

Nonlinear slow light propagation in photonic crystal slab waveguides: theory and practical issues

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ABSTRACT

In this paper, we consider the propagation of slow light optical pulses inside photonic crystal slab waveguides (PCSW) both from a theoretical and an application point-of-view. The numerical model used relies on a nonlinear envelope propagation equation that includes the effects of second and third order dispersion, optical losses and self phase modulation. Pulse propagation is examined both in the linear and nonlinear regime. It is numerically shown that for rates of 10Gb/s, the order of nanosecond delays can be achieved through the PCSW defect modes without excessive pulse broadening in the nonlinear regime. In the nonlinear case, it is shown that soliton pulses exhibit less broadening than pulses in the linear case. In comparing the linear and the non-linear case we consider launching pulses with the same initial full width at half maximum or the same RMS width. The influence of optical losses on the soliton pulse broadening factor is also incorporated and discussed providing a more practical perspective. The results demonstrate the potential of implementing a variety of linear and nonlinear signal processing applications in PCSWs, such as optical buffering.

Keywords: Photonic crystals, delay lines, optical waveguides, optical solitons

1. INTRODUCTION

All optical signal processing techniques may constitute a remedy for optoelectronic bottleneck encountered in modern optical networks [1]. However, several functionalities including optical buffering are very hard to implement in integrated form, necessitating the use of bulky components such as fiber delay lines [2]. Recent studies are supporting the increased attention to nanophotonic technologies, posing the idea of an all-optical buffering subsystem, using Photonic Crystals (PCs) [3]. PCs provide new means of manipulating light in a sub-wavelength scale opening up new paths in integrated optics design [4], [5]. PCs are formed by periodically modulating the refractive index of a material. Under certain conditions PC may exhibit photonic band gaps, i.e. frequency regions where no photons are allowed to propagate. By intentionally introducing a symmetry breaking, like a line defect, allows the formation of waveguides, which can guide light through sharp bends [6]. In these structures, propagation is supported by defect modes that appear inside the photonic band gap [3], [7] of the bulk crystal. This situation is unlike the index guiding mechanism which governs light propagation in conventional dielectric waveguides such as optical fibers [8]. PC slabs possess most of the optical properties of photonic crystals [6] and can be easily realized using standard lithographic techniques. PC slabs are periodic dielectric structures along two directions (x and y) and use index guiding to confine light along the vertical direction (z). Although there is a variety of line defects, that can be introduced inside the bulk PC, (Fig. 1a), most of the scientific attention is focusing on studying W1 photonic crystal slab waveguides (PCSW) (Fig. 1b), which are taken by simply removing a single row of holes in the bulk crystal. This is mostly due to the belief that these structures (Fig. 1a) sustain severe propagation losses. However some works seem to indicate that under certain design conditions such structures [9], [10] exhibit less propagation losses than the corresponding W1 waveguides. Propagation losses, under the light line, are primarily due to backscattering induced by fabrication imperfections. These losses can be combated by either improving the fabrication accuracy or applying some form of optical amplification.

Another fundamental limitation of PCSWs is the broadening of optical pulses due to the larger group velocity dispersion experienced with increasing group index, which is a result of the flattening of the guided mode's dispersion curve

$\beta=\beta(\omega)$. Given a PCSW of length L , it is therefore not enough to simply calculate the achieved delay $T_d=L/v_g$ but one needs to consider both the level of pulse broadening and the amount of propagation loss. At higher power levels, nonlinear phenomena can also affect signal propagation which may turn out quite different than in the linear regime. For instance, when soliton pulses [11] propagate inside a Kerr nonlinear media, self phase modulation (SPM) can balance the GVD-induced broadening, in the absence of optical losses and higher order dispersion. The properties of both linear and nonlinear pulse propagation have been previously analyzed and compared in purely 2D PhC waveguides [12] and coupled resonator optical waveguides [13]. In this work, we compare the use of W1 PCSWs as delay lines in the linear and nonlinear regime, taking into account the full 3D nature of the structure. Propagation losses as well as pulse broadening are taken into account. We will show that under certain conditions nonlinear soliton propagation is better suited for slow light propagation in this type of structures.

