STUDIES ON WATER JETS.[1]

BY K. PROSAD, M.Sc. (CAL.), B.A. (CANTAB.).

Professor of Physics

AND

B. N. GHOSH, M.Sc. (PAT.),

Government Research Scholar.

(Received for publication, 29th August, 1935.)

PLATE II.

ABSTRACT. The paper is divided for convenience into two parts. In Part I a sonorous water tap is described. The jet of water from this tap has a beaded appearance with an axial core. Photographs of the jet with time exposure of 20 secs. show the steadiness of the jet. Some earlier relations connecting λ, the wave-length of recurrent figures with the velocity of the jet and also the frequency of the note with velocity have been verified. There are still some interesting features in the structure of the jet which are not yet clearly understood. It is believed that these features escaped the observations of previous workers on liquid jets.

In Part II of the paper, from the stationary pattern of ripples formed on a silent jet, when an obstruction is interposed, a convenient method has been worked out for the determination of the value of surface tension of water. The mean value obtained is 71·6 dynes per cm. and is as good as can be expected.

PART I.

A SONOROUS WATER TAP.

§ 1. INTRODUCTORY.

The instability of cylindrical liquid jets has been studied mathematically by the late Lord Rayleigh.[1] He has shown that when the length of a liquid column is greater than 4·51 times its diameter, it becomes unstable and disintegrates. This condition is easily fulfilled by jets issuing horizontally from an orifice in the side of a tank, but when the jet issues vertically downwards the force of gravity, by augmenting the velocity from point to point, exerts a protective influence and the instability is lessened and disintegration now starts at a bigger distance.

[1] Read before the Indian Physical Society on 21st September, 1935.

When disintegration commences, there is a mixed procession of separate liquid drops and air bubbles with the result that the beginning of disintegration presents a frothy appearance. Further on, the jet possesses more or less an ill-defined beaded structure, partly due also to the presence of recurrent figures, through which each drop passes during its career downwards.

The problem of recurrent figures has been studied by Bidone, Magnus,[2] Buff [3] and the late Lord Rayleigh,[4] in the case of horizontal jets. They found that if the orifice be an ellipse with the major axis horizontal (as in the present case) the column at a certain distance becomes circular and further on elliptical, with the major axis vertical and this is repeated along the rest of the jet. The late Lord Rayleigh gave the following relation valid only for small amplitude of disturbance, for λ, the distance between successive similar recurrent figures in the jet, namely

$$\lambda = \frac{vA^{\frac{3}{4}}}{3.23\sqrt{n^3 - n}} \quad \ldots \quad (1)$$

where v = velocity of the jet,

A = cross section of the jet in the interval giving λ,

n = a constant depending on the nature of the aperture,

$n = 2$ for elliptic aperture,

$n = 3$ for triangular aperture,

$n = 4$ for rectangular aperture.

The disintegration can be further regularised if a sounding tuning fork is put into contact with the orifice when the beady structure of the jet becomes much more steady and well-defined. Instead of giving an initial impressed disturbance to the orifice, if the jet be allowed to fall into another vessel and if the reservoir of water is influenced by the shock due to the impact of the jet, the disintegration is accompanied by a musical sound. Plateau who first studied the phenomenon gave the following expression for the frequency ν of the note emitted, namely :—

$$\nu = \frac{v}{4{\cdot}51 \times d} \quad \ldots \quad (2)$$

where v is the velocity and d is the diameter of the jet. The frequency of the note in practice is hardly constant and fluctuates between certain limits. Moreover, a slight external disturbance acting on the reservoir of the liquid is liable to stop the sound altogether.

