or submarines the agreement is still worse.

The cause for this discrepancy between the theoretical deductions based on Newton's assumption and the experimental results is due mainly to the fact that according to Newton the condition of flow in front of the body is only taken into consideration, while what happens at the tail end of the object really governs the value of the drag. Let us suppose that a body is moving along a straight line with uniform velocity completely immersed in a homogeneous and incompressible fluid. Forces acting on the body are only viscous forces and inertia forces, the effect of gravity forces being completely eliminated as there is no free surface. The ratio between these two forces operating on the body is known as Reynolds' Number $l\omega/\gamma$ where w is the velocity of the body, l some length characteristics of the body and γ the kinematic viscosity of the fluid.

$$\frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{l\omega}{\gamma} = R_e, \text{ Reynold's Number} \tag{IV.2}$$

for very viscous fluids for which μ is very large, or for small body dimensions or fluid velocities, the value of R_e becomes very small. In such movement the body as if pushes through the fluid which is deformed by it, and the resistance to the motion is primarily due to the forces necessary for the deformation of the various fluid particles. For very small Reynold's Number the deformation action is felt over a long distance, whereas for large Reynolds' Number this is confined to the boundary layer only. In the latter case the viscous stresses produce what is know as "Skin Friction" or "Friction Drag". In the first case deformation resistance predominates and the skin friction and inertia action can be neglected.

In a motion with moderately high Reynold's Number such as in the movement of a sphere of 2" radius with a velocity of 3 ft/sc in water, the ratio between the Inertia forces to Viscous Forces is 50,000 : 1. Hence the magnitude of the viscous forces is negligible as compared to that of the Inertia Forces. But the value of the drag obtained by neglecting viscous forces altogether in the classical equations is zero. Hence the importance of viscous forces, even though very small and negligible, lies in the great influence that these forces exert in determining the flow pattern immediately round tbe hody. We have seen before how Prandtl by his assumption of the "Boundary Layer" had divided the flow medium into two regions so that excepting for the "Creeping Flow" the viscosity of the fluid produces two kinds of "drag" in the body, (1) the friction forces tangential to the surface of the body, the resultant of which is produced by the change in the pressure field due to the change in the flow pattern induced by viscous forces. For the "Creeping Flow" that is, for small Reynold's Number, the drag is called "Deformation Drag" whereas for larger Reynold's Number the drag consists of "Friction Drag" and "Pressure Drag".

The different types of resistance D enumerated above for different types of flow are functions of Reynold's Number R_e

1. For very small values of Reynold's Number, that is, $R_e < 1$

Stokes Law holds : $$D = C\mu l\omega = \frac{C}{R_e} A \cdot \frac{\rho\,\omega^2}{2} \qquad \text{(IV.3)}$$

where l is a characteristic length dimension of the body.

2. For Reynold's Number less than 5×10^5, Blassius derived a relationship

$$D = \frac{1.327}{\sqrt{R_e}} \; S.\rho. \frac{\omega^2}{2} \qquad \text{(IV.4)}$$

where S is the surface area of the plate.

3. For Reynold's Number greater than 5×10^6. Weiselberger derived

$$D = \frac{0.074}{\sqrt[5]{R_e}} \; S.\rho. \frac{\omega^2}{2} \qquad \text{(IV.5)}$$

4. For the intermediate range $R_e > 5.10^5$ and $< 5.10^6$. Prandtl has proposed

$$C = \frac{0.074}{\sqrt[5]{R_e}} - \frac{1.700}{\sqrt{R_e}} \qquad \text{(IV.6)}$$

From the above it will be seen that the laws governing Friction Coeff. 'C' of a body is a complicated function of Reynold's Number of the fluid flow and the exact value of the Coeff. can only be determined by model experiments.

IV. 5. Introduction of Foreign Substances in the Fluid Medium.

The most important of fluid dynamics problems that a Hydraulic Engineer has to deal with involve the presence of foreign substances such as sand, silt or clay that moving with the liquid medium affect the boundary condition, modify its viscosity and its turbulence characteristic also. The way these are effected and the extent to which these are affected are still very little understood. Prof. Vanoni of the California Institute of Technology, Pasadena, has shown in his most recent investigation that the presence of clay in suspension affect the viscosity of the fluid medium. It has not been possible as yet to assess this effect more quantitatively.

