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Theory of Extremely High Lapse-rates of Temperature very near the Ground

BY

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(Plate XIV.)

(*Received for publication, 7th November, 1931.*)

ABSTRACT.

The present paper concerns itself with the distribution of temperature on a clear day in the layer of air 10 to 20 cm. thick just above the ground. In this layer which may be called the "Surface layer" the temperature falls very rapidly with height and lapse-rates higher than 1 or 2°C per centimetre are common.

Temperature observations at suitable intervals of height above ground are described in section 2. In section 3, a mathematical solution of the "surface" problem has been advanced taking into consideration the balance in an elementary layer between the heat received by conduction or convection processes and the net loss of heat by radiation processes. The relation obtained

$$= \phi_0 \frac{\sinh a(h-z)}{\sinh ah}$$

is where ϕ is the variable part of the temperature, ϕ_0 is a constant and h the thickness of the surface layer ; a is a constant involving the constants of temperature radiation, convection and absorption of radiation by water vapour in the atmosphere. The observed temperature distribution agrees fairly well with the above solution.

In the last section a brief discussion of inferior mirages observed on asphalted roads is given.

1. *Introduction.*

The work of G. I. Taylor and others has shown that the distribution of temperature in the lower layers of the atmosphere is controlled by turbulence or large scale eddies in the air. In the literature on the subject of turbulence, it is usual to find that conditions in the layer of air very close to the ground are neglected. No attempts appear to have been made to elucidate the temperature distribution very near the ground.

On a clear day the air near the heated surface of the earth appears to boil or shimmer. This may be considered as due to the variations in the density and consequently the refractive index of the air near the ground arising from the upward streaming of the heated air and the descent of the relatively colder air from above, the situation at any one instant being roughly as indicated in Fig. 1.

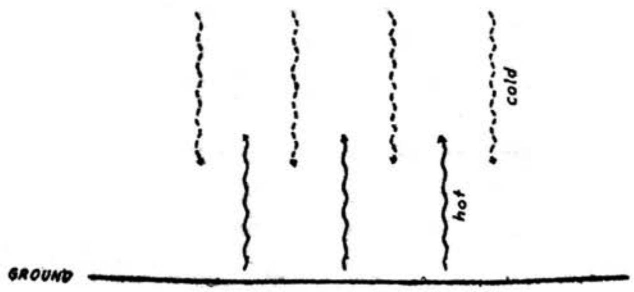


FIG. 1.

The shimmering effect caused by such a local circulation or exchange of warm and cold air is most vigorous very near the ground though sometimes one can see it extending up to a few feet above ground under favourable circumstances. We shall confine our attention to this surface layer and see how its properties differ from those of the super-incumbent layers in which the well-known large scale eddies prevail.

In the surface layer the air has a higher temperature and smaller density than the air above. It may be noted that the pressure which depends upon the weight of the superincumbent atmosphere does not sensibly alter on this account. It is sufficient to consider the variation of the actual, instead of the potential temperature. Measurements recently taken at Poona show that there is a very rapid fall of temperature with height in the surface layer of depth 10 or 15 cms. compared to which the variation of temperature in the free air is almost negligible.

In the present paper an attempt has been made to account for the observed distribution of temperature in the surface layer. In a later section the phenomena of inferior mirages which are usually associated with high lapse rates near the ground will be discussed in the light of data that have been obtained. Investigations in the laboratory confirm our picture of turbulence and temperature distribution near the surface. These results will be discussed in a later communication.

2. *Temperature observations near the ground.*

Temperature observations very near the ground are rare[1]; and where available the observations have been taken at such large height intervals that the variations in the surface layer are unfortunately not well brought out. We took a suitable series of measurements at short height intervals with a number of standard Assmann psychrometers of

[1] Some measurements of temperature at 5, 25 and 175 cms. above ground have been quoted by R. Geiger (Met. Zeit., Dec., 1929). These show roughly the nature of temperature variation near the ground.

In Met. Mag., July, 1928, p. 139, W. H. Bigg has mentioned that he took measurements near the ground at 1 inch, 3 inches, 1 foot, 3 feet and 6 feet respectively above tarred road. These data have not been published or discussed.