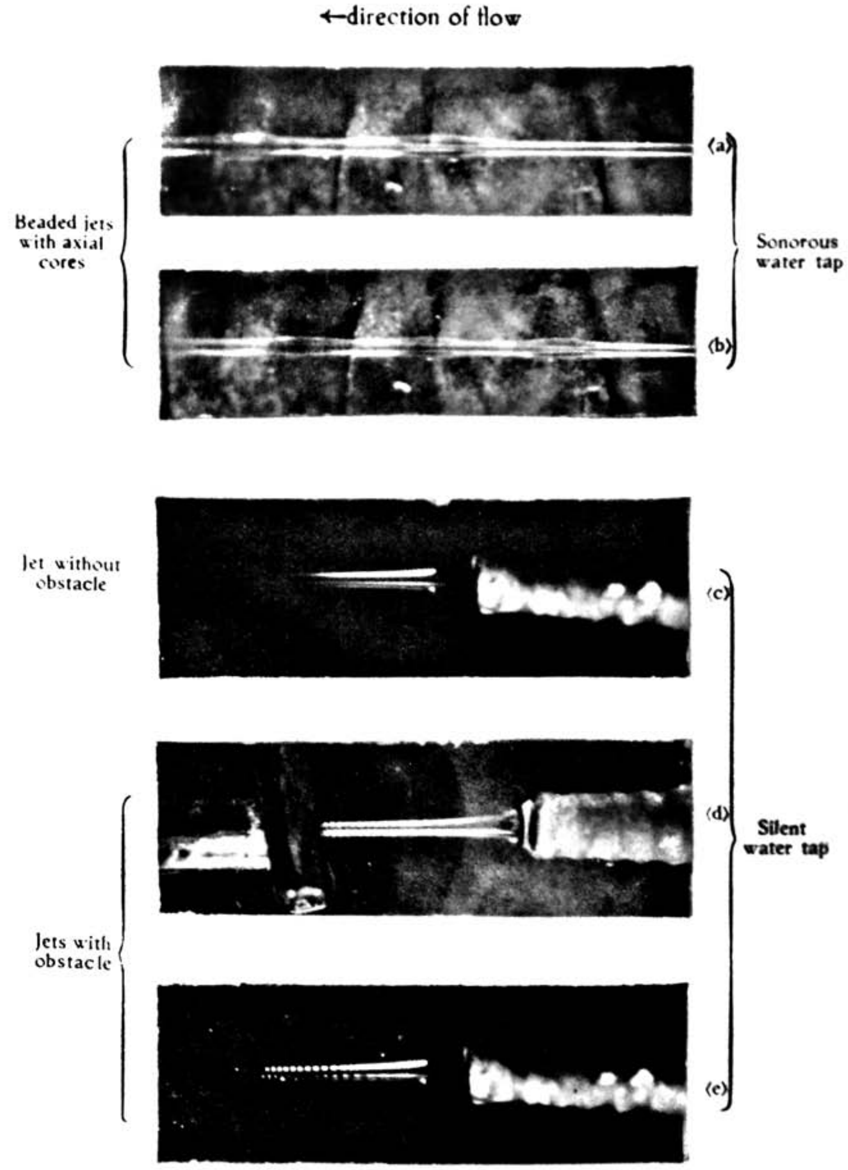
←direction of flow
Beaded jets with axial cores
(a)
(b)
Sonorous water tap
Jet without obstacle
(c)
(d)
Silent water tap
Jets with obstacle
(e)

Figure 1.

The object of the present investigation is to test in the first instance the application of the formulæ developed by earlier workers in the field, on a self-maintained sonorous water tap, and secondly to bring to notice certain new interesting features in the structure of the jet which, as far as we are aware, do not appear to have been recorded previously. In the second part of the paper, stationary ripples on a silent jet have been used to determine the surface tension of water.

§ 2. APPARATUS.

If a water-tap attached to a pipe fixed to a building has the frequency of disintegration of its jet in agreement with the natural frequency of oscillation of the water-pipe then the sound produced will be augmented by resonance, and will be maintained.

We accidentally discovered one such water tap in the Physics Department of this college. Later on, a search for others out of a large number of taps led to the detection of only another one which was also self-sonorous. The second one, however, is not so rigorous as the first and the experiments described in this paper were made with the latter, no special arrangement of reservoir and sounding arrangement being necessary.

