On the Working of a Rotating Commutator

By

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Plate XI.

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In conducting experiments with a rotating commutator of the kind described by the author [1] to find the capacity of a parallel plate condenser with a guard ring, certain apparent anomalies were encountered. Two sets of brushes were employed, one to charge the central segment of the condenser and discharge it through a galvanometer, the other to charge the guard ring and discharge it to earth. As explained in the above note, any desired phase difference between the operations of the two brush sets can be introduced by properly

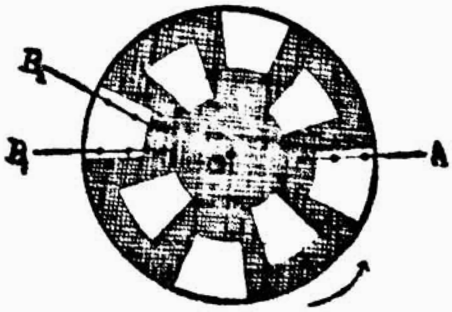

Fig. 1.

[1] Ind. Jour. Phys., Vol. 7, pp. 91-98 (1932).

locating them. First it was arranged that the charging and discharging processes by the two sets were as nearly simultaneous as possible, their positions being indicated by A and B_1 (Fig. 1). We may denote this arrangement as synchronisation. This caused a certain deflection in the galvanometer. On changing the relative positions of the brush sets as shown by A B_2 (Fig. 1), under certain circumstances, a considerable change in the deflection was observed. It will be seen that with the brushes in the positions A B_2, the charging and discharging action of B_2 is earlier than that of A by an amount proportional to the angle B_1OB_2, and the experiment showed that this departure from synchronism had a material effect on the deflection. Experiments were then conducted with the common motor-driven rotating commutator (Pye & Co.'s) to verify the nature and magnitude of the effect. It was possible to secure different degrees of departure from synchronism with this commutator by either,

(*i*) displacing the central brush of one set relative to that of the other, the commutator segments being aligned parallel as shown in Fig. 2 (*i*) or

(*ii*) by keeping the brushes parallel, but rotating one set of commutator segments round the axle relative to the other, as shown in Fig. 2 (*ii*).

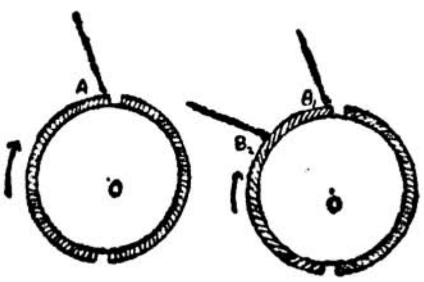

Fig. 2 (*i*).

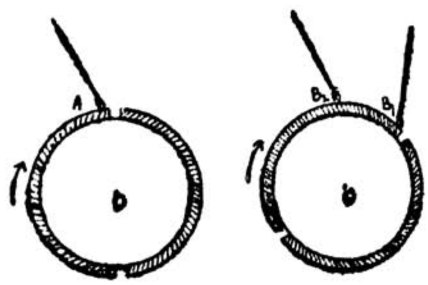

Fig. 2 (*ii*).

It is seen that in both cases, B_2 charges or discharges earlier than A, the phase difference being proportional to the angle B_1OB_2. In this case also, it was found that the deflection in the galvanometer varied with B_1OB_2 in precisely the same manner with newer commutator. This paper is intended to account for these variations, and incidentally to arrive at the conditions for correct operation of a rotating commutator.

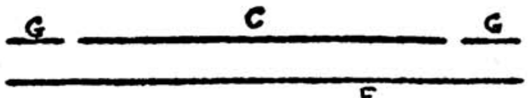

Fig. 3.

