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FINAL REPORT ON DOCUMENTATION OF STUDENTS' UNDERSTANDING

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Τίτλος Πρότασης

Epistemological and Didactical Aspects Related to the Concept of Periodicity Across Different School Subjects.

(Ελληνικά) **Επιστημολογικές και διδακτικές απόψεις σχετικές με την έννοια της περιοδικότητας σε διαφορετικά σχολικά μαθήματα.**

Φορέας Υποδοχής

ΑΝΩΤΑΤΗ ΣΧΟΛΗ ΠΑΙΔΑΓΩΓΙΚΗΣ ΚΑΙ ΤΕΧΝΟΛΟΓΙΚΗΣ ΕΚΠΑΙΔΕΥΣΗΣ (Α.Σ.ΠΑΙ.Τ.Ε.)

Ερευνητική μονάδα φορέα υποδοχής

ΓΕΝΙΚΟ ΤΜΗΜΑ ΠΑΙΔΑΓΩΓΙΚΩΝ ΜΑΘΗΜΑΤΩΝ (ΓΕΤΠΜΑ)

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ΤΡΙΑΝΤΑΦΥΛΛΟΥ-ΚΑΛΛΙΩΡΑ ΧΡΥΣΑΥΓΗ

Στοιχεία Επιστημονικής Υπευθύνου

ΣΠΗΛΙΩΤΟΠΟΥΛΟΥ ΒΑΣΙΛΙΚΗ, Καθηγήτρια

Chapter 1: INTRODUCTION

1.1 The overall aim of the research project

The present study is part of a research project that intends to take a close look at pedagogical practices adopted in mathematics and physics classrooms in Greek secondary schools on topics that are related to periodicity.

The notion of periodicity is very close to students' experiences since it appears in nature all around us (the annual motion of the earth around the sun, the tides etc.). Moreover, periodicity is a considerable part of the scientific culture of every student in his secondary and post-secondary studies. Particularly, students come to terms with this notion in different school subjects such as mathematics and science (oscillations in physics, periodic functions in trigonometry and calculus) and in post-secondary studies (Fourier series, signal processing etc.). Hence, connecting aspects of periodicity from different school disciplines is important for students' future studies in mathematics, science and engineering. In general, any motion that repeats itself identically at regular intervals is called 'periodic motion'. As we observe the periodic motion shown on a graph, we are looking at a function that repeats periodically and sinusoidal functions are of this type (King, 2009). Even though periodicity is central in a variety of disciplines, an extensive search of the literature shows that there is a limited number of studies that focus on its understanding. These studies conclude that most students' concept image of periodicity is based on time-dependent variations (Shama, 1998) while usually they consider any repetition as periodical (Buendia & Cordero, 2005).

1.2 Summative results from the first phase of our research

To meet the aims of our inquiry, in the first phase of our project we analyzed Greek textbooks taken from the subjects of physics, mathematics, astronomy and applied technologies. This analysis was carried on selected chapters in the topics of periodic motions and periodic functions respectively (Triantafillou, Spiliotopoulou & Potari, 2013a, b; Τριανταφύλλου & Σπηλιωτοπούλου, 2013; Τριανταφύλλου, Σπηλιωτοπούλου, Σιδερής & Κεχράκος, 2013).

Particularly, our analysis provided evidence about the reasoning and argumentation processes that the Greek mathematics and science textbooks adopted in the thematic units presenting the concept of periodicity and its properties. Four main categories of modes of reasoning: the empirical, the logical-empirical, the nomological and the mathematical. Each mode of reasoning plays a different role in conceptualizing aspects of periodicity. The nomological and logical-empirical seem to be the main syllogisms in all texts. Particularly, the high frequency of the main, and the initial claim in the category addressed as nomological modes of reasoning, means that Greek science and mathematics textbooks give emphasis on the teaching and learning of theoretical statements in the forms of definitions (main claim or initial claim modes of reasoning). In mathematics, these modes of reasoning lack any reference to time variations revealing the abstract profile of the subject. Physics texts try to relate features of the notion of periodicity with everyday life phenomena by employing empirical modes of reasoning. Even in this case, though, the recalling experience mode of reasoning was present only in early years at school, and was absent in mathematics. These results indicate that the above educational communities share traditional views on thinking as mental processes, while sensuous experiences are considered as less valuable in learning. All the above findings help us to identify what is valued as the most important rational processes in the Greek mathematics and science educational communities when aspects of periodicity are presented in the school texts. The analysis of the sequence of modes of reasoning in the two thematic units raises the following issues that seem to influence the argumentation developed in mathematics and physics texts: (i) Pragmatic considerations on the text understanding and scientific argumentation discourse and (ii) epistemological differences when ascending from observations to generalizations.

We also highlighted some common practices in the analysis of the proposed exercises among the subjects, for example, aspects of periodicity are tackled almost exclusively by means of sinusoidal functions and graph related practices were mostly on sketching graphs of particular situations. Furthermore, in physics, functions such as $f(x)=e^{-bx}\sin(\omega x)$ that fluctuate in a periodical way on the x-axis, are considered as functions that model periodic motions. This disciplinary understanding of periodicity could encourage incorrect generalizations, such as, any type of repetition is periodical.

1.3 The present report

Our aim in the present report

The aim of the present study is to document undergraduate students' understanding of aspects and properties of the notion of periodicity when confronted with various tasks mostly taken from their secondary school textbooks. In order to accomplish the above goals we designed and contacted three separate research activities (case studies). This was considered necessary in order to study different aspects of periodicity within a broad range of participants, as well as to integrate this kind of research activities in researcher's teaching experiences as in the case of Act 1.

Our research questions are

- How do students interpret and connect textual and visual representations of periodic motions taken from their school textbooks? (Act1&Act2&Act3)
- How do students/future engineering teachers transform the verbal and textual elements in providing teaching explanations on periodicity teaching plans? (Act1)
- What type of difficulties do students meet when they have to make connections between the visual representations of different aspects of a periodic phenomenon? (Act2)
- How do students understand certain characteristics of periodic motions in the case of sinusoidal functions? (Act2&Act3)
- How do students interpret graphs of periodic motions and do they distinguish them from graphs of repeated but non-periodic motions? (Act3)
- What type of examples of motions do they provide that could be represented by graphs of repeated functions? (Act3)

Activity 1

The researcher has taught the undergraduate course "Didactics of specific Subjects" in the department of "Mechanical Engineering Educators" of the School of Pedagogical and Technological Education (ASPETE), for two hours per week in the Spring Semester. During this course her main teaching goal was to introduce students with teaching techniques as future Engineering teachers.

A teaching experiment has been planned in order to document how a group of undergraduate engineering students understand different aspects of periodic phenomena, the level of their awareness of school textbooks elements on relevant topics and how they plan and develop teaching interventions on periodic phenomena. This teaching experiment is expected to help us identifying engineering students' level of awareness in order to make appropriate connections between the abstract scientific knowledge and the concrete applied cases. In this direction, before the final exam two engineering topics are introduced to the students. Particularly students were asked to design teaching activities on "The bicycle Dynamo" (Task 1/Topic 1) and "Cars' suspension and tire springs" (Task 2/Topic 2). Both topics were considered as real life applications of periodicity; were relevant to their undergraduate studies; and relevant to students' interest as future mechanical engineers and as future teachers. 86 undergraduate mechanical engineering students participated in this activity. The above extracts were given to the students before the final exam. The students were asked to study the texts and be prepared by identifying appropriate information (from internet or other resources) to support the understanding of the particular topics.

The main aim was to document how the students conceptualize periodic behaviors and if and how they develop ways to integrate mathematical models of periodic functions in their future teaching activities while teaching the specific topic.

Activity 2

A research questionnaire has been developed with open questions on the topic "Study of the motion of the linear spring". This questionnaire was distributed to 70 undergraduate engineering students from four different departments in two different Technological Institutions.

Activity 3

A research questionnaire with open questions has been formulated on the topic "Periodic motions and periodic functions". This questionnaire was distributed to undergraduate engineering students from two different departments in two different Universities and in two different Technological institutions. This questionnaire was distributed to 132 undergraduate students from two University and two technological Institutions.

Outline of this report

The theoretical framework will be presented in chapter 2 and in chapter 3 the methodology applied. In Chapter 4 the results of the qualitative and quantitative analysis will be presented and issues that emerged from this analysis will be highlighted. Finally, in Chapter 5 we present the general conclusions and issues we want to research in depth in the next phases.

Chapter 2: THEORETICAL FRAMEWORK

In this study we adopt a socio-cultural perspective where knowledge (about worldly objects and events) and linguistic knowledge (about sign forms) are mediated in and through the activities an individual engages in (Vygotsky, 1978). According to this point of view thinking about physical phenomena could enrich and promote the development of students' scientific knowledge (Buendia & Cordero, 2005).

Particularly, we adopt the activity theory perspective (ibid.) that recognizes mathematical and scientific school practices as different cultural activities, since they have different goals, purposes, and objectives. In general, mathematics deals with patterns and relationships (NCTM, 2000) while science deals with the understanding of every day and natural world phenomena (NRC, 1996). Thus, understanding the notion of periodicity and its properties involves creating a coherent framework where these ideas and practices are meaningful at an individual level.

Current emphases in mathematics and science studies around the world are on scientific and mathematical literacy though the interpretation of texts and textual elements (Norris & Philips, 2003; Roth, 2002). Hence, the analysis of how aspects and properties of a common scientific notion are presented, supported, and validated in science and mathematics texts transcends the two communities and is necessary for a meaningful conceptual understanding. In this direction, we take a literacy perspective which recognizes that mathematics and science as school subjects draw on a range of communities of meaning and on different type of genres that give to the members of the community the feeling of inclusion and identity (Gee, 2003). As genres we refer to abstract, socially recognised ways of using language (definitions, proofs, examples, visual representations as graphs and diagrams etc.). Olson (1994) emphasizes that school subjects belong to different textual communities, and to master a school subject is to develop the ability to manipulate different texts:

> To be literate it is not enough to know the words; one must learn how to participate in the discourse of some textual community. And that implies knowing which texts are important, how they are to be read and interpreted, and how they are to be applied in talk and action. (Olson, 1994, p. 273).

The notion of scientific literacy brings literary traditions, conventions and engagement in activities that mediate linguistic and socio-cultural practices to the foreground, and decreases the focus on individual knowledge deficits (Roth, 2002). Although a scientific theory or a scientific concept is independent of any given text, they both require the use of text so as to be expressed (Norris & Philips, 2003). Texts, although being fixed, invite and allow reader's interpretation and reinterpretation. Therefore, "the reader must take note of the very words, the very data, and other textual elements, and bears the burden of delivering interpretations" (ibid., p. 232). Love and Pimm (1996) noted that although the implied relation between the reader and the text is inherently passive, "the most active invitation to any reader seems to be to work through the text to see why the particular 'this' is so" (p. 371).

Interpreting texts and the accompanying visual representations (diagrammatic or symbolic images) in school textbooks becomes a major and distinguishing activity in science, mathematics and applied technology classrooms. This practice is classified as a component of mathematical literacy in the OECD's International Programme for Student Assessment (PISA) and is related to scientific literacy as well (Roth, 2002). In the engineering context, many studies ascertain professionals' need for continuous reflection and interrelation with the referential situation in order to achieve their goal and proceed with their working activity (Triantafillou, 2011). The PISA study determines scientific literacy in three dimensions First, *scientific concepts*, which are needed to understand certain real-life phenomena with a variety of applications such as science in life and health or science in technology. Second, *scientific processes*, which are centred on the ability to acquire, interpret and act upon evidence. Third, *scientific situations*, selected mainly from people's everyday lives (OECD, 2007). All the above support the need to consider the case of reading and interpreting verbal and visual data on a domain-specific situation as a crucial skill that has to be given special attention in the school curriculum in general and in engineering education in particular. Furthermore, the visual features in understanding aspects of periodicity are crucial and worth of being studied since the concept has multiple expressions in practical and theoretical situations (Buendia & Cordero, 2005).

Within the school curriculum, graphic competencies are central practices in mathematics and science classrooms (Roth & McGinn, 1997). Different theoretical perspectives have been adopted for analyzing students' making sense of graphs in the mathematical context. From a cognitive perspective, graph sense means "looking at the entire graph (or part of it) and gaining meaning about the relationship between the two variables and, in particular, of their pattern of co-variation" (Leinhardt, Zaslavsky & Stein, 1990, p. 11). Under the embodied cognition perspective, bodily activities are involved in conceptualizing graphical representations as dynamic processes (Nunez, 2007) while from a cultural-semiotic perspective, sensual experiences are important in making sense of motion graphs (Radford, Cerulli, Demers, & Guzman, 2004). Moreover, the conceptual movement from graphs to a situation that they represent is termed 'translation' which presupposes the practice of 'making sense out of graphs' (Roth, 2004, p. 77). In the engineering context, translating domain-specific graphs is a central action since graphs mediate collective scientific activities such as communicating and constructing facts (Roth & McGinn, 1997). In the present study, 'making sense out of graphs' means interpreting graphical features and describing a situation that could be represented by them.

The individuals in order to become aware, understand and communicate new cultural realities, such as translate the graphical representation of the periodic motions of helical springs in the context of the suspension system of cars (Task 1/Act1) use various 'semiotic means of objectification (Radford, 2003). Radford (ibid., p.41) defines objectification as a creative process of noticing something while the semiotic means that mediate this process are called "semiotic means of objectification". These means can be acknowledged in their texts and are the scientific or non-scientific signs they use in their responses when trying to transform the textbook extract into a learning activity.

Chapter 3: Methodology

All the research activities planned with open-ended questions. The data analysis was based on the grounded theory research perspective (Strauss & Corbin, 2007). In particular, we are looking for categories and patterns emerging from the analysis of the raw data. More specifically, inductive content analysis (Mayring, 2000) was applied on specific thematic units and a coding system of categories has been produced. Moreover, the technique of systemic networks (Bliss, Monk & Ogborn, 1983) has been adopted not only as a form of representing our scheme of categories, but also as an analytic tool. This scheme supported the quantification of the emerging categories in all the thematic units selected to be analyzed.