See also hourly observations by Sinclair (Monthly Weather Review, March, 1922, p. 143) and some observations of Steavenson in Egypt. (Quarterly Journal, Royal Meteorological Society, 1921, Vol. 47, pp. 15 *et seq.*)

the same size and make on a few days during the last summer and during the month of October, 1931, when skies were clear. The measurements were made over an asphalted road near the Meteorological Office, Poona, where inferior mirages could be seen usually. Both the dry and wet bulb thermometers were read. It was found that the variations in the vapour pressure with height were negligible. Temperature height curves were drawn for different occasions for study. A typical example is given in Fig. 2 which represents observations made on the 22nd October, 1931 at 14·00 hours.

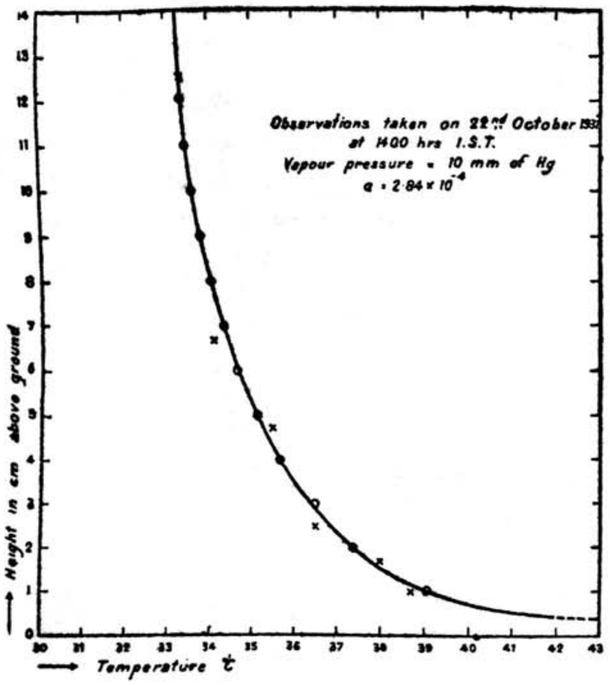


FIG. 2.

The curves show the variation of temperature with height during the steady conditions[2] prevailing at the time of "maxi-

All the observations were completed within an interval of about twenty minutes on each day.

mum temperature" (13·00-14·30 hours I.S.T.). It is seen that the temperature decreases very rapidly within a layer about 10 or 15 cms. thick. The maximum lapse-rates near the ground are of the order of 1 to 2°C per cm., *i.e.*, about ten times or more than the maximum lapse-rates predicted as possible by Brunt in a recent paper.[3] The variation of temperature above this layer is very small and attains an almost constant value within a small distance.

For the purpose of our problem the region of smaller lapse-rates is considered to be one of constant temperature. We have therefore to explain the temperature distribution in a shallow surface layer with a constant high temperature at its bottom (ground) and with a super-incumbent atmosphere which is also at a constant temperature.

3. *Theory of High Lapse Rates of Temperature in the Surface Layer.*

The distribution of temperature above the ground is controlled by (1) the radiative processes taking place near the ground and (2) the upward transfer of heat by conduction or convection. We shall consider these now.

(1) *Radiative processes.*—From the work of Hettner and Simpson[4] it is well known that a column of air containing more than 0·3 mm. of precipitable water completely absorbs all long wave or heat radiation except in the wave-length range 8 to 11μ. It would be therefore sufficient to consider radiations from layers above ground whose total thickness will not contain much more than 0·3 mm. of precipitable

[3] Proc. Roy. Soc. A., Vol. 130, 1930, p. 98.

[4] Mem. R. Met. Soc. No. 21 (1928). See also Brunt's contribution in Proc. Roy. Soc. A., Vol. 124 (1929), pp. 201-218 and the later contribution by O. F. T. Roberts in Proc. Roy. Soc. of Edin., 1930, Vol. 50, p. 225 *et seq.*

water. We neglect the effect of scattered infra-red radiation. We consider only the temperature radiation from the layers of air near the ground and that of the heated surface of the earth.

In the shallow surface layer one may assume that the amount of water vapour and hence the effective absorption a due to it is uniform. If a is defined as the effective absorption coefficient its value is given by[1]

$$a = \int a_\lambda J_\lambda(\theta) d\lambda / \int J_\lambda(\theta) d\lambda \quad \ldots \quad (1)$$

where a_λ is the absorption for the wave-length λ and $J_\lambda(\theta)$ is the radiation of wave-length λ from a black body at temperature θ. θ varies with z which is the height above the ground.

