A New Interferen e Phenomenon Observed with Crystalline Plates

By

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(Plates II and III.)

1. *Introduction.*

It is well known that when a crystalline plate, placed between two crossed Nicols, is viewed through a spectroscope, and a parallel beam of white light is allowed to traverse the optical system, the spectrum would be crossed by a number of dark and bright bands. The positions of these bands are defined by the relation $(\mu_1 - \mu_2)d = n\lambda/2$, where μ_1 and μ_2 are the principal refractive indices of the plate in its plane and d is the thickness, and n is any integer; the dark bands correspond to even values of n and the bright bands to odd values. The intensities at the minima would be zero and hence the visibility of the fringes would be best when the principal planes of the polarising and analysing Nicols make 45° with the two extinction directions of the crystalline plate. When the analyser, instead of being crossed with the polariser, is parallel to it, the fringe system observed through the spectroscope would be very similar, only the previous positions of maxima would now correspond to positions of minima of intensity, and *vice versa.* Such an optical arrangement is frequently used for studying the dispersion of birefringence of crystals.[1] In these experiments, the thickness of the plates is naturally so chosen as to give a closely spaced system of fringes, since the values of birefringence can then be determined accurately.

While conducting similar experiments with organic crystals where because of the very high double refraction thin crystals only had to be used, the writer came across a second set of fringes more closely spaced than, and quite different from, the birefringence fringes, described above. This subsidiary set of interference fringes, superposed on the interference system due to the birefringence, is not contemplated in the usual treatment of the subject. The purpose of the paper is to present an account of the experimental observations made in connection with this phenomenon together with a complete theory.

2. *Elementary Theory of the Birefringence Fringes.*

The explanation of the birefringence fringes described in the previous section is simple. Let us consider the case where the crystalline plate is placed between two parallel Nicols with its principal axes at 45° to those of the Nicols. The vibration incident on the crystal plate will resolve into two of equal amplitudes (*viz.*, $1/\sqrt{2}$ of that of the incident vibration), vibrating respectively along the two principal axes of the crystal. On emergence from the crystal plate, they will have a phase difference $\delta=\frac{2\pi}{\lambda}(\mu_1-\mu_2)d$, and for such wave-lengths for which δ is a multiple of 2π, they will compound into a vibration in the principal plane of the analyser, and will be wholly transmitted. On the other hand, whenever δ is an odd multiple of π, the re-

1. See for example K. S. Krishnan and A. C. Das Gupta, 'Ind. Jour. Phys.,' Vol. 8, p. 49 (1933).

sulting vibration will be completely cut out by the analyser. These two conditions define the positions of the bright and dark birefringence fringes observed through the spectroscope.

3. *Requirements of a More Complete Theory.*

In the above treatment, the amplitude of either of the vibrations transmitted by the crystal plate is taken to be $1/\sqrt{2}$ of the incident amplitude, which involves the tacit assumption that the amplitude is independent of wave-length. But actually, due to multiple reflections, the amplitude of each of these vibrations will vary with the wave-length, fluctuating rapidly through a series of maxima and minima. It is this that gives rise to the well known interference fringes observed with thin films placed in front of the slit of a spectroscope. Further the frequency of these fluctuations will be different for the two vibrations. When these vibrations pass through the analyser and interfere, there will naturally appear in the spectrum of the transmitted light, in addition to the birefringence fringes arising from the fluctuations with wavelength of the *phase difference* δ between the two vibrations, a system of secondary maxima and minima arising from the fluctuations in their *amplitudes*. Since these fluctuations of amplitude are very rapid, the latter set of maxima and minima in the spectrum will be more closely spaced than the former arising from the fluctuations of δ with wave-length. As a prelude to the theoretical investigation of these two sets of fringes, let us consider the simple case where the polariser alone is present, the analyser being removed, and the crystal plate is rotated in its plane so as to have one of its principal axes parallel to the incident vibrations.