3.1 The context

At undergraduate level, all students in scientific direction fields encounter aspects of periodicity in their first year Calculus and Fourier analysis courses. Fourier analysis is a prerequisite course for studying signal processing in the fields of Informatics and Electronics. Thus, for all the participants, periodicity is considered as an important scientific notion not only for their academic studies, but for their professional life as well.

3.2 The participants

The participants in all research activities were 288 undergraduate students from Greek University and Technological Institutions.

Activity 1

The participants were 86 undergraduate mechanical engineering students. The students were mostly in the 6the semester of their studies. The data are from their final exam on a Didactics course. During this course the students are introduced to teaching techniques, designing interventions and developing learning activities. They also obtain experiences in analyzing school textbooks and handling text and images for the enhancement of their students' learning. Two tasks have been given to students based on two school texts: "The bicycle Dynamo" (Task 1) and "Helical springs in the suspension system of cars" (Task 2). Data from the second task are discussed in this study.

Activity 2

The participants were 70 undergraduate engineering students. The students were from the following fields: 13 structural engineering, 16 mechanical engineering and 41 from the field of Electronics. The students were mostly in the second semester of their studies.

Activity 3

The participants were 132 undergraduate students from two different University Departments and two different Technological Institutions. 19 students were studying Mathematics, 70 were studying Informatics and 43 were studying Electronics. The students were at different stages of the courses (58 were in the second semester, 45 in the fourth semester and 29 in their sixth or remaining semesters). The data was collected by means of a questionnaire administered to the participants at the end of the academic year 2012-13. The questionnaire was completed in one teaching hour during a mathematics course in the case of the engineering students and during a course in mathematics education in the case of students in mathematics. The questionnaire was based on three different practices relating to periodicity (exemplifying; making sense out of graphs; and modelling periodic motions).

In Table 1 we present the number of participants in the different departments

Table 1: The number of participants from the different departments in different activities

Institutions	Departments	No of Activity	No of participants
Technological Institutions	Electronics	2	41
		3	43
	Informatics	3	31
	Structural Engineers	2	13
	Mechanical Engineers	1	86
		2	16
Universities	Informatics	3	39
	Mathematics	2	19
Total 288 participants			

3.3 The tasks

Activity 1

Two textbook extracts were given to the students before their final exam on their course on Didactics.

Task 1

This extract is taken from the textbook "Car systems". The topic they were specifically asked to prepare to teach in a class referred to "Helical springs" as parts of the car suspension system. A part of the textbook extract given to students is presented in Fig. 1.

Title: Cars' suspension system; *Subtitle:* Helical springs

Main text: Presents Hooks' Law and explains the visual representations as follows: In Fig. 4.27a we see two springs with different stiffness (different constant c). The spring with c=25000N/m [is named on the graph as stiff] is considered as more stiff than the other with c=7500N/m [is named on the graph as floppy]. In the common springs, the characteristic is a straight line and is called 'linear characteristic'. If the spring stiffness increases not linearly in respect to the change of the springs' deformation (Fig. 4.27b) then we have a progressive characteristic curve".

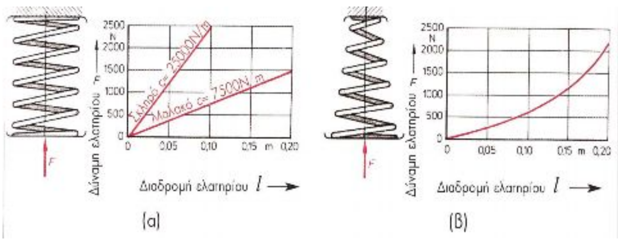


Fig.4.27: Characteristic lines of helical springs: (a) Linear; (b) Progressive

Fig. 1: A part of the textbook extract

Students' task was formed as follows: *"Describe in an analytical way how you will explain to your students the significance of the different characteristics of helical springs (linear and progressive) that are represented in Fig. 4.27 in controlling cars' vibrations".*

Task 2

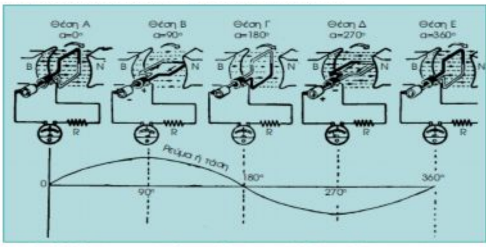
This extract is taken from the textbook "Car system". The topic they were specifically asked to prepare to teach in a class "The Bike Dynamo" as part of the Generators. A part of the textbook extract given to students is presented in Fig. 2.

Title: **Generators**

Subtitle: **Operation principles of AC generators**

Main text: [Describes systematically the materials and the conditions required in order a DC generator could operate. Then the main scientific entities (Magnetic Filed B (in Tesla), a coiled copper wire with length l (in meters) is rotated in the magnetic field, the velocity v (in m/s^2) of this rotation and the generated Voltage E in Volts) are introduced and their relation is described in a symbolic form as follows: **E =B l v sina (in Volts)**

In the following scheme (F. 2.2) the course of this phenomenon is presented as the coils of the copper wire is rotated relatively in the magnetic field. In the same scheme appears the alternate generated Voltage.



Σχ. 2.2: Πορεία της ανάπτυξης εναλλασσόμενης Η.Ε.Δ. σε πλαίσιο στρεφόμενου μέσα σε σταθερό μαγνητικό πεδίο.

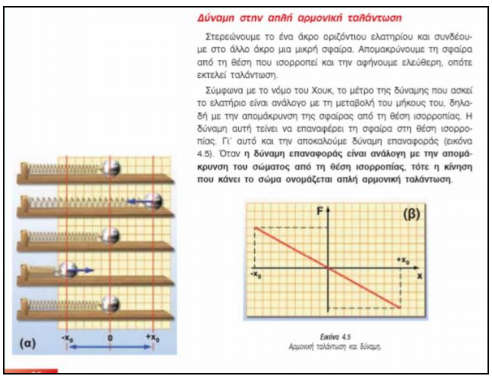
F. 2.2: The course of development of the generated Voltage in the ends of the rotating frame in a constant magnetic field.

Fig. 2: A part of the textbook extract

Students' task was formed as follows: Design proper teaching activities under the specific teaching goal: *"The student should be able to substantiate scientifically the relation between the rotation of the bicycle wheel and the brightness of the light bulb".*

Activity 2

The questionnaire was constituted of three main tasks. The two are based on extracts from physics textbook on the topic of "Studying the linear spring's motions as a specific case of harmonic oscillations", while the third was asking the students to conceptualize connections between the above tasks.



Δύναμη στην απλή αρμονική ταλάντωση

Στερεώνουμε το ένα άκρο οριζόντιου ελατηρίου και συνδέουμε στο άλλο άκρο μια μικρή σφαίρα. Απομακρύνουμε τη σφαίρα από τη θέση που ισορροπεί και την αφήνουμε ελεύθερη, οπότε εκτελεί ταλάντωση.

Σύμφωνα με το νόμο του Χουκ, το μέτρο της δύναμης που ασκεί το ελατήριο είναι ανάλογο με τη μεταβολή του μήκους του, δηλαδή με την απομάκρυνση της σφαίρας από τη θέση ισορροπίας. Η δύναμη αυτή τείνει να επαναφέρει τη σφαίρα στη θέση ισορροπίας. Γι' αυτό και την αποκαλούμε δύναμη επαναφοράς (εικόνα 4.5). Όταν **η δύναμη επαναφοράς είναι ανάλογη με την απομάκρυνση του σώματος από τη θέση ισορροπίας, τότε η κίνηση που κάνει το σώμα ονομάζεται απλή αρμονική ταλάντωση.**

Photo 1

Task 1

The book extract which is presented in Picture 1 shows an experimental arrangement that refers to the study of the movement of the spring and is included in the 3^{rd} Grade Secondary School Science book. The fist task (extract from Physics, Grade 9 textbook), was about Hooke's Law (i.e.

the magnitude of the restoring force is directly proportional to the spring’s deformation) as a sufficient condition of spring's Harmonic motion. The students were asked to interpret the graph showing force vs. displacement for a linear spring. The questions were as follows: *“(a) What is the purpose of the author? What does he want to show his students? Which way does he use in order to persuade them? How would you explain the connection of the phases of the experiment in Picture 4.5(a) with the graph in Figure 4.5(b)? More specifically, how would you explain the existence of negative values of the power as shown in Picture 4.5(b)?”*

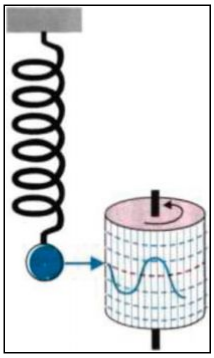


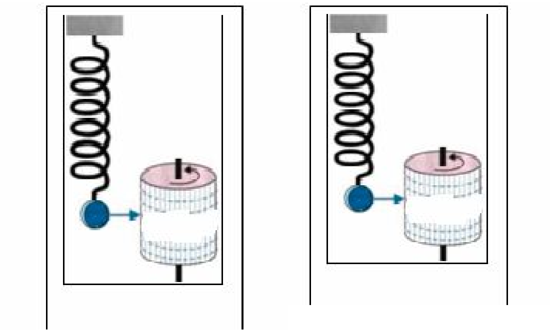
Photo 2

Task 2a

The second task (extract from Physics, Grade 11 textbook), was referring to the sinusoidal function that describes the x(t) displacement of a body attached on a linear spring. The students were asked to interpret the graph showing the x(t) displacement for a linear spring in a constructed situation. They had also to examine the conditions for this experiment to be successful.

Task 2b

The students were asked to depict the motion of the spring as follows: *“Try to depict the movement of the spring as is it written in the cylinder when its rotation takes place slower and quicker in relation to the movement of the spring. Justify your answer”.*



Much slower **Much faster**

Task 3

In Task 1 and 2 the students had to made connections between the real phenomenon and its visual representations while in the third task they required to relate the information provided in the above tasks since both task are refereeing to the same phenomenon. The question was as follows: *“The two graphs in Photos 1 and 2 refer to the same phenomenon. What relationships are depicted in each one of the two graphs? What are their differences? Analyze and justify your answer”.*

Activity 3

Task 1

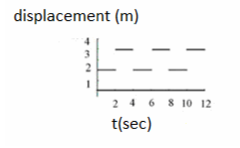
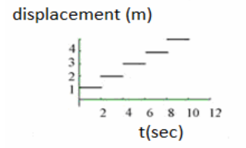
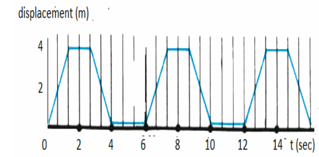
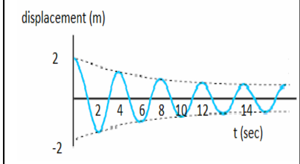
If there were asked you to justify which of the following motions is the best example of periodic behaviour which one will you choose? The motion of a swing; the motion of a ferryboat that travels between two ports; the rotation of the earth around the Sun

In the first task they were asked indicate and argue which of the following examples of motions have the best periodical behaviour under real conditions (a swing; a ferry that moves between two ports; the annual motion of the earth around the sun). All examples were used as examples of periodic motions in their school mathematics and science textbooks.

Task 2

In the second task, four graphs were given to the students that all represent displacement in meters versus time in seconds. Table 2 shows the four graphs and the resources used.

Table 2: The graphs in Task 2/Act 3

Graph 1 (Buendia & Cordero, 2005)	Graph 2 (Buendia & Cordero, 2005)	Graph 3 (Greek mathematics textbook)	Graph 4 (Greek physics textbook)
			

Graph 1 and Graph 3 represent periodic motions while Graph 2 and Graph 4 represent non-periodic motions. Moreover, Graph 1 and Graph 2 represent non-continuous motions. Two tasks were given to the students referring to each graph separately. Task 2a: *Does this graph represent a periodic motion? Justify your answer*. In this task, students are asked to focus on how the repetition is accomplished in order to distinguish the periodic from the non-periodic motions, as well as justifying their response. Task 2b: *Provide an example that could be described by this particular graph*. In this task the students were asked to assign to each graph a motion that could be represented by it.

Task 3

The students after a collective discussion on the motion of the ferry-wheel they had to answer the following questions: *"If we regard that you start from point A, can you define the function that describes the relation of the height of the wagon to time?"* and *"If the wagon moves without stopping for 4 minutes, try to graphically represent in Figure 1 the change of height in relation to time throughout its movement."*

General comment: Many of the above tasks are viewed as 'scientific literacy tasks' for the undergraduate students since they refer to a central 'scientific concept' that of periodicity and its applications in their area of expertise, is related with the 'scientific process' of interpreting domain-specific textual elements (visual and verbal) while most of the 'scientific situations' are placed on people's everyday life situations.

Data analysis

Inductive content analysis (Mayring, 2000) was applied on students' responses in all activities and tasks.

Specifically, in the Activity 1 a coding system of categories has been produced (Bliss et al, 1983). The unit of analysis is each student's response on the particular task. The final scheme of the categories emerging from the analysis of the students' responses is presented in the form of a systemic network. Subsequently, we reported the frequency and the valid percentage of students' responses on the categories that emerged. At the end, by taking into consideration of the semiotic means of objectification in students responses when we they were translating the textual information into learning activities we identified three levels of students' pedagogical content knowledge awareness.

The same category scheme was used to analyze data on Act 1/Task 2 and Act2/Task 1. The reason to use the same category scheme for all the three tasks are the following: (a) All tasks are addressed to engineering undergraduate students; (b) all tasks have similar goals (interpreting textual information in order to explain specific periodic phenomena); (c) Act1/Task 1& Task 2 are addressed to the same participants (86 undergraduate engineering students) who are asked to design teaching activities on the specific tasks; (c) Act2/ Task 1 is referring to the same phenomenon with Act 1/Task 1 (Hooks' Law). The difference is that the (Act 2/Task 1) is taking place on a school-based experimental context while the latter (Act 1/Task 1) in an applied context (car suspension system).

The use of the same category scheme could help us to identify differences in students' responses in the three tasks.