On opening the tap slowly, a note resembling the hum of a large transformer appears. The oscillations are sufficiently violent to produce a shaking of the building. These shakings can be felt very clearly on the first floor of the laboratory, although the tap itself is situated in a room on the ground floor.

§ 3. APPEARANCE OF THE JET AND THE INTENSITY OF SOUND.

As the tap is slowly opened, at first the intensity of the sound is feeble. The jet presents now a beady appearance—the separate water beads being very close to one another, yet very clearly separated by narrow connecting necks. Moreover, the first bead appears at a definite distance below the tap, depending on the actual rate of flow of water. As the rate of flow is increased slowly, the sound rises in intensity, the beads get larger in size and the distance λ separating the consecutive beads also increases. As the flow is further increased the intensity of the sound diminishes, λ increases, but the distance of the first bead from the tap diminishes, until a certain minimum is reached. On further increasing the flow, the intensity of the sound again rises up to a certain rate of flow after which it gradually dies down, as the flow is still further increased. During the latter stage λ continually increases as well as the distance between the tap and the position of the first bead. With the extinction of sound, the pronounced beady structure of the jet simultaneously vanishes. Figures I(*a*) and I(*b*) and in Plate II show the photographs of these beady jets. The plates were exposed for about 20 secs., the jet being illuminated by two ordinary eletric bulbs. The sharp outlines of the figures show very clearly how steady the appearance must have been. As will be seen from the photographs, the most interesting thing about the jet is the presence of an axial core running through all the water beads as if a thread were passing through them. The diameter of the core at the centre of the first bead is almost exactly half the diameters of the cores at the centres of the other beads. Another equally interesting feature is the shooting out of small droplets of water from the lower ends of the beads in all directions, while the jet as a whole seems to be spinning rapidly about the central core. Brinkworth [5] has demonstrated that beaded jets showing stationary vibrations which might be transverse or longitudinal, are perfectly stable.

§ 4. EXPERIMENTAL METHOD AND MEASUREMENTS.

In order to test equations (1) and (2) given earlier it is necessary to determine v, the velocity of flow of water at a particular point of the jet. This was done with the help of the relation:—

$$\frac{\pi d^2}{4} \cdot v = Q \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Where d is the diameter of the jet at the point considered and Q the quantity of water flowing per second and known from water collected in a given time. The diameter of the jet was measured by means of a cathetometer microscope having an eyepiece scale of which one division $= 7{\cdot}5 \times 10^{-3}$ cm. Q was estimated by a measuring cylinder and a stopwatch, the time of collection in most cases being one minute. At least three separate determinations were made for Q for each rate of flow and the mean taken. The value of λ, the distance in cms. between successive beads was determined by focussing a short focus telescope on the jet and a centimetre scale placed alongside the jet.

§ 5. VERIFICATION.

(*a*) Rayleigh's constant n :—

Having determined v, d, and λ for a given rate of flow of the jet, it is possible to calculate the value of the Rayleigh's constant n in equation (1). These values for different rates of flow are all given in Table I. The values of v and d for each rate of flow were determined just above the position of the first bead on the jet, *i.e.*, just before the commencement of disintegration.

(b) Frequency :—

The frequency of the note emitted by the jet was found slightly to vary with the rate of flow while the variation of the intensity of sound with flow has already been pointed out. There appeared to be two acoustical bands of sufficient intensity, but closely differing in frequency between which, for a particular rate of flow very loud and distinct beats could be heard. The frequencies of the notes at different rates of flow, as given by equation (2) have been calculated and are given in Table I. The frequencies were also directly determined by means of a sonometer and were found to lie between 165 and 185. It will be seen from Table I that these independent determinations are in close agreement with the values of the frequencies as determined by equation (2).