We proceed with the investigation as follows:—Let us assume brush A to be the central one of the set for charging and discharging through the galvanometer the central segment of the condenser, written simply as the condenser C (Fig. 3) while B is the corresponding brush for charging and discharging the guard-ring G. Diagrammatically we may represent each commutator by an endless series of metal strips, each of width a alternately at potential V and O, separated by insulating strips each of width b, the two sets moving parallel to each other with a constant velocity (Fig. 4). The brushes A and B are placed in fixed positions so as to come into contact with

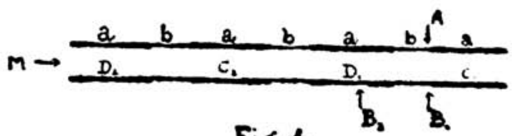

Fig. 4.

their respective sets of strips. In the positions A B_1, the two commutators act synchronously, while as A B_2, the action of B_2 is ahead of that of A by B_1B_2, which can be varied from 0 to PQ(=2a+2b), a complete cycle. To determine the discharge through the galvanometer per cycle of commutation, it is best to consider the amounts of charge carried by the condenser *viz.*,(*i*) Q_1, just before A reaches D and (*ii*) Q_2 just as it leaves, D. Then Q_1-Q_2 is the flow through the galvanometer per cycle. Now, the charge Q on the condenser at any instant depends on its own potential and the potential of the guard ring at that instant, the third plate being permanently at potential zero.

Thus Q_o (charge on condensor) $=q_{11}\ V_o+q_{12}\ V_g$

and Q_g (charge on guard ring) $=q_{22}\ V_g+q_{21}\ V_o$

where V_o and V_g are the potentials of the condenser and guard ring respectively, and q_{11} etc., are the co-efficients of the system, q_{11} and q_{22} being positive, while q_{12} and q_{21} are equal and negative. We shall write for shortness $q_{11}=q_1$, $q_{22}=q_2$, $q_{12}=q_{21}=q$.

Two cases arise, which require analysis, *viz.*, case I where $a>b$, and case II when $b>a$, the case $a=b$ being the limiting case of either, but conforming in practice to the case $a>b$.

Case I.

$a>b$ (Fig. 4.)

It is convenient to divide the possible range 0 to $2a+2b$ of $B_1\ B_2$ into four subranges 0 to b, b to $a+b$, $a+b$ to $a+2b$ and $a+2b$ to $2a+2b$. The following values of Q_1 and Q_2 as $B_1\ B_2$ is increased from 0 to $2a+2b$ are arrived at by inspection (see Fig. 4).

It is seen from these results that when the phase difference $B_1\ B_2$ ranges from 0 to b and $a+2b$ to $2a+2b$, the discharge and hence the deflection in the galvanometer would be proportional to (q_1+q), while, when $B_1\ B_2$ lies between b and $a+2b$ the deflection would be proportional to q_1-q.

Range of $B_1 B_2$.	Q_1	Q_2	$Q_1 - Q_2$
0 to b	If $b > B_1 B_2 > 0$, brush B will be on C_1 (pot. V) along with A for a space $a - B_1B_2$, B then gets insulated, and continues so till after A separates from C_1. So the charge on C at the instant A separates from C_1 and thus just before A reaches D_1 is $Q_1 = q_1V + qV = (q_1+q)V$.	A and B will be on D_1 for some time simultaneously, B gets insulated first, but does not reach C_2 till after A has left D_1. So the charge on C_1 when A is just leaving D is $Q_2 = q_1 0 + q0 = 0$.	$(q_1+q)V$.
b to $a+b$.	If $a+b > B_1 B_2 > b$, B will be on D_1 when A is just leaving C_1 and so the charge carried by C is $Q_1 = q_1V + q_2 0 = q_1 V$.	B will be on C_2 when A is just leaving D_1 $Q_2 = q_1 0 + qV = qV$.	$(q_1 - q)V$.
$a+b$ to $a+2b$	It $a+2b > B_1B_2 > a+b$, B is on D_1 when A is on C_1 for a little while, B is insulated first, but does not reach C_2 till after A leaves C_1. $Q_1 = q_1V + q0 = q_1V$	B will be on C_2 when A is on D_1, is insulated, but does not reach D_2 before, A leaves D_1. $Q_2 = q_1 0 + QV = qV$.	$(q_1 - q)V$
$a+2b$ to $2a+2b$.	If $2a+2b > B_1B_2 > a+2b$. B will be on C_2 when A is leaving C_1. Hence, $Q_1 = q_1V + qV = (q_1+q)V$	B will be on D_2 when A is leaving D_1. *i.e.*, $Q_2 = q_1 0 + q0 = 0$.	$(q_1+q)V$

Case II.