In the Act 2/ Task 2 & 3 we analyzed the data and two category schemes produced as well and we follow the same procedure with Act3/ Task 1&2&3. Particularly in the latter case we separated students' responses in Task1, in Task 2 we distinguishing periodical from non-periodical motions (Task 2a); justifying their responses (Task 2b); analysing the situations that were created by the students in respect of the salient features of the graphs they took into consideration (Task 2c); and categorizing the type of examples used by the students (Task 2d); and in Task 3.

The produced schemes of categories were tested by the two researchers (one specialized in Mathematics education and the other in Science education) through the whole set of data. It is considered to be general enough to describe the diversity of the students' responses in the different tasks.

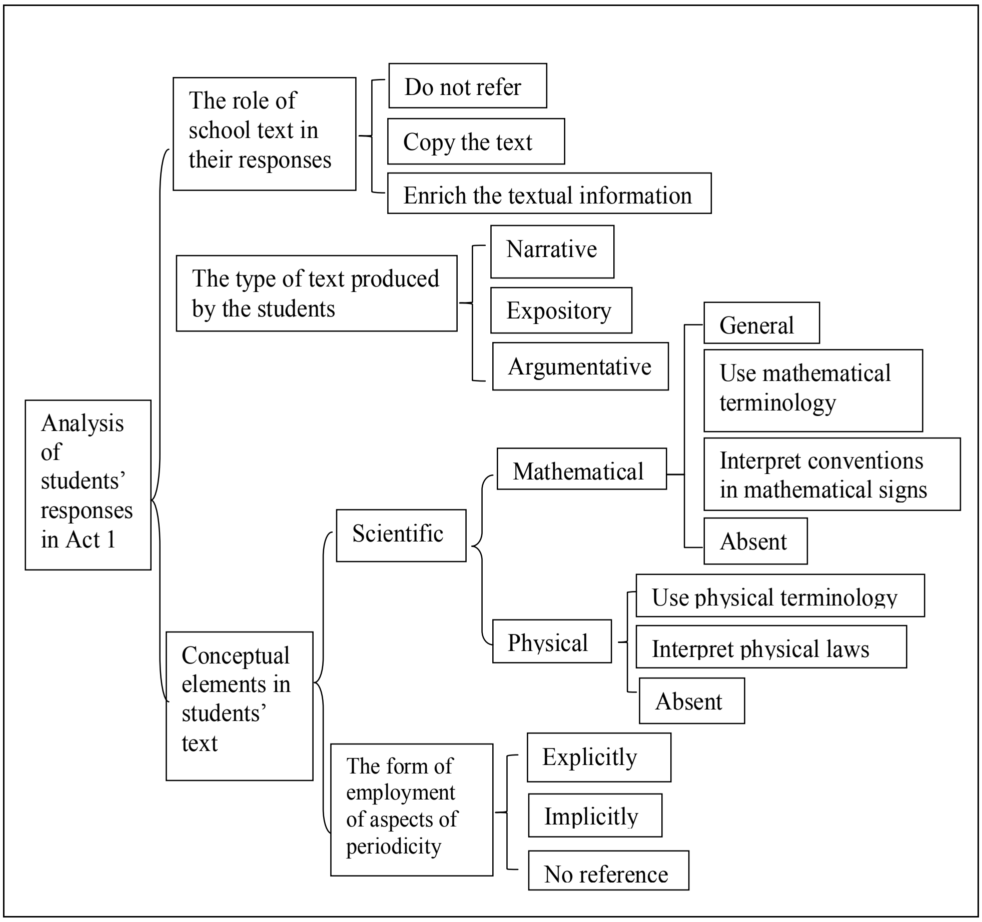
At the end, we reported the frequency and the valid percentage of students' responses and all categories that emerged from all activities and all tasks.

Chapter 4: RESULTS

4.1a The results of the qualitative and quantitative analysis

Act 1/ Task 1 & Task 2 and Act 2/ Task 1: interpreting textual information on specific topics on periodicity

The final scheme of the categories emerging from the analysis of students' responses in the Tasks 1 & 2 in Activity 1 is presented in the form of a systemic network (Bliss et al., 1983).



The BAR ([) notation signifies that the categories following are mutually exclusive, while the Bracket ({) notation signifies that a unit can be analyzed across all the following categories.

Three main categories were identified and further analyzed: *The role of the school text in their responses*; *the type of text produced by the students*; and *the conceptual elements identified in the students' text*. These categories capture the way the students are transform the school text into learning activities in activity 1.

The subcategories that emerged were the following: *The role of the school text* was discerned in the following subcategories: do not refer to any part of the textual element; copy or mimic the part that is relevant to the task; and enrich the textual information.

The type of text produced by the students was identified as narrative (tells a story), expository (informs or describes a learning situation) and argumentative (makes a claim and supports it with various types of warrants and/or backing).

The conceptual elements identified in the students' responses were recognized in the following dimensions: 'scientific' that include mathematical and/or physical elements, 'applied' (reference to the car suspension system) and the form of employing of aspects of periodicity.

The mathematical elements were recognized as general (general statements on variations); use of mathematical terminology (e.g., references on mathematical relations); interpreting conventions on mathematical signs (e.g., interpretation of the graphical mathematical entities); and absent when no trace of mathematical elements was identified. The physical elements identified were the use of physical terminology (e.g., the helical springs); interpreting and connecting physical entities with physical laws; and absent when no trace of physical elements was identified. Finally, the form of employment of aspects of periodicity was explicit, implicit or absent.

We present below and discuss the qualitative results of the above categories on the specific activities and the specific tasks. In all cases the students had to interpret visual and verbal elements in school texts.

The role of school text

Table 3: Percentages of students' responses in the category of the 'role of the school text'

Categories	Task 1/Act1: Explain Hooks' Law in a specific context (car suspension) (68 Participants) (%)	Task 2/Act1: Substantiate scientifically the relation between the rotation of the bicycle wheel and the brightness of the light bulb. (70 Participants) (%)	Task 1/Act2: Explain Hooks' Law in an experimental context (67 Participants) (%)
Do not refer to the text	32.4	52.9	23.0
Copy the text	39.7	24.3	42.4
Enrich the text	27.9	22.9	34.6

In the first and the last case (Hook's Law) the 'role of text' in the students' responses seems to be valued almost the same although the tasks are addressed to different students. In the second case (Bicycle Dynamo) half of the students leave the text aside. The reasons for not using the textual information could be either because the phenomenon was familiar to them or it was difficult for them to interpret the textual information. Maybe the rest of the results could help us to take a position on this issue.

Students' text produced

Table 4: Percentages of students' responses in the category of the 'students' text produced'

Categories	Task 1/Act1: Explain Hooks' Law in a specific context (car suspension) (68 Participants)	Task 2/Act1: Substantiate scientifically the relation between the rotation of the bicycle wheel and the brightness of the light bulb. (70 Participants)	Task 1/Act2: Explain Hooks' Law in an experimental context. (67 Participants) (%)

	(%)	(%)	
Narrative	39.7	60	46.2
Descriptive	19.1	27.1	42.3
Argumentative	41.2	12.9	11.5

Students' text when in explaining the different phenomena in the first case was mostly argumentative in relation to the other cases. The arguments could be based on the textual information or are developed by the students (see result on Table 1).

However, it is still an open question to be answered why students did not reason on their claims in the second and the third case.

Concerning the *conceptual elements identified in students' responses* we have the following results.

Table 5 shows the results in the subcategory of 'mathematical elements'

Table 5: Percentages of students' responses in the category of the 'mathematical elements'

Categories	Task 1/Act1: Explain Hooks' Law in a specific context (car suspension) (68 Participants) (%)	Task 2/Act1: Substantiate scientifically the relation between the rotation of the bicycle wheel and the brightness of the light bulb. (70 Participants) (%)	Task 1/Act2: Explain Hooks' Law in an experimental context. (67 Participants) (%)
Referred to variations in general	16.2	20	26.9
Used mathematical terminology without interpreting it	7.4	25.7	25.4
Interpreted conventions in mathematical signs	33.8	5.7	35.8
No traces of mathematical elements were identified.	42.6	48.6	11.9

The case of interpreting conventions in mathematical signs was mostly present in the first and the last case.

In the second case, the relation between the formula $E=Bl\upsilon sina$ and its graphical representation in a form of a sinusoidal function seems to be very difficult for the students to interpret (5.7% participants interpreted conventions in the above mathematical signs) according to the needs of the task. In this case we should note the formula used is not very common in mathematics and science classes since it is not so obvious what the independent variable is.

On the other side, linear relations that are represented by the formula $F=kx$ seems to be more familiar to students. In case 2 and 3 one out of four students used mathematical terminology without interpreting it.

Table 6 shows the results in the subcategory of 'physical elements':

Table 6: Percentages of students' responses in the category of the 'physical elements'

Categories	Task 1/Act1:	Task 2/Act1: Substantiate	Task 1/Act2:

	Explain Hooks' Law in a specific context (car suspension) (68 Participants) (%)	scientifically the relation between the rotation of the bicycle wheel and the brightness of the light bulb. (70 Participants) (%)	Explain Hooks' Law in an experimental context. (67 Participants) (%)
the use of physical terminology	35.3	38.6	43.3
interpret physical laws	29.4	27.1	41.8
No traces of physical elements were identified	44.1	34.3	14.9

Note: In the case of physical elements we do not include aspects of the applied context (e.g., the car suspension or the rotation of the wheels).

The above maybe explains why physical elements are mostly present in the third case which is more a school-like activity compared to the other two.

Table 7 shows the results in the subcategory of 'aspects of periodicity'

Table 7: Percentages of students' responses in the category of the 'aspects of periodicity'

Categories	Task 1/Act1: Explain Hooks' Law in a specific context (car suspension) (68 Participants) (%)	Task 2/Act1: Substantiate scientifically the relation between the rotation of the bicycle wheel and the brightness of the light bulb. (70 Participants) (%)	Task 1/Act2: Explain Hooks' Law in an experimental context. (67 Participants) (%)
Explicit	20.6	32.9	18.0
Implicit	52.9	45.7	67.2
No reference	26.5	21.4	14.8

Periodicity is mostly absent and implicit as an abstract notion in the students' answers. Periodicity when referring to Hook's law is explicit in almost 20% of the participants while in the Dynamo case was explicit in more than 30% of the participants. We interpret this outcome that aspects of periodicity seems to be more explicit in functions that involve sinusoidal curves than in functions that express linear relations as Hook's Law even though both are representing aspects of periodic phenomena.

4.1b Students' Levels of pedagogical content knowledge awareness

The analysis of the semiotic means of objectification (Radford, 2003) in students' responses in Act 1/ Task 1 when transforming the text material into learning activities revealed three categories of students' pedagogical content knowledge awareness on the connections of the textual domain-specific elements and the situation they refer to. The analysis of the semiotic means of objectification they use in their responses led us to consider their levels of awareness as *factual*, *contextual*, and *symbolic*.

Students are at the *factual level* when they copy or mimic the textbook extract (e.g., use scientific terminology but there are no signs of interpreting them into a meaningful understanding). Students are at the *contextual level* when enriching the textual material by interpreting features of the scientific context or the applied context, but without creating a new context where both are related. Finally, students' responses are at the *symbolic level,* when they are transforming the textual elements in a new integrated way where both the contextual and the applied elements are considered as a new inseparable unity.

Below we provide examples of the three categories of students' pedagogical content knowledge awareness.

> Example on the factual level of awareness in taken from Task 1*: "I could bring in the class two springs, a stiff and a floppy. In this case, they will have the chance to observe the consequences of the oscillation in the different springs. Hence, I could relate them with the car suspension and how they interrelate with the car motion"* (st78).

Table 8: Analysis of st78 response (Factual level of pedagogical content knowledge awareness)		
The role of school text		Do not use it as a tool
Students' text		Narrative (describes how will present the topic to the students)
Scientific elements	Math	Absent
	Physics	Use of physical terminology (stiff and floppy springs)
Applied		Present (car suspension)
Employ aspects of periodicity		Implicitly

We consider st78 response as factual since the student does not provide any clues of how the textual elements are connected with the specific situation.

The students that were characterized as falling in this level of awareness were N=26 or 38.2%. The analysis of their responses gave us the following results: In the case of *the role of school' text* most of them (N=16) did not referring to any part of the text and the rest (N=10) copy or mimic a part of it. In the case of *students' text* it was mostly narrative (N=20) and only in some cases it was expository (N=4) or argumentative (N=2). Concerning t*he conceptual elements* in students' responses in most cases the mathematical entities, the physical entities and the applied context were absent (N=19, N=15 and N=16 respectively). Finally, all student responses implicitly included periodicity and absent with the same frequency (N=12) and explicit in four cases.

Below we provide two examples of the students' responses that fall in the contextual level of awareness.

> Example1 *"Looking at the two diagrams I will explain to the students that if we use one spring that exhibits linear characteristic in a suspension, the spring will terminate and there will be no absorbance of the vibrations but if we use a spring with progressive characteristic then the vibrations will be absorbed and the suspension will be efficient"* (st24).

Table 9: Analysis of st24 response (contextual level of pedagogical content

knowledge awareness)		
The role of school text		Enriched
Students’ text		Argumentative (reasons about the use of one or the other type of springs)
Scientific elements	Math	Use of terminology (linear characteristics)
	Physics	Use of terminology (vibrations etc.)
Applied		Present
Employ aspects of periodicity		Implicitly (car vibrations)

Although St24 transforms the textual information according to the specific situation there is no indication of interpretation of scientific elements in his response.

> Example 2*: “The two springs have different structure, since one is cylindrical and the other conical. In the diagram (a) the spring displays a linear characteristic since it always has the same stiffness (constant c) in relation to the spring in diagram (b) where as the length of the spring increases its width decreases, so in the different width it has different stiffness so the diagram is a curve*” (st85).

Table 10: Analysis of st85 response (contextual level of pedagogical content knowledge awareness)		
The role of school text		Enriched
Students’ text		Argumentative
Scientific elements	Math	Interprets conventions on mathematical signs
	Physics	Entities
Applied		Absent
Employ aspects of periodicity		Implicit

St85, in contrast to st24, seems to be familiar with the scientific sign conventions but is not able to translate them in the applied context. Their inability to relate the two contexts, the scientific and the applied, was the reason to place them in the contextual level of awareness.

