THE ORIGIN OF MASS IN NEUTRONS AND PROTONS.*

By M. N. SAHA, D.Sc., F.R.S.

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§ 1. INTRODUCTION.

The use of 'mass' as a fundamental concept dates from the time of the rise of Galileo-Newtonian mechanics, but discoveries in physics within the last sixty years have shown the necessity of our revising the original concept. As early as 1885, J. J. Thomson showed from classical electrodynamics that a spherically charged body moving with a velocity v has its energy increased by the amount $\frac{2}{15}\frac{\mu e^2}{ac^2} v^2$, where a is the radius of the sphere and μ is the permeability of the medium. The arguments used were rather of a hydrodynamical nature. These studies were further continued by H. A. Lorentz and others, and brought to a close by Abraham. These results may be quoted here. When a sphere of radius a charged with the electricity 'e' moves with a velocity v, which is small compared to the velocity of light, it produces in the space,

$$\left.\begin{aligned} &\text{the } e.m. \text{ energy} = \frac{1}{3}\frac{e^2}{ac^2}v^2 \\ &\text{the } e.m. \text{ momentum} = \frac{e^2}{ac^2}v \end{aligned}\right\} \quad \ldots \quad (1)$$

We can say that the mass has increased by $m = \frac{2}{3}\frac{e^2}{ac^2}$ when we calculate it from the energy value. When, however, we calculate it from $e.m.$ momentum, the mass increment comes out to be $\frac{e^2}{ac^2}$. The explanation of this discrepancy has not been forthcoming.

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When v is comparable to the velocity of light, Lorentz showed that if we suppose that the length of the sphere is reduced in the ratio of $\sqrt{1-v^2/c^2}$ we get ... (2)

$$\text{The } e.m. \text{ energy} = \frac{1}{3}\frac{e^2}{ac^2}\frac{v^2}{\sqrt{1-v^2/c^2}} \quad \ldots \quad \text{(Hypothesis of contractile electron)}$$

Thus the increment in that part of the mass which is of electromagnetic origin follows the law, $m = m_0/\sqrt{1-v^2/c^2}$. When these theories were being developed, Kaufmann and J. H. Thomson showed experimentally that the mass of the electron increases with velocity and later experiments have shown that the variation is governed by the law

$$m = m_0/\sqrt{1-v^2/c^2} \quad \ldots \quad (3)$$

This proved that the whole mass is of electromagnetic origin, and we are justified in saying that the electron has a radius

$$a = \frac{2}{3}\frac{e^2}{mc^2} \quad \text{or} \quad = \frac{e^2}{mc^2}$$

according as we take the energy or the momentum for determining the mass.

Hypothesis of Rotating Electrons.

Abraham showed that if the spherical charge be supposed to rotate with the angular velocity ω it can be shown to possess the following properties :—

$$\left.\begin{aligned} \text{Rotational energy} &= \frac{1}{9}\frac{ae^2}{c^2}\omega, \\ \text{Mechanical moment} &= \frac{2}{9}\frac{e^2a}{c}\omega, \\ \text{Magnetic moment} &= \frac{a^2e}{3c}\omega, \end{aligned}\right\} \quad \ldots \quad (4)$$

Since the moment of inertia of a hollow spherical body of mass m about any diameter $= \frac{2}{3} ma^2$, the angular momentum $= \frac{2}{3} ma^2\omega$. We have, equating the two

$$m = \frac{2}{3}\frac{e^2}{ac^2}. \quad \ldots \quad (3')$$

Identity of Mass and Energy.

In the meantime, Einstein, proceeding from the assumptions of invariance of form of equations of motion when referred to two space-time co-ordinate systems moving with respect to each other with the velocity v, had arrived at the conclusion that energy and mass are identical, being connected by the expression

$$E = mc^2 \qquad \dots \quad (5)$$

where c = velocity of light. When the units of time and space are so chosen that $c = 1$, E becomes identical with m.

This theorem, which forms the corner stone on which all studies of nuclear reactions have been based, is independent of any hypothesis except the general assumption underlying the special theory of relativity. It is, however, in agreement with the Lorentz theory of electromagnetic origin of mass, as the variation of mass is given by

$$m = m_0 / \sqrt{1 - v^2/c^2}$$

In fact, if we accept this theorem, then to calculate the mass of any fundamental particle (electron, proton, etc.), we have to find out its energy of formation. This, when divided by c^2, gives us the mass of the particle.