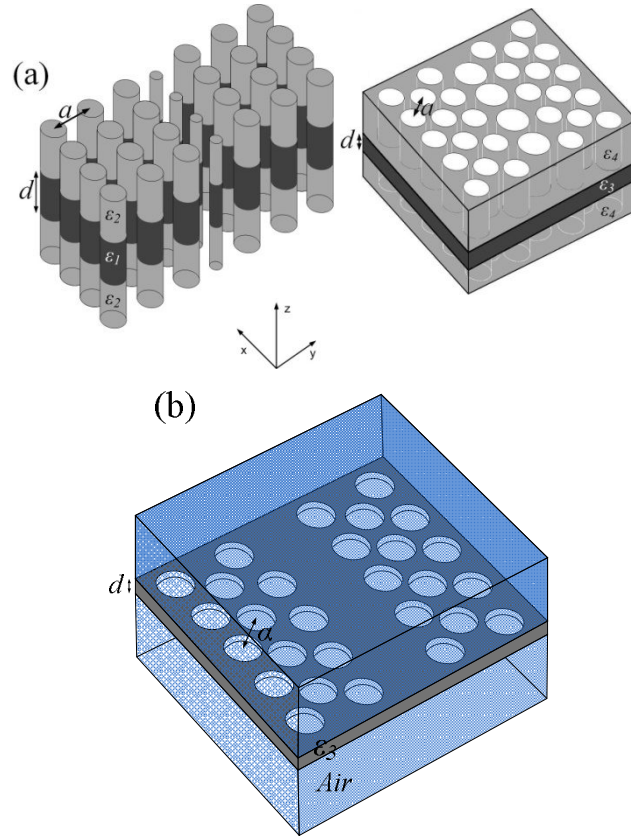


Figure 1. Different PCSW with introduced line defects a) a rectangular lattice of high index dielectric rods embedded in a low index material with a line defect of a single row with smaller radius $r_d < r_a$, and triangular lattice of low index dielectric rods embedded in a high index material with a line defect of a single row of holes with larger radius $r_d > r_a$ and b) a W1 PCSW membrane in air cladding.

2. PROPAGATION MODEL

2.1 Electromagnetic Field Expansion and PCSW defect modes

For narrowband signals, the total electromagnetic field inside the PCSW can be approximated by [14]:

$$\mathbf{E}(\mathbf{r}, t) \cong A(x, t)\mathbf{e}(\mathbf{r}, \omega_0)e^{-j\omega_0 t} + c.c. \quad (1)$$

$$\mathbf{H}(\mathbf{r}, t) \cong A(x, t) \mathbf{h}(\mathbf{r}, \omega_0) e^{-j\omega_0 t} + c.c. \quad (2)$$

where $A(x, t)$ is the slowly varying envelope function of the signal, \mathbf{E} and \mathbf{H} are the total electric and magnetic field inside the waveguide, x is the propagation direction, $\omega_0 = 2\pi c/\lambda_0$ is the central frequency of the signal, λ_0 the free space optical wavelength, $\mathbf{r} = (x, y, z)$ and c.c. stands for complex conjugate. The fields $\mathbf{e}(\mathbf{r}, \omega)$ and $\mathbf{h}(\mathbf{r}, \omega)$ are the electric and magnetic defect mode components. According to Bloch's theorem [15], these components can be expressed as $\mathbf{e}_0(\mathbf{r}, \omega) e^{j\beta(\omega)x}$ and $\mathbf{h}_0(\mathbf{r}, \omega) e^{j\beta(\omega)x}$ where $\mathbf{e}_0(\mathbf{r}, \omega)$ and $\mathbf{h}_0(\mathbf{r}, \omega)$ are the periodic vector functions. The propagation constant $\beta(\omega)$ and modal fields \mathbf{e} and \mathbf{h} are determined by the following Hermitian eigenproblem for the magnetic field component [7]:

$$\nabla \times \left(\frac{1}{\varepsilon_L(\mathbf{r})} \nabla \times \mathbf{h}(\mathbf{r}, \omega) \right) = \frac{\omega^2}{c^2} \mathbf{h}(\mathbf{r}, \omega) \quad (3)$$

where $\varepsilon_L(\mathbf{r})$ is the linear part of the relative permittivity of the PCSW. By expanding \mathbf{h} on a plane wave basis [15], eq. 3 is reduced into a matrix eigenvalue problem which can be solved using numerical techniques. We have implemented such a 3D plane wave expansion (PWE) mode solver based on the conjugate gradient minimization of the Rayleigh Ritz quotient [16]. For Kerr nonlinear media, the permittivity can be written as the sum of a linear and a nonlinear part:

$$\varepsilon(\mathbf{r}) = \varepsilon_0 \left(\varepsilon_L(\mathbf{r}) + \varepsilon_{NL}(\mathbf{r}) |\mathbf{E}|^2 \right) \quad (4)$$

The nonlinear part of the permittivity can be expressed in terms of the nonlinear refractive index of the material $n_2(\mathbf{r})$ as [17]:

$$\varepsilon_{NL}(\mathbf{r}) = \frac{\varepsilon_L(\mathbf{r}) n_2(\mathbf{r})}{Z_0} \quad (5)$$

where Z_0 is the free space impedance. Pulse broadening can be measured in terms of the root mean square (RMS) pulse width, [18] $\sigma(x)$, determined by:

$$\sigma^2(x) = \frac{\int (T - \bar{T})^2 |A(x, T)|^2 dT}{\int |A(x, T)|^2 dT} \quad (6)$$

where \bar{T} is the center of the pulse:

$$\bar{T} = \frac{\int T |A(x, T)|^2 dT}{\int |A(x, T)|^2 dT} \quad (7)$$

The broadening factor BF can be estimated through the following equation:

$$BF = \frac{\sigma(L)}{\sigma(0)} \quad (8)$$

where L is the waveguide length and $\sigma(0)$ corresponds to the initial pulse width. Values of the broadening factor below 1.33 (implying a 33% pulse broadening) are usually considered not to induce any serious ISI impairments [19].

2.2 Estimation of BF in the Linear and Non-Linear Regime.

In the linear regime ($\gamma=0$), the pulse broadening is determined solely by the group velocity dispersion (GVD) and the third order dispersion (TOD) coefficients β_2 and β_3 respectively. Assuming a Gaussian pulse at the waveguide input, i.e. $A(0,T)=\exp(-T^2/2T_0^2)$, then one can derive a closed form formula for the broadening factor [18] in eq. 8:

$$BF_L = \left(1 + (z / L_{D2})^2 + \frac{1}{4} (z / L_{D3})^2 \right)^{1/2} \quad (9)$$

where L_{D2} and L_{D3} are the dispersion lengths of the GVD and TOD respectively. In the nonlinear regime soliton pulses can be used to mitigate the effect of GVD induced broadening. In this case, broadening can still occur even in the lossless case because of the TOD. The soliton pulse shape depends on the sign of the GVD coefficient β_2 which as mentioned earlier can be either positive or negative, depending on the pulse launch frequency and waveguide parameters. If β_2 is positive (which is usually the case in the PCSWs considered here) then dark soliton should be launched in the structure, i.e. $A(0,T)=P_0^{1/2}\tanh(T/T_0)$. At $T=0$ one obtains $A(0,0)=0$ while for $|T/T_0| \gg 1$, $|A(0,T)| \cong P_0^{1/2}$ and hence this soliton pulse appears as a dip on a bright background. Pulse broadening in this case can be studied by numerically solving the propagation equation using the split step Fourier (SSF) method [18]:

$$\frac{\partial A}{\partial x} + \frac{\Gamma}{2} A + \frac{j\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} = j\gamma |A|^2 A \quad (10)$$

where β_2 and β_3 are the coefficients of GVD and TOD respectively.