The position now can be summed up as follows :—

The solution of fluid dynamics problems cannot be obtained with the help of mathematical analysis only excepting for a few simplified cases.

The problems of fluid dynamics can be divided broadly into two classes. One, in which the boundary is rigid with or without a free surface; two, in which the boundary is flexible *i.e.*, erodable, formed of the same material as is carried in suspension. These problems have generally a free surface.

Having come to realise that mathematics cannot help them in the solution of their everyday problems, Hydraulic Engineers turned to Nature for solving her own mysteries.

Experiments on flow in pipe and aqueduct were undertaken by Archimedes, Leonardo Da Vinci and others in very early days. Useful results were obtained and applied. It was however very soon realised that though these ad hoc experiments helped the practical engineers in their everyday life, the science of Hydraulics was not being advanced to any extent. Very soon Hydraulics became a science of coefficients. It was at this stage that Reynolds came out with his famous series of experiments on pipe flow. As is now well-known, Reynolds proved that the whole field of fluid dynamics can be broadly divided into two regions, Stream line flow and Turbulent flow; and the transition from one stage to another took place over a Zone of Instability. He also showed that a number now known as Reynold's Number involving a representative length and velocity of the flow and kinematic viscosity of the fluid $\gamma = \mu/\rho$

$$R_e = \frac{l\omega}{\gamma} \qquad \text{(IV.7)}$$

could very faithfully indicate the different conditions of the flow as it changed from the laminar to turbulent stage. The critical value of R_e has been confirmed to be in the neighbourhood of 2000–10,000 depending on the conditions of flow at the entrance.

As has been shown before

$$R_e = \frac{l\omega}{\gamma} = \frac{\text{Inertia forces}}{\text{Viscous forces}}$$

When the viscous forces are very large as compared to the Inertia forces, the value of R is small and the flow is laminar, when the viscous forces became very small, the magnitude of the Reynold's Number assumes very high value and the motion becomes turbulent. The same conclusion can be arrived at if we examine the values of 'Drag' as developed by Prandtl for increasing values of R . The conclusions therefore become clear that at very high Reynold's Number, the influence of viscous forces on the fluid movement tend to diminish and for sufficiently high values we can ignore it altogether. The only other forces that will be then operative will be "Inertia Forces" and forces of surface Tension.

It is well known that in fluid movement under natural condition such as canal or river flow, flow through pipes and aqueducts, discharges over spillway and undersluices in dams, the motion is turbulent and the value of the Reynold's Number is very high. As the difficulties of examining these movements in the prototype were found both great as well as expensive, attempts were made early this century to reproduce these phenomenon in scale models. Buckingham's analysis of the whole problem of scale models is too well known to bear repetition. The four non-dimensional parameters involving the various constants of fluid flow v velocity, a length, ρ density, μ friction, and e surface tension, are,

$$F = \frac{v^2}{ga}, R = \frac{va}{\mu}, W = \frac{va^2}{e/\rho}, C = \frac{v^2}{e/\rho}$$

In actual practice it is only on rare occasions that all these four parameters are operative. Buckingham's analysis has shown that in order that the movement in the model should be similar in all respects to that in the prototype, the values of these parameters should be the same both in the model and the prototype.

Let us examine the flow over *a spillway.* In this case the only parameters that will be operative are F and R unless the model is made too small to introduce surface tension. In the prototype the flow is certainly turbulent and the value of R is very high. We may therefore ignore the effect of viscosity and take into consideration the parameter F only which should be the same both for the model and the prototype.

$$F = \frac{v^2}{ga}$$

If therefore in the model we do not have very small scale so that R_e for the model is fairly large, velocity will be reduced in the same ratio as $\sqrt{a}$ and the value of F will remain unaffected. The problem that are treated by models of this class, but are influenced by other types of flow characteristics also, can be classified under

1) Scour below a fall.
2) Cavitation over a spillway.
3) Air Entrainment.