(c) Magnification M :—

It is well known from the work of Savart and C. A. Bell [6] that it is possible to magnify any small mechanical disturbance by applying it to an orifice from which a liquid jet is issuing. The latter has shown that if a watch is placed against a nozzle and a jet issuing from it is received on a membrane, the ticking of the watch can be rendered audible to a large audience. The problem of magnification has been treated theoretically by the late Lord Rayleigh [7] whose expression for the magnification M can be put in the following form, by assuming the surface tension of water to be 72 and its density equal to 1 :—

$$\log_{10} M = \frac{t}{0{\cdot}120\, d^{\frac{3}{2}}} \quad \ldots \quad \ldots \quad (4)$$

Where as before, d is the diameter of the jet and t is the time in seconds for all from the orifice to the point on the jet where disintegration starts, *i.e.*, the position of the first bead. Different values of M corresponding to different rates of flow and hence corresponding to different intensities of the note have been calculated from equation (4), and included in Table I.

TABLE I.

No. of obs.	Q in c.c. per sec.	Distance of 1st bead from tap in cms.	Frequency ν (calc.).	λ in cms. (obs.).	n (calc.).	M (calc.).	Intensity of sound.
1	1·355	3·60	162	1·77	1·72	$38{\cdot}8 \times 10^3$	Feeble
2	1·739	4·00	171	2·33	1·82	$21{\cdot}3 \times 10^3$	Do.
3	2·039	5·20	176	2·83	1·64	$11{\cdot}9 \times 10^3$	Moderate
4	2·600	7·40	178	3·43	1·32	$7{\cdot}5 \times 10^3$	Large
5	2·760	10·10	179	3·51	1·45	$9{\cdot}2 \times 10^3$	Moderate
6	2·918	9·80	186	3·60	1·25	$5{\cdot}1 \times 10^3$	V. large
7	3·060	10·40	192	3·73	1·30	$5{\cdot}1 \times 10^3$	Large
8	3·483	11·08	198	4·10	1·52	$1{\cdot}1 \times 10^3$	Moderate

In the following Table II the diameter of the jet, the diameter of the beads and those of the cores at the centres of the beads for different rates of flow are recorded.

TABLE II.

No. of obs.	Q in c.c. per sec.	Diameter of jet 2 cms. below tap in cms.	1st bead in cm.		2nd bead in cm.		3rd bead in cm.	
			diameter.	core.	diameter.	core.	diameter.	core.
1	1·225	0·176	0·327	0·038	0·346	0·076	0·340	0·082
2	1·433	0·195	0·359	0·050	0·359	0·101	0·353	0·107
3	1·783	0·211	0·390	0·063	0·384	0·145	0·386	0·146
4	2·133	0·227	0·409	0·075	0·422	0·157	...	...
5	2·367	0·239	0·422	0·088	0·416	0·189	...	...
6	2·567	0·252	0·441	0·095	0·435	0·195	...	...
7	2·783	0·271	0·466	0·104	0·453	...	...	...

Lastly curves 1 and 2 in figure 2 show the variation of the diameters of the sonorous and the silent jets respectively with the distance below the tap in cms. for a fixed rate of flow, *i.e.*, 1·757 c.c. per sec. The curves are plotted from observations on jet from a tap in the neighbouring room. The vibrations started by the particular sonorous tap could start forced vibrations in this tap also, while if the former was stopped the vibration and the beady nature of the jet disappeared from the latter.