2.3 Propagation Losses.

PCSWs are intrinsically loss-less, however the low level of sophistication of integrated optical devices causes severe propagation losses [20], [21]. Several fabrication imperfections (sidewall roughness, dislocation, etc) can cause light scattering. Backscattering due to the coupling between the counter propagating mode and out-of-plane losses due to coupling with radiation modes are considered the main parameters of the propagation losses observed in experiments [22]. In Ref. [22] is shown that for standard W1 waveguides, propagation losses coefficient a scale with respect to the group index n_g according to the following equation:

$$a = c_1 n_g + c_2 n_g^2 \quad (11)$$

where in c_1 and c_2 are including the out-of-plane and backscattering contribution respectively and $n_g=c/v_g$ is the group index. Backscattering is usually the dominant effect, indicating that in standard W1 waveguides, one can use a simpler n_g^2 scaling for describing propagation losses, i.e.:

$$a = a(n_{g0}) \left(\frac{n_g}{n_{g0}} \right)^2 \quad (12)$$

where $a(n_{g0})$ is the propagation losses for the fast light regime in dB/cm and n_{g0} is the minimum group index achieved in the fast light regime.

3. LINEAR OR NONLINEAR PROPAGATION IN PC-BASED DELAY LINES

When designing a PC-based delay line, one must take into account the amount of propagation losses and set a limit in the maximum allowable loss that can be experienced by the signal. This puts a limit in the maximum propagation distance L_w and since the loss factor depends on the group index n_g , one also expects that L_w will also depend on n_g . The maximum allowable loss L_{max} depends on the application at hand. If at some point, one wishes to use optical amplification to compensate for the optical losses, then L_{max} should not exceed 20dB, since this is a typical gain achieved

by a semiconductor optical amplifier (SOA). If optical amplification is not an option, then l_{\max} may have to be taken even smaller than that. In any case the propagation length is limited by

$$L_w \leq L_{\text{LOSS}} = \frac{l_{\max}}{a} \tag{13}$$

where a is the propagation losses described in dBcm^{-1} . Fig. 2, illustrates the maximum propagation limit L_{LOSS} induced by the losses as a function of n_g for various values of $a(n_{g0})$ in the case of a W1 PCSW for which $n_{g0} \approx 5$. The values assumed for $a(n_{g0})$ are 2dBcm^{-1} , 5dBcm^{-1} and 10dBcm^{-1} . It should be noted that certain loss levels allow for a propagation distance of the order of 1dm. However, since the delay line should be realized in integrated form, such propagation lengths may be unrealistic. We therefore choose the propagation length L_w to be fixed at 1cm if L_{LOSS} exceeds 1cm. Otherwise we assume $L_w = L_{\text{LOSS}}$.

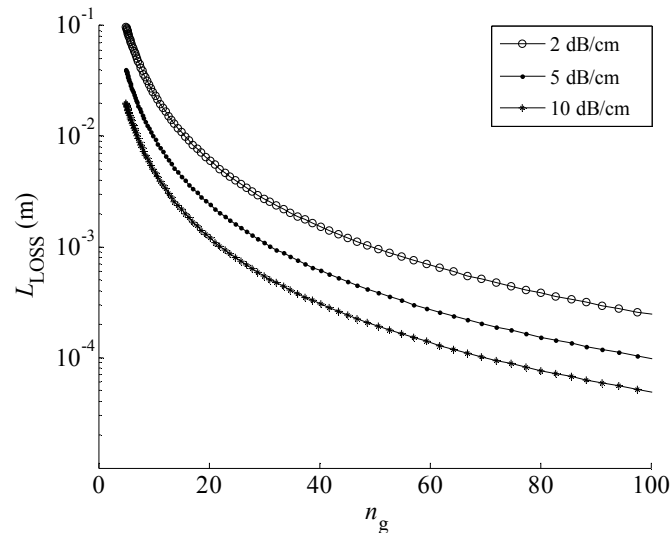


Figure 2. Propagation length limit as a factor of group index so that the output signal will be 20dB debased.