4. *Incident Vibration Parallel to One of the Principal Axes of the Crystal.*

Let a wave of unit amplitude represented by the expression $e^{i\omega t}$ be incident normally on the crystal plate. The expression for the transmitted wave just on emergence from the plate, will then be given by the usual expression

$$y=\frac{(1-r^2)\ e^{i(\omega t-\epsilon_1)}}{1-r^2\ e^{-2i\epsilon_1}} \qquad \dots \quad (1)$$

(which is obtained by taking into consideration, in addition to the directly transmitted wave, also those that are transmitted after suffering 2, 4, 6, etc., internal reflections), where r^2 is the reflecting power, *i. e.*, the ratio of the intensities of the reflected and incident waves, $2\epsilon_1=\frac{2\pi}{\lambda}\times 2\mu_1 d$ is the phase difference between two successive transmitted waves and μ_1 is the refractive index of the crystal corresponding to the particular direction of vibration of light. The intensity of the complete transmitted wave will evidently be given by

$$\mathrm{I}=\frac{(1-r^2)^2}{1+r^4-2r^2\cos 2\epsilon_1}. \qquad \dots \quad (2)$$

I will be a maximum or a minimum according as $2\epsilon_1=2n\pi$ *or* $(2n+1)\pi$, *i.e.*, $2\mu_1 d=n\lambda$ *or* $(2n+1)\lambda/2$. Hence the spectrum will be crossed by alternate bright and dark fringes. The intensities at the minima will of course be finite.

If the polarising Nicol is rotated by 90° so as to make the incident vibration parallel to the other principal axis of the crystal plate, the system of bands observed will be similar to the one described, being however differently spaced, the positions of the bright and dark fringes being now given by the conditions $2\mu_2 d=n\lambda$ and $2\mu_2 d=(2n+1)\lambda/2$ respectively.

5. *Incident Vibrations at 45° to the Principal Axes.*

If the incident vibrations are at 45° to the principal axes of the crystal, the intensity of the transmitted light would be given by the expression

$$\mathrm{I}=(1-r^2)^2\ \frac{1+r^4-2r^2\cos(\epsilon_1+\epsilon_2)\cos(\epsilon_1-\epsilon_2)}{(1+r^4-2r^2\cos 2\epsilon_1)\,(1+r^4-2r^2\cos 2\epsilon_2)} \qquad \dots \quad (3)$$

and hence the interference system observed will be the same as that obtained by a simple superposition of the two separate sets of fringes described in the previous section. Due to the difference in spacing of the two sets of fringes, there will naturally be alternating positions of maxima and minima of *visibility*, given by the relations $2(\mu_1-\mu_2)d=n\lambda$ and $2(\mu_1-\mu_2)d=(2n+1)\lambda/2$ respectively, these n's being of course different from the previous n's.

It may be mentioned here that the same system of fringes can also be obtained with incident *unpolarised* light.

6. *Transmitted Wave analysed by a Second Nicol.*

The results obtained when in addition to the first Nicol inclined at 45° to the principal axes of the crystal, there is introduced an analyser after the crystal, either parallel or crossed with the first Nicol, are highly interesting. Let us first consider the case when the Nicols are parallel.

Let us suppose that the incident wave is represented by $e^{i\omega t}$. The two vibrations in the crystal are then represented separately by $e^{i\omega t}/\sqrt{2}$, just when they enter the crystal. The first wave after transmission through the crystal plate and the analyser will be given by $\frac{1}{2}\frac{(1-r^2)e^{i(\omega t-\epsilon_1+\phi)}}{1-r^2e^{-2i\epsilon_1}}$ and the second wave by $\frac{1}{2}\frac{(1-r^2)e^{i(\omega t-\epsilon_2+\phi)}}{1-r^2e^{-2i\epsilon_2}}$, both vibrating in the same direction; ϕ is a constant which will depend on the path traversed after leaving the crystal.

