Indian J. Phys. **83** (3), 365-374 (2009)

Large amplitude double layers in a four component dusty plasma with non-thermal ions

Gurudas Mandal[1], Kaushik Roy[2] and Prasanta Chatterjee[2*]

[1]Department of APCE, East West University, Mohakhali, Dhaka
Bangladesh
[2]Department of Mathematics, Siksha Bhavana, Visva Bharati,
Santiniketan-731 235, West Bengal, India

E-mail : prasantachatterjee1@rediffmail.com

Received 16 September 2008, accepted 9 January 2009

Abstract : Dust acoustic double layers are studied in a four component dusty plasma. Positively and negatively charged mobile dust and Boltzmann distributed electrons are considered. The ion distribution is taken as nonthermal. The existence of compressive and rarefractive double layers is studied by pseudopotential approach. The effect of non-thermal ions on small amplitude and arbitrary amplitude double layers are also studied.

Keywords : Soliton; pseudo-potential; double-layers; non-thermal ions

PACS Nos. : 52.35.Sb, 52.35.We, 52.35.Fp

1. Introduction

Dust and plasmas are present together in the universe and they make dusty plasmas. Dusty plasmas are found in cometary tails, asteroid zones, planetary ring, interstellar medium, lower part of earth's ionosphere and magnetosphere [1-8]. These dusty plasmas play significant role in space plasma, astrophysical plasma, laboratory plasma and environment. It also plays an important role in low temperature Physics, radio frequency plasma discharge [9], coating and etching of thin films [10], plasma crystals [11] *etc.* Therefore, this subject has become more important to the investigators. Nonlinear wave phenomena like soliton, shocks and vortices in dusty plasmas have also been studied by several investigators for the last two decades [12-22] or so. For the recent discovery of dust acoustic wave (DAW) [15,16], dust ion-acoustic wave (DIAW) [17,18] and dust lattice

* Corresponding Author

(DL) wave [19,20] makes this dusty plasma research more important. However, most of the investigations are done by considering three component dusty plasma systems consisting of ions, electrons and negatively charged dust particles [27-29]. Recently, it has been suggested that positively and negatively charged dust grains can co-exist in space [23-25] and in laboratory [26] plasmas. Therefore, it is desirable to investigate the nonlinear properties of dust-acoustic waves in a four component plasma that consists of electrons, ions, and positively and negatively charged dust grains. Dust-acoustic solitary waves in the one dimension and unmagnetized plasma have also been investigated by several investigators. Recently, Sayed and Mamun [30] investigated solitary waves where they considered four components. To get the solitary wave solution they used Reductive Perturbation Technique (RPT). But Malfliet and Wieers [31] found that RPT is useful only to explain small amplitude soliton waves. Therefore, to study large amplitude solitary waves one should employ a non-perturbative technique. Sagdeev's [32] pseudo-potential method is one such method to obtain solitary wave solution. Using this method Sakanaka and Shukla [33], and Sakanaka and Iglika [34] studied Dust acoustic double layers (DADLs) in presence of charged dust grains. In their model they considered Boltzmann distributed ions. But from recent observations [35-37] it has been found that ion distribution does not follow Boltzmann distribution and in these cases the non-thermal distribution of ions is suggested. The Vela [38] satellite observed non-thermal ions from earh's bow-shock, *Phobos 2* [39] satellite observed the loss of energetic ions from the upper ionosphere of the Mars and *Nozami* [40] satellite observed very large velocity protons near the earth in the vicinity of the moon. Also Lundlin *et al* [39] have shown that for the planet having in not so strong magnetic field, the solar wind impacting with the planetary atmosphere results in non-thermal ion flux.

In this paper, we consider a four component unmagnetized dusty plasma system consisting of Boltzmann distributed electrons, non-thermal distributed ions and also positively (smaller size) and negatively (larger size) charged dust grains. Sagdeev's pseudo-potential technique is applied to study the existence of double layer. Small amplitude double layers are also studied by considering the expansion of pseudo-potential.

