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NGA Investments: A departure from the existing cost and demand structure assumptions

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Abstract

The two most significant factors that affect the deployment of Next Generation Access (NGA) networks are the cost of the investment and the expected demand for the new fibre-based services. The related literature is based on very simplified assumptions regarding cost and demand structures.

In particular, the investment cost is assumed to be increasing and convex reflecting the fact that fibre deployment becomes marginally more expensive as it is extended to rural, less populated areas. In addition, the demand for the new fibre-based services is estimated by assuming that a certain level of NGA investment leads all consumers to equally increase their willingness to pay for such services.

This article contributes to the emerging research on the investment in access infrastructures. In particular, the assumptions about cost and demand structures are modified in order to capture the access networks' underlying morphology complexity and the consumers' socioeconomic characteristics, respectively. Firstly, an empirical analysis is conducted for the 100 major municipal departments from urban to rural in Greece. Their street network data are analyzed as the basis of the NGA installation combining GIS technology and Graph Theory techniques and hence the main cost-drivers are derived. Using regression analysis a real-data-based cost function is obtained. Secondly, a novel model that takes into account socioeconomic characteristics affecting the impact of a certain level of NGA investment on consumers' willingness to pay is developed. The Pareto consumer distribution is used to reflect the greater (lower) positive impact of NGA investments on the willingness to pay of the consumers who live in more (less) populated areas.

The comparison of the existing models with the ones developed in this paper shows that: (i) the cost function used in the existing models always underestimates the investment cost of the higher populated areas and overestimates the investment cost of the lower populated areas; (ii) the demand for the new fibre-based services is higher under the proposed than the existing approach; and (iii) the level of NGA investment chosen by the investor is always much higher under the proposed than the existing approach.

1. Introduction

During the last decade, the number of Internet users, as well as, the capacity they demand have increased dramatically. As a result, the increasing transmitted volume of data made the traditional access copper networks incapable of providing end-users with the demanded bandwidth. On the contrary, access networks based on optical fibre are the only future proof solution capable to handle the future demands (Shumate, 2008), since the transmission capabilities of fibre are theoretically unlimited providing high data rates, low loss and low distortion. Such fibre-based access networks are widely known as Next Generation Access (NGA) networks. According to European Commission "NGA networks mean wired access networks which consist wholly or in part of optical elements and which are capable of delivering broadband access services with enhanced characteristics (such as higher throughput) as compared to those provided over already existing copper networks. In most cases NGA networks are the result of an upgrade of an already existing copper or coaxial access network" (EC, 2010a).

However, not only technical reasons but also economic ones make the need for investments in NGA networks imperative. In particular, it is found that investments in broadband infrastructure have an undisputable positive effect on economic growth and broadband diffusion (Czernich, Falck, Kretschmer and Woessmann, 2011; Katz, Vaterlaus, Zenhäusern and Suter, 2010; Reynolds, 2009). These results partially interpret why national governments rank among their top priorities the encouragement of investments in NGA networks. The US government's National Broadband Plan (FCC, 2010) and the European Commission's Digital Agenda for Europe (EC, 2010b) are examples of these perceived political priorities for the diffusion of broadband infrastructure access and services. According to EC (2010a):

"The EU single market for electronic communications services, and in particular the development of very high-speed broadband services, is key to creating economic growth and achieving the goals of the Europe 2020 strategy. The fundamental role of telecommunications and broadband deployment in terms of EU investment, job creation and overall economic recovery was notably highlighted by the European Council in the conclusions of its March 2009 meeting."

It can be thus concluded that very high-speed broadband services are widely accepted as strategically important not only because of their ability to accelerate the contribution of information and communications technology (ICT) to economic growth (Teppayayon and Bohlin, 2010), but also because network investments are potentially important targets of public investment during downturns as a way to increase demand and employment (Reynolds, 2009).

However, service providers are still reluctant to invest in NGA networks mainly due to the high investment cost and the ambiguity about the expected demand for the new fibre-based services. There are several theoretical economic approaches that aim to model the cost structure of investing in NGA networks, as well as, the impact of such investments on the future demand for the new fibre-based services. Most of these approaches assume that the investment cost is increasing and convex reflecting the fact that fibre deployment becomes marginally more expensive as it is extended to rural, less populated areas (Nitsche and Wiethaus, 2011; Foros, 2004). In addition, they also assume that the demand for the new fibre-based services is not affected by the relationship between the valuation that the consumers place to such services and their location (i.e. the population-affected type of the area they live in). Although these assumptions are useful for practical reasons; they fail to take into account the access networks' underlying morphology complexity and the consumers' socioeconomic characteristics, respectively.

This paper contributes to the emerging research on the investment in NGA networks by modifying the existing cost and demand structure assumptions in order to capture the access networks' underlying morphology complexity and the consumers' socioeconomic characteristics, respectively. Firstly, an empirical cost analysis is conducted for the 100 major municipal departments from urban to rural in Greece. Their street network data are analyzed as the basis of the NGA installation combining GIS technology and Graph Theory techniques and hence the main cost-drivers are derived. Using regression analysis a real-data-based cost function is obtained. Secondly, a novel model that takes into account socioeconomic characteristics affecting the impact of a certain level of NGA investment on consumers' willingness to pay is developed. The Pareto consumer distribution is used to reflect the greater (lower) positive impact of NGA investments on the willingness to pay of the consumers who live in more (less) populated areas.

The comparison of the existing models with the ones developed in this paper shows that: (i) the cost function used in the existing models always underestimates the investment cost of the higher populated areas and overestimates the investment cost of the lower populated areas; (ii) the demand for the new fibre-based services is higher under the proposed than the existing approach; and (iii) the level of NGA investment chosen by the investor is always much higher under the proposed than the existing approach.

The rest of the paper is as follows. In Section 2, the existing approach in terms of cost and demand functions is reviewed. Section 3 proposes a new approach to estimate the cost of deploying an NGA network based on real cost data from Greece and modifies the widely used demand model with network externalities in order to capture the fact that consumers who place a higher (lower) valuation to broadband subscription tend to live in higher (lower) populated areas. Section 4 compares the results of the two approaches in terms of the optimal investment level for the investor and the subsequent levels of subscribers, investment costs, revenues and profits. The last section summarizes the main results of this article and proposes the directions for future work.

2. Existing approach

This section provides the existing cost and demand functions that are widely used in the literature of NGA investments in order to estimate the investment level that maximizes the investor's profits.

2.1 Existing cost models

Currently, most telecom operators are reluctant to significantly upgrade their telecommunication access network due to the high investment cost. Upgrade to NGA networks is generally perceived as the Fibre to the Curb (FTTC), Fibre to the Building (FTTB) and, of course, Fibre to the Home (FTTH) which is the ultimate and most future-proof access solution. To make any profound decisions on replacing some or all copper cable with optical fibre, a reasonably accurate cost model is needed, with enough detail on differences between deployments in different regions.

