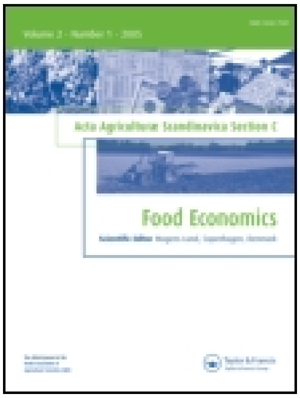

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Utility-derived supply function of sheep milk: The case of Etoloakarnania, Greece

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ORIGINAL ARTICLE

Utility-derived supply function of sheep milk: The case of Etoloakarnania, Greece

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Abstract
Dairy sheep farming is an important agricultural activity in Greece, since it contributes significantly to the country's gross agricultural production value. In this study, we suggest the use of multi-criteria analysis to estimate the supply response of sheep milk to price. The study focuses on the Prefecture of Etoloakarnania, located in Western Greece, where sheep farming is a common and traditional activity. A non-interactive technique is used to derive farmers' individual utility functions which are then optimised parametrically, subject to technico-economic constraints, to estimate the supply function of sheep milk. Detailed data from selected farms representing different farm types and management strategies have been used in the analysis. The results indicate that the multi-criteria model reflects the actual operation of the farms more accurately than the gross margin maximisation model and therefore leads to a more robust estimation of the milk supply.

Keywords: *Milk supply, multi-criteria, sheep farming, utility function.*

1. Introduction

Milk supply and its response to price changes has been the object of a number of economic studies (Papaioannou & Jones, 1972; Rayner, 1975; Papanagiotou, 1987; Roemen, 1993). The majority of these studies focus on the production of cow milk and the estimation of the supply response to price is achieved through econometric approaches. Unlike other developed countries, the production of sheep milk in Greece is as equally important as the production of cow milk (National Statistical Service of Greece (N.S.S.G.), 2006). Sheep farming is one of the most important agricultural activities in the country since it constitutes the main or side activity for a large number of farms (N.S.S.G., 2000). Greek sheep farms produce both milk and meat, but over 60% of their total gross revenue comes from milk (Hadjigeorgiou et al., 1999; Zioganas et al., 2001; Kitsopanides, 2006). Recently, the sheep farming activity has received further attention because of the certification of feta cheese, which consists mainly of sheep milk, as a protected designation of origin product.

The purpose of this study is to estimate the supply response of sheep milk to price through the use of mathematical programming. Specifically, a mixed integer programming model that incorporates detailed technico-economic characteristics of the sheep farms is used to simulate their operation. Linear programming models are commonly used to capture livestock farmers' decision-making process (Biswas et al., 1984; Conway & Killen, 1987; Alford et al., 2004; Veysset et al., 2005; Crosson et al., 2006). The common characteristic of these models is that they maximise gross margin assuming that this is the only objective of farmers. But the structure of the sheep farming activity in Greece indicates that this assumption is rather unrealistic.

The nature of the sheep farming activity and its ability to profitably utilise less fertile soil has caused its expansion in many agricultural areas of Greece, and traditionally its concentration in isolated and less favoured areas. In these areas the prevailing farm type is the small, extensive, family farm. According to the N.S.S.G. (2006), almost 63% of the Greek sheep farms have less than 50 sheep. Furthermore, almost 85% of the Greek sheep farms are extensive

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and have low invested capital (Hellenic Ministry of Rural Development and Food (H.M.R.D.F.), 2007). Apart from sheep farming found in mountainous and less favoured areas, more intensive and modern farms have appeared recently, especially in lowland areas. The different production systems identified in the country have different technical and economic characteristics and achieve different levels of productivity (Rancourt et al., 2006).

This high degree of diversification implies different management strategies developed according to farmers' individual preferences and combination of goals. The multiple goals of farmers and the development of different management styles and strategies have been the object of many studies (Harman et al., 1972; Cary & Holmes, 1982; Fairweather & Keating, 1994; Costa & Rehman, 1999; Solano et al., 2001; Vandermersch & Mathijs, 2002; Bergevoet et al., 2004). These studies indicate that farm level models that incorporate multiple goals can be more effective and can assist policy makers in developing more efficient and targeted policy measures and in adjusting the existing policy regime accordingly (Arriaza & Gómez-Limón, 2003).