Scour below a fall—This class of problem is very frequently met with in irrigation, agriculture, and land reclamation practices. It consists of a rigid structure over which the flow takes place but as it leaves the rigid downstream floor of the fall, it encounters the natural erodable soil on which the fall is built. The extent and depth of the scour produced by the flow on this soil which determine the safety of the structures, have to be determined from model experiments. The best devices to minimise the extent and intensity of the scour so as to make the fall safe are also investigated on this type of model.

Cavitation over a spillway—As we have mentioned before, with very high heads that are now common in hydraulic structures such as spillways, undersluices and power tunnels, the velocity of flow generated is also very high and danger due to cavitation is more frequent. One of the cases that may be recalled in this connection is the very serious damage to the spill way tunnel of the Hoover Dam that took place while a comparatively small discharge was being passed over the spillway—only a third of the designed discharge. Extensive investigations are being carried out in different laboratories to understand the dynamics of this phenomenon.

Air Entrainment—This is another phenomenon that occur in problems amenable to model experiments of the type under discussion. The difficulties of reproducing this flow characteristics under scale model conditions are recognised and efforts are being made to study this phenomenon in as big a scale as feasible but under regulated model condition.

Let us next examine the flow over a barrage to be constructed on the sandy bed of a river. Here also for sufficiently high Reynold's Number, turbulence will be high and viscous forces can be neglected. Even if the viscous forces are ignored both in the model and the prototype, the *tractive forces* cannot be neglected as otherwise the bed material in the model will not move in the same way as in the prototype, and the bed configuration will not be reproduced correctly in the model. The Tractive Force in the model is made the same as in the prototype by giving a greater depth of flow than that warranted by Froude Law. The scale ratio for the vertical dimension is therefore made different from the horizontal scale, so that the bed slope in the model is exaggerated and though the velocities in the model had been reduced according to Froude Law, the distortion in bed slope will introduce Tractive Forces that will move the bed materials in the same way as in the prototype.

Theory of "Tractive Force"—In the tractive-force formulas, the physical characteristics of the moving material are expressed in terms of the tractive force of the stream, namely, WRS, in which W is the unit weight of water, R is the hydraulic radius, and S is the energy-gradient. They are mainly empirical. Four such formulas, those of Schoklitsch, Krey, Kramer, and the United States Waterways Experiment Station are considered below.

The significance and applicability of tractive forces determined in an experimental flume of the usual dimensions are moot. Kramer defined the critical tractive force as that which effects a general movement of the material up to and including the largest component particles. He concluded from his experiments and other independent confirmations that the critical tractive force is constant for a given sand-mixture, that is, it is not determined by R alone, or by S alone, but is solely a constant product WRS. The *SFIT* investigations indicated no connection between tractive force and bed-load for a constant value of the former, the load was different for different depths of flow. Although these two findings are not incompatible, they show the need for further clarification of the concept of "tractive force".

In general, it may be said that little is known about the critical tractive force of very fine-grained material. The experimental results with coarser grains can be expressed in the following general form : Tractive force—cons $\times$ $(\text{diam})^n$ where n is an exponent. The tractive force formulas given below are of this form.

(a) *Schoklitsch tractive-force formula*

$$WRS = \sqrt{0{,}201\ W_1\ (W_1 - W)\ \lambda D^3}$$

in which W is the unit-weight of water in kg per cubic meter, R is the hydraulic radius in meters, S is the energy-gradient, W_1 is the unit-weight of the transported material, λ is a form-factor varying between 1.15 and 1.35 for sand and here assumed to be 1.25, and D is the diameter of grain in meters. The diameter is that of a particle having the same volume as the test-particle. The formula was originally given in terms of the volume.

This formula is for a single grain resting freely on a bed of uniform grains of the same size and gives the critical tractive force at which the particle just begins to move. In a natural stream the bed-load material (geschiebe) is not uniform, of course, but in most cases it is derived from the bed-material and, hence, is of the same size-range, though not uniform. For this reason, it is considered to be applicable to the stream-problem investigated here. This will be discussed again below in some greater detail.

It has been said that the condition for which the above formula was set up does not occur in nature. Inspection of Austrian mountain streams by Mr. Shulits during the dry season showed them to be covered with large stones and pebbles, all apparently of the same size, leaning downstream. They formed a protective or erosion pavement that had to be moved before the underlying bed could be scoured.

Transposition of above formula and conversion yields

$$D = 439\ (RS)^{2/3} \tag{8}$$

where D is in mm. and R is in feet.