The related literature on telecommunications investments (Nitsche and Wiethaus, 2011; Foros, 2004) is based on very simplified assumptions regarding the cost structure. The investor in the abovementioned approaches determines the extent of NGA deployment, R. The investment level is considered continuous and a larger R reflects a larger geographic coverage within a given market area (e.g. fibre to the outskirts rather than to the city centre, or to less populated cities). The NGA deployment is assumed to require investments of the following quadratic form:

$$C(R) = \frac{\varphi R^2}{2} \qquad (1)$$

where φ is an investment cost parameter. The convex form accounts for the assumption that deploying an NGA network becomes more expensive as the rollout is extended to rural, less populated areas indicated by a higher R.

In the case of investing in NGA networks in a nationwide level, R can be seen as continuous in $[1, R_{max}]$ implying that $R = 1$ corresponds to the highest populated area and $R = R_{max}$ corresponds to the lowest populated area. Therefore, the whole areas within a country have been ranked in a decreasing order according to their population.

2.2 Existing demand models

There are many economic models that aim to estimate the demand for a good. Most of them base their analysis on the market structure of the industry in order to derive the demanded quantity. Examples of such models are those proposed by Cournot, Bertrand, Stackelberg, etc. These models are widely used in conventional markets in which there is a negative relationship between the demanded quantity for a good and its price. However, network markets, such as telecommunications, computers, electricity and railroads, present an innate characteristic that make them differ from conventional markets. In particular, the utility which a given user derives from the network good depends upon the number of other users who are in the same network. According to Katz and Shapiro (1985), this fact implies a positive consumption externality, which is widely known in the literature as network externality or network effect.[1] Economides (1996) points out that this fact seems quite counterintuitive since it goes against the downward-sloping market demand. Thus, he proposes that a positive consumption externality signifies the fact that the value of a unit of the good increases with the expected number of units to be sold. In this case, the demand slopes downward but shifts upward with increases in the number of units expected to be sold. Therefore, when expectations are fulfilled, the derived demand curve for a network good is concave.

Based on these observations, Shy (2011) models the demand for a network good. In particular, he assumes that potential subscribers can be indexed in a decreasing order according to the valuation (or utility) that they place on the network good. In particular, potential consumers are indexed by x, $x \in [0,1]$, where consumers that are indexed by low values of x value the subscription highly, whereas consumers that are indexed by x close to 1 place a low valuation on this service. Therefore, the variation in their willingness to pay for the network good forms a continuum of types of consumers. The (expected) utility of a potential subscriber indexed by x is given by:

$$U_x = \begin{cases} (1-\beta x)\alpha q^e - p, \text{ if the consumer subscribes} \\ 0, \text{ if the consumer does not subscribe} \end{cases} \tag{2}$$

where p denotes the subscription fee, q^e the expected total number of subscribers and $\beta > 0$ captures the degree of consumer heterogeneity with respect to consumers' benefit from this service. The parameter $\alpha > 0$ measures the intensity of network effects. Higher values of α indicate that consumers place higher value on the ability to communicate with the q^e subscribers, whereas $\alpha = 0$ implies that there are no network effects.

A further assumption made by Shy (2011) is that the potential consumers are distributed uniformly in [0,1]. The uniform distribution of the consumers implies that in each type has

[1] Some authors distinguish between direct and indirect network externalities. For a discussion on this issue see Economides (1996), Katz & Shapiro (1985), Page and Lopatka (1999) and Shy (2011).

been assigned a fixed number of potential consumers. Therefore the market demand is derived by multiplying the number of types whose utility is positive (i.e. buy the product or subscribe) with the fixed number of consumers of each type. For example, let there be N potential subscribers of each type x, $x \in [0,1]$. Then, for a given subscription fee p, there is a consumer of type $0 \leq x(p) \leq 1$ who is indifferent between subscribing and not subscribing. Assuming perfect foresight, the total number of expected subscribers (or the demand for the network good) is $q_u^e = Nx(p)$.

However, in order to make investment costs and revenues comparable, we should transform the consumer type indexed by x, $x \in [0,1]$ into the range $\left[1, R_{max}\right]$. This implies that each consumer type x corresponds to a given geographic area R. Lower values of R imply that the consumer who lives in this area place a higher valuation to the network good. For this purpose, the normalization method with transformation $x = \frac{R - R_{min}}{R_{max} - R_{min}}$ is used. The notation R stands for the original dissimilarity and x for the normalized dissimilarity. Hence, the dissimilarity index R (respectively, x) lies between 1 and R_{max} (respectively, 0 and 1). Applying the above normalization method yields the original dissimilarity as a function of the corresponding normalized dissimilarity:

$$R = x\left(R_{max} - 1\right) + 1 \text{ , with } R \in \left[1, R_{max}\right] \tag{3}$$

Equation 3 shows that R is continuous in $\left[1, R_{max}\right]$. In addition, $x = 0$ corresponds to $R = 1$ and $x = 1$ to R_{max}. This implies that the number of the *potential consumers* is NR_{max}, whereas the utility function of Shy (2001) becomes $U_X = (R_{max} - \beta(x(R_{max} - 1) + 1))\alpha N(x(R_{max} - 1) + 1) - p$. Then, solving for x yields the indifferent consumer:

$$x_{L,H}^{U} = \frac{\alpha NR_{max} - 2\alpha\beta N \pm \sqrt{\left(\alpha NR_{max}\right)^2 - 4\alpha\beta Np}}{2\alpha\beta N\left(R_{max} - 1\right)} \tag{4}$$

Therefore, the expected number of *potential subscribers* is given by:

$$q_{L,H}^{U} = N\left(\left(\frac{\alpha NR_{max} - 2\alpha\beta N \pm \sqrt{\left(\alpha NR_{max}\right)^2 - 4\alpha\beta Np}}{2\alpha\beta N\left(R_{max} - 1\right)}\right)(R_{max} - 1) + 1\right) \tag{5}$$

Since there exist two indifferent consumers between subscribing and not subscribing, there also exist two consumer equilibria. At every given price p, either a low or a high demand level would be realized according to consumers' expectations for the demand level. If all consumers correctly anticipate low demand, only those who value this service highly ($0 \leq x \leq x_{L}^{U}$) will subscribe. If all consumers anticipate high demand, the gain from a larger anticipated network will also induce consumers with lower valuations ($x^U_L \leq x \leq x^U_H$) to subscribe. Note that q^U_L is an unstable equilibrium in the sense that a small increase in the number of subscribers would induce q^U_H consumers to subscribe. Therefore, the demand (number of subscribers) for the network good is given by:

$$q^U_H = N\left(\left(\frac{\alpha NR_{max} - 2\alpha\beta N + \sqrt{\left(\alpha NR_{max}\right)^2 - 4\alpha\beta Np}}{2\alpha\beta N\left(R_{max} - 1\right)}\right)(R_{max} - 1) + 1\right) \tag{6}$$

It should be noted that, in existing markets, R_{max} denotes the urban administrative divisions, i.e. municipal department (MD), of a country in which the consumers that place the lowest valuation on the new fibre-based services live. However, in the case of the NGA investments, this municipal department may not be covered by the investor. This implies that when a potential subscriber to the NGA services makes its decision to subscribe or not, s/he takes into account the expected number of subscribers to the NGA services rather than the whole population in a given country. Therefore, R_{max} should be replaced by R_{inv} which denotes the municipal department of a country that it is covered by the investor and in which the consumers that place the lowest valuation on the new fibre-based services live. In other words, R_{inv} denotes the optimal investment level chosen by an investor in NGA networks. Therefore, the investor maximizes the following equation with respect to R_{inv} .

$$\Pi^U = Pq^U_H - \frac{\varphi R^2_{inv}}{2} \Rightarrow$$

$$\Pi^U = PN\left(\left(\frac{\alpha NR_{inv} - 2\alpha\beta N + \sqrt{\left(\alpha NR_{inv}\right)^2 - 4\alpha\beta Np}}{2\alpha\beta N}\right) + 1\right) - \frac{\varphi R^2_{inv}}{2} \tag{7}$$

3. The proposed approach

This section provides a more realistic approach concerning the development of NGA networks. In particular, the proposed cost and demand functions depart from the existing ones since the proposed model captures the access networks' underlying morphology complexity and the consumers' socioeconomic characteristics.