Let us consider the radiation reaching the plane at z from a layer at x whose thickness is dx and temperature θ (see Fig. 3). By Kirchoff's law its value is $2aJ(\theta)e^{-a(z-x)} dx$ or

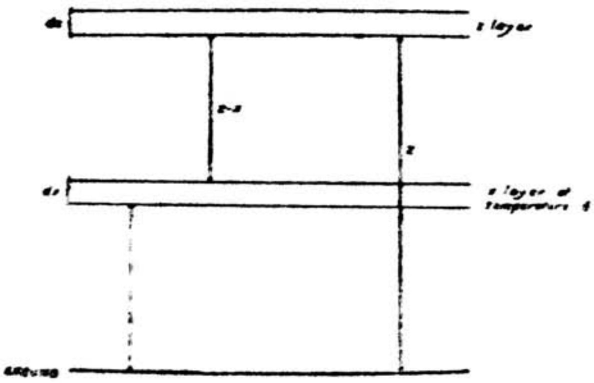


FIG. 3.

[1] The theory can be worked out in detail considering separately the absorption coefficients a_λ for different wave-lengths but the results will be sufficiently accurate and less cumbersome by the simpler method used above. The value of a in an atmosphere in which the water vapour pressure is 10 mm. of mercury is $2\cdot54 \times 10^{-4}$ per cm. for θ equal to 285° A.

$2aJ(\theta)\, e^{-a(x-z)}\, dx$ according as the x layer is below or above the z level.[6] The factor 2 is taken to account for the effect of diffuse radiation and the contribution from the layer at x is supposed to be vertical. The effect of all radiating layers below and above the z level are therefore respectively

$$2a\int_0^z J(\theta)e^{-a(z-x)}\, dx \text{ and } 2a\int_z^H J(\theta)e^{-a(x-z)}\, dx$$

where H is the maximum layer up to which we need to consider the effect of radiation. The radiation from the ground crossing the z plane is equal to $na'J(\theta_s)e^{-az}$ where θ_s is the temperature of the ground, a' is its absorption coefficient and 'n' is a numerical constant whose value will lie between 1 and π.

The layer at z of thickness dz will absorb some of the incident radiation. The amount is given by

$$\left\{2a\int_0^z J(\theta)e^{-a(z-x)}\, dx + 2a\int_z^H J(\theta)e^{-a(x-z)}\, dx + na'J(\theta_s)e^{-az}\right\}a.dz \qquad (2)$$

But the dz layer itself is radiating and the energy that leaves it on the top and bottom $= 4aJ(\theta)dz$. Hence the heat lost by the dz layer due to radiation is equal to

$$4aJ(\theta)dz - naa'J(\theta_s)e^{-az}.\, dz$$
$$-\left\{2a\int_0^z J(\theta)e^{-a(z-x)}\, dx + 2a\int_z^H J(\theta)e^{-a(x-z)}\, dx\right\}adz \qquad \ldots \qquad (3)$$

(2) As the temperature decreases with height conduction or convective forces come into play. To a first approximation

[6] It is easy to pass from the simpler theory to the more general one by replacement of $aJ(\theta)$ by $\Sigma a_\lambda J_\lambda(\theta)$ at any stage where Σ denotes summation over all wave-lengths.

the heat transfer across any layer can be assumed to be of the form $-k\frac{\partial\theta}{\partial z}$ where k is sensibly constant. The amount of heat that accumulates by this transfer in the layer dz

$$=k\frac{\partial^2\theta}{\partial z^2}.\ dz.$$

Owing to the changes in the caloric content in dz layer the temperature of air changes. Hence the equation of heat equilibrium can be written as

$$k\frac{\partial^2\theta}{\partial z^2}dz=\rho c_p\frac{\partial\theta}{\partial t}dz+4aJ(\theta)dz-naa'J(\theta_s)e^{-az}\,dz$$

$$-2a^2dz\left\{\int_0^z J(\theta)e^{-a(z-x)}\,dx+\int_z^H J(\theta)e^{-a(x-z)}\,dx\right\}$$

or

$$k\frac{\partial^2\theta}{\partial z^2}=\rho c_p\frac{\partial\theta}{\partial t}+4aJ(\theta)-naa'J(\theta_s)e^{-az}$$

$$-2a^2\left\{\int_0^z J(\theta)e^{-a(z-x)}\,dx+\int_z^H J(\theta)e^{-a(x-z)}\,dx\right\} \quad \dots (4)$$

where ρ is density and c_p the specific heat of the moist air at constant pressure.

