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The Unsymmetrical Hysteresis Loop

BY

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ABSTRACT.

This article describes an investigation to determine the magnetic properties of different grades of silicon sheet steel and 50 per cent. nickel steel when symmetrically and unsymmetrically magnetized; from the results a formula has been derived which should be of value in designing electrical apparatus used in circuits where alternating and direct currents are superposed.

1. *Introduction.*

Many investigators have found that when materials are carried through unsymmetrical hysteresis cycles, there is apparently more energy dissipated than when carried through the symmetrical loop of the same amplitude. By an *unsymetrical loop* is meant the hysteresis loop obtained when the magnetism is carried through a cycle in which the limiting values of flux are different in amount, or in other words, the mean value of the flux differs from zero. Such variation

* Based upon work done at the Research Laboratories of the Union Switch and Signal Company, Swissvale, Pennsylvania, U. S. A.

of magnetism occurs in many places, such as interstage and output transformers for vacuum tube amplifiers, modulator chokes for radio telephone transmitters, reactances for rectifier filter circuits, an induction generator, etc; in short, in all circuits where alternating and direct currents are superposed. There is a small alternating field superposed on a large constant one, thus giving a minor loop superposed on a major loop. The study of such loops is important to calculate hysteresis loss to be expected when the magnetism so varies and also for better understanding of magnetic phenomena.

It has been shown [1] that hysteresis loss due to loops superposed on maximum flux densities of 10,000 gausses or more cannot be represented accurately by equations previously published. The purpose of the investigation discussed in this article was (1) to study the magnetic properties of silicon sheet steel of different grades and 50 per cent. nickel steel when they are symmetrically and unsymmetrically magnetized, (2) to determine the effect of annealing on hysteresis loops, and (3) to derive an equation that would express the relation between hysteresis loss and the pulsating induction accurately enough to be useful in the economical design of such apparatus as mentioned above.

2. *Experimental Procedure.*

Data for both symmetrical and unsymmetrical hysteresis loops were obtained by the ballistic galvanometer method on laminations of sheet steel 0·014 in. thick and an iron section of 1 sq. in. They were assembled as a transformer using lapped laminations in form of a closed shell with the center leg or tongue cut once across for assembly purposes. The transformer was wound with 4 windings to make possible a high sensitivity for both major and minor loops. The error in the permeability caused by neglecting the slight air gap due to cutting of the tongues was carefully measured by comparing with laminations where the tongues had not been cut and was found to be not over 3 per cent. for permeabilities up to 11,000. This corresponds to an equivalent air gap of 0·00002 in.

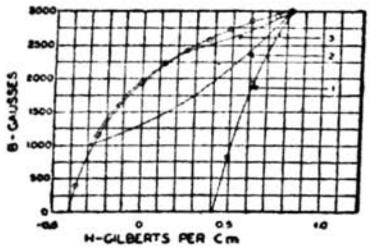
Samples of silicon steel were classified into grades A, B, and C. Grade A or high-silicon steel contained 4·65 per cent. silicon; grade B or medium-silicon steel, 3·25 per cent.; and grade C or low-silicon steel, 1·18 per cent.

Normal hysteresis loops were taken at various flux densities from 100 to 15,000 gausses for each grade, first unannealed and then annealed at 1,900 deg. F. Of course, the material called unannealed had received the ordinary anneal by the manufacturer before shipment. In addition, grade B was annealed at 1,300 deg. F and 1,475 deg. F before annealing at 1,900 deg. The annealing was done by heating the sample in a covered and sealed annealing box placed in an electric furnace at the required temperature for 4 hrs. and then letting it cool in the furnace.

Data for unsymmetrical loops were taken by keeping the maximum flux density constant at values of 3,000 and 10,000 gausses for each grade and in addition at values of 6,000 and 13,000 gausses for grade B. For 3,000 gausses maximum density, minor loops of ±100, ±300, and ±1,000 gausses were taken. The upper tip of the displaced loop corresponded with the upper tip of the major loop as shown by Figs. 1 and 2. This would mean that a minor loop of ±100 was superposed on an average of 2,900 gausses, a minor loop of ±300 on an average of 2,700 gausses, and a minor loop of ±1,000 on an average of 2,000 gausses. For 6,000 and 10,000 gausses maximum flux density, minor loops of ±100, ±300, ±1,000, and ±3,000 gausses were taken; and for maximum of 13,000 minor loops of ±300, ±1,000, ±3,000, and ±6,000 gauses.