3.1 Proposed NGA investment cost

In this case, an empirical cost analysis conducted on a nationwide NGA network provides insights on the cost form, so that the validity and the accuracy of the conventional cost form may later be explored.

An approach is presented in this section for getting a clear estimate of expenses for an NGA rollout, particularly the most future-proof access solution of FTTH deployment, as the investment level increases from areas with large number of households (HH) to areas with small number of households. Calculations are made for the main cost-drivers using real street network data as the basis of a fixed NGA installation combining GIS technology and Graph Theory techniques.

For simplicity, the focus in this study is on the most important expenses that are the outside plant (OSP) capital expenditures (CAPEX). Earlier studies (Colle et al., 2008) indeed indicate that the major part of the total investment in a telecommunication access network is the capital investment made in the lower part of the network that connects a subscriber by a physical link to its corresponding Central Office (CO) via intermediate network components. Possible expenses on the active equipment or the regional/national/global backbone are not considered here.

The architecture

The considered FTTH architecture is presented in Figure 1. The model consists of a CO where all the optical line terminals are located, the feeder part of the network connecting the CO with flexibility points (FP), and the distribution part from FP to the end-users. The FP (or splitting point or cabinet) plays a concentration role, allowing the merging of customer cables. There are two popular technologies used with FTTH. The Point to Point (P2P) technology which uses all active components throughout the chain and Point to Multi-Point (P2M) / Passive Optical Network (PON) technology which uses passive optical splitters at the aggregation layer. For the purposes of this study the PON technology has been envisaged since it has been proven that P2P technology requires a rather costly infrastructure (Chatzi and Tomkos, 2011).Gigabit-capable Passive Optical Networks (GPON) are standardized by ITU-T under the family of recommendations G.984 (ITU-T, 2009) and are already in use in several countries. Here, the deployment of GPON FTTH is considered with a centralized 1:128 splitting ratio. This means that per group of 256 customers covered by each FP, only 2 fibres are needed for the FP-CO feeder part connection. Each CO is assumed to cover up to 100.000 households.

Also, a greenfield deployment is assumed and no existing infrastructure is taken into account. The installation closely follows one street with the cable located at the middle of the street and connects all households along the street. Of course, savings are possible if part of the network can be installed by means of aerial deployment, e.g. in areas with small number of households.

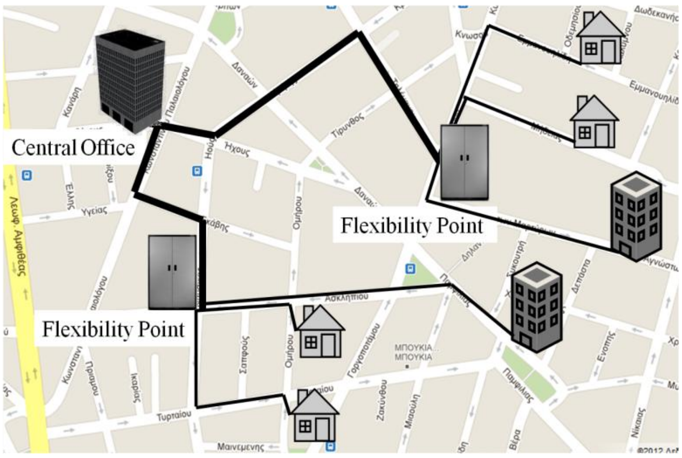

Figure 1. FTTH access network architecture

The regarded elements for the calculation of the total OSP CAPEX are the trenches, the ducts, the fibre cables, the splitters, the manholes, and the Y-branches. The individual components' costs were taken from (Chatzi and Tomkos, 2011). Any effects of changes in these prices to the total cost falls beyond the scope of this paper. The estimation of the volume of the material needed is described later on.

The use of geometric models is very often in techno-economics for analyzing the deployment area and estimating the OSP cost (Casier, 2009). Typically, these geometric abstractions of the installation region assume a regular grid-like structure where all lines have equal length and the same number of junctions. However, they cannot capture the complex details of the underlying urban street network in order to accurately estimate the key quantities for a cost evaluation of a fixed access network. In fixed access networks the cables run in trenches that use the road system as a natural guide to reach the customers. Access network nodes as well as connections strongly depend on the actual geography of the underlying urban street network and this has been proven to have a significant impact on the key quantities for estimating the deployment cost (Mitcsenkov et al., 2010; Maniadakis and Varoutas, 2012). For this reason a recently presented methodology (Maniadakis and Varoutas, 2012) that uses real GIS data is extended and applied for the cost analysis.

In this paper the 100 major MDs in Greece are selected in order to calculate and observe the form of the cumulative cost as the investment continues from the most populated down to the lowest populated MD in terms of households. The data are obtained from the collaborative project OpenStreetMap (OpenStreetMap, 2012) (GIS vector map) and the Hellenic Statistical Authority (Hellenic Statistical Authority, 2001) (number of HH, number of buildings, km^2 of area). The constructed dataset consists of 100 1-square-kilometer samples of street networks selected from the abovementioned municipal departments. Their data are imported in a GIS environment and are turned into spatial, weighted, undirected graphs using the Primal approach.

Figure 2. In the left it is the street network of the district sample of MD Nea Smirni, while to the right is the corresponding Primal graph split into square serving zones with buildings placed equidistant and FPs placed in the optimal locations

Methodology and results

Street networks are spatial, which is a special class of complex networks whose nodes are embedded in a two (or three) dimensional Euclidean space and whose edges do not define relations in an abstract space, but are real physical connections (Cardillo et al., 2006). Such a street network can be represented as a graph, which consists of a finite set of nodes and a finite set of edges. The graph nodes have precise position on the planar map, while the links follow the footprints of real streets and are associated a set of real positive numbers representing the street lengths.