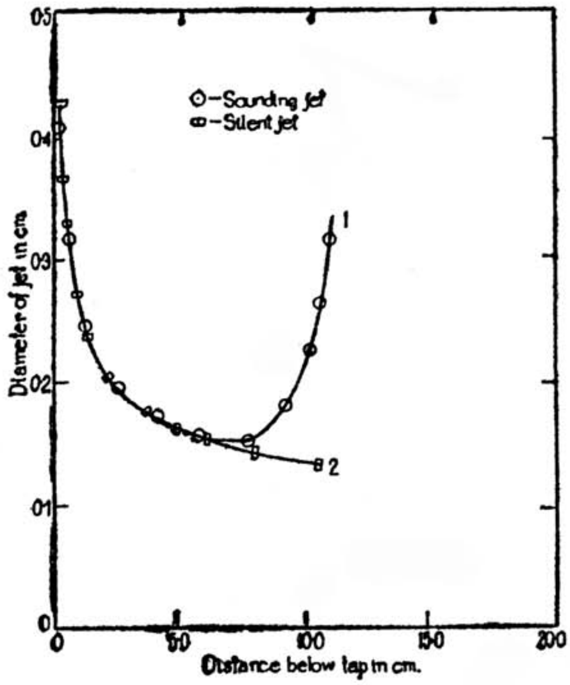

FIGURE 2.

It will be seen that the variation of diameters with distance of both the sonorous and the silent jets is almost indistinguishable up to a distance of 6·2 cms. below the tap. After this, the two jets separate. The sonorous jet remains cylindrical having a diameter of 0·1504 cm. over a distance of 1·6 cms. Beyond this the diameter begins to increase until the position of the first bead is reached. The diameter of the silent jet, on the other hand, continually decreases with distance from the tap, at any rate up to the point of disintegration. The rate of decrease of diameter with distance slows down gradually.

(*a*) The value of the Rayleigh number n recorded in Table I and calculated with the help of equation (1) making use of the observed values of λ at different rates of flow, show that the experimental values approach closely the theoretical value 2 for an elliptic orifice, when the intensity of the note emitted is feeble, that is, when the initial disturbance is correspondingly small. This is in line with the theoretical assumption made in establishing equation (1). Observation No. 6 in Table I gives the smallest value of $n = 1·25$ when the intensity of the sound is very large, while $n = 1·82$ (obs. No. 2) when the sound is very feeble and the initial disturbance is also small.

(*b*) The frequencies of the sonorous jet spread over a small range as calculated by equation (2) have also been directly verified. Observation No. 6 in Table I shows how the first bead shrinks up the jet inspite of the increasing rate of flow and it is at this stage that loud beats referred to earlier between the two acoustical bands were heard.

(*c*) Supposing an initial disturbance has been magnified a given number of times, the reciprocals $\frac{1}{M}$, where M is the magnification factor, will give us some idea of the relative magnitudes of the initial disturbances.

(*d*) A reference to Table II shows that within the limits of experimental errors possible in the present measurements the following conclusions may be drawn :—

(1) For a given rate of flow the diameters of the successive beads are equal.

(2) For a given rate of flow, the diameter of the core of the first bead is half the diameters of the cores of the other beads.

(3) As the rate of flow increases, both the diameters of the beads and of the cores also increase.

(*e*) The appearance and structure of the jet call for special attention. The axial core on careful examination appears to start from the point of the jet where it just becomes cylindrical. It tapers to a minimum diameter at the middle of the first bead. Air bubbles seem to be formed here and move in rapid succession downwards. The exact mechanism of formation of the axial core is not clear yet. A further detailed examination with the help of intermittent light is expected to give valuable information. It may be worth while to record the effect of an electrical field observed on the jet in question.

When an electrified ebonite rod is brought near the sonorous jet, it splits up vertically into two halves, the bifurcation always starting at the centre of the first bead, : one of the split portions of the jet is attracted towards the electrified rod in the form of a curve, while the other half which always contains the beads and cores is only slightly displaced toward the rod. If the electrified rod is between the tap and the first bead, the jet is attracted as a whole without any division. Further, the splitting is independent of the position of the electrified rod round the jet as is to be expected from the symmetry of the appearance of the jet.