2. Basic equations and pseudo-potential approach

The dynamics of the charged dust grains are governed by the following continuity and momentum equations, which are, respectively,

$$\frac{\partial N_p}{\partial t} + \frac{\partial (N_p V_p)}{\partial x} = 0 , \tag{1}$$

$$\frac{\partial V_p}{\partial t} + V_p \frac{\partial V_p}{\partial x} = -\frac{Z_p e}{M_p} \frac{\partial \Phi}{\partial x} \tag{2}$$

for positively charged dust grain, and

$$\frac{\partial N_n}{\partial t}+\frac{\partial (N_n V_n)}{\partial x}=0\,, \tag{3}$$

$$\frac{\partial V_n}{\partial t}+V_n\frac{\partial V_n}{\partial x}=\frac{Z_n e}{M_n}\frac{\partial \Phi}{\partial x} \tag{4}$$

for negatively charged dust grain.

The system of equations is closed with the Poission's equation

$$\frac{\partial^2 \Phi}{\partial x^2}=4\pi e\left(N_e - N_i + Z_n N_n - Z_p N_p\right) \tag{5}$$

where

$$N_e = N_{e0} e^{\frac{e\Phi}{Te}}\,, \tag{6}$$

$$N_i = N_{i0}\left(1+\beta\frac{e\Phi}{T_i}+\beta\left(\frac{e\Phi}{T_i}\right)^2\right)e^{\frac{-e\Phi}{Ti}}\,. \tag{7}$$

Here V_p, V_n, M_p, M_n are the fluid velocities and mass of the positively and negatively charged dust grains, respectively. Φ is the electrostatic potential and e is the magnitude of the electron charge. The quasi-neutrality at equilibrium is writen as N_{e0} + $Z_n N_{n0}$ = N_{i0} + $Z_p N_{p0}$. Here, N_{e0} and N_{i0} are the electrons and ions number density, Z_n and Z_p are the negative and positive dust particle charge, N_{n0} and N_{p0} are the dust particle number density, respectively. The dust particle are assumed to point charges and their sizes are much smaller than the effective Debye length. With the purpose of understanding the parametric space which limits the existence of DA solitons and double layers, we normalize all the parameters. Time and space are normalized by and λ_{Dd}. Velocities (V_n and V_p) are normalized by C_{da}. N_i, N_e, N_p and N_n are normalized by N_0. And Φ is normalized by . Where, $C_{da}=\sqrt{T_0/M_0}$, $\lambda_{Dd}=C_{da}/\omega_{pd}$, $\omega_{pd}^2=\left(4\pi e^2 N_0\right)/M_0$. Here C_{da} is the dust acoustic velocity, the effective Deybe length, ω_{pd} the dust plasma frequency, N_0, M_0 and T_0 are effective number density, the mass and the temperature, respectively. After normalizing all relevent parameters, eqs. (1) to (5) can be written as

$$\frac{\partial n_p}{\partial t}+\frac{\partial (n_p v_p)}{\partial x}=0\,, \tag{8}$$

$$\frac{\partial v_p}{\partial t}+v_p\frac{\partial v_p}{\partial x}=-\frac{Z_p M_0}{M_p}\frac{\partial \phi}{\partial x}\,, \tag{9}$$

$$\frac{\partial n_n}{\partial t}+\frac{\partial(n_n v_n)}{\partial x}=0\,, \tag{10}$$

$$\frac{\partial v_n}{\partial t}+v_n\frac{\partial v_n}{\partial x}=\frac{Z_n M_0}{M_n}\frac{\partial\phi}{\partial x}\,, \tag{11}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_e-n_i+Z_n n_n-Z_p n_p\,. \tag{12}$$

Eqs. (6) and (7) also can be written as

$$n_e=n_{e0}e^{a_e\phi} \tag{13}$$

$$n_i=n_{i0}\left[1+\beta a_i\phi+\beta a_i^2\phi^2\right]e^{-a_i\phi}\,. \tag{14}$$

In order to search for double layers, we introduced a linear substitution $\xi = x - Mt$ admitting only solution which depends on space and time in the form of wavy variable $x - Mt$. By substituting $(\partial/\partial x)=(d/d\xi)$ and $(\partial/\partial t)=-M(d/d\xi)$. From eqs. (8) and (9), with the conditions $\xi\to\infty$, $n_p\to n_{p0}$ and $V_p\to 0$. We obtain

$$n_p=\frac{n_{p0}}{\sqrt{1-2a_p\phi}} \tag{15}$$

where, $a_p=\dfrac{Z_p M_0}{M_p M^2}$.