A sample urban area is chosen and GIS data are collected without further GIS processing or analysis. The GIS data are then transformed to a spatial, weighted, undirected graph using the Primal approach (Porta et al., 2006) where intersections are turned into nodes and streets into edges, as shown in Figure 2. Depending on the number of buildings in the area, a new spatial network is made as an extension of the street network, with new nodes placed equidistance from neighbor nodes (inter building spacing - IBS) along the existing edges, so that the total number of nodes is equal to the number of buildings. In addition, depending on the number of HH in the area, a number of FPs is assigned at optimal locations applying the Closeness Centrality method. Each FP can serve a maximum number of households, e.g. 256, thus the total number of households is divided to this number to produce the required FPs. Then, the considered area needs to be split into serving zones, for example squares of equal size in order to serve approximately the same number of buildings/households. Each FP is associated with a serving zone such that the inscribed subnetwork that gathers all fibre lines between the FP and the subscribers displays a star structure that follows the underlying street network. The network can retain information in the edge weights, such as the trenching length, the size of the duct, the fibre length, etc. Then, the volume of the various network components may be computed with simple calculations on the graph weights, as described in (Maniadakis and Varoutas, 2012).

The cost is calculated for each 1-square-kilometer sample and then a cost/HH can be derived if the cost is divided with the number of HH in the sample, as shown in Figure 3 for the case of Greece. All costs are estimated for a project horizon of 20 years. The cost/HH per year is estimated to vary from 2,25€ to 265€.

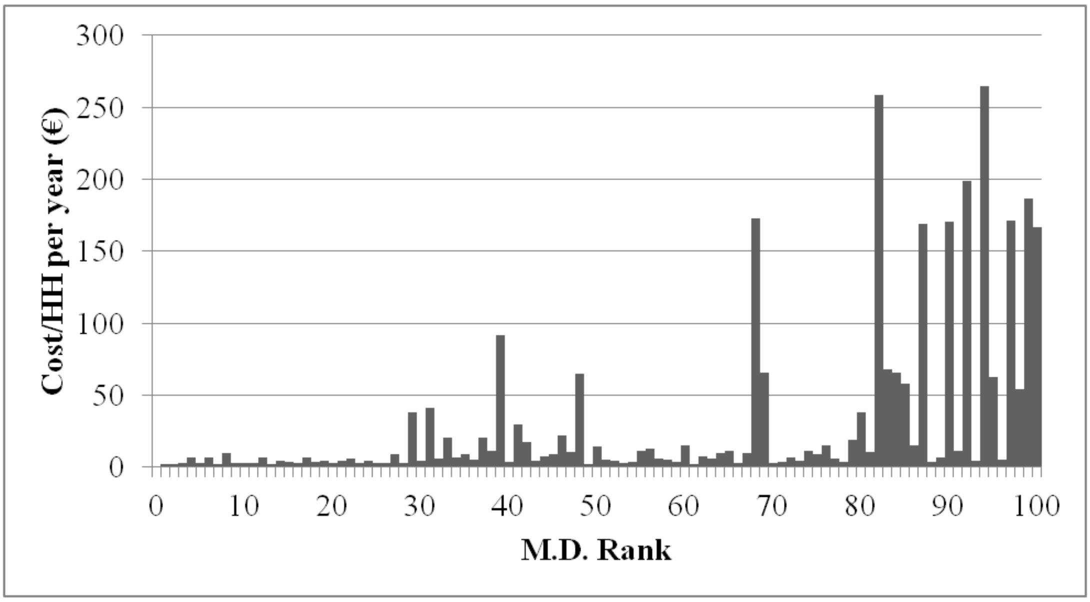

Figure 3. Cost per HH per year for the major MDs (sorted in number of HH) of Greece

The present study focuses on the 100 most populated MDs in terms of HH that vary from 301.566 HH to 5.953 HH and HH density that varies from 9.464 HH/km^2 to 24 HH/km^2. However, the vast majority of these MDs belongs to the high-dense HH MDs, as 67 out of 100 belong to the top-100 most densely populated MDs in HH. Thus, this means that either the investment grows from largest to lowest MD in terms of number of HH as described here, or from largest to lowest MD in terms of HH density, the cost results are similarly distributed. In total, the 100 MDs under investigation cover 2.089.992 HH or 57% of all HH in Greece.

Multiplying the above estimated cost per HH with the corresponding MD 's number of HH gives the cost per MD. Its cumulative distribution is depicted in Figure 4.

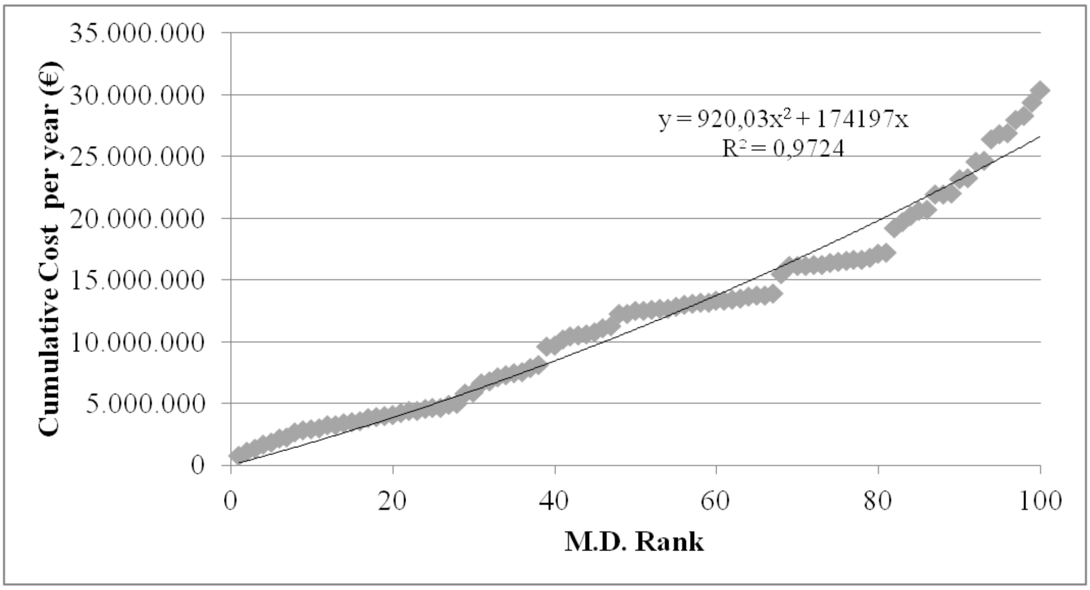

Figure 4. The cumulative cost per year for the major MDs (sorted in number of HH) of Greece

It is now convenient to derive the cost function that describes NGA investment as the investment level moves on greater level, covering less populated areas. Using regression analysis on the real cost data, the derived NGA investment cost function is given by:

$$C(R_{inv}) = 920,03 R_{inv}^2 + 174197 R_{inv} \quad (8)$$

3.2 Proposed NGA demand

Although the uniformity assumption is convenient for deriving analytical results, it has been fiercely criticized in the literature. The reason is that uniform consumer distribution may not be highly satisfactory in representing actual consumer distributions in many markets (Ansari, Economides and Ghosh, 1994) and hence it is more realistic to assume non-uniform consumer distributions (Anderson, Goeree and Ramer, 1997).