It can be easily shown that the resultant intensity of the light emerging from the analyser, obtained by compounding the above two vibrations, is equal to

$$(1-r^2)^2\cos^2\frac{\epsilon_1-\epsilon_2}{2}\;\frac{1+r^4-2r^2\cos(\epsilon_1+\epsilon_2)}{(1+r^4-2r^2\cos 2\epsilon_1)(1+r^4-2r^2\cos 2\epsilon_2)} \quad \ldots \quad (4)$$

The first factor $\cos^2 \frac{\epsilon_1 - \epsilon_2}{2}$ varies very slowly with wave-length. Considering the second factor

$$\frac{1 + r^4 - 2\, r^2 \cos\,(\epsilon_1 + \epsilon_2)}{(1 + r^4 - 2r^2 \cos 2\,\epsilon_1)(1 + r^4 - 2r^2 \cos 2\epsilon_2)},$$

and comparing it with the expression

$$\frac{1 + r^4 - 2r^2 \cos\,(\epsilon_1 + \epsilon_2) \cos\,(\epsilon_1 - \epsilon_2)}{(1 + r^4 - 2r^2 \cos\, 2\epsilon_1)(1 + r^4 - 2r^2 \cos\, 2\epsilon_2)}$$

in equation (3), it is seen the two expressions are practically the same, except for the presence of a slowly varying factor $\cos(\epsilon_1 - \epsilon_2)$ multiplying one of the terms in the numerator, so that the *positions* of the maxima and minima will be practically the same as those given by expression (3), *i.e.*, those obtained by the superposition of the two sets of interence fringes considered already. The positions of the maxima and minima of *visibility* will also be the same. But due to the presence of the first term, *viz.*, $\cos^2 \frac{\epsilon_1 - \epsilon_2}{2}$, the effect of which will be considered presently, the relative *intensities* at the different maxima will now be different from those given by (3).

Let us proceed to consider the effect of the first term, namely, $\cos^2 \frac{\epsilon_1 - \epsilon_2}{2}$. It is easily seen that whenever the visibility of the fringe system is a maximum, $2(\epsilon_1 - \epsilon_2) = 2n\pi$ and hence $\cos^2 \frac{\epsilon_1 - \epsilon_2}{2}$ is either a maximum or a minimum according as n is even or odd, the minimum value being zero. There will thus be absolute darkness whenever $(\epsilon_1 - \epsilon_2) = (2n + 1)\pi$, *i.e.*, $(\mu_1 - \mu_2)d = (2n + 1)\lambda/2$.

When the thickness of the crystal is large, the maxima and minima arising from the fluctuations in the second factor, may be too closely spaced to be resolved by the spectroscope, in which case, only the maxima and minima determined by the fluctuations of the first factor (which are much slower) will appear; these will be the birefringence fringes that are observed under normal conditions and that are explained by the elementary theory given in §2.

To sum up, there are closely spaced interference fringes, whose visibility fluctuates, the alternate positions of maximum visibility being completely dark.

The problem when the Nicols are crossed, can be treated similarly and the expression for the intensity of the transmitted light in this case comes out as

$$(1-r^2)^2 \sin^2 \frac{\epsilon_1-\epsilon_2}{2} \cdot \frac{1+r^4+2r^2\cos(\epsilon_1+\epsilon_2)}{(1+r^4-2r^2\cos 2\epsilon_1)(1+r^4-2r^2\cos 2\epsilon_2)}. \quad \ldots \quad (5)$$

It can be shown that the positions of maxima and minima arising from the fluctuation of the second factor

$$\frac{1+r^4+2r^2\cos(\epsilon_1+\epsilon_2)}{(1+r^4-2r^2\cos 2\epsilon_1)(1+r^4-2r^2\cos 2\epsilon_2)}$$

with wave-length will be practically the same as in the case of parallel Nicols considered before. The absolute minima of intensity, will now be very different since they are defined by the condition $\sin^2\frac{\epsilon_1-\epsilon_2}{2}=0$, instead of by the condition $\cos^2\frac{\epsilon_1-\epsilon_2}{2}=0$ of the previous case. This will correspond, in the case of observations with plates that are sufficiently thick to obliterate the interference system arising from the second term in (5), to an interchange of the positions of the bright and dark birefringence fringes, when the analyser is rotated from the position of parallelism with the polariser to the crossed position.

7. *Experimental Observations.*

The crystal plate was mounted suitably between two Nicols and white light from point source, collimated by an achromatic lens, was allowed to traverse the optical system axially. The crystal was mounted on a graduated turn-table, capable of rotation both about a vertical and a horizontal axis. The plate was adjusted to be normal to the path of light by the usual method. The transmitted light was allowed to fall on the slit of a spectrograph.