3. *Results.*

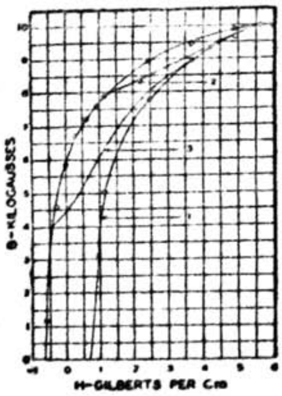
A typical set of hysteresis loops obtained for grade A silicon steel is shown by Figs. 1, 2, and 3; similar loops were obtained for grades B, C and 50 per cent. nickel steel. These curves show how the minor loops were superposed on major loops and how their area and shape vary with displacement. Altogether about 200 such curves were obtained.



1. Half of normal loop, 3,000 gausses
2. Loop ± 1,000 gausses superposed between 1,000 and 3,000 gausses
3. Loop ± 300 gausses superposed between 2,400 and 3,000 gausses

Fig. 1.

Hysteresis loops for grade A silicon steel annealed 4 hr. at 1,900 deg. F., and cooled in furnace.



1. Half of normal loop, 10,000 gausses
2. Loop ± 1,000 gausses superposed between 8,000 and 10,000 gausses
3. Loop ± 3,000 gausses superposed between 4,000 and 10,000 gausses

Fig. 2.

Hysteresis loops for grade A silicon steel annealed 4 hr. at 1,900 deg. F., and cooled in furnace.

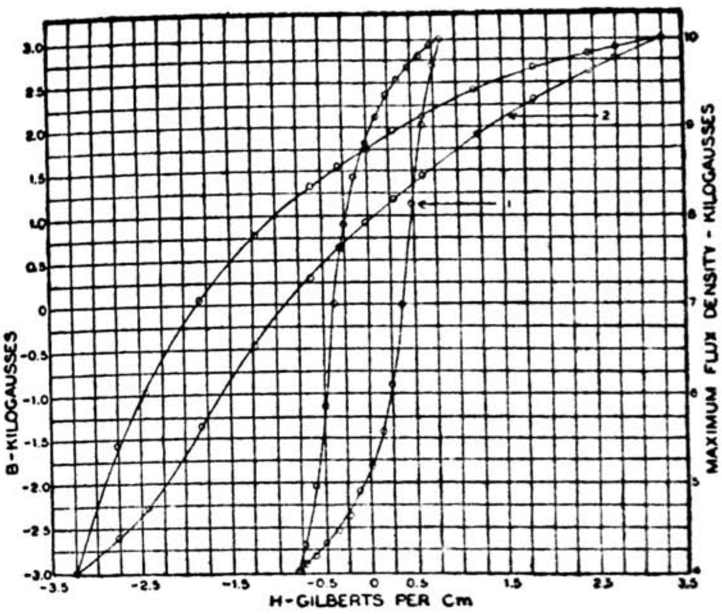


Fig. 3.

Hysteresis loops for grade A silicon steel unannealed.

1. Normal loop, 3,000 gausses.
2. Loop 3,000 gausses superposed maximum flux density 10,000 gausses.

For maximum flux densities upto and including 6,000 gausses, the loss due to superposed loops can be represented by the Steinmetz formula expressed in the form

$$W_h = Kvf\, B^x \times 10^{-7}$$

where

W_h = watts hysteresis loss

K = constant

v = volume of iron in cubic cm.

f = frequency in cycles per second

B = flux variation superposed (lines per sq. cm.)

A separate formula was calculated for each test value of maximum flux density; these formulae are given in Table I at the bottom of the tabulated data to which they refer. It can be seen that the average value of exponent x in this case is much nearer to 2·0 than 1·6 as given by Steinmetz (exponent c).

From loops superposed on maximum flux densities of 10,000 gausses or more, curves of type 1, Fig. 4, were obtained, when log W_h was plotted against log B, to calculate Steinmetz's formula

$$W_h = KB^c$$

or

$$\log W_h = \log K + c \log B$$

Since this formula is based upon a straight line relation, naturally it was found inadequate to express the results given by these curves. Not only the exponent c, but also the co-efficient K for each point was different. It was found that if log W_h log B is plotted against log B as shown by curve 2, Fig. 4, a straight line can be drawn which passes much closer to the plots of measured points than if log W_h is plotted against log B (Steinmetz's formula).