Indeed, in many network markets, such as telecommunications, the valuation that the consumers place to the network good is significantly affected by their location (i.e. the population-affected type of the area they live in). The related literature studies the determinants of broadband availability, adoption and usage depending on location. For example, Flamm and Chaudhuri (2007) find a positive urban and suburban role in stimulating both dialup and broadband adoption. They attribute this result to social characteristics that make the Internet and broadband use more attractive to urban and suburb dwellers than to rural folk. A more conclusive study that includes the main results of the related literature is Preston, Cawley and Metykova (2007) which analyze the status of broadband in rural areas in the EU. They look at availability, adoption and use of broadband, taking a policy perspective drawing from the results of the BEACON research, which provides an analysis of the broadband situation in each of the 25 EU countries. Their main findings are: a geographic broadband divide; lower investment in infrastructure in rural areas; where broadband is available, lack of competition in infrastructure and services; the fact that the rural broadband divides go along with other traditional divides; the fact that rural areas suffer from declining and aging population; the fact that the rural dwellers tend to be slower adopters; the fact that the rural areas have less technical support; the circumstance that social factors that facilitate broadband use (such as education, profession, economic status and cultural practice) can be less favourable in rural communities. The main take-away of the above studies is that consumers who place a higher (lower) valuation to broadband subscription tend to live in higher (lower) populated areas.

This conclusion signifies the fact that uniform consumer distribution fails in representing the actual demand in telecommunications markets in which the consumers' valuation for the good varies according to the population of the location (area) they live in. Thus, the aim of this section is to estimate the demand for the new fibre-based services when the relationship between location and valuation for the good is taken into account. This implies that the distribution of the consumers to their different types is not uniform but follows a certain non-uniform distribution that captures the fact that consumers who place a higher (lower) valuation to the network good tend to live in areas with higher (lower) population.

A particular type of non-uniform distribution in literature that can describe the population allocation in urban divisions is the power law or Pareto distribution (Soo, 2005). This distribution, when plotted on double logarithmic axes, shows a remarkable linear pattern where the slope of the line is usually close to -1, corresponding to the well-known Zipf's law distribution (Zipf, 1949). This type of non-uniform distribution is quite appropriate to chose since it states that a size is inversely proportional to its rank in a sorted order. For example, in the case of populations, the population size of each city in a country appears to be inversely proportional to the city rank. Therefore, the variation in the population of each area forms a continuum of areas. This fact is in full accordance with the model proposed by Shy (2011) and hence they can be easily compared.

In Figure 5 there are presented the most populated municipal departments of Greece in terms of households. Specifically, there are included MDs until 100 HH (3679 MDs in total). Their HH-Rank distribution fits a power law (R^2>0,99) with a power law exponent near -1, indicating a Zipf distribution.

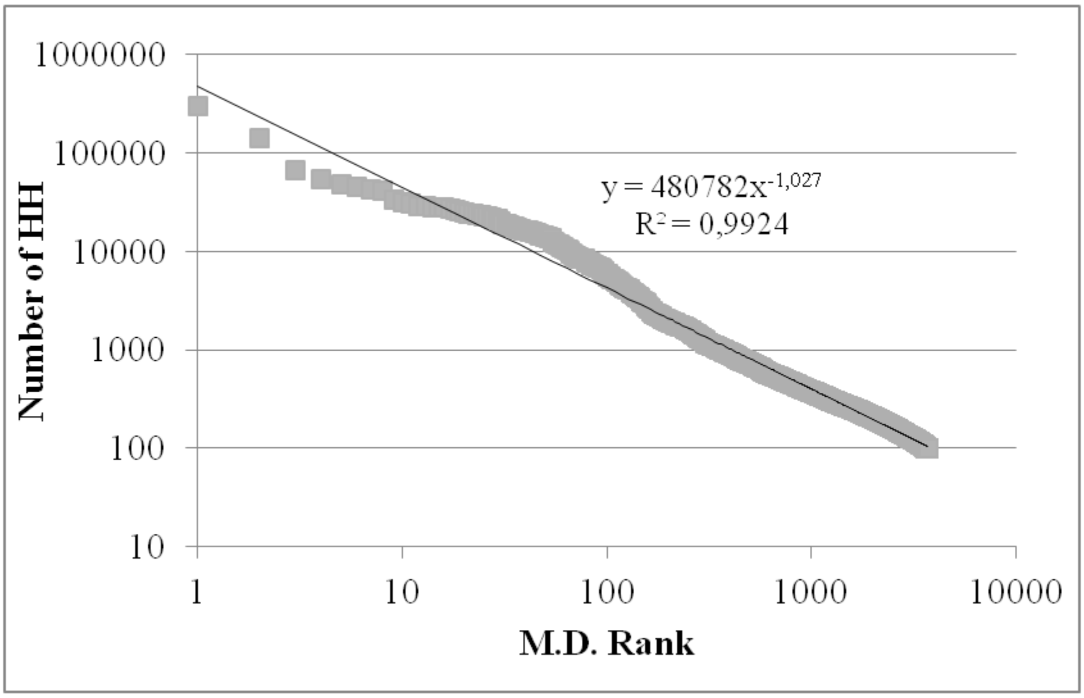

Figure 5. HH distribution is a Zipf distribution; the case of Greece (2001)

As mentioned by Kyriakidou, Michalakelis and Varoutas (2011), Zipf's law can be described by the following equation:

$$Rank\ x\ Population = Constant \Leftrightarrow Population = \frac{Constant}{Rank} \Leftrightarrow pop(R) = \frac{C}{R} \qquad (9)$$

Let rank the MDs according to their HH number in a decreasing order. Then, $R_{min} = 1$ denotes the area with the highest population and $R = R_{max}$ denotes the area with the lowest population. Figure 6 plots the non-uniform distribution of consumers according to their willingness to pay.

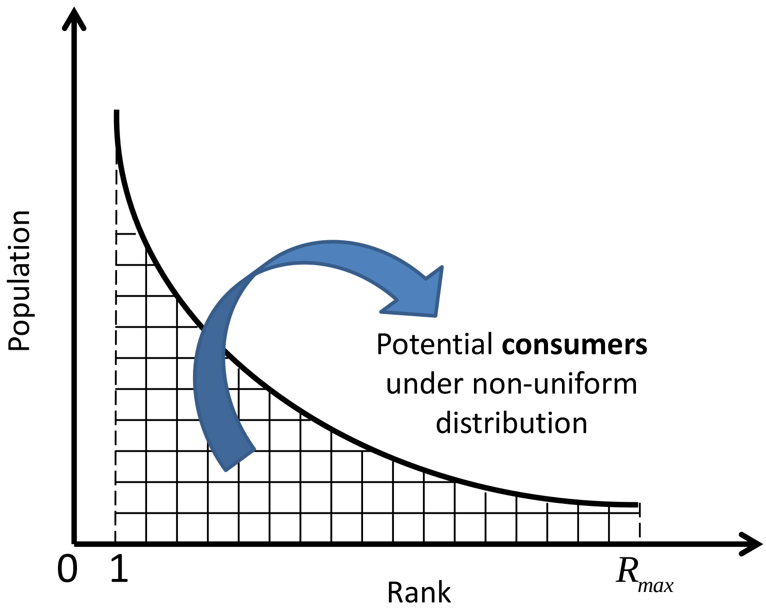

Figure 6. Non-uniform distribution of consumers according to their willingness to pay