Given the waveguide length L_w we can calculate the broadening factor as a factor of the resulting delay time for several bit rates and various levels of fast light regime losses $a(n_{g0})$. In Fig. 3, we have calculated the broadening factor for a bit rate of 60Gbps. We can see that the nonlinear soliton propagation by using the fundamental soliton power ($P_0 = |\beta_2|/\gamma T_0^2$), sustain slightly less broadening than the linear Gaussian propagation. In this case we assume an even smaller optical loss margin of 10dB. In Fig. 4 the fundamental soliton power is shown for every n_g assumed here. The results improves when solitons are launched with higher than the fundamental power ($P > P_0$) as the soliton will experience an initial compression [24]. This improvement is less than 0.1nsec for a bit rate of 60Gbps, translating in an additional 5 bits storage capacity from the linear case.

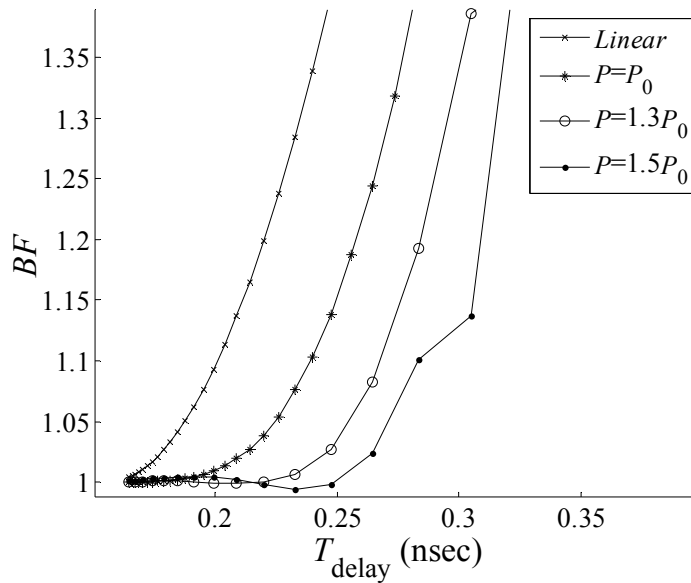


Figure 3. Broadening factor vs delay time in nsec at 60Gbps. In this case we take into account fast regime losses 2dB/cm ($n_g=5$) and waveguide length limited by 10dB absorption.

In Fig. 5, the losses at the fast regime ($n_g=5$) losses were set higher at 5dBcm^{-1} . This figure shows that as we moving to more lossy cases the difference between the linear and nonlinear signal propagation is now smaller. As fabrication accuracy is increased in the future, nonlinear soliton signal propagation seems to be more suited for delay line applications.

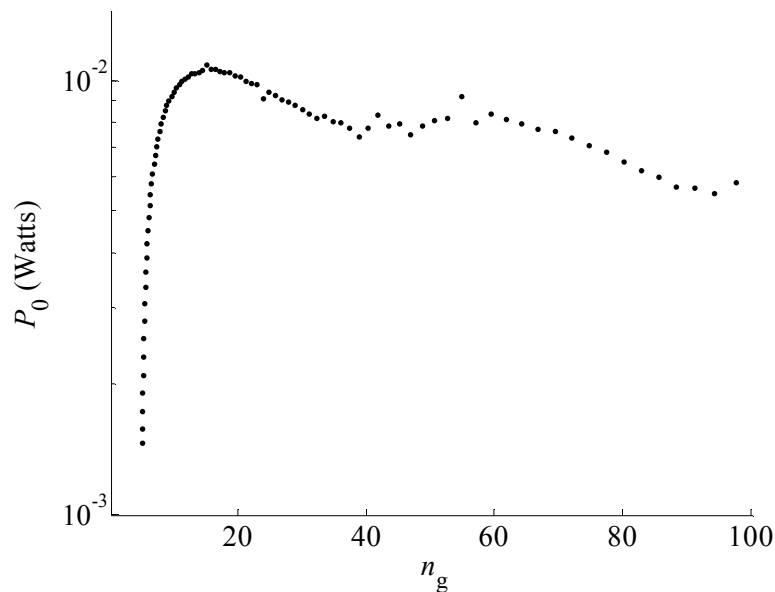


Figure 4. Fundamental soliton power P_0 vs n_g for the bit rate of 60Gbps. As seen here the calculated soliton power varies at a miliwatt power level for the majority of the group indices.