A thin plate of the monoclinic crystal chrysene (1, 2 benzo-phenanthrene) parallel to its (001) face was used to obtain the spectrograms reproduced in Figs. 1 to 5 in Plate II. Fig. 1 corresponds to the case where the incident light vibrations are parallel to one of the principal axes of the crystal plate (namely, the '*b*' axis) and the analyser is removed from the path of the transmitted light. Fig. 2 is a similar spectogram obtained with the incident vibrations along the other principal axis, *viz.*, the '*a*' axis, of the crystal. The refractive index and also the dispersion of the crystal for vibrations along the '*b*' axis are much larger than for vibrations along '*a*'; consequently, the first fringe system is more closely spaced than the second, besides also having a slightly better visibility, since due to the higher value of the refractive index the reflectivity is also higher.

Fig. 3 was obtained with incident unpolarised light, which may be taken to be equivalent to two vibrations of equal intensity along the first and the second principal axes, respectively, of the crystal. The fringes obtained may therefore be treated as though they were obtained by superposing those in Figs. 1 and 2. The frequent alternation in the degree of visibility are clearly brought out. Fig. 4 was obtained with the crystal between parallel Nicols, at 45° to the crystal axes, and Fig 5 corresponds to the Nicols crossed. In both the figures, namely, 4 and 5, the positions of maximum and minimum visibility are clearly the same as in Fig. 3, and further every alternate posi-

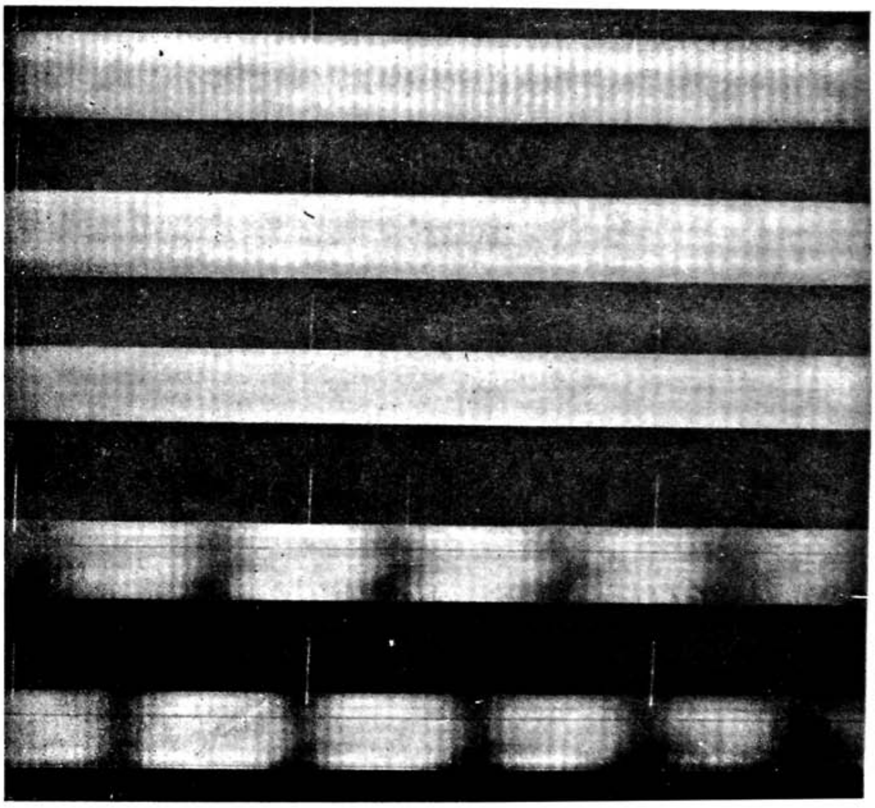

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

tion of maximum visibility is dark, as required by theory. For all the figures, a Cu arc spectrum has been given for reference. Fig. 6 (Plate III) is a microphotometric record of a small portion of the negative corresponding to Fig 3. The regions of maximum and minimum visibility are brought out even more clearly than in Fig. 3. Fig. 7 is similarly a microphotometric record corresponding to Fig. 4. The figure shows clearly that the regions of maximum visibility are alternately dark and bright.