Figure 6 reflects the fact that consumers with high willingness to pay live in areas with high population. However, the distribution of consumers according to their willingness to pay (or equivalently the rank of the areas they live in) is not uniform, but follows the Zipf's law. Therefore, the maximum number of *potential consumers* is represented by the shaded region of the above figure, which is given by:

$$q_{n-u}^{max} = \int_{1}^{R_{max}} pop(R)\,dR = \int_{1}^{R_{max}} \frac{C}{R}\,dR = C\left[ln|R|\right]_{1}^{R_{max}} = C ln(R_{max}) \qquad (10)$$

Similar to Shy (2011), only those types of consumers whose valuation for the network good is positive buy the good or subscribe. Therefore, for a given subscription fee p, there is a consumer of type $0 \leq x(p) \leq 1$ who is indifferent between subscribing and not subscribing. This indifferent consumer is affected by the expected total number of subscribers which is given by:

$$q^{e}_{n-u} = \int_{1}^{x(R_{max}-1)+1} pop(R)\,dR = \int_{1}^{x(R_{max}-1)+1} \frac{C}{R}\,dR = C\left[ln|R|\right]_{1}^{x(R_{max}-1)+1} \Rightarrow$$

$$q^{e}_{n-u} = Cln\left(x(R_{max} - 1) + 1\right) \tag{11}$$

Substituting Eq. (11) into Eq. (2) and using Eq. (3) gives the (expected) utility of a *potential subscriber* indexed by $x \in [0,1]$ when the distribution of consumers according to their willingness to pay follows the Zipf's law:

$$V_X = \begin{cases} (R_{max} - \beta(x(R_{max} - 1) + 1))\alpha Cln(x(R_{max} - 1) + 1) - p, \text{ if the consumer subscribes} \\ 0, \text{ if the consumer does not subscribe} \end{cases} \tag{12}$$

Solving Eq. (12) with respect to x yields the indifferent consumer:

$$x^{V}_{L,H} = \frac{\alpha CR_{max} - \alpha\beta C \pm \sqrt{(.)}}{2\alpha\beta C\left(R_{max} - 1\right)} \tag{13}$$

where

$$\sqrt{(.)} = \sqrt{\left(\alpha\beta C\right)^{2} + \left(\alpha CR_{max}\right)^{2} - 2\alpha^{2}\beta C^{2}R_{max} - 4p\alpha\beta C} \tag{14}$$

Then, by substituting Eq. (13) into Eq. (11), the expected number of *potential subscribers* is derived:

$$q^{V}_{L,H} = Cln\left(\frac{\alpha CR_{max} - \alpha\beta C \pm \sqrt{(.)}}{2\alpha\beta C} + 1\right) \tag{15}$$

Once again, it is proven that q^{V}_{L} is an unstable equilibrium in the sense that a small increase in the number of subscribers would induce q^{V}_{H} consumers to subscribe. Therefore, the demand (number of subscribers) for the network good is given by:

$$q^{V}_{H} = Cln\left(\frac{\alpha CR_{max} - \alpha\beta C + \sqrt{(.)}}{2\alpha\beta C} + 1\right) \tag{16}$$

which is represented by the shaded region in Figure 7.

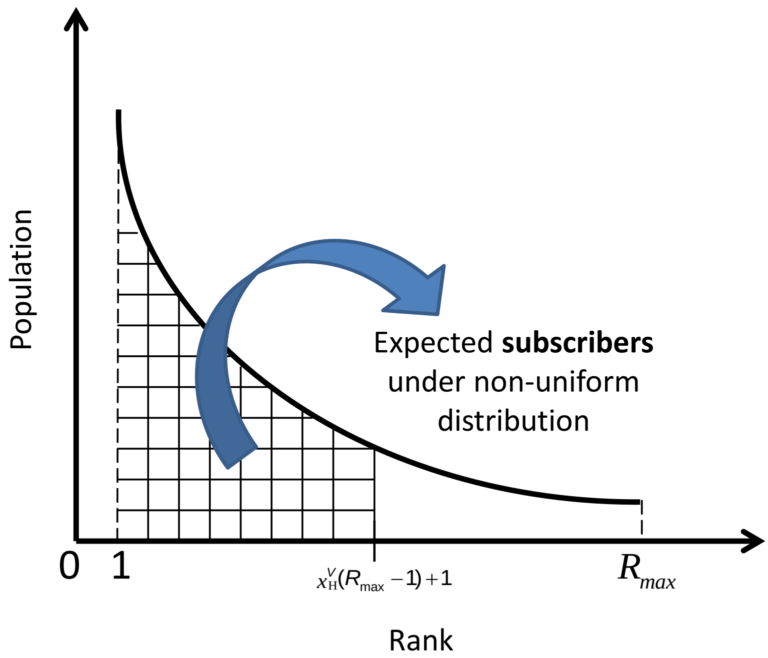

Figure 7. Demand with non-uniform consumer distribution

As in the case of the existing approach, R_{max} should be replaced by R_{inv} in order to capture the fact that a potential subscriber to the NGA services makes its decision to subscribe or not based on the expected number of subscribers to the NGA services rather than the whole population in a given country. Therefore, the investor maximizes the following equation with respect to R_{inv}.

$$\Pi^V = Pq_H^V - 920{,}03R_{inv}^2 + 174197R_{inv} \Rightarrow$$

$$\Pi^V = PCln\left(\frac{\alpha CR_{inv} - \alpha\beta C + \sqrt{(.)}}{2\alpha\beta C} + 1\right) - (920{,}03R_{inv}^2 + 174197R_{inv}) \tag{17}$$

where

$$\sqrt{(.)} = \sqrt{(\alpha\beta C)^2 + (\alpha CR_{inv})^2 - 2\alpha^2\beta C^2 R_{inv} - 4p\alpha\beta C} \tag{18}$$

4. Comparison of the two approaches

This section compares the outcomes of the two approaches in terms of the optimal investment level (R_{inv}^i) and the subsequent levels of subscribers (q_H^i), investment costs (C_{inv}^i), revenues

(R^i) and profits (Π^i), where the superscript $i = U,V$ stand for the existing and the proposed approach, respectively. The main part of the analysis that follows is conducted via numerical simulations due to the complexity of closed-form solutions for the endogenous variable R_{inv} in Eqs. (7) and (17). The optimal investment levels for the investor under both approaches are derived for 9 different scenarios concerning different values of the independent parameters β and φ. These parameters are chosen because the sensitivity analysis conducted showed that β and φ have the more powerful impact on total profits. In order to define the other independent parameters actual data from Greece are used. In particular, $C = 400000$ denotes the most populated municipal department in Greece (see Figure 5) and $P = 480$ denotes the annual average price per household. In addition, the level of α is chosen arbitrarily to 1 since sensitivity analysis shows that α does not significantly affect the final results.

A very significant observation is that the total number of HH is the same either if they are uniformly or non-uniformly distributed to the different municipal departments. In the former case the total number of HH is NR_{max}, whereas in the latter case the total number of HH is $Cln\left(R_{max}\right)$. Equating the two populations and solving with respect to N gives the fixed number of HH assigned by Shy (2001) to each municipal department:

$$N = C\frac{ln\left(R_{max}\right)}{R_{max}} \tag{19}$$

In the case of Greece, $R_{max} = 6122$ since there are 6122 municipal departments. Therefore, Shy (2001) assigns 570 HH to every of 6122 municipal departments. The final results can be summarized in Tables 1 and 2.