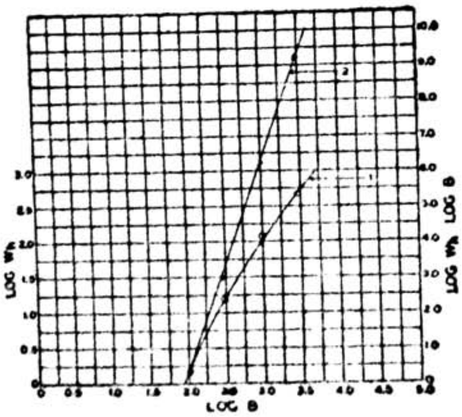


FIG. 4.

Relation between (1) log W_h and log B and (2) log W_h log B and log B.

This relation may be expressed mathematically as follows :

$$\log W_h \log B = A + C \log B \tag{1a}$$

or

$$\text{Log } W_h = \frac{A}{\log B} + C$$

$$= C - \frac{-A}{\log B} \tag{1b}$$

or

$$W_h = \frac{x}{\text{antilog } (-A/\log B)} \tag{1c}$$

where W_h and B have their former meanings, X is the antilog of the slope C of the line graph log W_h log B *vs.* log B, and A is the value of the Y-intercept. The Y-intercept A is below the X-axis, so A is negative number. In order to avoid dealing with antilogs of negative numbers, the term + (A/log B) in the numerator of equation *1b* is changed to -(A/log B) so that (-A) becomes a positive number. This equation gives the loss in ergs for one cubic centimeter at a frequency of one cycle per second. To get the loss in watts for any structure, worked at uniform density at any frequency simply multiply by $(vf \times 10^{-7})$ where v is volume of iron in cubic centimeters and f is the frequency in cycles per second. Then equation *1c* may be written in the form

$$W_h = \frac{x\, vf \times 10^{-7}}{\text{antilog}(-A/\log B)} \tag{1d}$$

within the limits of the test, the average value of A was found to be $-12{\cdot}3$ and of C $6{\cdot}15$, which happens to be just half of the absolute value of A. These formulae are given in Table II at the bottom of the tabulated data to which they refer.

TABLE I—Hysteresis Losses in Silicon Steel for Displaced Loops.

Hysteresis Losses in Ergs per c.c. per Cycle

Flux Density, Gausses		Grade A : Silicon, 4·65%		Grade B : Silicon 3·23%		Grade C : Silicon 1·18%	
Maximum	Superposed	Unannealed	Annealed at 1,900° F	Unannealed	Annealed at 1,900° F	Unannealed	Annealed at 1,900° F.
3,000	100	0·53	0·381	0·52	0·494	1·0	0·732
3,000	300	4·72	4·37	4·82	4·54	9·35	6·82
3,000	1,000	43·9	47·2	42·8	41·2	82·0	62·0
W_h in watts = $86 \times 10^{-13} v/B^{2.05}$		$33 \times 10^{-13} v/B^{2.05}$	$88 \times 10^{-13} v/B^{1.90}$	$55 \times 10^{-13} v/B^{1.97}$	$120 \times 10^{-13} v/B^{1.96}$	$124 \times 10^{13} v/B^{1.9}$	

TABLE II—Hysteresis Losses in Silicon Steel for Displaced Loops.

Hysteresis Losses in Ergs per c.c. per Cycle

Flux Density, Gausses		Grade A : Silicon 4·65%		Grade B : Silicon 3·23%		Grade C : Silicon, 1·18%	
Maximum	Superposed	Unannealed	Annealed at 1,900°F.	Unannealed	Annealed at 1,900°F.	Unannealed	Annealed at 1,900°F.
10,000	100	0·945	1·05	1·0	1·06	1·47	1·55
10,000	300	11·05	11·4	11·5	12·35	18·45	15·7
10,000	1,000	92·4	103·0	94·4	101·5	164·0	118·5
10,000	3,000	406·0	440·0	404·0	328·0	618·5	447·6
W, in watts =		$\frac{0·16\,vf}{\text{antilog}\,(12·6/\log B)}$	$\frac{0·192\,vf}{\text{antilog}\,(12·74/\log B)}$	$\frac{0·151\,vf}{\text{antilog}\,(12·5/\log B)}$	$\frac{0·276\,vf}{\text{antilog}\,(12·7/\log B)}$	$\frac{0·141\,vf}{\text{antilog}\,(11·9/\log B)}$	