Similarly, from equations (10) and (11) we obtain

$$n_n=\frac{n_{n0}}{\sqrt{1+2a_n\phi}} \tag{16}$$

where, $a_n=\dfrac{Z_n M_0}{M_n M^2}$.

Now, eq. (12) can be written

$$\frac{d^2\phi}{d\xi^2}=n_{e0}e^{a_e\phi}-n_{i0}\left(1+\beta a_i\phi+\beta a_i^2\phi^2\right)e^{-a_i\phi}+\frac{Z_n n_{n0}}{\sqrt{1+2a_n\phi}}-\frac{Z_p n_{p0}}{\sqrt{1-2a_p\phi}} \tag{17}$$

It is convenient to obtain the so-called Sagdeev potential, such that

$$\frac{d^2\phi}{d\xi^2} = -\frac{d\psi}{d\phi} \tag{18}$$

where

$$\psi(\phi) = \frac{n_{e0}}{a_e}\left(1 - e^{a_e\phi}\right) + \frac{n_{i0}}{a_i}(1 + 3\beta)\left(1 - e^{-a_i\phi}\right) - 3\beta n_{i0}\phi e^{-a_i\phi}$$

$$-\beta n_{i0}a_i\phi^2 e^{-a_i\phi} + \frac{Z_n n_{n0}}{a_n}\left(1 - \sqrt{1 + 2a_n\phi}\right) + \frac{Z_p n_{p0}}{a_p}\left(1 - \sqrt{1 - 2a_p\phi}\right). \tag{19}$$

Multiplying both sides of eq. (18) by $d\phi/d\xi$ and integrating w.r.t. with the boundary conditions and $(d\phi/d\xi) \to 0$ we get

$$\psi(\phi) + \frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 = 0\,. \tag{20}$$

Eq. (20) can be considered as a motion of a particle (whose pseudo-potential is ψ at
$\xi\phi\phi\psi\xi \to \infty$, $\psi \to 0$ pseudo-time) with pseudo-velocity ( ) in a pseudo-potential well . That is why Sagdeev's potential is called pseudo-potential. Hence the conditions for the existence of double layers solution are

(i) $\psi(\phi) = 0$ at $\phi = 0$ and $\psi(\phi)$ has a double root at $\phi = 0$. Moreover $\psi(\phi)$ has local maximum at $\phi = 0$, so the condition is $\psi(\phi) = 0$, $d\psi/d\phi = 0$ and $d^2\psi/d\phi^2 < 0$ at $\phi = 0$.

(ii) $\psi(\phi_m) = 0$ for some $\phi_m \neq 0$ and $\psi(\phi)$ has a double root at $\phi = \phi_m$. Moreover $\psi(\phi)$ has local maximum at $\phi = 0$, so the condition is $\psi(\phi) = 0$, $d\psi/d\phi = 0$ and $d^2\psi/d\phi^2 < 0$ at $\phi = \phi_m$.

(iii) If ϕ_m is positive the double layer is called compressive double layer and if ϕ_m is negative the double layer is called rarefractive double layer.

(iv) $\psi(\phi)$ is negative in the interval (0, ϕ_m).