Table 1

		R_{inv}^U	R_{inv}^V	x_H^U	x_H^V	q_H^U	q_H^V
$\beta = 1,00$	$\varphi = 5.000$	54,70	279,14	0,99	0,99	31.160	2.252.684
$\beta = 1,00$	$\varphi = 10.000$	27,37	279,14	0,99	0,99	15.580	2.252.684
$\beta = 1,00$	$\varphi = 15.000$	18,27	279,14	0,99	0,99	10.387	2.252.684
$\beta = 1,45$	$\varphi = 5.000$	37,75	279,14	0,68	0,68	14.821	2.252.684
$\beta = 1,45$	$\varphi = 10.000$	18,92	279,14	0,67	0,68	7.140	2.252.684
$\beta = 1,45$	$\varphi = 15.000$	12,67	279,14	0,66	0,68	4.940	2.252.684
$\beta = 1,90$	$\varphi = 5.000$	28,84	279,14	0,51	0,52	8.632	2.252.684
$\beta = 1,90$	$\varphi = 10.000$	14,50	279,14	0,49	0,52	4.316	2.252.684
$\beta = 1,90$	$\varphi = 15.000$	9,76	279,14	0,46	0,52	2.878	2.252.684

Table 2

		C_{inv}^{U}	C_{inv}^{V}	R^{U}	R^{V}	Π^{U}	Π^{V}
$\beta = 1{,}00$	$\varphi = 5.000$	7.482.694	120.312.585	14.956.961	1.081.288.267	7.474.267	960.975.682
$\beta = 1{,}00$	$\varphi = 10.000$	3.747.679	120.312.585	7.478.489	1.081.288.267	3.730.810	960.975.682
$\beta = 1{,}00$	$\varphi = 15.000$	2.505.512	120.312.585	4.985.685	1.081.288.267	2.480.173	960.975.682
$\beta = 1{,}45$	$\varphi = 5.000$	3.563.062	120.312.585	7.113.898	1.009.948.067	3.550.836	889.635.482
$\beta = 1{,}45$	$\varphi = 10.000$	1.790.753	120.312.585	3.556.988	1.009.948.067	1.766.235	889.635.482
$\beta = 1{,}45$	$\varphi = 15.000$	1.204.186	120.312.585	2.371.439	1.009.948.067	1.167.253	889.635.482
$\beta = 1{,}90$	$\varphi = 5.000$	2.079.628	120.312.585	4.143.218	958.052.323	2.063.590	837.739.738
$\beta = 1{,}90$	$\varphi = 10.000$	1.051.994	120.312.585	2.071.725	958.052.323	1.019.731	837.739.738
$\beta = 1{,}90$	$\varphi = 15.000$	715.171	120.312.585	1.381.484	958.052.323	666.313	837.739.738

The values of β are chosen in order to ensure that $0 \le x(p) \le 1$, whereas the values of φ represent three different scenarios concerning the relationship between the investment cost function given by Eq. (1) and the real-cost-data-based investment cost function given by Eq. (8). Figure 8 shows that regardless of the particular value of φ, Eq. (1) always underestimates the investment cost of the higher populated MDs and overestimates the investment cost of the lower populated MDs. In particular, the lower the value of φ, the more underestimated (overestimated) the investment cost of the higher (lower) populated MD becomes. This implies that the lower the value of φ, the lower populated is the MD that the cost functions of Eqs. 1 and 8 result to the same deployment cost.

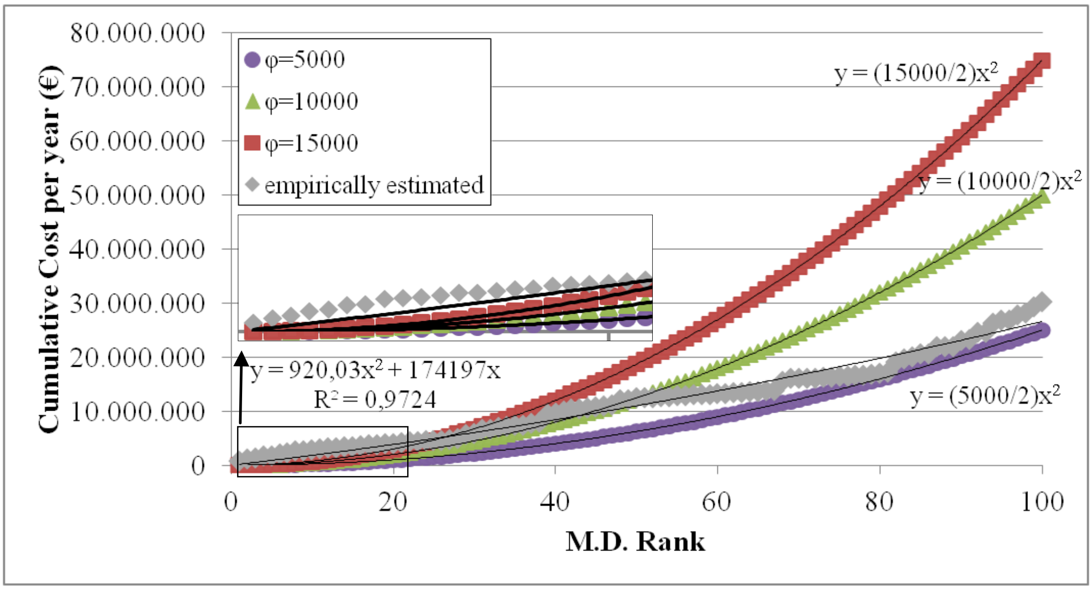

Figure 8. Cumulative investment cost per year

A number of observations derived by the analysis of Table 1 and 2 are instructive. An increase in φ and/or β leads R_{inv}^{U} to decrease, whereas R_{inv}^{V} is not affected by a change in φ and/or β. The comparison between R_{inv}^{U} and R_{inv}^{V} shows that R_{inv}^{V} is always much greater than R_{inv}^{U}. Since the indifferent subscriber is almost the same in both approaches, it is reasonable that the number of subscribers is higher under the proposed than the existing approach. This, in turn, results in higher revenues under the proposed than the existing approach since it is assumed that the price of the service is the same under both approaches. Concerning the deployment cost, it is shown that an increase in φ and/or β leads C_{inv}^{U} to decrease, whereas C_{inv}^{V} is not affected by a change in φ and/or β. The comparison between C_{inv}^{U} and C_{inv}^{V} shows that C_{inv}^{V} is always much greater than C_{inv}^{U}.

Another very significant finding is that an increase in β negatively affects the investor's profits under both approaches. However, the investor's profits under the proposed approach are much greater than the investor's profits under the existing approach. The main reason for this result is that the uniformity assumption underestimates the number of households in the most populated areas where the cost per household is lower. This partially interprets the investor's decision to limit its investment level under the existing approach. This, in turn, decreases the expected number of subscribers, which also decreases the actual number of subscribers.