Observations were also made with other crystals, *e.g.*, potassium chlorate, mica, which can be obtained in the form of natural thin plates, and similar results were obtained.

8. *Comparison with the Observations of Chinmayanandam on Haidinger's Rings in Crystalline Plates.*

Following Lord Rayleigh[2] who first drew attention to the complication in the Haidinger rings observed with mica, arising from its doubly refracting properties, Chinmayanandam[3] and later, Schaefer[4] have studied the phenomenon in great detail, both theoretically and experimentally. They find in the observed interference system, there are curves of minimum visibility (which may be considered as arising from the superposition of two sets of Haidinger fringes), coinciding with the isochromatic lines that would be obtained with a plate of double the thickness under crossed nicols in "convergent light." It is easily seen that the phenomenon treated in this paper is complementary to that observed by these authors. In their experiments, the incident light is monochromatic and the interference system observed arises from the different points in the field of view corresponding to different angles of observation and therefore to different

[2] Lord Rayl igh, 'Phil. Mag.,' Vol. 12, p. 186 (1906).

[3] T. K h mayanandam, 'Proc. Roy. Soc.,' A, Vol. 95, p. 176 (1919).

[4] O. Sc er and K. Fricke, 'Zeits. f. Physik,' Vol. 14, p. 253 (1923); C. Schaefer, *ibid*, Vol. 17, 1 (1923).

lengths of path in the crystal. On the other hand in the present experimental arrangement, the direction of observation is always the same, namely, normal to the crystal plate, and hence the distance traversed inside the crystal is the same for all the rays. The differences in optical path that give rise to the interference system in this case arise from *the differences in refractive index for different wave-lengths*, the incident light being now white. In both Chinmayanandam's experiments and in ours, because of the *double refraction* of the plate, there will be two sets of interference fringes which would have different spacings and the occurrence of maxima and minima of visibility in both the cases has a similar explanation. Indeed the analogy will be even closer if the Haidinger fringe system of the crystalline plate is observed in the transmitted light instead of by reflection as in Chinmayanandam's experiments. The curves of maximum visibility in this case would coincide with the isochromatic lines that would be observed between crossed nicols with a plate of the same crystal of double the thickness. The birefringence fringes in the present experiment will be analogous to the isochromatic curves described above. Further their positions are the same as the positions of maxima of visibility obtained with the crystal plate, when its thickness is halved. With crystal plates of sufficiently large thickness, the individual fringes may be too close to be resolved by the spectroscope, and only the birefringence fringes may then be visible.

9. *Analogy with Other Phenomena.*

According to the theory developed in this paper, the birefringence fringes are viewed as arising from the superposition of the two sets of closely spaced fringes having a small difference in spacing, every alternate position of maximum visibility of the superposed system corresponding to the dark band of the birefringence system. The frequency of occurrence of the