Mass of the Electron and the Proton.

The energy of formation of a spherical charge e distributed over a sphere 'a' is given by $\frac{e^2}{2a}$ and hence its mass is $\frac{1}{2}\frac{e^2}{ac^2}$ but it is usual to multiply it by $4/3$ which is supposed to represent the action of forces which prevent the electron from exploding. We have no method of knowing a directly, but as e and m are known, it is customary to use the term "electronic radius" to denote the quantity $\frac{2}{3}\frac{e^2}{mc^2}$, or omitting $\frac{2}{3}$, simply to denote $\frac{e^2}{mc^2}$ as the electronic radius. Its value is

$$\frac{e^2}{mc^2} = 2{\cdot}83 \times 10^{-13}\,\text{cm}.$$

and we have

$$\frac{e^2}{mc^2} = \frac{h^2}{4\pi^2 e^2 m}\left(\frac{2\pi e^2}{ch}\right)^2 = r\alpha^2 \qquad (6)$$

where r = fundamental Bohr radius, and α = Sommerfeld fine-structure constant.

The difficulties in the above theroy of origin of mass have not yet been overcome; in fact they have been accentuated after the rise of quantum mechanics, and increase in our knowledge of the physical properties of the electron.

§ 2. THE MASS OF THE PROTON.

Before 1932, the other fundamental particle was the proton.

The task of the accounting for the mass of the proton on the above basis presented greater difficulty. It had to be assumed that the radius of the proton is nearly 1847 times smaller, *i.e.*, nearly 10^{-16} cm. While there is nothing against the hypothesis of such a small diameter for the proton, it does not help us much, for it merely accepts the situation; the reason why the proton mass is so much heavier than the electron, though the charge is the same, remains unexplained.

In recent years, the discovery of the neutron has put the whole question in a new light and has shown that the energy of formation of the proton cannot be of electrical origin alone. The neutron has no electrical charge; still it has a mass which is 1852 times heavier than that of the electron. If we wish to account for the mass of the neutron, we can no longer seek for its origin in the electromagnetic theory as the neutron is uncharged. We have, therefore, to calculate its energy of formation in a different way from that of the electron. When we have been able to account for the mass of the neutron, that of the proton may be next attempted, as the proton is most probably a neutron which has lost an electron or a neutron which has acquired a positron.

Mass of the Neutron.

According to the recent measurements, the neutron is $1847 \times \frac{1.084}{1.081} = 1852$ times heavier than the electron. It has a spin of $\frac{1}{2}$ and obeys, as Heisenberg has shown, Fermi-Dirac statistics. We cannot say what its magnetic moment in the free state is, but in the nucleus its magnetic moment is certainly of the same order as that of the proton. This is proved by two known results. The nitrogen nucleus N^{14} is most probably composed of three α-particles, one proton and one neutron. Its spin is known from measurements of intensity data of N_2^+ bands to be one. But Bacher has shown that lines of N show no hyperfine structure. Hence the magnetic moment of the nucleus is zero. Now the α-particles have their spin $=0$ and magnetic moment $=0$ and therefore, we have to assume that the proton and the neutron have their spin in the same direction, but their magnetic moments cancel each other, *i.e.*, the neutron behaves like the anti-proton as far as the magnetic moment is concerned. The spin and the magnetic moment of the deuteron tell us the same story. The spin is one, but the magnetic moment has been found to be ·4 times that of the proton, *viz*, $\frac{5}{2}\,\frac{eh}{4\pi cM}$, hence that of the neutron is $\frac{3}{2}\,\frac{eh}{4\pi cM}$, where M is the mass of the proton.

§ 3. FREE MAGNETIC POLES.

It was Dirac[1] who first showed that quantum mechanics demands the existence of free magnetic poles, having the pole strength (or magnetic charge) $\frac{ch}{4\pi e}=\frac{e}{2\alpha}$, where α = Sommerfeld fine-structure constant. Recently, the present author deduced the existence of free magnetic poles from very simple considerations. If we take a point charge e' at A and a magnetic pole μ at B, classical electrodynamics tells us that

A.______________.*B*
(e) (μ)

the angular momentum of the system about the line AB is just $e\mu/c$. Hence, following the quantum logic, if we put this $=\frac{1}{2}\,\frac{h}{2\pi}$, the fundamental unit of angular momentum, we have $\mu=\frac{hc}{4\pi e}=\frac{e}{2\alpha}$ which is just the result obtained by Dirac.