(*f*) It appears probable that the origin of the sonorous vibrations is due to the presence of loose washers in the tap. As water flows down, the loose washer begins to oscillate and it then throws the whole pipe system into forced vibration. It is also possible to decide by a reference to curves 1 and 2 in Figure 2 whether the oscillations are mainly longitudinal, or transverse. Since the diameters of the sonorous and the silent jets for the same rates of flow (*i.e.*, $Q = 1{\cdot}757$ c. c. per sec.) are almost exactly the same at different depths for some distance below the tap, it seems that the oscilllations must be mainly longitudinal. If any appreciable transverse vibration had also existed, its presence could have been detected by the bulging out or contraction of the sonorous jet at different portions and no length of the jet would have its diameter at different points on it agreeing almost exactly with that for a silent jet. In direct support of this view we might mention here that a shining point on the water tap system focussed by a microscope was drawn out into a vertical straight line as soon as the jet began sounding.

(*g*) There are other finer features in the structure of the jet that have not been revealed by the photographs reproduced. It is intended to study these in detail as soon as opportunity permits.

PART II.

A Silent Water Jet.

§ 1. INTRODUCTORY.

It is well known that when an obstacle is placed in a running stream of water or any other liquid, a beautiful stationary wave pattern is formed next to the obstacle on the liquid surface. These have been described and figured by Scott Russel.[8] Lord Kelvin [9] was the first to show that this formation was principally governed by the surface tension of the liquid. Recently Hopfield [10] has succeeded in photographing a similar stationary pattern formed at the junction of two colliding jets of water issuing at a small angle. The late Lord Rayleigh gave the following relation connecting the wave-length λ of a wave with its velocity, namely

$$v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda}$$

where the various letters have their usual significance.

A similar pattern was observed by us when an obstacle was placed in the path of a jet of water falling from a tap and the object of this paper is to record some measurements made on the pattern from which the surface tension of water could be easily calculated.

§ 2. APPEARANCE OF THE PATTERN.

On interposing an obstacle (in our case a small wooden platform was used) in the path of the jet, the portion of the jet between the obstacle and the tap gets covered by the stationary pattern referred to above. The pattern is most prominent in the region next to the obstacle. Using two ordinary electric bulbs to illuminate the jet the pattern was clearly seen as a series of alternate bright and dark rings piled on one another. The appearance also resembles the threads of screw cut on a tapering rod. Figures. 1 (*d*), 1 (*e*) and 1 (*c*) on Plate II show the photograph of the jet with and without the stationary pattern, respectively, with an exposure of 20 secs.

Since the velocity of ripples of different wave-lengths are different as also the velocities of the jet at different points along its length, the ripples of shorter wave-length possessing higher velocities are rendered stationary lower down the jet where the velocities are correspondingly greater. While the ripples of longer wave-lengths are rendered stationary at points higher up the jet where their own smaller velocities are in agreement with the smaller velocities of the jet.

§ 3. THEORY AND MEASUREMENT.

In the case under consideration, since ripples are propagated mainly by surface tension we can write down the Rayleigh relation as

$$v^2 = \frac{2\pi T}{\lambda\rho} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

If we can find v and λ for the ripples, we can determine T the surface tension of water.

Measuring d, the diameter of the jet corresponding to a bright ring on the pattern by means of the cathetometer microscope used in Part I of the paper, the velocity of the jet at this point could be calculated by means of equation (3) of the same paper. Since the pattern is stationary, the velocity of the ripple of wave-length λ given by the distance between the two dark rings, one above and the other below that bright ring whose diameter has been measured above, is also the velocity of the jet. In practice, v need not be calculated but merely eliminated between equations (3) and (5), so that we get,

$$\frac{16Q^2}{\pi^2 d^4} = \frac{2\pi\tau}{\lambda\rho}$$

$$\text{or} \qquad d^4 = \frac{8Q^2\rho}{\pi^3\tau}.\,\lambda.$$

$$\text{or} \qquad d = \frac{2^{\frac{3}{4}}.\,Q^{\frac{1}{2}}.\,\rho^{\frac{1}{4}}}{\pi^{\frac{3}{4}}\tau^{\frac{1}{4}}}.\,\lambda^{\frac{1}{4}}$$