There is another critical tractive-force formula by Schoklitsch which will be given for general purpose, although it is not used here. It is for the critical tractive force of a single stone of volume V resting on a bed of similar stones of volume $V_1 < V$. This tractive

$$WRS = \sqrt{0.385\ W_1\ (W_1 - W)\ \lambda v} \Big/ 1 + \sqrt{10E\left[\frac{V}{V_1} - 1\right]} \tag{9}$$

The formula is for metric units, the volumes being in cubic meters, and the other symbols as defined in the above formula.

(b) *Krey tractive-force formula.*

$$WRS = (0.045 \text{ to } 0.07)\ (W_1 - W)\ D \tag{10}$$

which is dimensionally correct. The symbols have the same meaning as in the Schoklitsch tractive force formula. The diameter is the median diameter, that is, 50 per cent of the mixture is larger than this size.

The numbers 0.045 to 0.07 represent a variable co-efficient, for the determination of which no rule is given. It will be assumed equal to 0.06, the mean of the limiting values. This formula finally becomes

$$D = 3080\ RS \tag{11}$$

in which D is in mm. and R is in feet.

The formula is based on tests with sands that were practically uniform in size. The use of the "median" diameter is, therefore, somewhat misleading, since mixtures were not employed.

The critical tractive force given by this formula is that at which noticeable or lively movement occurs and, therefore, for a given size it will be higher than that obtained with the Schoklitsch tractive-force formula. This means that for a given tractive force the Krey formula should yield generally a smaller diameter than the Schoklitsch formula.

The reason for the criterion, "noticeable or lively movement", was that the purpose of Krey's tests was to determine the conditions necessary to insure such a movement in river-models with movable beds.

(c) *Kramer-tractive-force formula.*

$$WRS = (100/6)\,(D/M)\,(W_1 - W) \tag{12}$$

which is a dimensionally correct formula. D is the mean diameter obtained mathematically from the mechanical-analysis curve and M is the "uniformity-modulus". The latter is computed in the following manner: A mechanical-analysis curve is plotted with the percentages passing as ordinates and the grain-diameters as abscissas. The area bounded by the curve, the ordinate axis, and the horizontal line through 100 per cent is divided into two parts by a horizontal line through 50 per cent. The area below this 50 per cent line divided by that above it is the uniformity-modulus.

This formula is based on tests with three sands by Kramer himself, and the results obtained with other sands by Schaffernak, Schoklitsch, Krey, Prussian, Experimental Institute for Hydraulic Engineering and Naval Architecture, Engels, and Gilbert.

For a uniform-grain sand, M has the value unity; for mixtures it is always less than unity; the greater the range between maximum and minimum grain-sizes, the closer does the modulus approach zero. As will be explained below and was intimated above under the Schoklitsch tractive force formula, a uniform-grain sand will be used as a basis of comparison. Hence, $M=1$, and formula (12) becomes finally

$$D = 11170\,RS \tag{13}$$

where D is in mm. and R is in feet.

Kramer defines his critical tractive force as that which effects a general movement of the material up to and including its largest component particles. This definition indicates that, in the last analysis, the purpose of his tests was the same as Krey's.

Although the range of critical tractive force of the data Kramer investigated was 0.00492 to 0.1115 ℔. ft^{-2}, the chart on which the data were plotted and which gives the line represented by his formula shows that the maximum tractive force actually employed was 113 m^{-2} (0.0232 ℔. ft^{-2}). Only four points have a higher value.

(d) *United States Waterways Experiment Station tractive-force formula*—

$$WRS = 0.0039\sqrt{D(W_1 - W)/M} \qquad (14)$$

in which R is in feet, W and W_1 are in ℔. ft^{-3}, and D is in *inches*. The diameter and M, the uniformity-modulus, are as defined for the Kramer formula.

In its final form for use in this study, this equation becomes—

$$D = 66{,}600{,}000\ (RS)^2 \qquad (15)$$

in which D is in mm. and R is in feet.

The basis of the United States Waterways Experiment Station formula is the data Kramer used plus the results of tests with eight mixtures conducted in the Experiment Station. The remarks made above under the Kramer formula relative to the range of the critical tractive force apply here also.