4. *Discussion of Results.*

Among previous investigators on hysteresis losses due to displaced loops, John D. Ball[2] was the only one who formulated any definite law from his data although each of the others gave several general conclusions. The general equation given by Ball is of the form

$$W_h = (N + aB_m^{\,y})B^x$$

where N and x are constants similar to those of Steinmetz formula; a, is a co-efficient depending upon the material; y, a power of the mean density B_m; and B, the superposed flux density. The results and analyses of Ball's tests show that

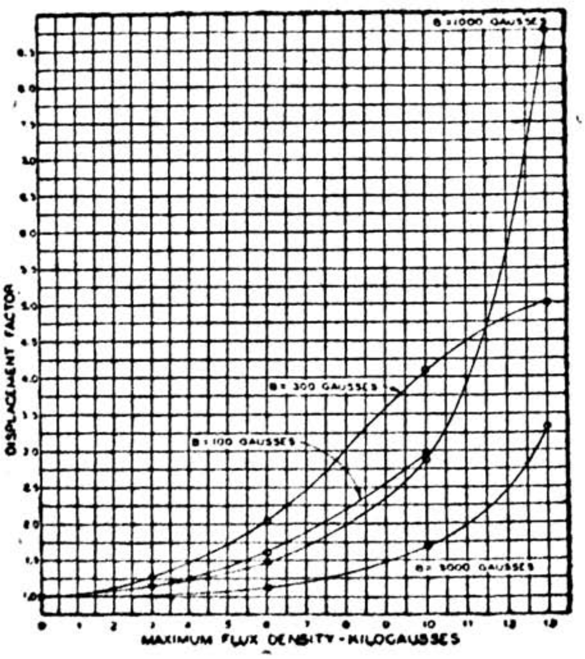


Fig. 5.

Maximum flux density *vs.* displacement factor for grade B silicon steel annealed at 1,300 deg. F.

Superposed flux densities indicated on curves.

N, a, x, and y all vary with induction. Although he expressed the hysteresis loss for a displaced loop as a function of the mean induction, the form of the equation has not been changed materially from that of Steinmetz.

Chubb and Spooner[8] also have pointed out that the hysteresis loss in sheet steel does not follow the Steinmetz law when the material is unsymmetrically magnetized, since both the co-efficient and exponent of the familiar equation $W = n(B/2)^{1.6}$ are found to change with displacement; the co-efficient increasing and the exponent decreasing with increase of displacement.

This investigation, besides bearing out almost all the important conclusions of the previous investigators on hysteresis losses in sheet steel due to displaced loops, brings out several new points of importance and interest. Within the range of the test it is found that the hysteresis loss of loops superposed on maximum flux densities up to and including 6,000 gausses can be represented by an equation of the same form as that of Steinmetz, since the plot of log W_k against log B (pulsating induction) on ordinary cross section paper gives approximately a straight line. The co-efficient and exponent are both nearly constant for each maximum flux density. They are found to change, however, when the same pulsating inductions are superposed on different maximum flux densities. As seen in Table III, the co-efficient increases and the exponent decreases as the maximum flux density is increased.

The hysteresis loss due to loops superposed on maximum flux densities of 10,000 gausses or more cannot be represented by Steinmetz's equation. Equation (1) was derived to express the results from all 3 grades of silicon sheet steel, both unannealed and annealed, and 50 per cent. nickel steel. As seen in Table IV, the values calculated with this equation check much closer with the observed values, than those calculated with Steinmetz equation or Ball's equation.