42.7.9% of the students considered as falling in this level of awareness (N=29). In the case of *the role of school text* most of them (N=17) copied or mimicked, while with the same frequency (N=6) either they do not use it at all or enriched it. In the case of *students’ text* it was mostly argumentative (N=13), expository in N=9 and narrative in N=7. Concerning t*he conceptual elements* in students’ responses the frequencies are as follows: In many cases (N=10) students interpret conventions in mathematical signs, but in some cases mathematical signs are absent (N=8). The students either refer to physical elements (N=13) or connect them with physical laws (N=5). The cases of using the applied situation was almost equally distributed to present or absent (N=15 and N=14 respectively). Finally, the periodicity was mostly implicit (N=19), absent (N=6) or explicit (N=4).

At this level of awareness the students use arguments taken mostly of the textual material and value the scientific elements, but do not acknowledge the importance of the specific situation.

An example of the case of symbolic awareness follows

> *If we want slow response on the absorbance of car' vibrations and consequently on car road holding we choose the cylindrical spring. The reason for this choice is that its deformation (l) is related linearly to the load (Force F) imposed on it. We will not have the same effect if we choose a conical spring with a progressive characteristic since the relation between the deformation (l) and the force (F) which is imposed on it is not linear. For small loads we will have immediate response but for larger forces we will not have the analogous deformation and the forces will be transferred to the passengers' cabin"* (st36).

Table 11: Analysis of st36 response (symbolic level of pedagogical content knowledge awareness)

The role of school text		Enriched
Students' text		Argumentative
Scientific elements	Math	Interprets mathematical signs conventions
	Physics	Connect physical Entities & Laws
Applied		Present
Employ aspects of periodicity		Implicit

St36 starts his response by making an argument. He claims that cylindrical springs have slow response on car vibrations. Moreover, he relates the cars vibrations to the holding capacity of the car and the load with the Force F in Hooks' Law and he uses the linear relation as warrant for slow car response on vibrations. St36 interprets symbolic mathematical relations and connects the physical entities with the physical laws.

His convenience to move back and forth from the scientific to the applied context and his familiarity with scientific sign conventions helped him to see through the graphical representations the situation they refer to. For all the above reasons we classify his response in the 'symbolic level of awareness'.

The students that belonged to this level of awareness were N=13 or 19.1%. They all enrich the textual information and use arguments in their responses. Almost all (N= 12) were interpreted conventions in mathematical signs and connected physical entities with Laws (N=10). Finally, in all their responses periodicity was expressed implicitly (N=13) or explicitly (N=8).

Issues emerging from the analysis of interpreting textual information on specific topics on periodicity

Aspects of periodicity seems to be more explicit in functions that involve sinusoidal curves than in functions that express linear relations as Hook's Law even though both are representing aspects of periodic phenomena.

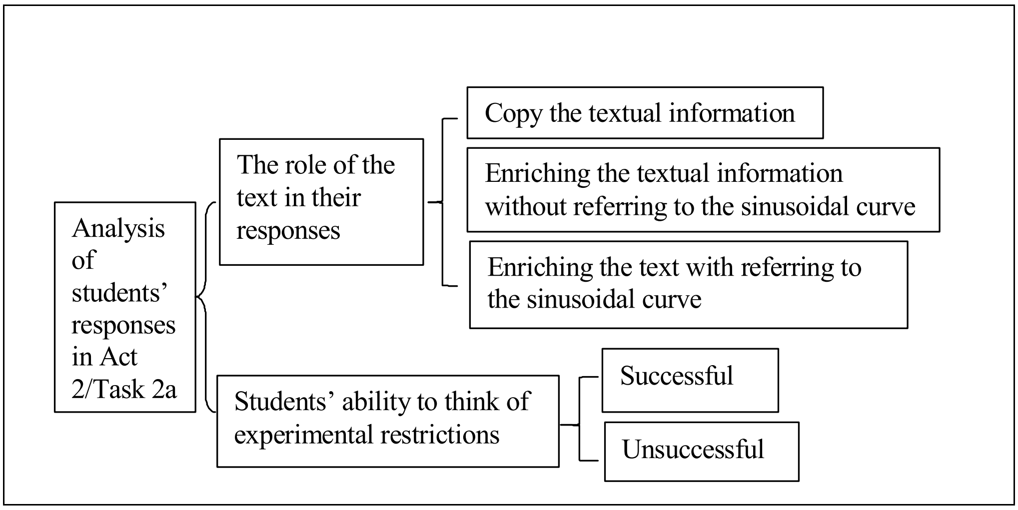
It seems that the most difficult textual information for the students to interpret is the Bicycle Dynamo case. In this case the physical phenomenon is easy to realize on the one side but difficult to interpret scientifically on the other. Besides, the mathematical representation ($E=Bl\upsilon \sin a$) is not typical in math and science classes. Moreover, the students seem to be more familiar with proportional relations expressed on linear graphs but not with sinusoidal graphs (e.g., the relation of the physical quantities E, υ to the formula $E=Bl\upsilon \sin a$).

In general, three levels of pedagogical content knowledge awareness were identified in the analysis of the students' responses in Task 1 when they transformed textual elements in learning activities: the factual, the contextual and the symbolic. Our analysis indicates that as students move from one level of awareness to the next, they employ and enrich the information in the textbook extract, while their text becomes less narrative or expository and more argumentative. Moreover, their reasoning becomes more scientific-oriented by employing elaborated semiotic means when objectifying new relations (e.g., interpreting conventions in mathematical signs according to the needs of the specific-domain task).

4.2 The results of the qualitative and quantitative analysis (Act 2/ Task 2 & Task 3)

Activity 2/Task 2a: Realize the goal of an experimental activity and think of its restrictions

The final scheme of the categories emerging from the analysis of students' responses in Task 2a in Activity 2 is presented in the form of a systemic network (Bliss et al., 1983). The main categories are *the role of textual information* provided by the researcher in the specific task in the students' responses and students' *ability to set up constraints* in the specific experimental situation. The *role of textual information* in the students' responses is characterized as (a) Copying the text; (b) interpreting the textual information in common terms; (c) using scientific terms when interpreting the textual information.



Students' *ability to think of experimental restrictions* is either successful (state at least one constrain) or not successful.

The qualitative results of the above categories on the specific activities and the specific tasks are presented and discussed below.

How the students use the textual information provided by the researcher in the specific task.

Table 12: Percentages of students' responses in the category of the 'use of the textual information'

Categories	Act 2/ Task 2a: State students' conclusions by interpreting a system of an oscillating spring and a rotating cylinder (N=67) (%)
Copy the textual information	31.3
Enrich the textual information without referring on the produced graph	42.2
Enrich the textual information by referring to the produced graph	26.6

Most of the students (almost 70%) enrich the text given to them but only 26.6% referred specifically to the produced graph. Identified the sinusoidal curve was considered as the goal of this textual activity. So, students referring to this curve are considered as important in the meaning-making process.

For example, in the case of 'copying the textual information' st2 states:

> "*The writer wants to show the connection between the graphical representation and the oscillation*".

While st10 who 'enriches the textual information by referring to the produced curve' states:

> "*The system represented in Photo 2 tries to show the students how the sinusoidal function is produced while the oscillated body is moving. The student can see through this the physical connection between the oscillating sphere system and the produced graph*".

The goal of this experimental activity seems not to be so clear for one out of four students.

Students' *ability to think of experimental restrictions* in an experimental activity

Table 13: Percentages of students' responses in the category of the 'think of experimental restrictions'

Categories	Act 2/ Task 2a: Think of an occasion that the experiment in Task 2a may not work. (N=60) (%)
Successful	61.7
Unsuccessful	38.3

Students' answers in this question could be related to their ability to visualize the situation presented in the graph in a dynamic form (see the movement and make suggestions in order for the experiment to be successful). In this case, the students should take into consideration the given data (an ideal spring, a cylinder rotating) realize the goal of the activity (the produced graph) and suggest some restrictions in order for the experiment to succeed.

Six out of ten students manage to think of an experimental restriction. An example of a successful case is st15's answer:

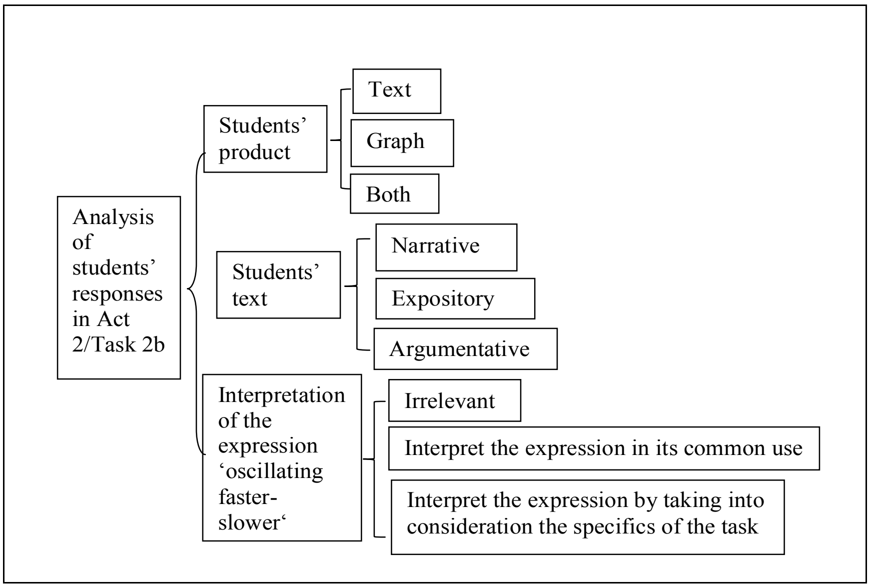
> *"In order for this experiment to succeed the oscillation of the spring and the rotation of the cylinder should have the same frequency".*

An example of an unsuccessful case is st10's answer:

> *"The environmental conditions could prevent the experiment from being successful".*

Activity 2/Task 2b: Realizing properties of periodic motions (frequency and period) by relating two periodic motions

The final scheme of the categories emerging from the analysis of students' responses in the Tasks 2b in Activity 2 is presented in the form of a systemic network (Bliss et al., 1983) as follows.



The main categories are *the genre of students' product* (only a text; only a graph; a text and a graph)*, the type of students' text* (narrative; expository; and argumentative) *and students' interpretation of the expression 'faster-slower'* (irrelevant; interpret the expression in its common use; interpret the expression by the specifics of the task into consideration.

The qualitative results of the above categories on the specific task are presented and discussed below.

Students' product

Table 14: Percentages of students' responses in the category of 'students' product'

Categories	Act 2/Task 2b: depict periodic motions on a paper (N=63) (%)
Only text	25.4
Only graph	12.7
Text & Graph	61.9

Most students (almost 60%) sketch the graph and explain their graph verbally. One out of four (almost 25%) prefer only to express their depiction verbally.

The type of students' text

Table 15: Percentages of students' responses in the category of the 'the type of students' text'

Categories	Act 2/Task 2b: depict periodic motions on a paper (N=56) (%)
Narrative	1.8
Expository	76.8
Argumentative	21.4

Although most students (almost 77%) describe the situation they are presenting graphically only one out of five (almost 20%) justify their depiction.

Students' interpretation of the expression 'oscillating faster- slower'

Table 16: Percentages of students' responses in the category of the 'students'

interpretation'	
Categories	Act 2/Task 2b: depict periodic motions on a paper (N=63) (%)
Irrelevant	20.6
Interpret the expression 'oscillating faster-slower' without taking into consideration the specifics of the task	42.9
Interpret the information by taking into consideration the information given	36.5

Most students (80%) know how the varying of frequency affects the form of a sinusoidal curve. For them 'oscillating faster' means little period or higher frequency and 'oscillating slower' means exactly the opposite. However, only half of them (36.5%) take the specifics of the task into consideration as when the rotation of the cylinder is slower than the oscillation of the spring then the sinusoidal curve at the same time writes quicker (higher frequency) than the opposite case.

We present below two examples of students' responses in this task and their analysis according to the scheme we present above.

Example 1: st52 states:

> *"Due to the slower or quicker motion of the cylinder, the form of the wave is presented either more dense or sparse respectively"*

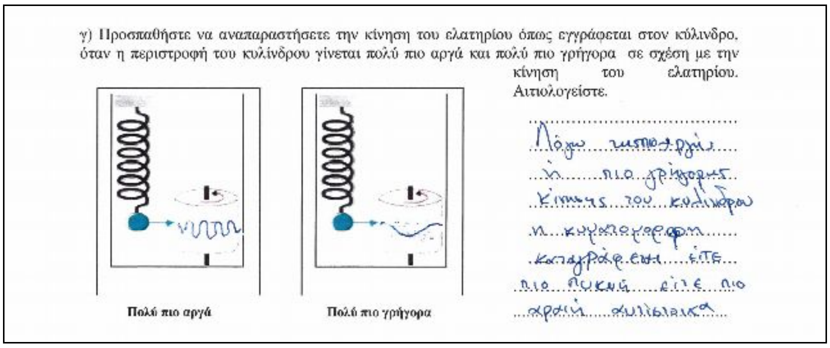


Table 17: Analysis of st52 answer	
Genre of students' product	Text and graph
Type of text	Argumentative
Type of interpretation	Interpret the expression 'oscillating faster-slower' by taking into consideration the specifics of the task

Example 2: st1 states:

> "*If the cylinder moves faster it will present more dense oscillations than if it moved slower"*

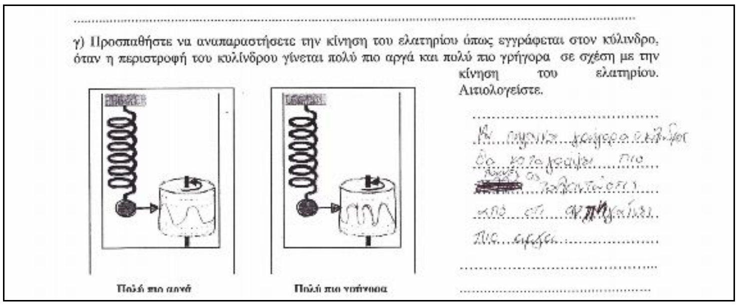


Table 18: Analysis of st1 answer	
Genre of students' product	Text and graph
Type of text	Descriptive
Type of interpretation	Interpret the expression 'oscillating faster-slower' in their common use

Both students realize an oscillation with high frequency as depicted on a 'dense sinusoidal curve' while an oscillation with low frequency as depicted on a 'sparse sinusoidal curve'. The only difference is that st52 interprets the expression 'oscillating faster-slower' by taking into consideration the specifics of the task while st1 does not. Moreover, st52 justifies his answer by taking into consideration the relation of the two periodic motions.