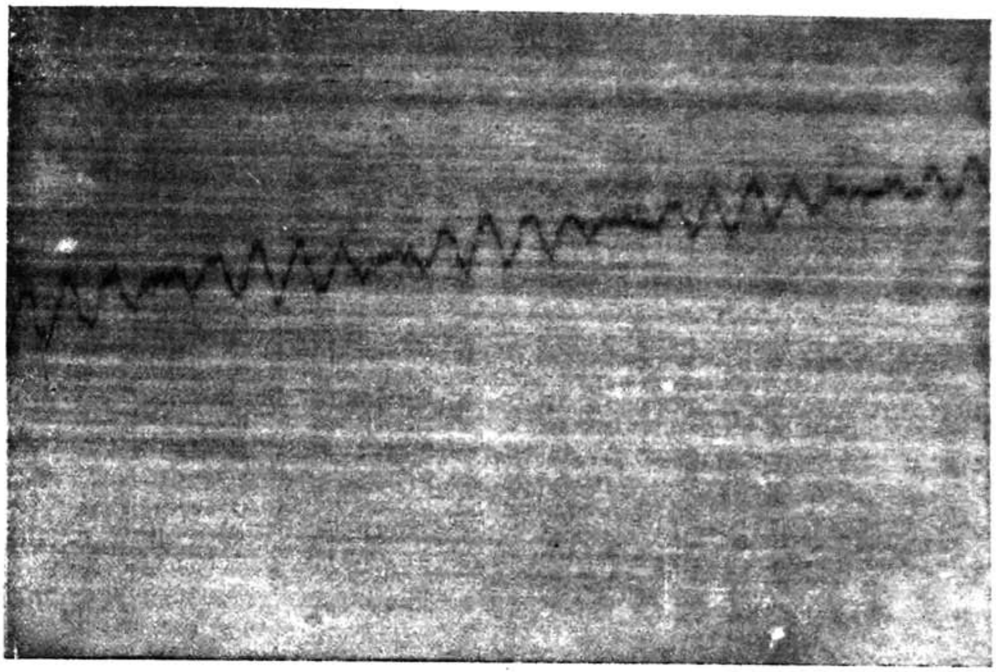

Fig. 6.

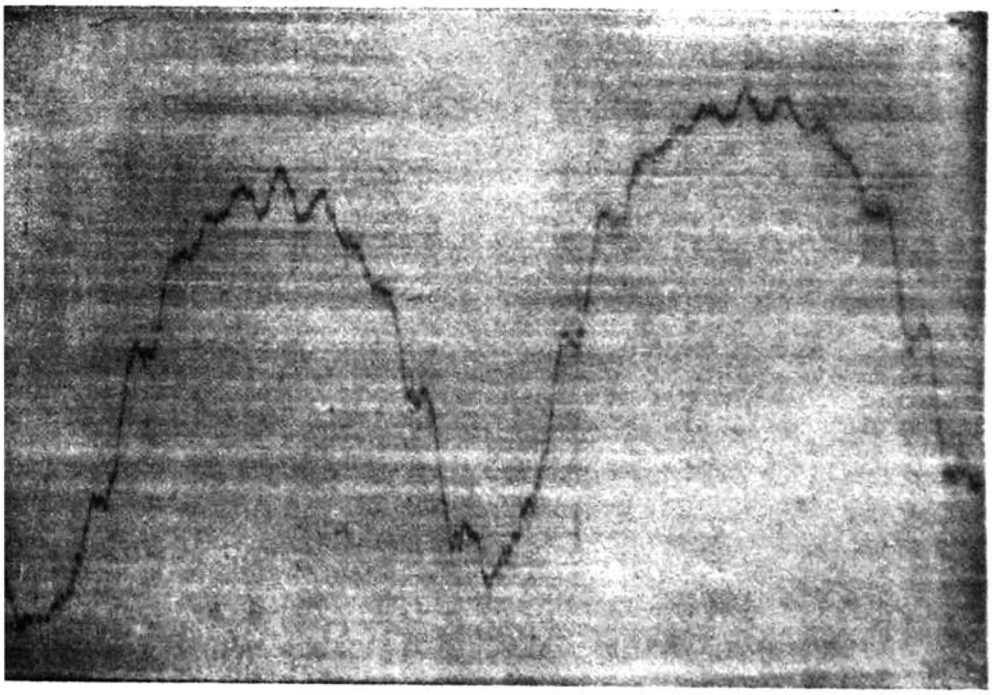

Fig. 7.

birefringence fringes in the spectrum will therefore be much smaller than that of either of the two original systems which give rise to it, by superposition (actually half the difference between the two original frequencies). Thus it is possible to observe the birefringence fringes even under conditions where the original systems of fringes may be too closely spaced to be resolved by the spectroscope. The analogy with the 'heterodyne' beats in wireless, which are detectable, while the individual waves that continue to produce the beats are not, is obvious.

It should be mentioned here that the above view of the birefringence fringes as arising from the differential combination of two sets of closely spaced fringes is again analogous to the explanation suggested some years back by Schuster [5] for the origin of Brewster's bands as due to the superposition of two sets of Haidinger rings due respectively to the two plates. The analogy may be easily followed in detail.

In conclusion, I express my heartfelt thanks to Prof. K. S. Krishnan, for his valuable suggestions and the keen interest he has taken in my work throughout. My thanks are also due to Mr. P. K. Seshan for helping me in many of the experiments.

[5] Sir Arthur Schuster, 'Phil, Mag.,' Vol. 48, p. 609 (1924).