$b > a$ (Fig. 5.)

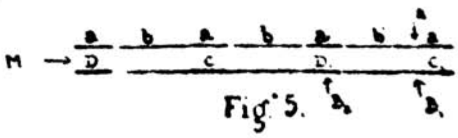

Fig. 5.

Here we have to divide the range $PQ = 2a + 2b$ into six sub-ranges 0 to a, a to b, b to $a+b$, $a+b$ to $2a+b$, $2a+b$ to $a+2b$, and $a+2b$ to $2a+2b$. The ranges a to b and $2a+b$ to $a+2b$ require special treatment, while the remaining four give results as in case I. The arguments and results for 0 to a, b to $a+b$, $a+b$ to $2a+b$ and $a+2b$ to $2a+2b$ in this case are respectively exactly the same as for the ranges $0-b$, b to $a+b$, $a+b$ to $a+2b$ and $a+2b$ to $2a+2b$ of case I as can be deduced by inspection using Fig. 5 for the purpose.

The peculiarity of the other two ranges of case II arises from the fact that A and B are here at no time simultaneously on metal segments (*i.e.*, at known potentials V or 0), each leaving a metal segment before the other gets on to one. But we can show that a steady state of cyclic changes of charges and potentials will be attained after a few revolutions of the commutator, after which the following analysis will apply

$$b > B_1B_2 > a.$$

Let G (connected to B) be at a potential V_1 when A is leaving C_1. Then $Q_1 = q_1V + qV_1$. This charge is unaffected as A travels between C_1 and D_1, but the potential of A (*i.e.*, of C) is changed, say to V_2, when B reaches D_1 *i.e.*, the guard ring is earthed. This gives $Q_1 = q_1V_2 + q0 = q_1V_2$. The guard ring however has at this stage a charge $Q' = q_20 + qV_2 = qV_2$ which it carries as B leaves D_1 before A touches D_1. This charge gives it a potential V_3 when A is earthed (touches D_1) such that $Q' = q_2V_3 + q0 = q_2V_3$, C which is earthed at this stage will have a charge $Q_2 = q_10 + qV_3$ which it carries as A leaves D_1 before B arrives at C_2. The potential of C due to this charge is changed to V_4, when B touches C_2, where $Q_2 = q_1V_4 + qV$. The charge Q'' on G at this stage $= q_2V + qV_4$ which it carries as B leaves C_2. Lastly A now comes up to C_2 when a new cycle starts, A being at V and B at V_1, such that the charge Q'' on G is given by $Q'' = q_2V_1 + qV$. We have thus four equations:

$$Q_1 = q_1V + qV_1 = q_1V_2. \qquad Q' = qV_2 = q_2V_3$$

$$Q_2 = qV_3 = q_1V_4 + qV. \qquad Q'' = q_2V + qV_4 = q_2V_1 + qV.$$

Solving these we get

$$Q_1=\frac{q_1q_2(q_1+q)}{q_1q_2+q^2}V, \qquad Q_2=\frac{q^2(q_1+q)}{q_1q_2+q^2}V.$$

$$\text{i.e., } Q_1-Q_2=(q_1+q)\ V\frac{q_1q_2-q^2}{q_1q_2+q^2}$$

$$2b+a>B_1B_2>2a+b.$$

Here when A is on C_1, B will lie between D_1 and C_2, say at a potential V_1'. Starting with this condition, and proceeding as in the previous para., we obtain using parallel nomenclature the following equations.