Observed Natural Tractive Forces.

Certain agencies have observed in nature the critical tractive forces necessary to move given sizes of particles. The information available is of two types : (1) The critical tractive force necessary to move particles of stated sizes or between certain size-limits, to which class belong the data of the Nuernberg Agricultural Bureau (Kulturamt Nuernberg); (2) the critical tractive force required to move a size described in general terms, such as sand, coarse gravel, etc.. In the latter group are the data given by Franzius and Kreuter.

(a) *Nuernberg Kulturamt critical tractive forces*—Table I gives the critical tractive forces at which the stated sizes just begin to move. These

TABLE 1.

Critical tractive forces observed by Nuernberg Kulturamt.

Description of material	size	Critical tractive force
	mm	kgm^{-2}
Ordinary quartz-sand	0.20—0.40	0.18—0.20
Ordinary quartz-sand	0.4 —1.0	0.25—0.30
Ordinary quartz-sand	Up to 2.0	0.40
Round quartz-gravel	5—15	1.25
Loamy soil	—	1.0 —1.2
Coarse quartz-pebbles	40—50	4.8
Flat limestone	(10—20 thick) (40—60 long).	5.6

are the results of measurements by the Nuernberg Kulturamt with material in its natural condition in the river.

A plot of tractive force against particle-size was constructed from the tabulated data. There were two lines representing the size-limits as given. Since the tractive force for various river-discharges could be computed, the diameter of grain transportable by different flows could be read from the chart and plotted. As the maximum transportable size was desired, the branch of the chart corresponding to the upper limit was read.

I have stated here the theory of "Tractive Force" on which all the arguments of movable bed models rest. The theory is not yet complete and hence "Proving of the Model" is necessary before we can proceed to apply findings from the model to prototype conditions.

Proving of the model.

This is one of the most important step before the model can be used for testing the effect of any proposed alterations or training works in a hydraulic structure. It must be remembered that a scale model is only an apparatus which must be proved to reproduce known results before it can be used for predicting what will happen in future after certain changes have been made in the object under investigation. In proving a model the following points have to be closely reproduced :—

1. Gauge, Gauge-discharge relationship, Tidal Curves.
2. Velocity and Current direction, both surface and bottom and
4. Bed configuration.

In non-tidal rivers or flow over spillways, weirs or falls, the reproduction of gauge-discharge relationship is of the first and foremost consideration. Unless this is done correctly at least within the range of accuracy of prototype measurements, the model can not be accepted as a reliable instrument. There may be various reasons why the reproduction of gauge-discharge relationship as obtained from the prototype observation is not satisfactory at least within the limit of prototype accuracy. In river models the choice of the scale distortion for the sand used might not have been correct. Under such circumstances instead of changing the bed material which may mean considerable expense and time, a suitable change in the scale distortion very often gets over the difficulty. It should be mentioned here that different circumstances demand different treatment.

In tidal rivers again the reproduction of tidal cycles with proper gauge values and time lag as in the prototype is of the first consideration. It very frequently happens that due to the operation of the tide generating gates a "hunting movement" in the values of the gauges is noticed at different points of the channel. This is undesirable and should be avoided as much as possible. It is likely that this "hunting effect" on the gauges which must have been produced by similar "hunting" in the tidal flow will produce movement of bed materials different from those in nature.

From our previous analysis of the formation and propagation of "bores", it will be apparent that the reproduction of the phenomenon connected with "bores" must follow in the model if the tidal cycles are correctly reproduced.

The next flow characteristics that demand correct reproduction in the model are the magnitude of the velocity and direction of the current. In tidal estuary models unless these are done satisfactorily the flow conditions in the upper reaches of the channel will not be correctly obtained. In the Seine Estuary model that is being operated in the Neyrpic Laboratory at Grenoble, France, for the correct reproduction of the flow lines in the estuary, the inlet to the model has been divided into seventeen compartments each provided with an automatic sluice valve to control the tidal flow from the sea end.