Thus, in this study a farm level model that incorporates multiple goals is built to replace the traditional single objective model. In most multi-criteria studies the elicitation of the individual utility function is accomplished through the implementation of interactive techniques. But the use of interactive techniques comes with many problems and often leads to ambiguous results (Patrick & Blake, 1980; Sumpsi et al., 1996). To overcome interaction problems we have used a non-interactive technique to derive farmers' individual utility functions, proposed by Sumpsi et al. (1996) and further extended by Amador et al. (1998). The individual utility functions are then optimised parametrically, subject to the technico-economic constraints of the farms to estimate the supply response of sheep milk to price. Kazakçi et al. (2007) minimise maximum regret instead of maximising gross margin for better approximation of supply response curves of energy crops in France and a number of studies use multi-criteria analysis for the estimation of the demand for irrigation water since it leads to a more accurate reflection of the actual operation of the farms and therefore to a more robust estimation of supply response (Gómez-Limón & Berbel, 2000; Gómez-Limón & Riesgo, 2004; Latinopoulos, 2008).

For the purpose of this paper detailed data from selected farms, representing different farm types have been used. The study focuses on the Prefecture of Etoloakarnania, where sheep farming is a well known and traditional activity. Results of our analysis support the point of view expressed in previous studies regarding the usefulness of the methodology to researchers and policy makers.

In the following section the methodology, used in this analysis, is described. Section 3 presents the case study and the model specification. Finally, the last two sections contain the results of the analysis and some concluding remarks.

2. Methodology

The methodology used for the estimation of the milk supply function, in this study, can be analysed in three distinct parts. First, for each of the selected farms, a mixed integer programming model that reflects its operation is built. The technico-economic constraints and decision variables are defined according to the data collected from the selected farms. Secondly, the set of farmers' goals to be used in the analysis is determined and the multi-criteria technique is applied to derive the individual utility function of each farmer. Then, the estimated utility function is optimised parametrically (various price levels) and the individual (disaggregated) supply function for each farmer is extracted. Finally, the total supply function of sheep milk is estimated, using the number of farms represented by each farm type.

2.1. Mixed integer livestock farm detailed model

Optimisation models taking into account interrelationships, such as resource and agronomic constraints as well as synergies and competition among activities, usually select the most profitable activity plan and have been extensively used in agriculture. They allow for a technico-economic representation of production units (farms) containing a priori information on technology, fixed production factors, resource and agronomic constraints, production quotas and set aside regulations, along with explicit expression of physical linkages among activities.

Livestock mathematical programming models are in general more complicated than arable cropping ones. They include a large number of decision variables and resource, agronomic and policy constraints (Alford et al., 2004; Crosson et al., 2006). The model used in this analysis uses similar decision variables and constraints, though it is in fact a mixed integer programming model, since some variables are constrained to receive only integer numbers. These variables refer to the number of ewes. The mixed integer programming models are commonly used, when livestock, crop livestock and aquaculture farms are studied (Engle, 1987; Shaftel & Wilson, 1990).

2.2. Non-interactive multi-criteria methodology

Multi-criteria approaches mainly goal programming and multi-objective programming, are most common in agricultural studies (McGregor & Dent, 1993; Piech & Rehman, 1993; Siskos et al., 1994; Berbel & Rodriguez-Ocana, 1998). In most of these multi-criteria approaches, the goals incorporated in the model and the weights attached to them are elicited through an interactive process with the farmer (Dyer, 1972; Rehman & Romero, 1993). This interaction with the farmer and the self-reporting of goals has limitations, since farmers often find it difficult to define their goals and articulate them. Another problem associated with this interactive process is that individuals feel uncomfortable when asked about their goals or are often influenced by the presence of the researcher and adjust their answers to what they feel the researcher wants to hear. The above problems denote the need to employ a different method to determine farmers' objectives in multi-criteria studies.

In this study, we apply a well known, non-interactive methodology to derive the utility function of each farmer (Sumpsi et al., 1996). The basic characteristic of this methodology is that the farmer's actual and observed behaviour is used for the determination of the objectives and their relative importance. Assume that:

- x = vector of decision variables (see Appendix 1);
- F = feasible set (see Appendix 1);
- $f_i(x)$ = mathematical expression of the ith objective (Equations (6)–(10) in Section 3);
- w_i = weight measuring relative importance attached to the ith objective;
- f_i^* = ideal or anchor value achieved by the ith objective;
- f_{*i} = anti-ideal or nadir value achieved by the ith objective;
- f_i = observed value achieved by the ith objective;
- f_{ij} = value achieved by the ith objective when the jth objective is optimised;
- n_i = negative deviation (underachievement of the ith objective with respect to a given target) and
- p_i = positive deviation (overachievement of the ith objective with respect to a given target).