Therefore, the departure from the uniformity assumption allows capturing the fact that subscribers who place a higher valuation to broadband subscription tend to live in higher populated areas where the cost per household is lower. It is thus obvious why the proposed approach leads to much higher investment level than the existing approach.

5. Conclusions

The aim of this paper was twofold; firstly to investigate whether the traditional quadratic convex cost form is suitable for being used in NGA investments; and secondly to propose a more realistic demand model. Thus, (i) an empirical cost analysis was conducted for a real case of NGA deployment and a real-data-based cost function was obtained; and (ii) the Pareto consumer distribution was used to reflect the greater (lower) positive impact of NGA investments on the willingness to pay of the consumers who live in more (less) populated areas.

In the case of the investment cost, the existing assumption of the quadratic convex cost form was found inaccurate when compared with the cost estimation conducted for the 100 major municipal departments in terms of HH in Greece. The methodology used for the cost estimation took into account the underlying street morphology complexity that the classic approaches ignore due to the use of the simple geometric models. In particular, the cost function used in the existing models always underestimates the investment cost of the higher populated areas and overestimates the investment cost of the lower populated areas.

Concerning the demand for the new fibre-based services, it was found that the existing demand models with network externalities always underestimate such demand since they assume uniform consumer distribution. The reason is that the uniformity assumption underestimates the number of households in the most populated areas where the cost per household is lower. Therefore, the optimal investment level from an investor's perspective is always much higher under the proposed approach than the traditional one.

Although this article provided some very useful results, there are many directions to be extended in order to overcome its limitations. First, the derived cost structure is based on actual cost data from Greece and hence its robustness should be investigated by using cost data from other countries. Second, this article neglects the impact of competition on the retail price, as well as, regulatory issues concerning the access price that an access seeker should pay to the investor in order to have access to the new fibre-based infrastructure.

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References

Anderson, S. P., Goeree, J. K., & Ramer, R. (1997). Location, Location, Location. *Journal of Economic Theory*, *77*(1), 102-127.

Ansari, A., Economides, N., & Ghosh, A. (1994). Competitive positioning in markets with nonuniform preferences. *Marketing Science*, *13*(3), 248–273.

Cardillo A., Scellato S., Latora V., and Porta S. (2006). Structural properties of planar graphs of urban street patterns. *Phys Rev E Stat Nonlin Soft Matter Phys*, *73*, p. 066107.

Casier K., (2009). Techno-Economic Evaluation of a Next Generation Access Network Deployment in a Competitive Setting. *PhD degree at the Faculty of Engineering of the Ghent University*, Ghent, Belgium.

Chatzi, S. and Tomkos, I. (2011). Techno-economic study of high-splitting ratio PONs and comparison with conventional FTTH-PONs/FTTH-P2P/FTTB and FTTC deployments. *Optical Fiber Communication Conference and Exposition (OFC/NFOEC), 2011 and the National Fiber Optic Engineers Conference*, 1-3.

Czernich, N., Falck, O., Kretschmer, T., & Woessmann, L. (2011). Broadband Infrastructure and Economic Growth. *The Economic Journal*, *121*, 505-532.

Colle D., Casier K., and Pickavet M. et al. (2008). A clear and balanced view on FTTH deployment costs. *The Journal of the Institute of Telecommunications Professionals*, *2*(3), 27-30.

EC (European Commission) (2010a). European Commission Recommendation on regulated access to Next Generation Access Networks (NGA). Available at: <http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2010:251:0035:0048:EN:PDF>.

EC (European Commission) (2010b). *A Digital Agenda for Europe*. Available at: <http://ec.europa.eu/information_society/digital-agenda/publications/index_en.htm>.

Economides, N. (1996). The Economics of networks. *International Journal of Industrial Organization*, *14*(6), 673-699.

FCC (Federal Communications Commission) (2010). *Connecting America: The national broadband plan*. Available at: <http://www.broadband.gov/download-plan/>.

Flamm, K., & Chaudhuri, A. (2007). An analysis of the determinants of broadband access. *Telecommunications Policy*, *31*(6-7), 312-326.

Foros, O. (2004). Strategic investments with spillovers, vertical integration and foreclosure in the broadband access market. *International Journal of Industrial Organization*, *22*(1), 1-24.

Hellenic Statistical Authority (2001). Population Census. Available online at: <http://www.statistics.gr/>

ITU-T (2009). G.984.1: Gigabit-capable passive optical networks (GPON): General characteristics. Available online at: <http://www.itu.int/rec/T-REC-G.984.1/>

Katz, M. L., & Shapiro, C. (1985). Network Externalities, Competition, and Compatibility. *The American Economic Review*, *75*(3), 424-440.

Katz, R., Vaterlaus, S., Zenhäusern, P., & Suter, S. (2010). The impact of broadband on jobs and the German economy. *Intereconomics*, *45*(1), 26-34.

Kyriakidou, V., Michalakelis, C., & Varoutas, D. (2011). Applying Zipf's Power Law Over Population Density and Growth as Network Deployment Indicator. *Journal of Service Science and Management*, *4*(2), 132-140.

Maniadakis D. and Varoutas D. (2012). Structural Properties of Urban Street Networks for FTTH deployment. *11th Conference of Telecommunication, Media and Internet Techno-Economics (CTTE 2012)*, Athens, Greece.

Mitcsenkov A., et al. (2010). Geographic Model for Cost Estimation of FTTH Deployment: Overcoming Inaccuracy in Uneven-populated Areas. *Communications and Photonics Conference and Exhibition (ACP)*, Shanghai.

Nitsche, R., & Wiethaus, L. (2010). Access regulation and investment in next generation networks — A ranking of regulatory regimes. *International Journal of Industrial Organization*, *29*(2), 263-272.

OpenStreetMap (2012). Available online at: <http://www.openstreetmap.org/>

Page, W. H., & Lopatka, J. E. (1999). Network externalities. *Encyclopedia of Law and Economics*, 952-980.

Porta S., Crucitti P., and Latora V. (2006). The network analysis of urban streets: a primal approach. *Environment and Planning B: Planning and Design*, *33*, 705-725.

Preston, P., Cawley, A., & Metykova, M. (2007). Broadband and rural areas in the EU: From technology to applications and use. *Telecommunications Policy*, *31*(6-7), 389-400.

Reynolds, T. (2009). *The role of communications infrastructure investment in economic recovery (DSTI / ICCP / CISP (2009) 1 / FINAL)*. *Policy*. Paris. Available at: <http://www.oecd.org/dataoecd/4/43/42799709.pdf>.

Shumate P. W. (2008). Fiber-to-the-home: 1977-2007. *J Lightwave Technol*, *26*(9-12), 1093-1103.

Shy, O. (2011). A Short Survey of Network Economics. *Review of Industrial Organization*, *38*(2), 119-149.

Soo K.T. (2005). Zipf's Law for cities: a cross-country investigation. Regional Science and Urban Economics, *35*, 239-263.

Teppayayon, O., & Bohlin, E. (2010). Broadband Universal Service in Europe: Review of Policy Consultations 2005-2010. *Communications and Strategies*, *80*, 21-42.

Zipf G.K. (1949). Human Behavior and the Principle of Least Effort. Cambridge, Massachusetts: Addison-Wesley.