$$Q_1=q_1V+qV'_1=q_1V'_2+qV. \qquad Q'=q_2V+qV'_2=q_2V'_3$$

$$Q_2=qV'_3=q_1V'_4 \qquad Q''=qV'_4=q_2V'_1+qV$$

from which we get

$$Q_1=\frac{q^3+q_1^2q_2}{q_1q_2+q^2}V \qquad Q_2=\frac{q_1q(q+q_2)}{q_1q_2+q^2}V \text{ and}$$

$$Q_1-Q_2=(q_1-q)V\ \frac{q_1q_2-q^2}{q_1q_2+q^2}$$

The results for case II can be tabulated as follows :—

B_1B_2 →	0 to a	a to b	b to a + b	a + b to 2a + b	2a + b to 2b + a	2b + a to 2a + 2b
Q_1	$(q_1+q)V$	$\frac{q_1q_2}{q_1q_2+q^2}(q_1+q)V$	q_1V	q_1V	$\frac{q^3+q_1^2q_2}{q_1q_2+q^2}V$	$(q_1+q)V$
Q_2	0	$\frac{q^2}{q_1q_2+q^2}(q_1+q)V$	qV	qV	$\frac{q_1q(q+q_2)}{q_1q_2+q^2}V$	0
Q_1-Q_2	$(q_1+q)V$	$\frac{q_1q_2-q^2}{q_1q_2+q^2}(q_1+q)V$	$(q_1-q)V$	$(q_1-q)V$	$\frac{q_1q_2-q^2}{q_1q_2+q^2}(q_1-q)V$	$(q_1+q)V$

To verify the above analysis, experiments were conducted with the new commutator, wherein the set of brushes connected with the guard ring was held in a fixed position, while

