Steady-state rarefaction waves in magnetized flows and their application to gamma-ray burst outflows

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Abstract: We investigate the characteristics of a relativistic magnetized fluid flowing around a corner. If the flow is faster than the fast-magnetosonic speed the non-smooth boundary induces a rarefaction wave propagating in the body of the flow. The subsequent expansion is accompanied with a very efficient increase of the flow speed and bulk Lorentz factor γ. We apply this "rarefaction acceleration mechanism" to the collapsar model of gamma-ray bursts, in which a relativistic jet initially propagates in the interior of the progenitor star, before crossing the stellar surface with a simultaneous drop in the external pressure support. We integrate the steady-state equations using a special set of partial (r self-similar) solutions. The use of these solutions degrades the system of the complex, non-linear, 2nd order partial differential equations into a system of two 1st order ordinary differential equations whose integration is straightforward. For the conditions expected in a gamma-ray burst, a fully analytical solution can be obtained. The aim of this work is to better understand the results of recent time-depended numerical simulations and show that rarefaction acceleration is a plausible mechanism in gamma-ray burst outflows.

1   Introduction

If an outflow passes from an acute angle and its velocity exceeds the velocity of the fastest propagating disturbances the information of the non-smooth boundary propagates in the body of the flow forming a weak discontinuity, ie a discontinuity on the derivatives of the quantities describing the flow rather than on the quantities themselves. This discontinuity is called rarefaction wave and in general three regions are formed: i) unperturbed flow, ii) perturbed - rarefied region, iii) void space (see Fig. 1) Rarefaction is a common phenomenon from classical (see for example [1] for the classical hydrodynamic case) to the most relativistic and magnetized flows; see also the numerical simulations by [2],[3].

In the context of GRBs rarefaction acceleration was proposed by [3] as a mechanism producing jets with half-opening angles significantly larger that $1/\gamma$, allowing the appearance of achromatic breaks in the afterglow light curves. Especially in the collapsar model rarefaction is a plausible scenario, since the outflow is partly accelerated up to superfast magnetosonic velocities in the interior of the progenitor star and crosses the surface of the star to the zero pressure area (void space).

2   Model and Results

We solve the steady-state, relativistic, ideal MHD equations for a polytropic plasma, assuming planar symmetry $\partial/\partial y = 0$ in the y-direction (see Fig. 1), flow in the $x - y$ plane, and magnetic field along y. We consider solutions in which all quantities have a dependence $r^{F_i} f(\theta)$ in polar coordinates on the $x - y$ plane, and determine the various exponents F_i in order for the equations to be separable and under which the system is degraded in a system of two ordinary differential equation. We refer the interested reader to the full analysis in [4].

Below we present the results for two cases (see Fig. 1):
I. A cold flow with energy to mass flux ratio $\mu = 600$, initially splitted into Poynting and matter parts with ratio $\sigma_i = 4$ (thus the bulk Lorentz factor is $\gamma_i = \mu/(1 + \sigma_i) = 120$).
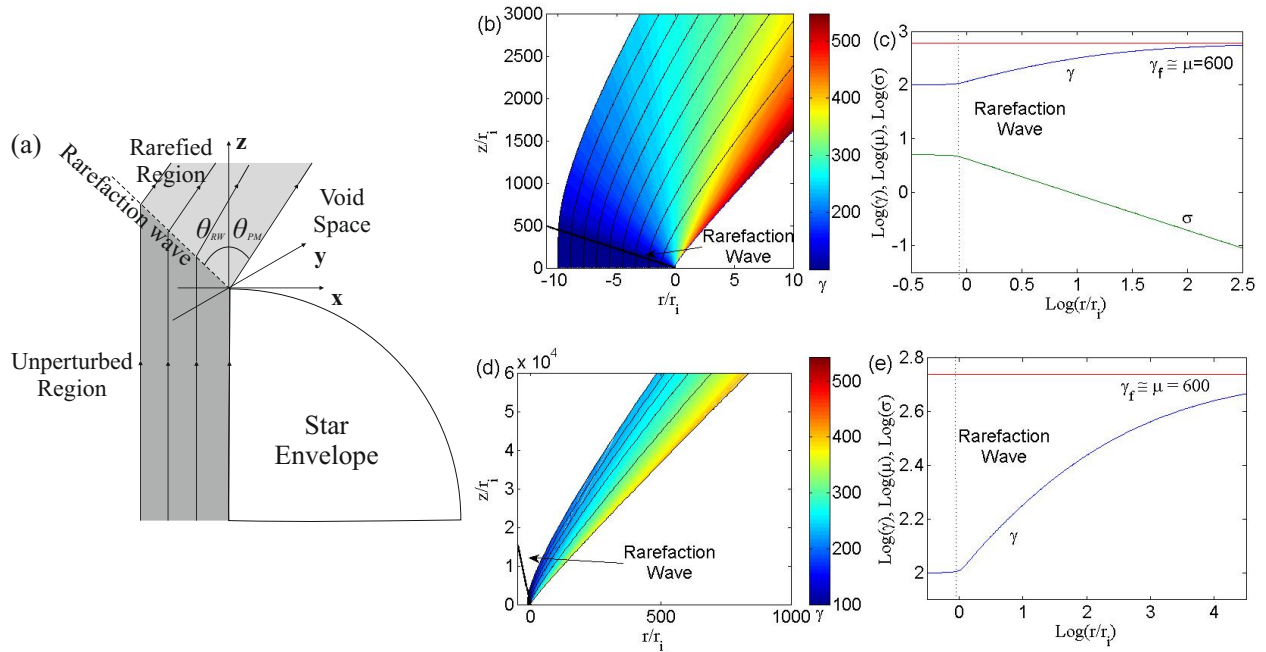
Figure 1: Model geometry (panel a) and results for the cold (panels b,c) and unmagnetized (panels d,e) cases. Panels b and d show the streamlines and γ. Panels c and e show the acceleration (increasing γ).

II. An unmagnetized warm flow with energy to mass flux ratio $\mu = 600$, initially splitted into enthalpy and matter parts with ratio $h_i - 1 = 4$ (thus the bulk Lorentz factor is $\gamma_i = \mu/h_i = 120$).

In case I the acceleration is very efficient reaching the maximum values ($\gamma \approx \mu$) in relatively short distances: in each fluid parcel, 90% of the final value is completed in $10^4 r_i$, where r_i the distance from the corner when that parcel crosses the $z = 0$ plane. The rarefaction starts at a distance $z \approx 50 r_i$, corresponding to the envelope surface of all fast magnetosonic waves emitted from the point of the non-smooth boundary. In the case of a cold, ultrarelativistic flow the angle of the head of the rarefaction wave is given by the expression $\theta_{RW} = \sqrt{\sigma}/\gamma_i$.

These results can be compared to the unmagnetized case II. Acceleration is still very efficient with 90% percent of the final value ($\gamma = \mu$) obtained at a distance of $10^7 r_i$, while rarefaction wave occurs at $z \approx 300 r_i$.

From the results presented we see that rarefaction is a plausible mechanism for the GRB acceleration. Though this is the case in both (cold or unmagnetized) situations, in cold flows the acceleration is completed in much shorter distances.

References

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