Spin and Mass of the Magnetic Particle.

But the concept of a fundamental particle requires that we should have also precise knowledge about their rest-mass their spin, as well as the statistics they obey.

Mass of the Free Magnetic Poles.

We can calculate the mass of the free magnetic poles in the same way as for electric charges by using classical electrodynamics. It is useless to repeat the mathematical working. If the poles are spherical, and the magnetic charges are distributed over a radius b we have the mass M given by

$$M=\frac{2}{3}\,\frac{\mu^2}{bc^2}=\frac{2}{3}\,\frac{e^2}{bc^2}\,\frac{1}{4\alpha^2} \quad \ldots \quad (7)$$

Thus the ratio of the mass of the magnetic poles to that of the electron is

$$\frac{M}{m}=\frac{q}{b}\cdot\frac{1}{4\alpha^2} \quad \ldots \quad (8)$$

We have no method of determining b as the free magnetic pole is still undiscovered and its mass is not known. But let us assume with Eddington[2] that the radius of fundamental particle in the sense used here, is given by some universal principle and is the same for all particles. Thus we take $a=b$. We have then

$$\frac{M}{m} = \frac{1}{4\alpha^2} = \frac{(137 \cdot 29)^2}{4} = 4712 \cdot 1. \qquad \dots (9)$$

Thus on these assumptions the free magnetic pole is $4712/1852 = 2 \cdot 540$ times heavier than the neutron. Its radius is now $r\alpha^2$, where α = Sommerfeld constant, and r is the fundamental Bohr radius. The objection may legitimately be raised against the hypothesis that the radius of the *Magnetron* (free magnetic pole) should be the same as that of the electron, but if we assume a smaller radius, the particles become proportionately heavier. The existence of such heavy particles is not yet known.

Why have we not been able to observe the free magnetic pole?

This question was tackled by Dirac. He thinks that the force of attraction between the poles is so great, that in Nature, a positive and a negative pole always occur in pairs forming a dipole, secondly, Tamm [3] tried to calculate the 'eigen'-energy of a system consisting of a free magnetic pole and an electron. No 'eigen'-values were found, but it was pointed out by the present writer that the assumptions underlying these mathematics were probably faulty.

Identification of Magnetic Dipoles with Neutron.

It was suggested by D. S. Kothari [4] that the neutron or *the skeleton of it*, may possibly be the dipole composed of two equal and oppositely charged free magnetic poles. This suggestion may be given a trial. As we have already shown, the magnetic moment of the neutron is of the order $\frac{eh}{4\pi Mc}$ and let us suppose that it is given by

$$J = \frac{eh}{4\pi Mc}\,\theta = l.\,\mu \qquad \dots (10)$$

where l = distance between the centres of the two poles. We get

$$l = \frac{e^2}{M_p c^2} = \frac{e^2}{mc^2} \cdot \frac{m}{M_p} = a.\ 4\alpha^2 = 4r\alpha^4. \qquad \dots (11)$$

Thus while we assume the dimensions of the magnetic poles to be of the order of $r\alpha^2$, the distance of their centres when they form dipoles appears to be of the order $r\alpha^4$ *i.e.*, α^2-times less. Let us see whether we can obtain any justification for this apparent contradiction.

The Dirac Equations for Free Magnetic Poles.

For this purpose, we can study a system consisting of two Dirac oppositely charged poles. Let their masses be M_p and the magnetic charge be μ. Our problem is to write out the relativitistic Dirac Equations for the system and to find out 'eigen'-values.

This is a problem of two bodies, for which special relativity has as yet found no solution, as each particle has its own individual space and time. But we can reduce the present problem to an one body one, by assuming that the bodies are always at the opposite ends of a diameter passing through the centre of gravity, and their motions are equal and opposite. We can also formulate the equations of motion in the same way as in the case of the electron, only we have to use $\frac{e}{2\alpha}$ for e, and the potential four-vector is now the magnetic potential four vector, *i.e.*, they act upon a magnetic pole.

We have $a_x = a_y = a_z = 0$

and $a_t = -\frac{i\mu}{2r}$; hence the potential energy

$$= -\frac{\mu^2}{2r} = -\frac{e^2}{8\alpha^2 r}$$ where r is the distance of any particle from the C. G.