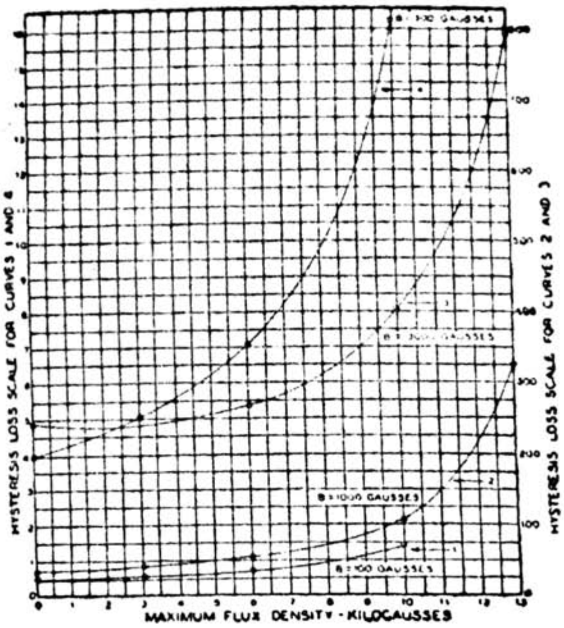


FIG. 6.

Maximum flux density *vs.* hysteresis loss due to pulsating induction in grade B silicon steel annealed at 1,300 deg. F.
Superposed flux densities indicated on curves.

The displacement factor (the ratio of the area of displaced loop to the area of a symmetrical loop of equal amplitude) for low-, medium-, and high-silicon steels, both unannealed and annealed, varies greatly between different steels at the same pulsating induction and maximum flux density. In other respects it follows certain marked regularities, e.g., for the same sample and the same pulsating induction, it increases consistently with maximum flux density as shown in Fig. 5. This means that the hysteresis loss due to pulsating induction increases with maximum flux density, which is quite in agreement with the results shown in Fig. 6. Also, as shown in Fig. 7, for the same maximum flux density and all three grades of

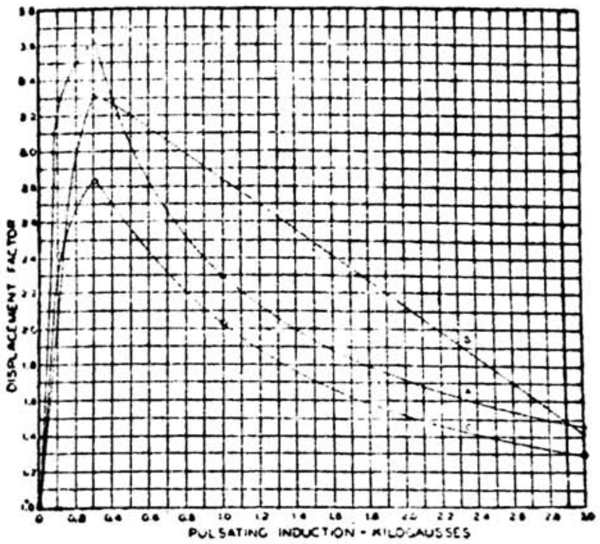


FIG. 7.

Pulsating induction *vs.* displacement factor for maximum flux density, 10,000 gausses, grades A, B and C silicon steel.

steel, both unannealed and annealed, the displacement factor increases rapidly at low pulsating inductions, reaches a maximum, and then decreases as the inductions are increased. The general shape of the curves appears similar to that of permeability curves. For maximum flux densities up to and including 10,000 gausses, the displacement factor showed a maximum for about 300 gausses superposed flux density. In other words, for maximum flux densities up to and including 10,000 gausses, the hysteresis loss due to pulsating inductions increases rapidly, reaches a maximum at about 300 gausses, and then decreases as the inductions are increased.

As the displacement factor in all cases is found to be greater than one, the hysteresis losses in silicon sheet steel due to displaced loops varying from ±100 to ±6,000 gausses are

TABLE III—Change in Co-efficient and Exponent of Steinmetz's Formula With Maximum Flux Density for Grade B (3·23 per cent.) Silicon Steel.

Heat Treatment	Max. Flux Density* 3,000 Gausses		Max. Flux Density* 6,000 Gausses	
	Co-efficient	Exponent	Co-efficient	Exponent
Unannealed	88·0	1·90	144	1·86
Annealed at 1,300°F.	62·5	1·96	100	1·94
Annealed at 1,900°F.	55·0	1·97	73	1·96

* Superposed Flux Densities ± 100, ±300, ±1,000 gausses.