3. Small amplitude approximation

To obtain small amplitude approximation to double layer structure let us expand $\psi(\phi)$ (about $\phi = 0$) up to ϕ^4 and get

$$\psi(\phi) = A\phi^2 + B\phi^3 + C\phi^4 \tag{21}$$

where

$$A = \frac{1}{2}\left[n_{i0}a_i(\beta - 1) - n_{e0}a_e + z_n n_{n0}a_n + z_p n_{p0}a_p\right]$$

$$B = \frac{1}{6}\left[n_{i0}a_i^2 - n_{e0}a_e^2 - 3z_n n_{n0}a_n^2 + 3z_p n_{p0}a_p^2\right]$$

$$C = \frac{1}{24}\left[-n_{i0}a_i^3 - n_{e0}a_e^3 + 15z_n n_{n0}a_n^3 + 15z_p n_{p0}a_p^3\right].$$

Hence from Eq. (18) we get

$$\frac{\partial^2\phi}{\partial\xi^2} = A_1\phi + A_2\phi^2 + A_3\phi^3 \tag{22}$$

where $A_1 = -2A$; $A_2 = -3B$; $A_3 = -4C$.

Solution of the eq. (22) is

$$\phi = \left[-\frac{A_2}{3A_1} - \sqrt{\frac{A_2^2}{9A_1^2} - \frac{A_3}{2A_1}}\cosh\left(\sqrt{A_1}\xi\right)\right]^{-1} \tag{23}$$

is the modified KdV Soliton provided $A_1 > 0$. Again if $A_2^2 = 9/2A_1A_3$, the above soliton solution would not exist. In that case a shock wave solution is obtained from eq. (23) which is given by

$$\phi = -\frac{3A_1}{2A_2}\left[1 + \tanh\alpha\left(\xi + \xi_0\right)\right] \tag{24}$$

where ξ_0 is the integration constant and $\alpha = -A_2\big/\left(3\sqrt{2A_3}\right)$. Eq. (24) is the small amplitude double layer solution.

4. Result and discussions

Figure 1 shows that the vs ϕ for different values of *viz.* and 0.102. Other parameters are n_{i0} = 0.405, n_{e0} = 0.4534, n_n = 1.48, n_p = 1.4788, a_n = 0.2, a_p = 0.02, a_i = 1.995, a_e = 0.1. It is clearly seen from this figure that for β = .102, Sagdeev's pseudo-potential satisfies the conditions (i) to (iv) for existence of double layer and hence double layer exists for β = 0.102. But for = 0.0 the condition (i) is satisfied but the condition (ii) is not satisfied and hence for = 0.0 double layer does not exist for this set of parameters. Hence the presence of , non-thermal parameter ensures the existence of double layer for this set of parameters. Since , the maximum value of ϕ is positive so the double layer is called Compressive double layer. Figure 2 shows that the vs ϕ plots for different values of *viz.* = 0.0 and 0.1. Other parameters

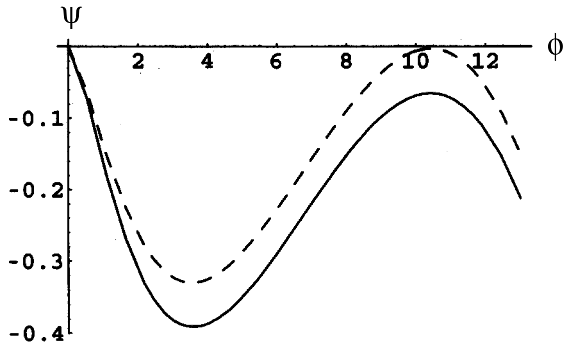

Figure 1. Plot of vs ϕ for β = 0.102 and 0.0 and other parameters are n_{i0} = 0.405, n_{e0} = 0.4534, n_n = 1.48, n_p = 1.4788, a_n = 0.2, a_p = 0.02, a_i = 1.995, a_e = 0.1.