In general, the high frequency of students' participation in this task (63/70) indicates students' will to be involved in concrete sensory-motor experiences.

Activity 2/Task 3: Making connections among representations of the same periodic phenomenon

The analysis of students' responses in Act 2/Task 3 (identifying that the Hook's law display the linear relation of the x-F while the sinusoidal curve display the relation x-t and both representations are referring to different aspects of the same phenomenon) provided the following information.

Table 19: Percentages of students' responses in the category of the 'connection different VRs'	
Categories	*Act 2/Task 3*: Connect the different graphical representations of the same phenomenon (the linear relation and the sinusoidal curve as VRs depicted different aspects of the spring motion) (N=37) (%)
Not making a connection	45.9 (N=17)
Interpret one of the two successfully	18.9 (N=7)
Connecting both as graphical representations of the same phenomenon that are depicted different mathematical relations	35.1 (N=13)

Only 37 out of the 70 participants replied in this task. This low participation indicates students' reluctance to participate in activities of making connections.

From those who participated, the majority interpreted the different graphs due to the different spatial arrangement of the spring-in horizontal and vertical position. Only some of them managed to connect the graphical representations as representing different aspects (mathematical relations) of the same phenomenon (N=13 of the 70 participants). The last outcome addresses students' compartmentalization in knowledge.

Below we present an example of a student who realized the different graphical representations as representing different aspects (mathematical relations) of the same phenomenon. St17 answer is as follows:

> "*Fig. 1 [represents the relation] (F, displacement) & Fig. 2 [represents the relation] (displacement, time)*
>
> *or in Fig. 1 we have F = -Dx while in Fig. 2 we have F=-Fmaxsin(ωt+φ)*".

St17 identifies that the linear relation of the x-F and the sinusoidal curve (relation x-t) both are referring to different aspects of the same phenomenon.

Issues emerging from the analysis of students' understanding of aspects of periodicity expressed graphically

The graphs of sinusoidal functions are the main graphical models of periodic motions in school textbooks. This explains why most students' familiarity with the sinusoidal curves. Particularly, most students enrich the textual information and they provide cases where the experimental phenomenon, whose out come is this curve, is successful in Task 2a. In this task many of them relate the sinusoidal curve to simple harmonic oscillations. Moreover students easily comprehend issues of how the varying of certain characteristics of periodic motions (e.g., its frequency) could affect properties of the curve (Task 2b). Besides, more than 7 out of 10 students sketch sinusoidal curves with relatively different frequencies in Task 2b.

Students' high participation in some of the above tasks indicates their will to be involved in concrete sensory-motor experiences (think of experimental restrictions or depict periodic motions on a paper). On the other side, justification of their answer is not a common practice to students. Usually, students make a claim without justification (Task 2b). The lowest students' participation was in the task of connecting different VRs of the same phenomenon (Task 3). On the latter two issues (no arguing on their claim and difficulty in making connections) has to be taken serious attention in the school activities.

4.3 The results of qualitative and quantitative analysis (Act 3)

Act 3/ Task 1: Periodic motions with the best periodical behaviour

In the first task the majority of the students in all Institutions considered the earth as the best example of periodic motion under real circumstances. Particularly, 123 students participated in this task.

Almost 83% (82.9) considered the motion of the earth around the sun as the best example of periodic motion under real circumstances, 13.8% the swing and only 1.3% the Ferry-boat traveling between two ports.

Students' main arguments for their choice were as follows.

Earth represents the best periodical behaviour since "*It has the same period of 365 days or 1 year;* or "*It is a natural phenomenon (i.e. there are no conditions and losses; it happens all the time for centuries; people have no control on it or nothing could interfere".*

The swing represents the best periodical behaviour since: *"It has all the characteristics of a periodic motion (equilibrium position, amplitude, frequency etc.)"* or *"It is not a complex phenomenon to imagine and graph".*

The ferry represents the best periodical behaviour since: "*It is not a complex phenomenon to be explained".*

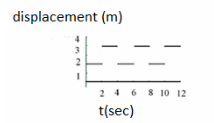
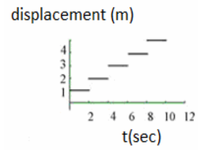
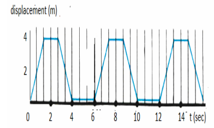
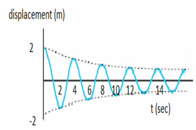
When students argue on their choice in most cases (71.3%) refer to the "periodic characteristics" of the periodic motions. 22.5% of participants' justifications for accepting the earth as the example with the best periodical behaviour or rejecting the other two are "no-human interference on the motion".

Act 3/ Task 2a: Distinguish periodic from non-periodic graphs

We present the results of our analysis of the categories that emerged from the students' responses on the two tasks in the case of each graph and we provide some characteristic examples.

Does this graph represent a periodic motion? Two categories emerged from the analysis of this task: the graph represents a *non-periodic motion;* and the graph represents a *periodic motion*.

Table 20: Percentages of students' responses across categories and across graphs on 'distinguishing periodic from non-periodic graphs'

Categories	Graph 1	Graph 2	Graph 3	Graph 4
				
	113 Participants	107 Participants	109 Participants	114 Participants
Non-Periodic	23.89	74.77	7.34	32.46
Periodic	76.11	24.30	92.66	67.55

Almost three out of four students identified periodicity in Graph 1 and non-periodicity in Graph 2 while this percentage increases in the case of Graph 3 since more than nine out of ten students identified it as a periodic graph. Graph 4, which represents a repeated but a non-periodic motion, seemed to confuse students a lot since almost seven out of ten considered it to be a periodic graph.

Justify your answer. The following categories emerged from the analysis of students' responses: *Referring to general patterns of repetition*, *relating variations in x-y axis*, *focusing on continuity issues*, *using the formal definition of periodic functions*, and *reasoning on a specific situation.*

Table 21: Percentages of students' responses across categories and across graphs on "justifying"

Categories	Graph 1	Graph 2	Graph 3	Graph 4
	59 Participants	46 Participants	58 Participants	63 Participants
Referring to general patterns of repetition	23.73	23.91	32.76	15.87
Relating variations in x-y axis	37.29	26.09	41.38	61.90
Focusing on continuity issues	13.56	15.22	1.72	1.59
Using a formal definition	0.00	2.17	1.72	0.00
Reasoning on a specific situation	25.42	32.61	22.41	20.63

In both cases of periodic and non-periodic graphs, students preferred to justify their answers by relating patterns of repetitions between the x-axis and y-axis rather than referring to general patterns of repetition. It is interesting that the same type of the above justifications were used for conflicting answers for the same graph.

Two characteristic responses of the type of *General pattern* in the case of Graph1 are the following:

> *"It is periodic because we have a repetition"* (st91_elec) and, *"It is not periodic since there is not any harmony"* (st59_elec).

In the following examples we could identify inconsistencies in the students' responses in the case of Graph 4 when *relating patterns of variations in the x-axis and the y-axis*:

> *"It is periodic but we can see that as the time passes it dwindles and we are led to a standstill"* (st101_elec); or *"It is a periodic motion that decreases (its amplitude diminishes) all the time"* (st68_elec).

The contradictions in students' responses were not realized by them.

Focusing on the continuity issue is used as a warrant to take the stance that Graphs 1 and 2 are both non-periodic.

For example, st19_math notices:

> *"I do not know if this graph preserves a periodic behaviour because in its second position it has different values from left and right".*

Only st6_math reasons by *using the definition of periodic functions* in order to accept that Graph 3 is periodic and Graph 2 is non-periodic. For example, Graph 3 *"is* *periodic with period T=6 seconds since f(x+T)=f(x) for every x in the interval [0,14].* In the case of Graph 4, the same student changes his argument as follows: "*It is periodic since any sinusoidal function is periodic"*. In this case misconceptions that every curve that distributes a fluctuation is a sinusoidal curve therefore periodic are predominant even in students with their major on mathematics.

The last category is *reasoning on a specific situation.* These situations, in most cases, were the examples they provided in Task 2. This type of situated justification was common in students' responses in all graphs and ranged from 20% to 30%.

Some characteristic examples are:

> (Graph 1) *"the body of the graph diverges from the starting point of motion and then always returns within 4 seconds, therefore the graph is periodic"* (st99_elec);
>
> (Graph 2) "*the graph shows a person who, as time passes, only draws away from a point 'a,' therefore non-periodic"* (st107_inf);
>
> and (Graph 4) "*it is periodic because it represents the motion of the swing"* (st129_inf).

All the above responses indicate students' need to set up a background for their justifications.

Act 3/Task 2b: *making sense out of graphs*

The following categories emerged from the analysis of the salient features that were taken into consideration when the students were asked to provide examples that could be represented by each graph: *Enriched repeated motion* when students considering the repeated behaviour and other characteristics emerging from the graphs (periodicity, piece-wise continuity, and the relation between the variables), *Only repeated motions* when students took into consideration only the repeated behaviour, *Non-repeated motions* when there was no-indication of a repeating motion in students' responses, and *no-motion* when the example was not representing a motion at all.

Table 22: Percentages of students' responses in the categories and across graphs on "providing an example of motion that could be described by each graph"

Categories	Graph 1	Graph 2	Graph 3	Graph 4
	84 Participants (%)	83 Participants (%)	85 Participants (%)	102 Participants (%)
Enriched repeated motions	9.52	21.69	7.06	6.86
Only repeated motions	55.95	31.32	67.06	79.41
Non-repeated motions	15.48	35.55	12.94	0.98
Non-motions	19.05	8.43	12.94	12.75

Creating a motion example of a piece-wise continuous function is very difficult but a few students managed to provide examples that could satisfy all the graphical features in these graphs. In this case, students used their kinesthetic experiences of 'jumping' or 'climbing stairs' in order to respond to this task. Some typical examples of *enriched repeated motions* in the case of Graph 1: "ascending and descending jumps between uneven steps (st1_math)"; in the case of Graph 2 is: "someone who is climbing stairs" (st57_inf). Noticing the resemblance of Graph 2 with stairs and visualize the motions helped almost 22% of the participants to provide enriched examples. However, Graphs 3 and 4 were more complicated since the students had to take into consideration the type of co-variance of the two variables in order to provide enriched examples. Particularly, Graph 3 refers to an object's motion that moves with constant speed in different directions and makes a few seconds stops. A significant example for Graph 3 is: "someone who is using a piece of gym equipment which is going to and fro with constant speed and stops for a few seconds" (st59_elec). Although many students used the swing example for Graph 4, a typical example in their physics classes, only a few managed to specify what the x-axis and the y-axis represent in this graph. This is the reason we have the least percentage of enriched cases.

The number of students who provided *examples of repeated motions* but did not consider other graphical features was high (more than one out of two students) for all graphs besides Graph 2. Two characteristic ideas were met in their answers and the corresponding examples for the graphs follow: (a) discontinuity was not taken into consideration (Graph 1) "*it represents an elevator that is trapped going up and down between the second and fourth floor"* (st3_math); (b) not specifying the x-y co-variance (Graph 3) *"two people who are throwing a ball to each other"* (st50_inf). The amount of students who provided examples of *non- repeated motions* was higher in the case of Graph 2, as for example: "*a dog that goes hunting and increases its speed*" (st71_elec). Some students provided *examples that do not represent motions* at all. These examples are mostly taken from their academic signal processing courses. Finally, students' high participation in this task indicates their willingness to assign meaning to abstract mathematical entities.

Kinesthetic students' experiences seem to play a significant role in providing examples of repeated motions. So, we further analyzed the type of kinesthetic experiences they refer to and the categories emerged were: *bodily actions* when a human agent performs the motion (an athlete running or a frog jumping), *physical tools motions* (a car is accelerating or a swing is oscillating), and *vibrations of natural objects* (a sea wave or a sound wave).

Table 23: Percentages of students' responses in the categories and across graphs on "providing an example of motion that could be described by each graph"

Categories	Graph 1	Graph 2	Graph 3	Graph 4
	70 Participants (%)	81 Participants (%)	74 Participants (%)	90 Participants (%)
Bodily actions	24.29	32.50	13.51	7.78
Physical tools' motions	72.14	63.80	81.09	64.44
Vibrations of natural objects	3.57	3.70	5.40	27.78

Physical tools' motions provided the context used by most students to translate the graphs to situations. The highest percentage of *bodily actions examples* was in the case of Graph 2 (32.5%). The highest percentage of *physical tools' motions* (81%) was in the case of Graph 3. We interpret this result that most students consider that human actions are very difficult to model this type of motion graphs so they have changed the context of their example from bodily actions to physical tools' motions. More than one out of four students used examples of *vibrating natural objects* (e.g. sea waves) in describing the case of Graph 4.

The graphical image resemblance with traveling sinusoidal waves was the reason to use them as the context of their examples. We note that waves are functions of two variables, the displacement x and the time t (King, 2013).

Act 3/ Task 3: Modeling a periodic motion

During the collective discussion most students seem to have problems in modeling activities. The collective discussion helps almost half of them to continue the tasks.

Most of them (88.1%) identify the periodic characteristics of the motion (i.e., period, frequency and amplitude). But not all of them (72.9%) use the periodic characteristics when expressing the sinusoidal function that models this motion. 12.9% refer to the sine function without taking its periodic characteristics into consideration. Also 12.9% refer to linear relations. In the case of sketching the graph, almost 80% sketched a periodic graph but only half of them 38.7% sketched the sinusoidal curve by indicating its periodic characteristics.