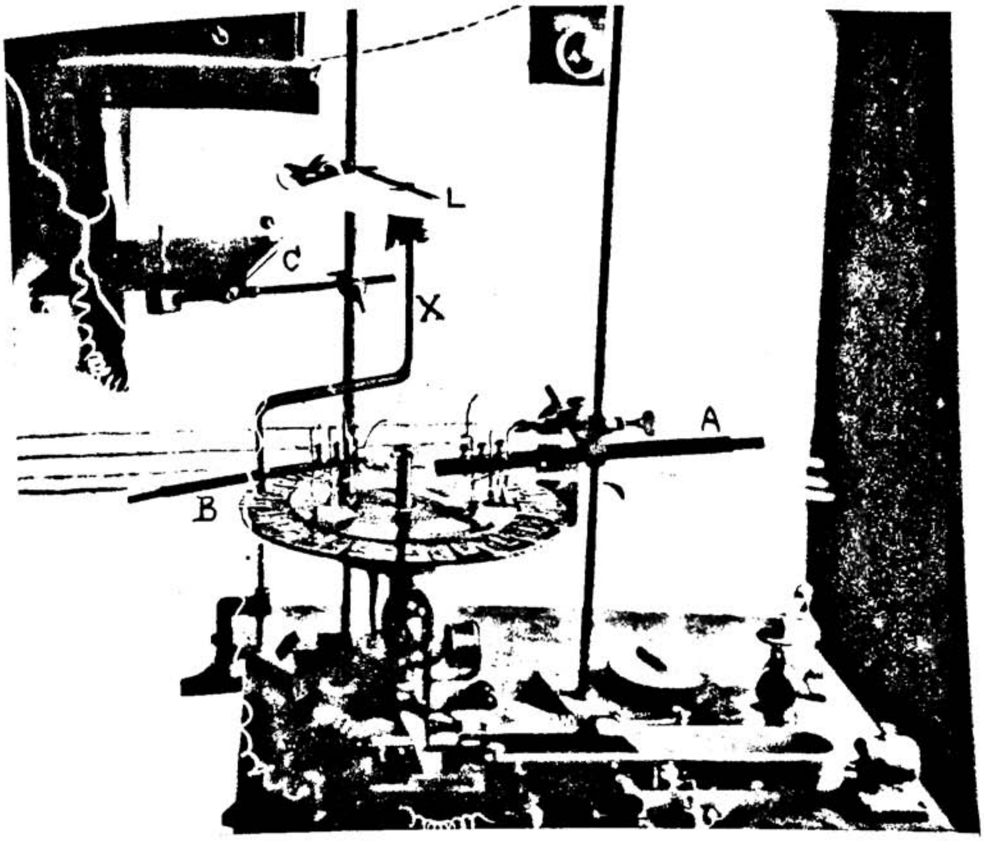

Fig. 6.

Arrangement of commutator and brushes.

A fixed set of brushes. B—second set of brushes mounted on an arm X so as to be movable round the same axis as the disc. L—is the pointer fixed to X moving on a circular scale. C—The condenser.

the other was mounted so that it could be turned round an axis coinciding with that of the commutator plate (Fig. 6). This plate was one containing four sets of elements, so that each cycle of commutation occupied, 90°. By having on one side of the sheet sectors wherein $a=31°$ and $b=14°$ while on the other side $a=17\frac{1}{2}°$ and $b=27\frac{1}{2}°$ nearly, it was possible to provide for cases I and II above. The condenser was an air condenser with tin foil coated on two vulcanite sheets each 20 cms. square with small vulcanite separators each about 3 mms. thick. The condenser part is a circle about 16 cms. diameter, the outside being the guard ring. About 90 volts were used for charging. First, observations were made to determine the values of q_1, q_2 and q. For this the deflections were obtained

(*i*) when the condenser was charged and discharged through the galvanometer, and synchronously, the guard ring was charged and discharged to earth.

$$Q_1 - Q_2 = (q_1 + q)\, V = a\, X_1 \quad \text{where X is the deflection.}$$

Thus $q_1 + q = \frac{a}{V} X_1 = K\, X_1$

(*ii*) When the guard ring was charged and discharged through the galvanometer, and synchronously, the condenser was charged and discharged to earth.

$$Q_1 - Q_2 = (q_2 + q)\, V = a X_2$$

$$q_2 + q = K X_2$$

(*iii*) The guard ring was permanently earthed and the condenser was charged and discharged through the galvano. meter, only one set of brushes being used.

$$Q_1 - Q_2 = q_1 V = a\, X_3$$

$$q_1 = K\, X_3$$

(*iv*) The condenser was permanently earthed and the guard ring was charged and discharged through the galvanometer.

$$Q_1 - Q_2 = q_2 V = a X_4$$
$$q_2 = K X_4$$

The following were the results :—

$X_1 = 9{\cdot}1$ cms.	*i.e.*	$q_1 + q = 9{\cdot}1$ K
$X_2 = 12{\cdot}1$	,,	$q_2 + q = 12{\cdot}1$ K
$X_3 = 12{\cdot}0$	,,	$q_1 = 12{\cdot}0$ K
$X_4 = 15{\cdot}0$	,,	$q_2 = 15{\cdot}0$ K

from which $q = -2{\cdot}9$ K, and $\dfrac{q_1 q_2 - q^2}{q_1 q_2 + q^2} = \left(\dfrac{19}{21}\right)$ nearly.

Also $q_1 - q = 15{\cdot}0$ K. $\quad (q_1 + q)\dfrac{q_1 q_2 - q^2}{q_1 q_2 + q^2} = 8{\cdot}2$ K.

$$(q_1 - q)\frac{q_1 q_2 - q^2}{q_1 q_2 + q^2} = 13{\cdot}6 \text{ K}.$$

Next, the original experiment was conducted, with the two sets of brushes, one fixed, to charge the guard ring and discharge it to earth, and the second, to charge the condenser and discharge it through the galvanometer. The second was mounted so as to move round a vertical axis coinciding with that of the commutator plate, arrangements being made to read the position of the brush set on the commutator plate correct to about a degree (Fig. 6). The deflections in the galvanometer for different positions of this brush were obtained. The readings are plotted in Fig. 7 for case I and Fig. 8 for case II. The thick lines in the graphs show what the readings would be according to the explanation worked out in this paper, taking the values of q_1, q_2 and q already