are n_{i0}= 0.0105, n_{e0} = 0.99, $^n{}_n$ = 0.712, n_p = 1.6917, a_n = 0.9999, a_p = 0.1, a_i = 10.4, a_e = 0.9. It is clearly seen that for = 0.0, Sagdeev's pseudo-potential satisfies the conditions (i) to (iv) and hence double layer exists for β = 0.0. For = 0.01 the condition (i) is satisfied but the condition (ii) is not satisfied and hence for = 0.01 double layer do not exists. Since , the maximum absolute value of ϕ is negative so the double layer is called rarefractive double layer. In this case it is seen that for this set of parameters the double layer do not exists for > 0. Figure 3(a) shows that the small approximation vs ϕ for the values of = 0.5566. Other parameters are n_{i0} = 0.405, n_{e0} = 0.4534, n_n = 1.48, n_p = 1.4788, a_n = 0.2, a_p = 0.02, a_i = 1.995, a_e = 0.1, and Figure 3(b) shows that the small amplitude approximation vs ϕ for the values of = 0.65. Other parameters are n_{i0} = 0.405, n_{e0} = 0.4534, n_n = 1.48, n_p = 1.4788, a_n = 0.2, a_p = 0.02, a_i = 2.76, a_e = 0.1. From figure 3(a) and 3(b) increases the value of and a_i,

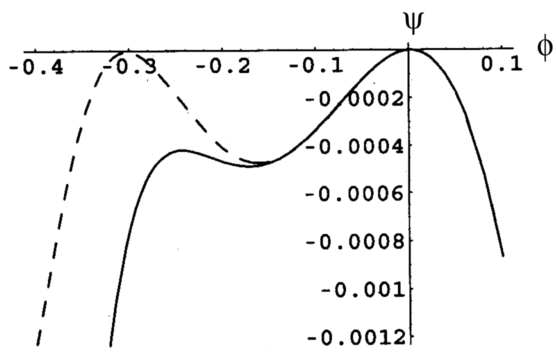

Figure 2. Plot of $\psi(\phi)$ vs ϕ for β = 0.0 and 0.01 and other parameters are n_{i0}= 0.0105, n_{e0} = 0.99, n_n = 0.712, n_p = 1.6917, a_n = 0.9999, a_i = 10.4, a_e = 0.9.

decreases the amplitude of the double layer. Figure 4(a) shows that the small amplitude approximation $\psi(\phi)$ *vs* ϕ for the values of = 0.9233. Other parameters are n_{i0} = 0.0105, n_{e0} = 0.99, n_n = 0.712, n_p = 1.6917, a_n = 0.9999, a_p = 0.1, a_i = 10.4, a_e = 0.9 and Figure 4(b) shows that the small amplitude approximation *vs* ϕ for the values

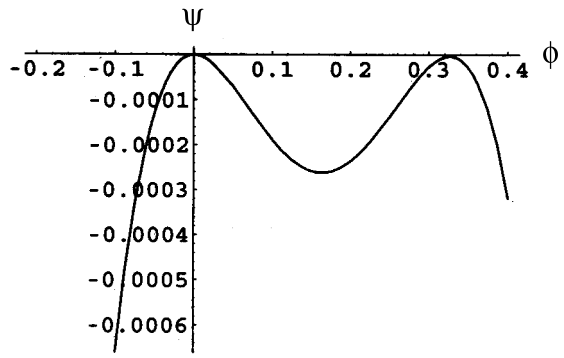

Figure 3(a). Plot of vs ϕ for β = 0.5566 and other parameters are n_{i0} = 0.405, n_{e0} = 0.4534, n_n = 1.48, n_p = 1.4788, a_n = 0.2, a_p = 0.02, a_i = 1.995, a_e = 0.1.

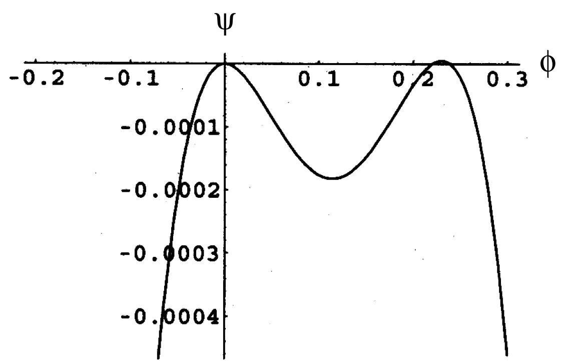

Figure 3(b). Plot of $\psi(\phi)$ vs ϕ for β = 0.65 and other parameters are n_{i0} = 0.405, n_{e0} = 0.4534, n_n = 1.48, n_p = 1.4788, a_n = 0.2, a_p = 0.02, a_i = 2.76, a_e = 0.1.