The equations of motion for one particle can therefore be written as :—

$$\frac{i}{hc}\left(E + \frac{e^2}{8\alpha^2 r} + E_0\right)U_1 + \frac{dU_3}{dz} + \frac{dU_4}{dx} - i\frac{dU_4'}{dy} = 0 \qquad \ldots \quad (12)$$

and three other similar equations. (For the notation, see, Bethe, Handburch der Physik, 24, p. 311.) $\left(h = \frac{h}{\pi}\right)$.

We have $E_0 = M_{p_0} c^2$, $E = M_p\, c^2$

where M_{p_0} is rest mass of the particles $= m/4\,\alpha^2$,

M_p = mass under present conditions, which we have to calculate.

As shown by Bethe, the equations can be reduced to the forms :—

$$\frac{dF}{dr} - K\frac{F}{r} = \left[\frac{M_0 c}{h}\left(1 - \frac{E}{E_0}\right) - \frac{1}{8\alpha r}\right]G \qquad \ldots \ (13)$$

and

$$\frac{dG}{dr} + K\frac{G}{r} = \left[\frac{M_0 c}{h}\left(1 + \frac{E}{E_0}\right) + \frac{1}{8\alpha r}\right]F$$

our F is Bethe's χ_1, G is his χ_2.

The equation differs from (9·12) of Bethe's only in having $M_0 = m/4\alpha^2$ in place of m in Bethe's and in place of α which is $\frac{1}{137 \cdot 29}$, we have $\frac{1}{8\alpha} = \beta = 17 \cdot 16$.

We thus find that β is no longer a small quantity but is equal to 17·16.

We have also

$$\frac{Mc}{h} = \frac{mc}{h} \cdot \frac{1}{4\alpha^2} = \frac{1}{4a\alpha^3}. \qquad \ldots \ (14)$$

For solving this equation, let us put $F = Ae^{-\lambda r}$, $G = Be^{-\lambda r}$ and $\lambda = \frac{M_0 c}{h}\sqrt{1-\epsilon^2}$ where $\epsilon = \frac{E}{E_0}$ and we introduce a new variable $\rho = 2\lambda r$.

Then the equations reduce to

$$\frac{dA}{d\rho} - \frac{A}{2} - \frac{k}{\rho}A = \left[\frac{c}{2} - \frac{\beta}{\rho}\right]B$$

$$\frac{dB}{d\rho} - \frac{B}{2} + \frac{k}{\rho}B = \left[\frac{1}{2c} + \frac{\beta}{\rho}\right]A \qquad \ldots \ (15)$$

where $\epsilon = \sqrt{\frac{1-\epsilon}{1+\epsilon}}$.

This equation can be solved exactly as in Bethe's article, by the polynomial method, and we obtain,

$$\epsilon = \frac{1}{\sqrt{1 + \left(\frac{\beta}{n_r + \sqrt{k^2 - \beta^2}}\right)^2}} \qquad \ldots \ (16)$$

We have $\beta = \frac{1}{8a}$ but unfortunately there is at present great divergence of opinion regarding the exact value of $a = 2\pi e^2/ch$ as the values of c, h and particularly of e obtained from different experimental methods, do not agree within reasonable limit. Birge has discussed the problem in a recent note to the Physical Review (Vol, 48, 1935). If we follow his directions, $1/\alpha$ is found to vary between 137·29 to 136·26. If $\alpha = \frac{1}{136}$, β would be exactly 17, but if it is 137·29, $\beta = 17{\cdot}16$: it is not an integral number. But k must be integral and $\geq \beta$.

If $\beta = 17$, we could have put $k = \beta$ and $n_r = 0$ we would get $\epsilon = 0$, *i.e.*, this would correspond to the case of complete annihilation; the whole mass energy is given out as radiation.

But there is very little likelihood that $\beta = 17$; we take $\beta = 17{\cdot}16$ and k can be given integral values > 17. If $k = 18$ and $n_r = 0$ we obtain,

$$\epsilon = {\cdot}302.$$

i.e., the mass of the dipole is now reduced to $2 \times {\cdot}302 \times \frac{1}{4\alpha^2} = 2846$-times the mass of the electron $= 1{\cdot}52$-times the mass of the neutron. Thus the mass-ratio does not come out correctly. When $k > 18$ and tends to infinity, ϵ tends to unity.