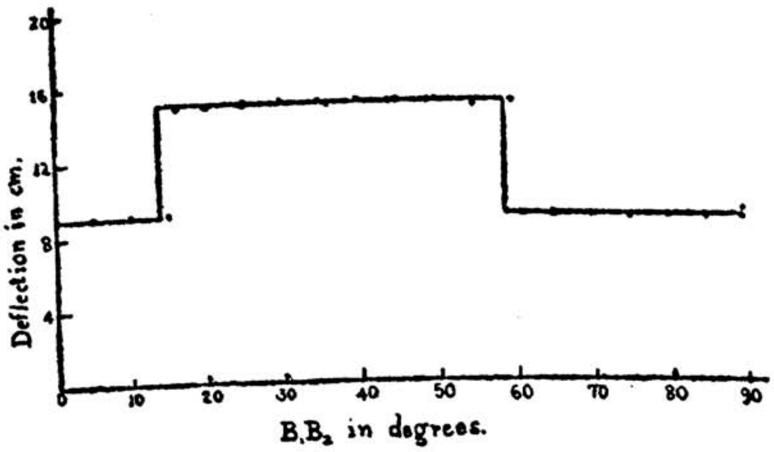

FIG. 7.

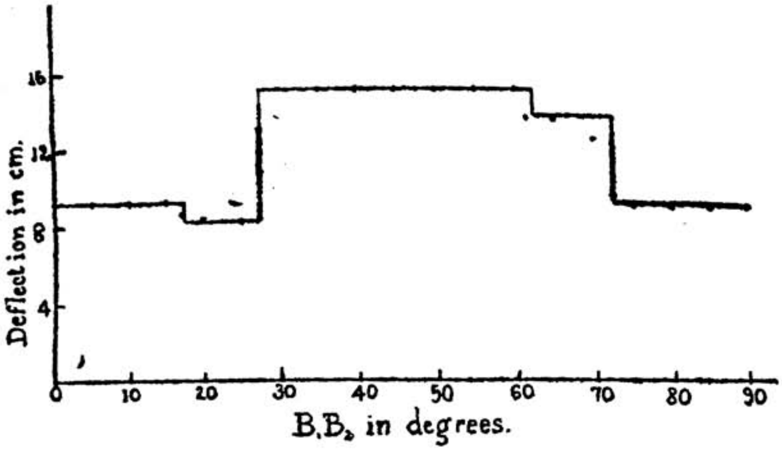

FIG. 8.

given. It will be found that practically all the points fall on the thick line graphs indicating the correctness of the explanation offered. On account of the form and size of the brushes it is difficult to determine their positions accurately and to obtain and judge the position of exact synchronism. An error of 2 or 3 degrees is quite possible in this respect.

It now remains to consider the conditions for the satisfactory operation of commutators with two sets of brushes for simultaneous use. It is seen from the graph that starting from synchronism, the discharge changes from $(q_1 + q)$ V to $(q_1 - q)$ V by a phase change b on one side and a on the other. In the common or older type of commutator b is usually quite small, being, for example, less than 4 mms. on a cylinder 4 cms. in diameter in a Pye motor driven commutator. This makes b about 3% of a cycle. Considering the nature of the brushes used and the methods of fixing them, it is quite easy to have a phase difference of the above magnitude by accident, unless one is very careful. The indication is therefore that b should be increased.

I would also draw attention to the possibility of utilising the above investigation to find the values of the induction coefficients experimentally in various arrangements of conductors.

In conclusion I wish to thank Mr. T. Tirunarayanachar, M.Sc., for the valuable help rendered in conducting the above experiments.

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