In Fig. 6 an intuitive comparison between the lossless linear and nonlinear case is shown for a lower bit rate of 10Gbps. Of course the lossless case cannot be achieved in any structure, however we can deduce that as the fabrication accuracy

improves the closest to the lossless case we will be. As seen from Fig. 6 the delay limitations of such structures reaches an order of nanosecond delays for the bit rate of 10Gbps.

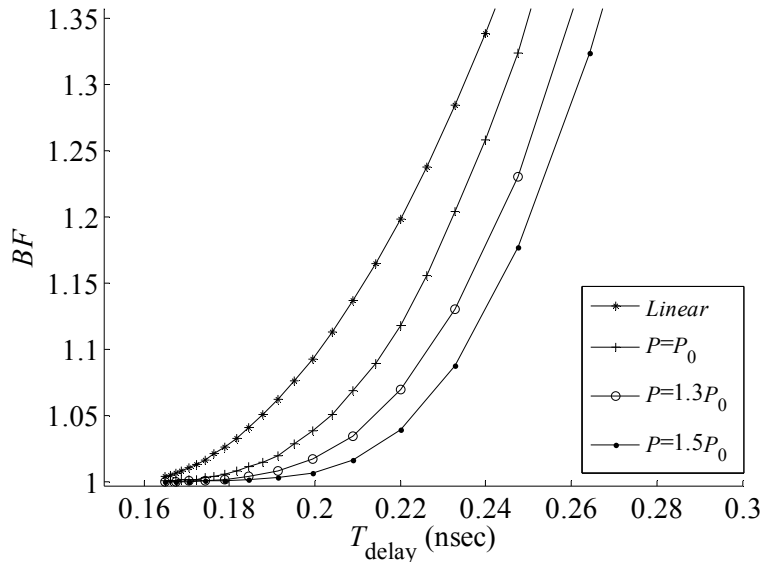


Figure 5. Broadening factor vs delay time in nsec at 60Gbps. In this case we take into account fast regime losses 5dB/cm ($n_g=5$) and waveguide length limited by 20dB absorption.

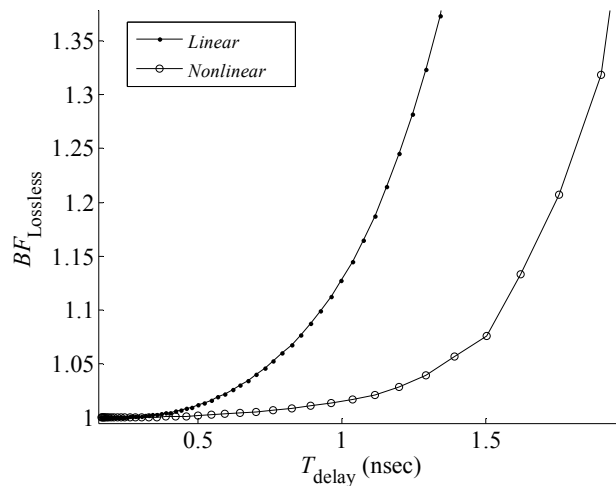


Figure 6. Broadening factor vs delay time in nsec at bit rate of 10Gbps assuming a lossless regime. It is expected that the nonlinear soliton propagation will sustain less broadening than the linear case.

4. CONCLUSIONS

In this paper, we studied and compared the linear and nonlinear propagation in PCSWs from a delay time point of view. The dispersion-induced broadening of an optical signal pulse as well as the propagation losses are taken into account. We have shown that the nonlinear case may provide important improvements to the delay time as long as the propagation losses are kept relatively low (corresponding to 2dBcm^{-1} in the fast light regime). As the fabrication

accuracy improves over time and losses are reduced the nonlinear propagation may therefore find important applications in PCSW-based delay lines, improving the delay-bandwidth product achieved by such devices.

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