Thus, if d is plotted against $\lambda^{\frac{1}{4}}$ a straight line will be obtained, and if m be the tangent of the angle which the straight line makes with the axis of $\lambda^{\frac{1}{4}}$, then

$$m = \frac{2^{\frac{3}{4}}.\,Q^{\frac{1}{2}}.\,\rho^{\frac{1}{4}}}{\pi^{\frac{3}{4}}.\,\tau^{\frac{1}{4}}}.$$

Thus τ the surface tension can be determined. Since all other quantities are known in the expression for m corresponding to each rate of flow, d was plotted against $\lambda^{\frac{1}{4}}$ and m determined from the best straight line drawn through the points. Thus varying the rate of flow, different independent graphs were drawn and in each case the corresponding m was determined. From the values of m, τ was calculated. The results are given in the following table.

TABLE III.

Series of observations.	Q in c.c. per sec.	m from graph.	τ in dyne (calc.) per cm.	Mean τ.	Temp.
1	1·167	0·2665	69·6		
2	1·300	0·2764	74·7	71·6	30°C. nearly.
3	1·633	0·3120	72·6		
4	2·433	0·3842	70·1		
5	2·950	0·4213	71·2		

The straight lines in figure 3 are the plots of series of observations 3, 4 and 5 in the above table. The other series namely 1 and 2 could not be conveniently plotted on the same scale.

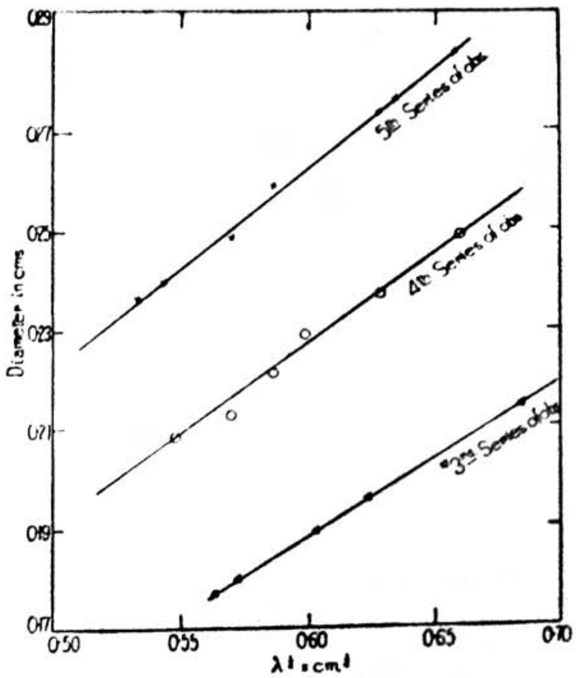

FIGURE 3.

The measurements of λ on the jet only some distance away from the obstacle give good results owing to the fact that water accumulates on the obstacle before flowing out, and thus produces disturbing effect on the pattern immediately next to it.

In conclusion, one of us (B. N. Ghosh) would like to record his grateful thanks to the Government of Bihar and Orissa for the award of a scholarship that enabled him to take part in the present investigations.

PHYSICS DEPARTMENT,
SCIENCE COLLEGE, PATNA.

REFERENCES.

[1] Rayleigh, *Proc. Math. Soc. London*, **10**, 7 (1878).
[2] Magnus, *Pogg. Ann.* **95** (1855).
[3] Buff, *Pogg. Ann.* **100** (1857).
[4] Rayleigh, *Proc. Roy. Soc.*, **29** 71, (1879).
[5] Brinkworth, *Proc. Phy. Soc. London*, **44**, 225 (1932).
[6] Bell, *Phil. Trans.*, **177**, 383 (1886).
[7] Rayleigh, Sound **2**, page 362.
[8] Scott Russel, *Brit. Ass. Rep.* (1844), p. 362.
[9] Kelvin, *Proc. Lond. Math. Soc.*, **15**, 69 (1883).
[10] Hopfield, *Nature*, Nov. 10, 737 (1934).