From the above equation by an easy transformation the effect due to a slow uniform horizontal current of air can be deduced ; but for our purpose we take no horizontal movement of air.

The equation as given above looks complicated but for actual purposes we may simplify it considerably.

We may take temperatures under the steady conditions which exist at the time of maximum temperature of the day and so put $\frac{\partial\theta}{\partial t}=0$. Further the absolute temperature θ is of the order of 325°A while the variations of θ are of the order 15° to 20°. To study the small variations of temperature it is inconvenient to use the total value of θ. So we put $\theta=\theta_0+\phi$ where θ_0 is some large constant temperature (to be chosen later) of the order of 300°A and ϕ is the varying part. After substituting for θ we may neglect squares and higher powers of ϕ.

So we have

$$k\frac{\partial^2\phi}{\partial z^2}=4aJ'(\theta_o)\phi-naa'J(\theta_s)e^{-az}$$

$$+4aJ(\theta_o)e^{-a\,H/2}\cosh a(H/2-z)$$

$$-2a^2J'(\theta_o)\left[e^{-az}\int_o^z\phi e^{ax}\,dx+e^{az}\int_z^H\phi e^{-ax}\,dx\right] \qquad (5)$$

where $J'(\theta)$ represents $\frac{\partial J(\theta)}{\partial\theta}$.

The equation (5) is a linear differential equation of the second order except for the terms in the square brackets. We will show later that the contribution to the lapse-rate due to these terms is negligible. We solve the equation neglecting them.

As the temperature decreases with height and becomes sensibly constant after a certain height we may put the solution without loss of generality as

$$\phi=\phi_o\sinh\bar{a}\,(h-x)/\sinh\bar{a}h$$

$$+\left\{naa'J(\theta_s)e^{-az}-4aJ(\theta_o)e^{-a\frac{H}{2}}\cosh a\left(\frac{H}{2}-z\right)\right\}\Big/4aJ(\theta_o)-ka^2 \qquad (6)$$

if $z\leq h$; where $\bar{a}^2=4aJ'(\theta_o)/k$; h may be taken to be the height up to which temperature varies sensibly for our purpose. So returning to $\theta = \theta_o + \phi$ we have

$$\theta = \theta_o + \left\{ naa'J(\theta_s)e^{-a z} - 4aJ(\theta_o)e^{-a\frac{H}{2}} \cosh a\left(\frac{H}{2} - z\right) \right\} / 4aJ'(\theta_o) - ka^2$$

$$+\phi_o \sinh a(h-z)/\sinh ah \qquad \ldots \qquad \ldots \quad (7)$$

It may be mentioned that from actual calculations in our case 'a' was of the order of 10^{-4} in C. G. S. units for layers of the order of 'h' equal to 15 or 20 cms.; the contribution due to the second term on the right hand side in (7) for the lapse-rate is negligible, *i.e.*, the term acts like a constant. So, for values of z above h we may put

$$\theta = \theta_o + \left\{ \frac{naa'J(\theta_s)e^{-a z} - 4aJ(\theta_o)e^{-a\frac{H}{2}} \cosh a\left(\frac{H}{2} - z\right)}{4aJ'(\theta_o) - ka^2} \right\} \quad (8)$$

while for the variation of the temperature height curve for values of $z \leqq h$ with which we are concerned at present we may simply consider

$$\phi = \phi_o \sinh a(h-z)/\sinh ah \qquad \ldots \qquad \ldots \quad (9)$$

which becomes zero at $z = h$ and is equal to ϕ_o at $z = 0$. [7]