Issues emerged from the analysis of interpreting periodic motions

Our findings indicate that conceptions such as "every repeated motion is periodic" or "any sinusoidal graph, even with decreasing amplitude, represents a periodic motion" dominate students' understanding. Almost all undergraduate students do not seem to realize the contradictions in their responses. The formal mathematical tools, such as the definition of periodic functions, seem not to be enough to change such perceptions even in the case of students who study mathematics. Moreover, conceptions such as that a periodic motion is possible only when no human interferes on it gives periodicity a 'supernatural' meaning.

However, students' strong willingness to assign meaning to abstract mathematical entities is proved both by their high participation in providing situations that could fit motion graphs (Task 2b) and by the fact that they use these situations as warrants for their justifications (Task 2a). In this case, the role of students' kinesthetic experiences proved central both when they provided enriched examples of motions represented by the particular graphs and when they take the stance to change the context of the examples according to their perception of the graphical features represented.

Finally, modeling periodic motions seems to be a very difficult task for all undergraduate students.

Chapter 5: GENERAL CONCLUSIONS

In this study we take the position that understanding the notion of periodicity and its properties involves creating a coherent framework and integrating ideas and educational practices in different school subjects in order to be meaningful at an individual level. By adopting the perspective that periodicity, as an abstract notion, is realized through specific situations where it takes its meaning (Radford, 2003), we design three different research activities (case studies) where different aspects of the notion are involved. Our main interest is how undergraduate students perceive periodic motions and their graphical representations. The resources used with students are taken mostly from mathematics, physics and engineering secondary school textbooks.

We highlight below the main of our findings.

Conceptions on periodic motions

Almost 83% of 123 participants (from two University departments and two Technological Institutions Departments) in Task 1/Act 3 considered the motion of the earth around the sun as the best example of periodic motion under real circumstances when comparing it with the motion of a swing or the motion of a Ferry-boat traveling between two ports. Most students' justification on the above claim is based on the idea of an ideal periodic motion, which is considered as a situation with "no-human interference on the motion". Conceptions, such as that a periodic motion is possible only when no human interferes on it, give periodic motions a 'supernatural' meaning.

The vast majority of students easily identified the periodical property in periodic no sinusoidal graphs. It is interesting though that a Graph, which exhibits a sinusoidal fluctuation (looks like the sinusoidal curve but with decreasing amplitude) (see Fig. 3), seems to confuse students a lot since almost seven out of ten considered that represents a periodic motion.

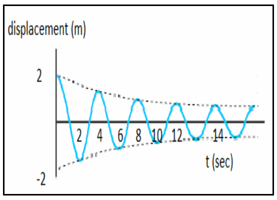


Fig. 3: A non-periodic motion graph.

Students' overgeneralizations such as any repeated function is periodical is also highlighted in Buendia and Cordero study (2005). We can explain this students' misunderstanding since in physics texts functions such as $f(x)=e^{-bx}\sin(\omega x)$ are considered as functions that model 'periodic motions'. As a result, conceptions such as "every repeated motion is periodic" or "any sinusoidal graph, even with decreasing amplitude (e.g., damping oscillation) represents a periodic motion" seems to dominate students' understanding (Task 2a/Act3).

When the above participants justify their answer on which graph represents or not a periodic motion (Task 2a/ Act 3), they mostly relate variations in x-y axis. But even in this case students' confusion of what is periodic is evident. We support our claims with the following examples of students' responses in the case of the graph in Fig. 3.

> *"It is periodic but we can see that as the time passes it dwindles and we are led to a stand still"* (st101_elec); or *"It is a periodic motion that decreases (its amplitude diminishes) all the time"* (st68_elec).

Although st101 expresses a doubt, he/she still names the motion 'periodic'. In st68 response the reference to the decreasing amplitude 'fits' with periodicity.

In terms of formal justifications, only one participant (st6_math), who studies mathematics, reasons on what is periodic by using the 'definition of periodic functions'. For Graph in Fig. 4, the participant claims that *"it is periodic with period T=6 seconds since f(x + T) = f(x) for every x in the interval [0, 14].*

In the case of Graph in Fig. 3, the same student changes his/her argument as follows: "*It is periodic since any sinusoidal function is periodic"*. Why st6_math avoids using the definition of periodic function in order to argue on the non-periodical behaviour of Graph in Fig. 3? Maybe his/her conception that any function that looks like the sinusoidal is always periodic is valued as more important than a mathematical definition, or perhaps he/she distinguishes periodic functions and periodic motions.

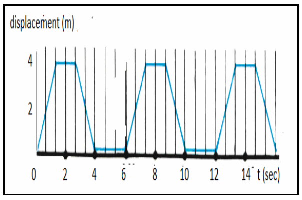


Fig. 4: A periodic motion graph

Graphing and making sense out of sinusoidal functions (y(t) = Asin(ωt))

Since sinusoidal graphs are the main graphical models of the behavior of periodic motions in school textbooks (Spiliotopoulou & Triantafillou, in press), we used a variety of tasks to examine how students interpret this type of graphs. In general, all students were familiar with sinusoidal graphs. We argue on this issue as follows: (a) almost 75% of the participants sketched sinusoidal graphs when asked to (Task 2b/ Act2); (b) Most participants (80%) know how the variation of frequency affects the form of a sinusoidal curve (Task 2b/Act2). For most of them 'oscillating faster' means small period or higher frequency and 'oscillating slower' means exactly the opposite; (c) Many of the participants relate the curve with simple harmonic oscillations (Task 2a/Act2) by visualizing the movement of an ideal spring and make suggestions in order for the experiment to be successful (62%). To answer this task the students had to take into consideration two periodic motions performing simultaneously (an oscillating spring and a cylinder rotating) and take into account that realizing the produced sinusoidal graph is the goal of this activity.

On the other side, undergraduate students from engineering and mathematics departments (132 participants) did not answer to tasks which required to sketch the sinusoidal graph in a modeling activity (Task 3/ Act3) by taking into consideration realistic data. Particularly, only half of the participants (N=62) replied in this task. From them only 21% (N= 13) sketched the sinusoidal curve by indicating its periodic characteristics (amplitude, period, angular velocity ω). Besides, it seems that interpreted proportional relations in the case of the sinusoidal function E=Blυsina (relations between E-υ) are a difficult task for students (Task2/Act1).

Making connections between different graphical representations of the same periodic phenomenon

In Task 3/Act 2, participants (70 undergraduate engineering students from the Departments of Electronics, Mechanical and Structural engineering) asked to connect the following representations of the same phenomenon (the oscillating motion of an ideal spring). The two graphical representations are taken from their secondary school textbooks in physics (Fig. 5).

Most students interpreted the linear and sinusoidal graphical relations as due to the different spatial arrangement of the spring - in horizontal and vertical position - without paying attention on the different variables represented (displacement and time). Only 13 out of 37 participants realized that both graphs represent different aspects of the same periodic phenomenon.

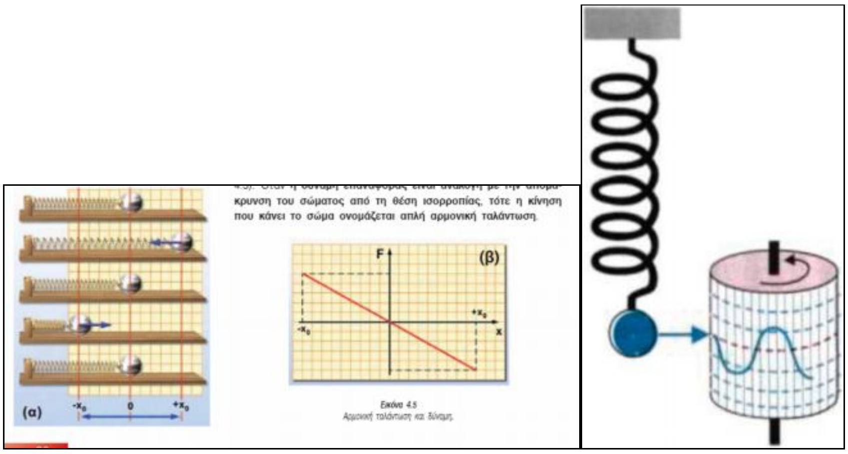


Fig. 5: Two different functions that model the same periodic motion

Argumentation and reasoning in students' responses when interpreting visual representations of periodic motions

Many studies argue on the interrelationship between students' engagement in argumentation and their conceptual understanding (Aydeniz, Pabuccu, Cetin & Kaya, 2012; Ogan-Bekiroglou & Eskin, 2012). For this reason we trace argumentation and reasoning elements when students asked to interpret and connect verbal and visual textual elements on topics related to periodicity.

In general, the justification of undergraduate students' arguments seems to be a non-familiar practice for them. Evidence for this issue is students' low participation in tasks where justification was necessary (e.g., Task 2b in Act 2 or Task 2a in Act 3). Students' reluctance to argue on their claims could be either because argumentative-pedagogy is not a common practice in the Greek educational system, or because argumentation is from its nature a more demanding task.

Furthermore, from the analysis of students' responses to Tasks 1 & 2/Act 1 (the participants were 86 mechanical engineering students and the tasks were taken from their final exam in a course on Didactics) in respect to the text produced (narrative or descriptive or argumentative) the following divergent outcomes emerged: When in Task 2/Act 1 undergraduate engineering students had to plan specific teaching actions in order their students to be able *"to substantiate scientifically the relation between the rotation of the bicycle wheel and the brightness of the light bulb",* only 13% argue on their responses and only 5.7% realized that the proportional relation of the quantities E-υ on the formula E=Blυsina could be a warrant for the above scientific claim. The same participants, though, dealing with another task (relation of Hook's Law and car suspension in Task 1/Act 1) argue in a more consistent way; in this case a frequency of 41.2% produced an argumentative text, while 34% of them interpreted proportional relations on the formula F=kx (relations between F-x). Why the same participants in one Task argue meaningfully and in the other they cannot articulate basic justification on proportional relations? Maybe in conceptualizing proportional relations of the quantities E (in Volts) - υ (in m/s^2) on the formula E=Blυsina is more difficult than conceptualizing proportional relations of the quantities F-x on the formula F=kx. The second formula is the typical formula of examining linear relations in mathematics, while the first one is a sinusoidal one. This may mean that for students linear relations are more difficult to be handled when they are located in sinusoidal functions.

Three levels of pedagogical content knowledge awareness were identified in the analysis of students' responses in Task 1/Act1 when they transformed textual elements in learning activities on a topic that models a periodic behaviour in a real situation: the factual, the contextual and the symbolic. Students are at the *factual level* when they copy or mimic the textbook extract (e.g., use scientific terminology but there are no signs of interpreting them into a meaningful understanding). Students are at the *contextual level* when enriching the textual material by interpreting features of the scientific context (e.g., reason on the different characteristics of the springs) or the applied context (e.g. reason on choosing the one or the other type of springs on the car suspension system), but without creating a new context where both are related. Finally, students' responses are at the *symbolic level,* when they are creating a new object where elements from the scientific and the applied context co-exist (e.g., interpret the role of different spring characteristics according to the needs of the specific situation

Our analysis indicates that as students move from one level of awareness to the next, they employ and enrich the information in the textbook extract, while their text becomes less narrative or expository and more argumentative. Moreover, their reasoning becomes more scientific-oriented by employing elaborated semiotic means when objectifying new relations (e.g., interpreting conventions in periodic graphs and formulas according to the needs of the specific-domain task).

The complexity of integrating contextual and scientific salient elements is apparent in the above task since only a small number of participants manage to reach the symbolic level of pedagogical content knowledge awareness.

An example of this case is the following:

> *If we want slow response on the absorbance of car' vibrations and consequently on car road holding we choose the cylindrical spring. The reason for this choice is that its deformation (l) is related linearly to the load (Force F) imposed on it. We will not have the same effect if we choose a conical spring with a progressive characteristic since the relation between the deformation (l) and the force (F) which is imposed on it is not linear. For small loads we will have immediate response but for larger forces we will not have the analogous deformation and the forces will be transferred to the passengers' cabin"* (st36).

St36 starts his response by making an argument when he/she claims that cylindrical springs have slow response on car vibrations by relating the car vibrations to the holding capacity of the car and the load with the Force F in Hook's Law. His/her convenience to move back and forth between the scientific and the applied context and his/her familiarity with scientific sign conventions helped him/her to see through the graphical representations the situation they refer to.

For all the above we conclude that students' engagement in reasoning actions is affected by the conceptual elements involved in the situation. The last outcome could point to an interrelationship between conceptual understanding and engaging in argumentative activities expressed in textual form.

Students' high and low participation on specific tasks over others

Since students participated voluntarily in two of the three activities (Act 2 and Act 3, we consider that the same students' high or low participation in specific tasks over other tasks signifies students' readiness to be involved with them.

Particularly, we identified low participation in Task 3/ Act 2 with main goal of making connections between graphs of the same phenomenon, where only 37 out of 70 students participated in this task and we notice high participation in Task 2a and 2b/ Act 2. The main goals of latter Task was thinking of restrictions of an experimental activity and relating visually two periodic motions that occur simultaneously where high participation occurred, 63 out of 70 and 60 out of 70 respectively.

We identified low participation of students in Act3/Task 2a (from 58-63 from 132 participants) (justify which graph represents periodic or non-periodic motion) and high participation in Act3/Task 2b (from 83-102 from 132 participants) (Provide an example that could be described by this particular graph). This fact is surprising since the second task is considered to be more time-consuming and effort-required than the first one. This may mean that the second task seems to be more interesting for students and such an activity could be a useful teaching tool.

Hence, difference in students' participation in tasks could provide us with information concerning students' preference to work in tasks requiring visualization and imagination than reasoning and abstracting, and using their own experiences that justifying at an abstract level.

Students' strong willingness to participate in concrete sensory-motor tasks

Participants' strong willingness to assign meaning to abstract mathematical entities is proved both by their high participation in providing situations that could fit onto motion graphs (Task 2b/ Act3); by the fact that they use these situations as warrants for their justifications (Task 2a/ Act3); and visualizing situations that an experiment to be successful (Task 2b/ Act2). In this case, the role of students' kinesthetic experiences proved central both when they provided enriched examples of motions represented by the particular graphs and when they took the stance to change the context of their reasoning according to their perception of the graphical features represented.