The ψ-functions for the above solution have their maximum value at

$$r = \frac{\sqrt{k^2 - \beta^2} - 1}{\lambda} = 4a\alpha^3\{\sqrt{2\beta + 1} - 1\} \qquad \ldots \quad (17)$$

i.e., at distances of the order of $4a\alpha^{\frac{5}{2}}$ since $\beta = \frac{1}{8\alpha}$. Thus the nuclear distance does not come out to be of the order $a\alpha^4$ as demanded by physical considerations, but is much larger.

The solutions we have treated are real only for $k > \beta$ but it is just possible that we may have solutions which hold for $k < \beta$ but a search for such solutions has not yet yielded any positive result.

§ 4. A REVIEW OF OTHER ATTEMPTS FOR EXPLAINING THE PROTON-ELECTRON MASS RATIO.

It is now recognised that the explanation of the proton-electron mass ratio forms one of the outstanding fundamental problems of physics, and in recent years, a number of attempts has been made by distinguished scientists to solve it.

Sir A. S. Eddington published between 1929 and 1932 a number of papers in the Proc. Roy. Soc. on this subject. He believes that $\frac{1}{\alpha} = \frac{ch}{2\pi e^2}$ is exactly

137 and $136 = 137 - 1$ represents the number of degrees of freedom of the Dirac-electron; and that 10 represents the number of degrees of freedom of a particle in Riemannian space. From these assumptions he writes out the following equations for particles in Riemannian space:—

$$\left\{10\left(iE_{\mathrm{I}}\frac{\partial}{\partial\theta_{\mathrm{I}}}\right)^2 + 136\left(iE_{\mathrm{I}}\frac{\partial}{\partial\theta_{\mathrm{I}}}\right) + 1\right\}\psi = 0 \quad \dots \quad (18)$$

The mass m of the particle satisfying this equation is given by the roots of

$$10m^2 - 136m + 1 = 0 \quad \dots \quad (19)$$

The ratio between the two roots is 1847·60 which is almost the proton electron mass-ratio.

No comment is needed on this interesting speculation, but physicists will probably like to have some theory which will make a more direct appeal to their experience.

The second attempt has been made by Born and Pryce. They suppose that the proton and the positron are different quantum states of the same particle, the position being defined by $s=\frac{1}{2}$, $l=0$, $j=\frac{1}{2}$ and the proton by $s=\frac{1}{2}$, $l=1$, $j=\frac{1}{2}$. The analogy to the Goudsmit-Uhlenbeck explanation of the states of the H-atom is apparent. The spin motion is supposed to give rise to the electrostatic energy formation, *viz.*, $\frac{2}{3}\frac{e^2}{a}$, or rather Born and Infeld's modification of the above expression in which the difficulty of an infinite energy with $a=0$ is avoided. In the state $l=1$, $s=\frac{1}{2}$ the particle receives an increment of energy due to the l-motion, which is identified with the mass of the proton. This is calculated as follows:—The motion endows the particle with the magnetic moment $l.\frac{eh}{4\pi m^2}$ or since $l=1$ with the moment $\frac{eh}{4\pi mc}$. This gives rise to a rotating magnetic field in space. If this body be supposed to be a sphere of radius a, the energy of the field is $\frac{1}{2}\frac{\mu^2}{a^3}$ on the analogy of classical electrodynamics. We find therefore the energy of formation of the particle

$$Mc^2 = \frac{1}{2}\frac{\mu^2}{a^3} = \frac{1}{2}\left(\frac{eh}{4\pi cm}\right)^2 \cdot \frac{1}{a^3} = \frac{1}{2}\frac{e^2}{4a^2}\cdot\frac{1}{a}.$$

Since $\frac{e^2}{mc^2} = a$, we have $\frac{M}{m} = \frac{1}{8a^2} = 2340.$ (20)

The experimental value is 1847.

Born is of opinion that though the ratio has not come out correctly, the investigation has made it clear that the ratio M/m should be a simple function of the Sommerfeld fine-structure constant.

In spite of the great ingenuity displayed in the above working, it is doubtful whether the theory will carry much conviction. First, the identification of the energy of s-motion with electrostatic energy will find few supporters amongst physicists and is opposed to the accepted explanation of s-motion. Secondly, there is no experimental evidence that the positron and proton are different quantum states of the same particle. It will be noticed that *the neutron is altogether ignored* in this investigation. Thirdly, the calculation of energy of formation has been made only for $s=\frac{1}{2}$, $l=1$. But what about the states $l=2$, $3, \ldots\ldots\ldots$? On the above logic, they are likely to give rise to nuclei of masses 2^2, $3^2 \ldots\ldots$ times that of the proton. No experimental evidence has yet been found for the existence of such nuclei.