To express the above result in more physical terms, we find that, on putting the proper values in the complete equation, we obtain an approximate solution that is the same as the solution of the following simple differential equation

$$k\frac{\partial^2\theta}{\partial z^2} = 4aJ(\theta) = 4aJ'(\theta_o)\,\phi.$$

This tells us that, when we are considering the temperature lapse-rate within a fraction of a metre above ground, the two predominant terms determining the equilibrium between

[7] *Our recent experiments in the laboratory show that the surface turbulence sho[wn] roughly in Fig.* (1) starts from a level *slightly above* the hot surface. These results will [be] discussed later.

gain and loss of heat in each layer are the gain of heat by conduction or convection and the loss of heat by radiation. All other terms which comprise gain of heat by radiation from other air layers and from the ground can be neglected, because they contribute more or less equally to all layers and so do not affect the lapse-rate.

From an analysis of the data that we have obtained a is usually found to be about 0·25. From this value of a and known value of $a\,J'(\theta_0)$[*] the value of k can be estimated.

In Fig. 2, points indicated by circles show the theoretical distribution of temperature and the actual values are given by the crosses. The agreement is fairly close.

With the value obtained in (7) for ϕ

$$\left.\begin{aligned} \phi &= C_1 + \phi_0 \sinh \alpha(h-z)/\sinh \alpha h; \quad h > z \\ &= C_1 \; ; \qquad z > h \end{aligned}\right\} \quad \ldots (10)$$

where C_1 is sensibly constant we can calculate the effect of the neglected terms in (5) by substituting in the integrals the value of ϕ.

The constant term C_1 gives rise to an expression of form

$$A \cosh \alpha(H/2 - z)$$

whose variation with height is small, *i.e.*, this part may be considered to be an addition to the constant part of the temperature. The other part gives rise to

$$2a^2 J'(\theta_0)\, \phi_0 \,\{e^{-\alpha z} \sinh \alpha h - e^{-\alpha(h-z)} + O(a/\alpha)\}/\alpha \sinh \alpha h \quad \ldots \quad \ldots \quad (11)$$

where $O(a/\alpha)$ means terms of order a/α.

[*] $J'(\theta_0)$ may be taken for purposes of calculation as equal to $4\sigma\theta_0^3$ where σ is Stefan Boltzmann Constant and θ_0 is the absolute temperature.

The value of the term inside the brackets is finite and is of the same order as sinh $a(h-z)$ and so comparing with $4aJ'(\theta_0)\phi_z$

$$=4aJ'(\theta_0)\phi_0 \sinh \bar{a}(h-z)/\sinh \bar{a}h \quad \ldots (12)$$

The term (11), which is a/α times th) term (12), can be neglected in our calculations.

4. *Inferior Mirages.*

Inferior mirages are common in the tropics over level stretches of sand, as in the deserts of Africa and Arabia or over wide stretches of prairie wastes as in America.[9] Recently they have been observed on asphalted roads[10] in many modern cities and also on level stretches of sand near the sea-coast.[11] Some photographs of the phenomenon have been published in an interesting note by W. H. Steavenson.[12]

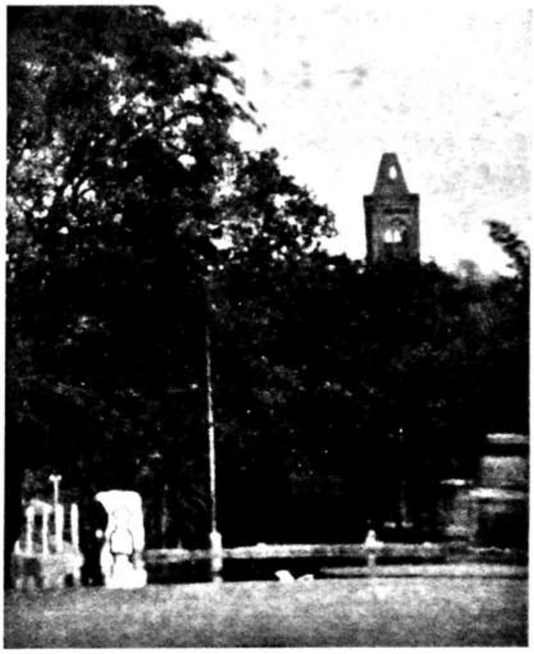
A photograph of the road mirage near the Meteorological Office, Poona, taken with a camera with telephoto attachment on the 30th May, 1931 is shown in Plate XIV. When the variation of temperature and therefore of the refractive index of the air is confined to a shallow transition layer near the ground (see Fig. 2), the rays of light from an object AM (see Fig. 4) will travel practically in straight lines till they reach the transition layer and then curve upward at some point C very close to the ground. Thus AC undergoes refraction at the surface layer, becomes horizontal at C and is later refracted

[9] In a recent interesting note "The passing of the mirage locally" A. A. Justice (Monthly Weather Review, October, 1930, p. 414) mentions that with the coming of extensive settlements over areas which once consisted prairies, the glorious scenes simulated by mirages are becoming rare.