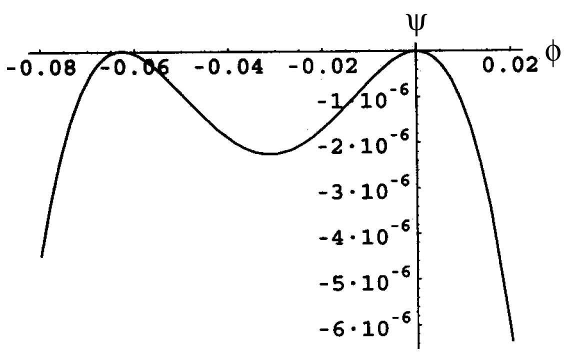

Figure 4(a). Plot of $\psi(\phi)$ *vs* ϕ for β = 0.9233 and other parameters are n_{i0} = 0.0105, n_{e0} = 0.99, n_n = 0.712, n_p = 1.6917, a_n = 0.9999, a_i = 10.4, a_e = 0.9.

of β = 0.1. Other parameters are n_{i0} = 0.0105, n_{e0} = 0.99, n_n = 0.712, n_p = 1.6917, a_n = 0.9999, a_p = 0.1, a_i = 8.10, a_e = 0.9. From figure 4(a) and 4(b) increases the value of (in this case decreases the value of a_i) shows that the amplitude of the double layer increases.

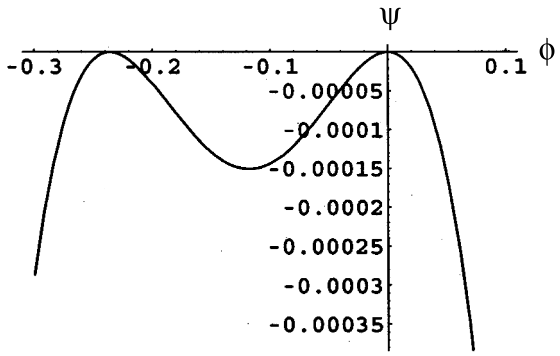

Figure 4(b). Plot of vs ϕ for β = 0.1 and other parameters are n_{i0} = 0.0105, n_{e0} = 0.99, n_n = 0.712, n_p = 1.6917, a_n = 0.9999, a_i = 8.1, a_e = 0.9.

5. Conclusions

Large amplitude compressive and rarefractive double layer is studied in four component dusty plasma. The ions are considered to be non-thermal. Small amplitude double layers are also studied by considering the expansion of pseudo-potential. It is seen that β, the non-thermal parameter which has a significant effect on the existence of both large and small amplitude of the compressive double layer is large, so this type of problem should be treated by a non-perturbative approach like Sagdeev's pseudo-potential approach. We can and stress that the results of the present investigation should lead to laboratory experiments which deal with the demonstration of DAW waves in a four component dusty plasma with both the positive and negative grains. Moreover, our parametric studies of double layers should be useful in identifying coherent nonlinear structures in the Earth's mesosphere when data from the forthcoming rocket missions are available. In addition non-stationary double layers could be potential accelerators for dust particles in space plasmas (see ref. [33]). This work can be extended to study the double layers in four compnent dusty plasma in presence non-thermal electrons or vortex like ions or vortex like electrons. Work is progress in this direction.

Acknowledgment

This work is supported by UGC under SAP (DRS) programme. Authors are grateful to the referee for his suggestions which helped to improve the paper.