These findings show the embodied nature of mathematical thinking and the genetic relationship between the sensual and the conceptual in knowledge formation (Nunez, 2007; Radford et al., 2004). Translating graphs into describing situations (Roth, 2004) seems to be an activity that attracts undergraduate students' attention. Maybe such activities help students to cope with the contradictions that arise between their divergent conceptions on periodicity. The formal mathematical tools, as the definition of periodic functions, seem to be not enough to change such perceptions even in the case of students who study mathematics.

As a general conclusion, we consider that meaning of periodicity is not made within a single graph alone or a text alone, or by one subject alone. Hence, teachers need to provide students with activities required analysing of both written and corresponding visual texts, enabling in-depth meaning to be constructed from their textual experiences, using knowledge from all subjects and providing them with opportunities for contradictions, reasoning, visualization and imaginative acts.

REFERENCES

Aydeniz, M., Pabuccu, A., Cetin, P. S., & Kaya, E. (2012). Impact of argumentation on college students' conceptual understanding of properties and behaviors of gases. *International Journal of Science and Mathematics Education,* 10, 1303-1324.

Bliss, J., Monk, M., & Ogborn, J. (1983). *Qualitative data analysis for educational research*. London: Croom Helm.

Buendia G. & Cordero F. (2005). Prediction and the periodical aspect as generators of knowledge in a social practice framework. *Educational Studies in Mathematics 58*(3)*,* 299– 333.

Gee, J. P. (2003). Social linguistics and literacies. Ideology in Discourse. New York: Routledge Falmer.

King, G. C. (2013). *Waves and Vibrations*. Willey publishers.

Kynigos C., & Gavrilis, K. (2006). Constructing A Sinusoidal Periodic Covariation. In Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. (Eds.). *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 9-16. Prague: PME.

Love, E., & Pimm, D. (1996). 'This is so': A text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer Academic Publishers

Leinhardt, G, Zaslavsky, O, and Stein M. (1990). Functions, Graphs and Graphing: Tasks, Learning and Teaching. *Review of Educational Research*, 60(1), 37-42.

Mayring, P. (2000). Qualitative Content Analysis [28 paragraphs]. Forum Qualitative Sozialforschung / Forum: Qualitative Social Research. 1(2). Art. 20. http://nbn-resolving.de/urn:nbn:de:0114-fqs0002204.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Research Council. (NRC). (1996). *National science education standards.* Washington, DC:National Academy Press.

Norris, S. P., & Phillips, L. M. (2003). How literacy in its fundamental sense is central to scientific literacy. *Science Education, 87*(2), 224–40.

Nunez, R. (2007). The cognitive science of mathematics: Why is it relevant for mathematics education? In R. A. Lesh, E. Hamilton & J. Kaput (Eds.), *Foundations for the future in mathematics education*. London: Lawrence Erlbaum Associates.

Ogan-Bekiroglou, F. & Eskin, H. (2012). Examination of the relashionship between engagement in scientific argumentation and conceptual knowledge. *International Journal of Science and Mathematics Education*, *10*(6), 1415-1443.

Olson, D. R. (1994). *The world on paper: The conceptual and cognitive implications of reading and writing*. Cambridge: Cambridge University Press.

Organisation for Economic Cooperation and Development. (2007). *Assessing scientific, reading and mathematical literacy: A framework for PISA 2006.*

Radford, L. (2003). Gestures, speech and the sprouting of signs. A Semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning, 5*(1), 37-70.

Radford, L., Cerulli, M., Demers, S., & Guzmán, J. (2004). The sensual and the conceptual: Artefact mediated kinesthetic actions and semiotic activity. In M. J. Høines, & A. B. Fuglestad (Eds.), *Proceedings of the 28 Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 73–80). Norway: Bergen University College.

Roth, W. (2002). Reading graphs: contributions to an integrative concept of literacy. *Journal of Curriculum Studies*, 34 (1), 1-24.

Roth, W. M. (2004). What is the meaning of "meaning"? A case study from graphing. *Journal of Mathematical Behavior*, 23, 75-92

Roth, W., & McGinn, M.K. (1997). Graphing: Cognitive ability or practice? *Science Education*, 81, 91–106.

Shama, G. (1998). Understanding periodicity as a process with a gestalt structure. *Educational Studies in Mathematics*, 35, 255-281.

Triantafillou, C. (2011). Mathematical Literacy skills in a Workplace Context. The Case of Reading and Interpreting Data. *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education, Ankara, PME. Vol.* Aydeniz, M., Pabuccu, A., Cetin, P. S., & Kaya, E. (2012). Impact of argumentation on college students' conceptual understanding of properties and behaviors of gases. *International Journal of Science and Mathematics Education,* 10, 1303-1324.

Triantafillou, C., Spiliotopoulou, V., & Potari, D. (2013). Periodicity in textbooks. Reasoning and visual representations. In Lindmeier, A. M. & Heinze, A. (eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 297-304. Kiel, Germany: PME.

Triantafillou, C., Spiliotopoulou, V., & Potari, D. (2013). The nature of argumentation in school texts in different contexts. In Ubuz, B., Haser, Ç., & Mariotti, M. A. (Eds.). *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education*, pp. 256-265. Antalya, Turkey: CERME.

Spiliotopoulou V., & Triantafillou, C. (in press). Visual representations of the concept of periodicity across subjects: their context and genre. e-proceedings of the 10th Conference of the European Science Education Research Association (ESERA).

Vygotsky, L. S. (1978). *Mind in society: the development of higher psychological processes*. Cambridge: Harvard University.

REFERENCES IN GREEK

Τριανταφύλλου Χ., & Σπηλιωτοπούλου, Β. (2013). Μια μέθοδος ανάλυσης της επιχειρηματολογίας σχολικών βιβλίων: Η περίπτωση της περιοδικότητας σε κείμενα Φυσικής. Στο Δ. Βαβουγιός & Σ. Παρασκευόπουλος, *Πρακτικά 8ου Πανελλήνιου Συνέδριου Διδακτικής των Φυσικών Επιστημών και Νέων Τεχνολογιών στην Εκπαίδευση*, Παν. Θεσσαλίας, σελ. 264-272.

Τριανταφύλλου, Χ., Σπηλιωτοπούλου, Β., Σιδερής, Ε., & Κεχράκος, Δ. (2012). Η Έννοια της Περιοδικότητας στα Σχολικά Βιβλία και οι Αντιλήψεις των Σπουδαστών ΤΕΙ. *Πρακτικά 29ου Συνεδρίου ΕΜΕ: Μαθηματικά: Θεωρία -Πράξη-Προεκτάσεις*. ΕΜΕ, σελ.841-855.

APPENDIX

ACTIVITY 1

Task 1

Title: Cars' suspension system; *Subtitle:* Helical springs

Main text: Presents Hooks' Law and explains the visual representations as follows: In Fig. 4.27a we see two springs with different stiffness (different constant c). The spring with c=25000N/m [is named on the graph as stiff] is considered as more stiff than the other with c=7500N/m [is named on the graph as floppy]. In the common springs, the characteristic is a straight line and is called 'linear characteristic'. If the spring stiffness increases not linearly in respect to the change of the springs' deformation (Fig. 4.27b) then we have a progressive characteristic curve".

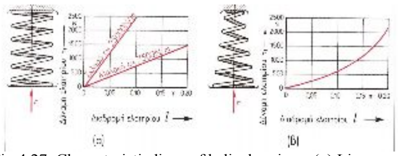


Fig.4.27: Characteristic lines of helical springs: (a) Linear; (b) Progressive

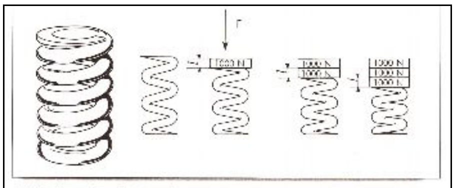
"Describe in an analytical way how you will explain to your students the significance of the different characteristics of helical springs (linear and progressive) that are represented in Fig. 4.27 in controlling cars' vibrations".

Task 1-The original text

Χαρακτηριστικά ελατηρίων

Το ελατήρια, οποιασδήποτε μορφής και αν είναι, όταν πιέζεται στατικά από μία δύναμη F, υποχωρεί, μέχρις ότου η συνισταμένη των εσωτερικών τάσεων γίνει ίση με την εξωτερική δύναμη F, οπότε επέρχεται ισορροπία και το ελατήριο παύει να υποχωρεί.

Εφόσον οι εσωτερικές τάσεις δεν υπερβούν τα όρια της αναλογίας δυνάμεων και παραμορφώσεων (Νόμος HOOK) σε κανονικές συνθήκες - πάντα μέσα στα όρια αυτά - και εφόσον η δύναμη F πάψει να ενεργεί, το ελατήριο θα επανέλθει στο αρχικό του μήκος, χωρίς να υποστεί καμία μόνιμη παραμόρφωση.



Μέσα, λοιπόν, στα όρια αυτά υπάρχει απόλυτη αναλογία δυνάμεων και παραμορφώσεων, οπότε διπλάσια δύναμη προκαλεί και διπλάσια παραμόρφωση, κ.ο.κ. *(Σχ. 4.26)*.

Η δύναμη F, όταν εφαρμόζεται σε ένα συγκεκριμένο ελατήριο και προκαλεί την παραμόρφωσή του κατά 0,01 m (1 cm), ονομάζεται "σταθερά c" του ελατηρίου αυτού και όσο μικρότερη είναι αυτή η "σταθερά c", τόσο το ελατήριο είναι πιο εύκαμπτο (περισσότερο μαλακό).

Όταν μάλιστα επιβληθεί φορτίο (δύναμη) π.χ. 1000 N (Νιούτον), το αρχικό μήκος του ελατηρίου ελαττώνεται κατά το ύψος ℓ, ενώ εάν διπλασιαστεί το φορτίο, το μήκος μειώνεται κατά 2ℓ, κ.ο.κ. Είναι δηλαδή $c=F/\ell$ (σε N/m -Νιούτον ανά μέτρο).

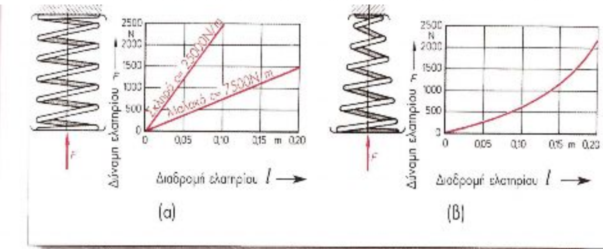
Στο Σχ. 4.27(α) φαίνονται δύο ελατήρια με διαφορετική σκληρότητα (διαφορετική "σταθερά c"). Το ελατήριο με c=25.000 N/m, είναι σκληρότερο από το δεύτερο με c=7500 N/m. Στα συνήθη αυτά ελατήρια, η χαρακτηριστική γραμμή είναι ευθεία "γραμμική χαρακτηριστική".

Εάν, όμως, η σκληρότητα αυξάνεται δυσανάλογα με τη μεταβολή του μήκους του ελατηρίου *[Σχ.4.27(β)]*, τότε έχουμε την "προοδευτική χαρακτηριστική" και η γραμμή της είναι καμπύλη.

Επίσης, τα ελικοειδή κωνικά ελατήρια, όπως και τα ημιελλειπτικά, δίνουν προοδευτική χαρακτηριστική καμπύλη.

Ταλάντωση

Ας υποθέσουμε ότι ο τροχός ενός οχήματος *[Σχ. 4.28(α)]* κινείται σε λείο δρόμο (θέση Α) και έχει, ήδη, το ελατήριο ανάρτησής του φορτωθεί με μία δύναμη, που προέρχεται από το στατικό φορτίο του βάρους του οχήματος. Εάν, λοιπόν, ο τροχός δεχθεί μία δεύτερη στιγμιαία συμπίεση, π.χ. από ένα κρουστικό φορτίο (βαθμίδα-σκαλοπάτι), θα υποχωρήσει

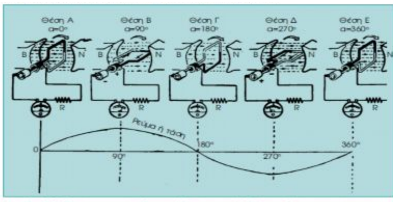


Σχ.4.27 Χαρακτηριστικές ελικοειδών ελατηρίων
(α) Γραμμική χαρακτηριστική. (β) Προοδευτική χαρακτηριστική

Task 2

Title: **Generators**
Subtitle: **Operation principles of AC generators**
Main text: [Describes systematically the materials and the conditions required in order a DC generator could operate. Then the main scientific entities (Magnetic Filed B (in Tesla), a coiled copper wire with length l (in meters) is rotated in the magnetic field, the velocity v (in m/s^2) of this rotation and the generated Voltage E in Volts) are introduced and their relation is described in a symbolic form as follows: **E =B l v sina (in Volts)**

In the following scheme (F. 2.2) the course of this phenomenon is presented as the coils of the copper wire is rotated relatively in the magnetic field. In the same scheme appears the alternate generated Voltage.

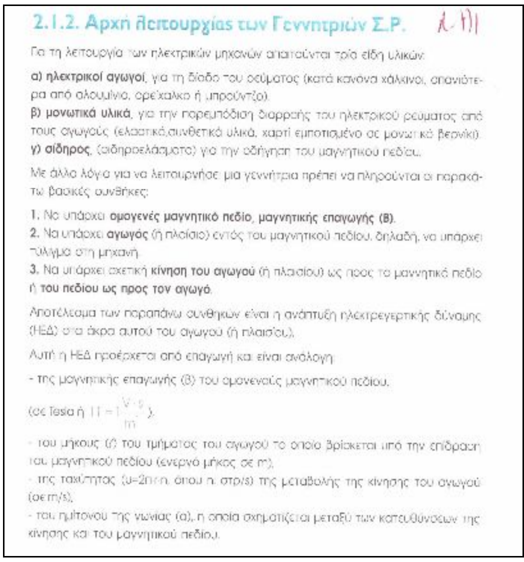


Σχ. 2.2: Πορεία της ανάπτυξης εναλλασσόμενης Η.Ε.Δ. σε πλαίσιο στρεφόμενου μέσα σε σταθερό μαγνητικό πεδίο.