The reproduction of bed configuration correctly is one of the most difficult part of "Proving a model". Changes in the bed configuration take place in nature due to the scouring and silting process that are continually going on in a flowing stream. If the bed material in the model and the slope distortion had been correctly chosen the scouring can be faithfully reproduced; but the silting which is produced by the following factors will be qualitative :—

1. Deposition of the scoured bed materials which mostly consist of coarse sand;
2. Sedimentation of finer materials that always remains in suspension except in stagnant waters;
3. Coagulation of colloidal materials such as clay which is brought down by upland discharges as they come in contact with salt water from the sea.

In highly distorted models however even the depth of scour will be affected if the angle of repose of the model bed material is exceeded. If the silting process is due to the first factor alone quantitative reproduction of the bed configuration can be expected to be more faithful.

In the above I have tried to deal in a general way with the question of "Proving a Model". The "Proving of a model" however depend on the type of problems that the model is required to examine. If it be the design of a spillway whose coefficient of discharge only is to be ascertained, the model need be a geometrically similar one and rigid in structure. The surface should be fairly smooth and the scale need not be very large. If however the problem of cavitation or air-entrainment is to be studied on a spillway, the scale of the model should be very large so that very high velocities are met with and at the same time arrangements should be made to reduce the atmospheric pressures under which the model will operate.

Let us take the case of a *tidal channel* like the river Hooghly. The problem to be studied on this model is that of keeping a navigable channel of sufficient draft open all throughout the year. Here "Proving of the model" will entail both the reproductions of tidal cycles, bores as well as bed configuration. Only the reproductions of the tidal cycles or bores will not be sufficient guarantee that the results obtained from the study about the maintenance of a navigable channel will be applicable to the prototype.

If however the problem is that of draining a country side by tidal channels, the "Proving" need only show that the tidal cycles and velocities have been reproduced correctly and sufficient high velocities will be attained to maintain a channel of sufficient discharge capacity to carry the drainage of the countryside. Tollys Nullah Model now under investigation in the Tidal Model Station of River Research Institute, West Bengal, is a model of this type. The problem that is being tackled by this model is that of maintaining a drainage channel for the area south of Calcutta. The Nullah was siltcleared and excavated in 1943 but silted up immediately. The proposal is to re-excavate it. The model has however shown that under the existing condition, the channel will silt up unless some amount of pumping is resorted to at the head end of the excavated reach. Experiments are now in progress to see how much pumping will be required and whether this would be successful all throughout the year.

Interpretation of Model Results.

Interpretation of the results obtained from a model have to be carefully done keeping in view the above limitations of these experiments. It must have become clear to the audience by now that though hydraulic models are useful and powerful tools for the engineering profession, designing of these experiments require great skill and experience. The science of Hydraulic Model Experimentation is still in its infancy and will require the nursing co-operation of the Scientists and Engineers to make it an exact and useful science. Many basic experiments will have to be undertaken to explain some of the little understood aspects of fluid movements in models and the prototype. Efforts will have to be made to correlate some of the results obtained from these experiments with those that occur in nature.

When a problem for model experiment is reported to a Research Station, the following informations among others are generally asked for :—

1. History and statement of the Problem.
2. Value and Nature of the property at stake.
3. Hydraulic data relating to the problem. These will be different for different types of problems. It will always be advisable to give precise and exhaustive instructions about the collection of these data. There should be no possibility of misunderstanding the instructions.

A clear history of the problem has been found to help considerably in finding a solution to the problem. In a river problem particularly it has been found that difficulties increasingly experienced in a certain stretch of the river have been due to changes that had taken place some distance upstream. In such cases treatment at the source has been found more effective and cheaper than treatment at the spot of trouble. The circumstances vary from problem to problem and it is not possible to enumerate them in detail here.

Having by now got a clear picture of the problem and its nature, the type of the model and its scales will have to be decided upon. Though the broad principles on which these are to be selected have been stated before, their actual determination require great care and experience. Further the facilities for experiments that are available at a model station will also determine to some extent the dimensions of the models.

The last though the most important part of model experimentation is the interpretation of the results obtained. As I have stated before this is not an exact science as yet. Apart from the various limitations of model experiments references to some of which have already been made, the difficulty of interpreting hydraulic phenomenon even in nature according to recognised principles of Hydrodynamics will have to be always kept in mind when examining model results. I will now close my lectures with the remarks that though much has been achieved in this field of Hydraulic Model Experimentation, much still remains to be done.