[10] See (a) "Mirage on the Queensbury Road" by A. G. Ramage, Proc. Roy. Soc. of Edinburgh, Vol. 38, p. 166. (b) "Road Mirages" by T. W. Vernon Jones, Met. Mag. Dec., 1927, p. 261. (c) "Road Mirages" by W. H. Bigg, Met. Mag., July, 1928, p. 138.

[11] "Sand Mirages' by L. G. Vedy, Met. Mag., Dec., 1928, p. 249.

[12] Note on the Mirage, as observed in Egypt by W. H. Steavenson, Quarterly Journal, Roy. Met. Soc., London, Vol. XLVII, 1921, p. 15.



The arrow indicates the objects on the left whose mirage reflections can be seen. To the right of these is visible what appears to be a pool of water on the road.

upwards as CB. On placing the eye at B one can see A′ the mirage of A.

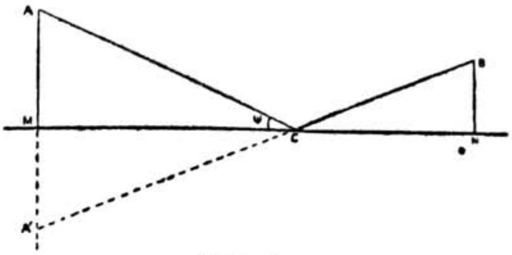


FIG. 4.

Then we have[13] $_A\mu_M$ the refractive index of air at M or C with respect to that at A, equal to $\sin(90-\psi)\ 1 \sin 90 = \cos\psi$. For a change of temperature $\triangle\theta$ in passing from C to A we have $\mu-1$ changing from 29×10^{-5} to $29\times10^{-5}\ (1-\Delta\theta/273)$ where μ is the refractive index of the air with respect to vacuum.

$$\text{So, } \triangle\theta = (1-\cos\psi)\times 273/29\times10^{-5}$$

$$= 273/29\times10^{-5}\times 1/2(\text{AM}+\text{BN})^2/\text{MN}^2$$

as ψ is small.

From actual observations on the road near the Meteorological Office, Poona, on 9th June, 1931, we found that when AM was 13″, BN was 12″ and MN, the distance between the object and the place from which its mirage was viewed, was 184 yards.

This gives

$$\triangle\theta = 6{\cdot}7°\text{C}$$

The observations of temperature made simultaneously show that $\triangle\theta$ was actually 6·7°C from ½″ above ground. We are therefore justified in assuming that the variation of μ is almost entirely due to variation of temperature.

[19] This method is fully worked out by L. G. Vedy, *loc. cit.*

There are a few other points that may be emphasised here. We find, as some others have found that the phenomena of road mirage persisted even when (1) moderate to strong breeze was blowing, (2) when the surface air was disturbed by frequent traffic, (3) when the skies clear to start with began to get cloudy later. Previous writers like Vernon Jones have emphasized the difficulty of explaining the persistence of the mirage even when the surface air is disturbed. It may be pointed out that as shown by us in section 3 the temperature distribution near the ground depends on conduction as well as radiation processes. Any stirring up of the air will not materially alter these processes since radiation will remain practically undisturbed and the surface turbulence also will not be materially altered; all that forced convection, whether due to prevailing wind or to passage of traffic, does is to cause a more or less rapid horizontal drift of the pattern shown in Fig. 1. Experimental studies on these and related phenomena will be discussed in a later paper.

In conclusion, we have great pleasure in expressing our best thanks to Dr. C. W. B. Normand, Director-General of Observatories and to Dr. K. R. Ramanathan for their helpful criticisms while the above note was being written.