Another objection is that the magnetic moment of the proton has been observed by Stern and Eastermann to be $\frac{5}{2}\frac{eh}{4\pi cM}$ and this is about $\frac{1}{750}$ times the moment ascribed to the l-motion of the particle, *viz.*, $\frac{eh}{4\pi cm}$. Born says that this moment is due to s-motion which is also $\frac{eh}{4\pi cm}$, but acts in the opposite direction, leaving a small residue $5/2\,\frac{eh}{4\pi cM}$. But the assumption is frankly arbitrary, and further it is illogical to regard 's' as being of electrostatic origin and then to suppose that it gives rise to a magnetic moment.

While criticising other views, it is not the author's intention to conceal the insufficiency of his own investigation. First, the mass ratio has not come correctly. This may be partly due to a faulty formulation of the problem of relativitistic wave-mechanics of two bodies, and partly due to the fact that the Dirac equation has other solutions which have not yet been discovered. But a more potent reason seems to be the assumption that the magnetostatic attraction between the two particles is given by the law of inverse square. The size of the particles has been assumed to be of the order $r\alpha^2$, whereas the nearest distance of approach when the particles from a neutron is of the order $r\alpha^4$. Hence it appears that we shall have to assume a different law of attraction. Besides, we have to account for the spin-value, the magnetic moment and the statistics obeyed by the dipole. The spin of the free magnetic pole is probably *zero*, for we have assumed that the spin of the combination magnetic pole-electron is $\frac{1}{2}$ about the joining line AB, while that of the electron is also $\frac{1}{2}$. Considerations of equilibrium also require that the electron axis would be parallel to the line joining the two particles. Hence the spin of the magnetic pole should be *zero*.

It is doubtful, if the spin of the free magnetic pole be zero, whether Dirac's equations of motions can be applied to it, for in Dirac's theory, the resultant angular momentum which comes as an integral of the equations of motion is always half-valued. The other possibility is Schrödinger's treatment of the relativistic wave equation, but even this does not give us the correct result.

The only positive result is that the large value of the mass-ratio M/m is ascribed to the fact that the mass of the neutron arises from an entirely different cause than the mass of the electron. It is due to free magnetic poles.

The whole investigation is based upon the tacit assumption of the existence of free magnetic poles and since these have not yet been discovered, we have to show that they are not figments of the imagination. Their existence has been deduced from straightforward quantum logic, and hence it is difficult to throw doubt on their existence. We rather discuss why the poles have not so far been discovered. According to our hypothesis the magnetic poles can never occur in free state in our universe. When two magnetic poles combine to form a neutron, nearly eighty per cent. of the energy is radiated away in the form of radiation of energy $3{\cdot}7 \times 10^9$ e. volts, hence it is almost impossible to split up the neutron. It is just possible that when a neutron lying within a nucleus is bombarded by a cosmic ray of suitable energy, it is split up into free magnetic poles which produce intense disturbance in the nucleus as they are liberated. May not the mysterious phenomena of cosmic ray *bursts* be due to this cause ?

I wish to express my thanks to Dr. D. S. Kothari, and Mr. Ramnivas Rai, with whom the contents of the paper were discussed.

[Note added :—In course of a discussion on the paper, Prof. D. M. Bose raised the point that if the same mathematics were to be applied to the motion of a positron and electron about each other, we should get corresponding solutions, where 80% of the mass would be radiated away. We know of no such radiation or of particles. I have since given some thought to Prof. Bose's point but find that the electron-positron case cannot give rise to the kind of solutions contemplated by Prof. Bose . For we should have

$$\epsilon = \frac{1}{\sqrt{1 + \left(\dfrac{a/2}{\sqrt{n_r + k^2 - a^2/4}}\right)^2}} \qquad \text{(A)}$$

where $\frac{a}{2}$ takes the place β in (16). Now β is a large number > 17, while $\frac{a}{2}$ is a small fraction. The lowest allowable value of k in (A) is unity. It may be easily verified that this leads to values of $\epsilon = 1 - \frac{\alpha^2}{8n^2}$, $n = n_r^2 + k$, and the radiation emitted is

$$\nu = \frac{Ry}{2}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

i. e., they should have double the wave-length of ordinary hydrogen lines. Such lines were looked for in the spectrum of the corona (see Observatory, 56), but none has been so far obtained.]

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