References

[1] M Horanyi and D A Mendis *J. Astrophysics* **294** 357 (1985)

[2] P K Shukla, D A Mendis and T Desai *Advances in Dusty Plasmas* (World Scientific : Singapore) (1997)

[3] C K Goertz *Rev. Geophysics* **27** 271 (1989)

[4] P K Shukla and A A Mamun *Introduction to Dusty Plasma Physics* (Institute of Physics : Bristol) (2002)

[5] A Bouchule *Dusty Plasmas* (Wiley : New York) (1999)

[6] F Verheest *Waves in Dusty Space Plasmas* (Kluwer) (2000)

[7] D A Mendis and M Rosenberg *Annual Rev. Astronomy and Astrophysics* **32** 419 (1994)

[8] F Verheest *Space Sci. Rev.* **77** 267 (1996)

[9] J H Chu, J B Du and I Lin *J. Phys.* **D 27** 296 (1994)

[10] G S Selwyn *Jpn. J. Applied Phys. Part 1* **32** 3068 (1993)

[11] H Thomas, G E Morfill and V Dammel *Phys. Rev. Lett.* **73** 652 (1994)

[12] D P Sheehan, M Carilo and W Heidbrink *Rev. Scient. Instrum.* **61** 3871 (1990)

[13] R N Carlile, S Geha, J F O'Hanlon and J C Stewart *Appl. Phys. Lett.* **59** 1167 (1991)

[14] P V Bliokh and V V Yarashenko *Sov. Astron.* **29** 330 (1985)

[15] N N Rao, P K Shukla and M Y Yu *Planet Space Sci.* **38** 543 (1990)

[16] A Barkan, R L Merlino and N D'Angelo *Phys. Plasmas* **2** 3563 (1995)

[17] P K Shukla and V P Silin *Phys. Scripta,* **45** 508 (1992)

[18] R L Merlino, A Barkan, C Thompson and N D'Angelo *Phys. Plasmas* **5** 1607 (1998)

[19] F Melandso *Phys. Plasmas* **3** 3890 (1996)

[20] B Farokhi, P K Shukla, N L Tsindsadze and D D Tskhakaya *Phys. Lett.* **A264** 318 (1999)

[21] P K Shukla and R K Verma *Phys. Fluids* **B5** 236 (1993)

[22] P K Shukla, M Y Yu and R Bharuthram *J. Geophys. Res.* **96** 21343 (1992)

[23] Y Nakamura and I Tsukabayashi *Phys. Rev. Lett.* **52** 2356 (1984)

[24] S Watanabe *J. Phys. Soc. Japan* **53** 950 (1984)

[25] T E Sheridan *J. Plasma Phys.* **60** 17 (1998)

[26] P K Shukla *Phys. Plasmas* **1** 1362 (1994)

[27] S Mahmood and H Saleem *Phys. Plasmas* **10** 47 (2003)

[28] S Maitra and R Roychoudhury *Phys. Plasmas* **10** 1 (2003)

[29] P Chatterjee and R K Jana *J. Naturforschung* ***60*a** 275 (2005)

[30] F Sayed and A A Mamum *Phys. Plasmas* **14** 014501 (2007)

[31] W Malfliet and E Wieers *J. Plasma Phys.* **56** 441 (1996)

[32] R Z Sagdeev in *Rev. of Pl. Phys.* Edited by M A Leontovich (Consultants Bureau : New York) **Vol. 4** 23 (1966)

[33] P H Sakanaka and P K Shukla *Phys. Scr.* **T84** 181 (2000)

[34] P H Sakanaka and I Spassovska *Brazilian J. Phys.* **33** 1 (2003)

[35] J R Asbridge, S J Barne and I B Strong *J. Geophys. Res.* **73** 5777 (1968)

[36] R Lundlin, A Zakharov and R Pelinenn *et al. Nature* (London) **341** 609 (1989)

[37] Y Futana, S Machida and Y Saito *et al J. Geophys. Res.* **108** 151 (2003); *ibid* 406 (1999)

[38] J R Asbridge, S J Barne and I B Strong *J. Geophys. Res.* **73** 5777 (1968)

[39] R Lundlin, A Zakharov and R Pelinenn *et al, Nature (London)* **341** 609 (1989)

[40] Y Futana, S Machida and Y Saito *et al, J. Geophys. Res.* **108** 151 (2003)