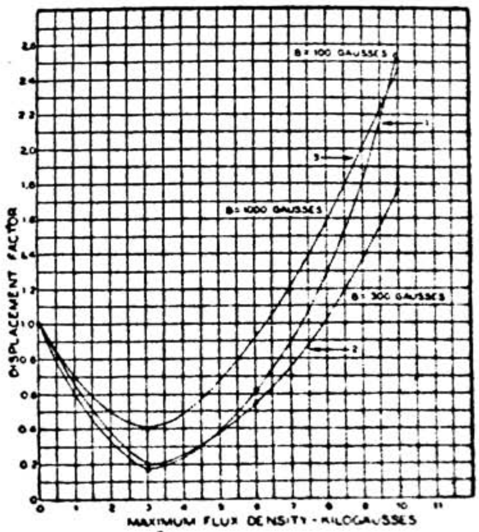
TABLE IV—Comparison between Observed and Calculated Values. Grade C (1·18 per cent.) Silicon Steel.

Observed lues	Calculated Values					
	Author.		Steinmetz.		Ball.	
W_h.	W_h.	Per cent. error.	W_h.	Per cent. error.	W_h.	Per cent. error.
1·47	1·26	−14·3	2·024	+37·6	3·81	+159·0
18·45	20·42	+10·82	13·88	−24·8	21·62	+17·2
164·00	162·20	−1·1	114·53	−30·15	137·63	−16·0
618·50	628·1	+1·55	785·9	+27·1	642·4	+3·9

Maximum Flux Density 10,000 gausses.

Superposed Flux Densities, ± 100, ± 300, ± 1,000, ± 3,000 gausses.

evidently greater than those due to symmetrical loops of the same amplitude. This, however, is not true of 50 per cent. nickel steel. As shown in Fig. 8, the displacement factor for minor loops of ±100, ±300, and ±1,000 gausses is less than one; above this flux density it is greater than one. That is, the hysteresis losses in 50 per cent. nickel steel due to such pulsating inductions, when superposed on maximum flux densities at least up to 6,300 gausses, are less than those due to normal loops of the same amplitude. This is very interesting, and adds to the unusual qualities of this material as pointed out by T. D. Yensen.[4]



Superposed flux densities indicated on curves

FIG. 8.

Relation between displacement factor and maximum flux density for 50 per cent. nickel steel annealed at 1,650 deg. F. for 2 hr. cooled in furnace to 975 deg. F. and then in air.

5. *Conclusions.*

Principal findings of this investigation are as follows:

1. In common with the results of other investigators, the data here shown indicate that Steinmetz's theory for hysteresis loss in many instances is inaccurate and that its greatest errors occur at very high and very low flux densities.

2. Hysteresis loss in silicon sheet steel due to loops superposed on maximum flux densities up to and including 6,000 gausses can be represented by an equation of the same form as that of Steinmetz, the co-efficient and exponent are both nearly constant for each maximum flux density; they are found to change only when given pulsating inductions are superposed on different maximum flux densities. For the displaced loops the value of exponent c is much nearer 2·0 than 1·6 as given by Steinmetz.

3. Hysteresis loss due to loops superposed on maximum flux densities of 10,000 gausses or more cannot be represented accurately by equations previously published. Equation 1 was derived to express the relation between hysteresis loss and pulsating induction superposed on flux density of 10,000 gausses or more and was successfully applied to all results presented here. The values of constants A and C derived for the samples annealed at 1,900 deg. F. apply to material which was somewhat oxidized. Where the material tested was unannealed, the oxidation should be negligible. Also, the samples tested differ in form from those that have been used by most of the other investigators; this investigation was made on laminations in form of a closed shell with the centre leg cut once across for assembly purposes, whereas the previous investigations had been made mostly on samples in form of a ring.

4. The displacement factors for low-, medium-, and high-silicon steels, both unannealed and annealed, differ widely

from one another at the same pulsating induction and maximum flux density.

5. For the same grade of steel and the same pulsating induction, the displacement factor increases with an increase in the maximum flux density.

6. For the same maximum flux density and all 3 grades of steel, both unannealed and annealed, the displacement factor increases rapidly at low pulsating inductions, reaches a maximum, and then decreases as the inductions are increased. For maximum flux densities up to and including 10,000 gausses, the displacement factor showed a maximum for about 300 gausses superposed flux density.

7. For all grades of silicon steel, both unannealed and annealed, and all superposed loops, the displacement factor is greater than one.

8. For 50 per cent. nickel steel and superposed loops of ±100, ±300, ±1,000 gausses, the displacement factor first decreases, reaches a minimum, and then increases as the maximum flux density is increased. It is less than one up to 6,300 gausses and greater than one above that density.

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