F. 2.2: The course of development of the generated Voltage in the ends of the rotating frame in a constant magnetic field.

Students' task was formed as follows: Design proper teaching activities under the specific teaching goal: *"The student should be able to substantiate scientifically the relation between the rotation of the bicycle wheel and the brightness of the light bulb".*

Task 2-The original text



2.1.2. Αρχή λειτουργίας των Γεννητριών Σ.Ρ.

Για τη λειτουργία των ηλεκτρικών μηχανών απαιτούνται τρία είδη υλικών:

α) ηλεκτρικοί αγωγοί, για τη δίοδο του ρεύματος (κατά κανόνα χάλκινοι, σπανιότερα από αλουμίνιο, ορείχαλκο ή μπρούντζο).
β) μονωτικά υλικά, για την παρεμπόδιση διαρροής του ηλεκτρικού ρεύματος από τους αγωγούς (ελαστικά, συνθετικά υλικά, χαρτί εμποτισμένο σε μονωτικό βερνίκι).
γ) σίδηρος, (σιδηροελάσματα) για την οδήγηση του μαγνητικού πεδίου.

Με άλλο λόγια για να λειτουργήσει μια γεννήτρια πρέπει να πληρούνται οι παρακάτω βασικές συνθήκες:

1. Να υπάρχει **ομογενές μαγνητικό πεδίο, μαγνητικής επαγωγής (Β)**.
2. Να υπάρχει **αγωγός** (ή πλαίσιο) εντός του μαγνητικού πεδίου, δηλαδή, να υπάρχει τύλιγμα στη μηχανή.
3. Να υπάρχει σχετική **κίνηση του αγωγού** (ή πλαισίου) ως προς το μαγνητικό πεδίο ή **του πεδίου ως προς τον αγωγό**.

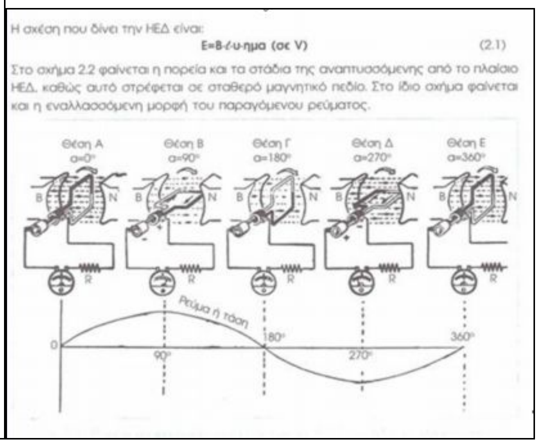
Αποτέλεσμα των παραπάνω συνθηκών είναι η ανάπτυξη ηλεκτρεγερτικής δύναμης (ΗΕΔ) στα άκρα αυτού του αγωγού (ή πλαισίου).

Αυτή η ΗΕΔ προέρχεται από επαγωγή και είναι ανάλογη:

- της μαγνητικής επαγωγής (Β) του ομογενούς μαγνητικού πεδίου.

(σε Tesla ή $1T = 1\frac{V \cdot s}{m}$).

- του μήκους (ℓ) του τμήματος του αγωγού το οποίο βρίσκεται υπό την επίδραση του μαγνητικού πεδίου (ενεργό μήκος σε m).
- της ταχύτητας (υ=2π·r·n, όπου n: στρ/s) της μεταβολής της κίνησης του αγωγού (σε m/s).
- του ημίτονου της γωνίας (α), η οποία σχηματίζεται μεταξύ των κατευθύνσεων της κίνησης και του μαγνητικού πεδίου.



Η σχέση που δίνει την ΗΕΔ είναι:

$$E = B \cdot \ell \cdot u \cdot \eta\mu\alpha \text{ (σε V)} \qquad (2.1)$$

Στο σχήμα 2.2 φαίνεται η πορεία και τα στάδια της αναπτυσσόμενης από το πλαίσιο ΗΕΔ, καθώς αυτό στρέφεται σε σταθερό μαγνητικό πεδίο. Στο ίδιο σχήμα φαίνεται και η εναλλασσόμενη μορφή του παραγόμενου ρεύματος.

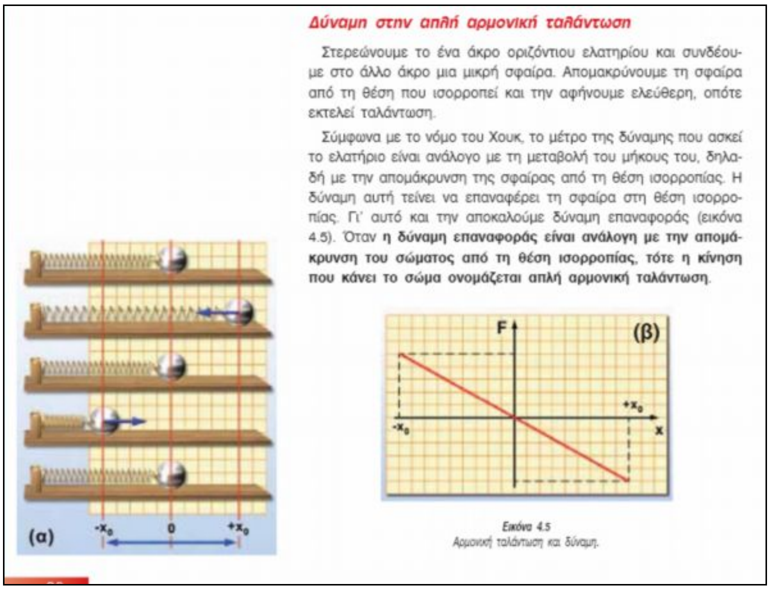
ACTIVITY 2

Department……………………………….	Semester: ………………………….
Sex: ❑ Male ❑ Female	High school Graduation: General ❑ Vocational ❑

STUDY THE MOTION OF THE SPRING

Task 1

The book extract which is presented in Picture 1 shows an experimental arrangement that refers to the study of the movement of the spring and is included in the 3rd Grade Secondary School Science book.


Δύναμη στην απλή αρμονική ταλάντωση

Στερεώνουμε το ένα άκρο οριζόντιου ελατηρίου και συνδέουμε στο άλλο άκρο μια μικρή σφαίρα. Απομακρύνουμε τη σφαίρα από τη θέση που ισορροπεί και την αφήνουμε ελεύθερη, οπότε εκτελεί ταλάντωση.

Σύμφωνα με το νόμο του Χουκ, το μέτρο της δύναμης που ασκεί το ελατήριο είναι ανάλογο με τη μεταβολή του μήκους του, δηλαδή με την απομάκρυνση της σφαίρας από τη θέση ισορροπίας. Η δύναμη αυτή τείνει να επαναφέρει τη σφαίρα στη θέση ισορροπίας. Γι' αυτό και την αποκαλούμε δύναμη επαναφοράς (εικόνα 4.5). Όταν **η δύναμη επαναφοράς είναι ανάλογη με την απομάκρυνση του σώματος από τη θέση ισορροπίας, τότε η κίνηση που κάνει το σώμα ονομάζεται απλή αρμονική ταλάντωση.**

Εικόνα 4.5
Αρμονική ταλάντωση και δύναμη.

Photo 1

(a)What is the purpose of the author? What does he want to show his students? Which way does he use in order to persuade them?

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How would you explain the connection of the phases of the experiment in Picture 4.5(a) with the graph in Figure 4.5(b)?

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More specifically, how would you explain the existence of negative values of the power as shown in Picture 4.5(b)?

Task 2

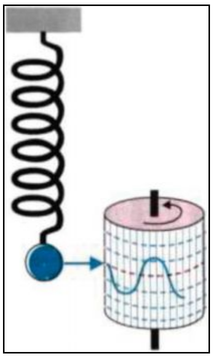

Photo 2

Photo 2 is included in the 2nd Grade Senior High School Core Science book. In this photo the author presents an ideal spring with a sphere attached to its lower edge which oscillates. The sphere is connected with a pen that writes on a cylinder's paper while the cylinder is rotating at a constant speed so as to show the movement of the spring.

Task 2(a) What is the purpose of the author? What does he want to show his students? What conclusions can the students reach?

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Can you think of an occasion that the experiment may not work? Justify your answer.

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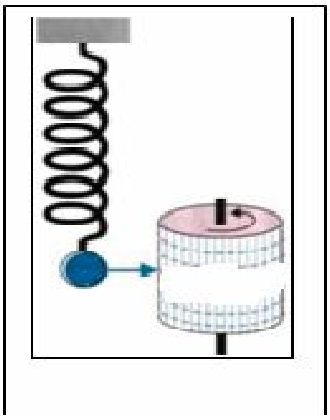
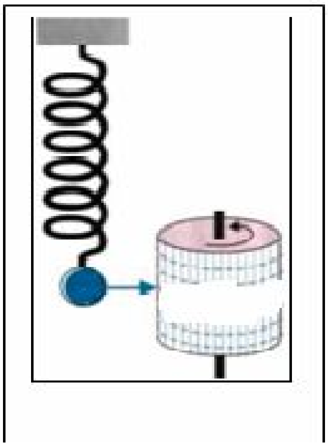
Task 2(b) Try to depict the movement of the spring as is it written in the cylinder when its rotation takes place slower and quicker in relation to the movement of the spring. Justify your answer.



Much slower



Much faster

TASK 3

The two graphs in Photos 1 and 2 refer to the same phenomenon. What relationships are depicted in each one of the two graphs? What are their differences? Analyze and justify your answer.

ACTIVITY 3

Department: ……………………………….	Semester: ………………………….
Sex: ❑ Male ❑ Female	High school Graduation: General ❑ Vocational ❑

Periodic Motions

Task 1

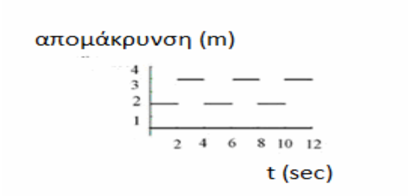
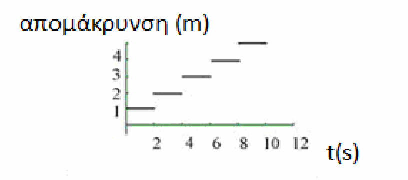
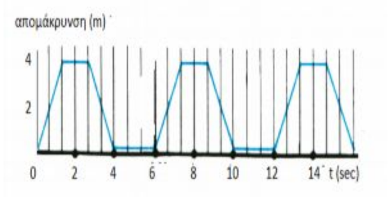
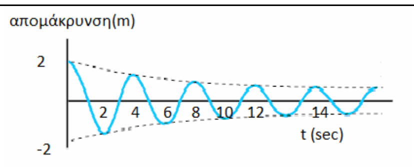
Consider the following 3 movements. Define the elements that characterize the periodical behaviour in each case.

MOVEMENT	PERIODICAL BEHAVIOUR ELEMENTS
The motion of a ship traveling back and forth two ports	
The motion of the swing	
The motion of the earth around the sun	

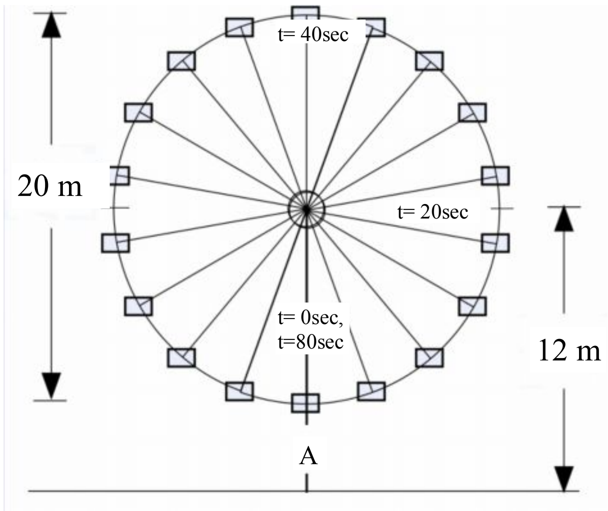
If you were asked to show which of the above motions is the best example of periodical behaviour in real-life situations, which of the three situations would you choose? Justify your choice in comparison to the other two.

Task 2

The graphs presented below show the removal of a body from the starting point in relation to time.

GRAPH	Write an example of a movement described in each graph	Which of the graphs do you think show periodical movement? Justify your answer.
		
		
		
		

Task 3



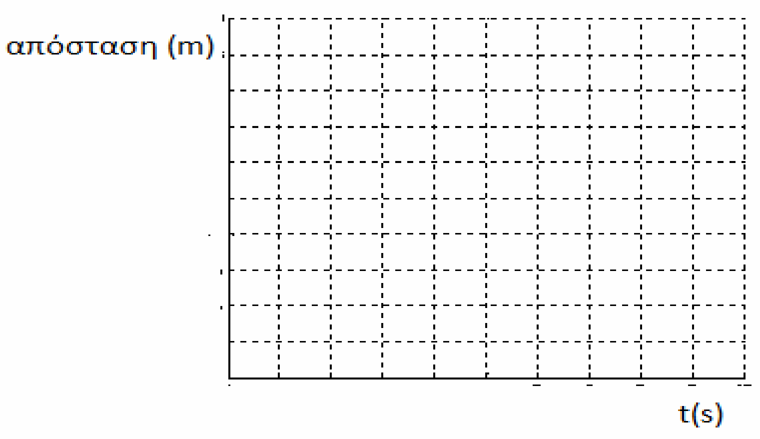
Εικόνα 1. Ferris wheel

Supposing your are on the ferris wheel of an amusement park.

(a) Which elements concerning the movement of the wheel are given or we can deduct from Picture 1?

(b) If we regard that you start from point A, can you define the function that describes the relation of the height of the wagon to time?

(c) If the wagon moves without stopping for 4 minutes, try to graphically represent in Figure 1 the change of height in relation to time throughout its movement. Use explanations in your figure to clarify your answer as well as your way of working with it.



Σχήμα 1