The first step of the methodology involves the definition of an initial set of objectives $f_1(x),\ldots, f_i(x),\ldots,f_q(x)$. The researcher can define this initial set of objectives according to previous research and related literature or through preliminary interviews with the farmers. In the second step, each objective is optimised separately over the feasible set. At each of the optimal solutions the value of each objective is calculated and the pay-off matrix is determined (Sumpsi et al., 1996). Thus, the first entry of the pay-off matrix is obtained by:

$$\text{Max}\, f_1(x), \text{ subject to } x \in F \qquad (1)$$

since $f_1^* = f_{11}$. The other entries of the first column of the matrix are obtained by substituting the optimum vector of the decision variables in the remaining q–1 objectives. In general, the entry f_{ij} is acquired by maximising the $f_j(x)$ subject to $x \in F$ and substituting the corresponding optimum vector x^* in the objective function $f_i(x)$.

The elements of the pay-off matrix and the observed (actual) values for each objective are then used to build the following system of q equations. This system of equations is used to determine the weights attached to each objective:

$$\sum_{j=1}^{q} w_j f_{ij} = f_i \qquad i = 1, 2, \ldots, q \qquad (2)$$

$$\sum_{j=1}^{q} w_j = 1.$$

The non-negative solution generated by this system of equations represents the set of weights to be attached to the objectives so that the actual behaviour of the farmer can be reproduced $(f_1, f_2,\ldots,f_q)$. Usually the above system of equations has no non-negative solution and thus the best solution has to be alternatively approximated.

To minimise the corresponding deviations from the observed values, the entire series of L metrics[1] can be used. In our analysis, we have used the L_1 criterion that minimises of the sum of positive and negative deviational variables (Sumpsi et al., 1996; Amador et al., 1998). The L_1 criterion assumes a separable and additive form for the utility function. Alternatively, the L_∞ criterion according to which the maximum deviation D is minimised can be used (Appa & Smith, 1973). Both criteria are commonly used in agricultural studies, partly because they can be managed through an LP specification. The L_∞ criterion corresponds to a Tchebycheff utility function that implies a complementary relationship among objectives (Amador et al., 1998). Nevertheless, in this first attempt to explore the behaviour of sheep farmers in Greece we use the L_1 criterion and assume the separable and additive utility function (Equation 4), often used in agricultural studies (Sumpsi et al., 1996; Gómez-Limón et al., 2003).

To solve the minimisation problem (minimisation of the sum of positive and negative deviational variables) we use the weighted goal programming technique (Appa & Smith, 1973; Sumpsi et al., 1996). The formulation of the weighted goal programming technique is shown below:

$$\text{Min} \sum_{i=1}^{q} \frac{(n_i + p_i)}{f_i}$$

subject to:

$$\sum_{j=1}^{q} w_j f_{ij} + n_i - p_i = f_i \qquad i = 1, 2, ..., q \qquad (3)$$

$$\sum_{j=1}^{q} w_j = 1$$

As mentioned above the L_1 criterion corresponds to a separable and additive utility function. The form of the utility function is shown below:

$$u \sum_{i=1}^{q} \frac{w_i}{k_i} f_i(x) \qquad (4)$$

k_i is a normalising factor (e.g. $k_i = f_i^{*} - f_{i*}$). It is essential to use the normalising factor, to avoid overestimating the weights of goals with high absolute values in the utility function, when goals used in the analysis are measured in different units (Rehman & Romero, 1993; Sumpsi et al., 1996; Tamiz et al., 1998). After estimating the farmers' individual utility function, we maximise it subject to the constraint set (see Appendix 1) and the results of the maximisation are compared to the actual values of the q goals. This way the ability of the utility function to accurately reproduce farmers' behaviour is checked and the model is validated. Namely, the following mathematical programming problem is solved:

$$\text{Max} \sum_{i=1}^{q} \frac{w_i}{k_i} f_i(x)$$

subject to:

$$f_i(x) + n_i - p_i = f_i \qquad i = 1, 2, ..., q. \qquad (5)$$

$$x \in F$$

If the estimated function gives results for each goal close to the actual values then it is considered the utility function that is consistent with the preferences of the farmer. On the other hand, if the above utility function cannot reproduce farmer's behaviour, other forms of the utility function should be examined (Sumpsi et al., 1996; Amador et al., 1998). However, it should be noted that the utility function has to represent the actual situation accurately, not only against alternative objectives, but also against decision variables.

2.3. Parametric optimisation to estimate supply response at the farm and the sector level

The microeconomic concepts of supply curve and opportunity cost could be approximated in a satisfactory way by using mathematical programming models, called supply models, based on a representation of farming systems. Thanks to supply models, it is possible to accurately estimate these costs by taking into account heterogeneity and finally to aggregate them in order to obtain the raw material supply for industry. It is postulated that the farmers choose among crop and animal activities so as to maximise the agricultural income or gross margin. Variables take their values in a limited feasible area defined by a system of institutional, technical and agronomic constraints. To estimate the individual supply function for each farmer the above optimisation problem can be solved for various levels of milk price. Moreover, the total supply function can be estimated by aggregating the individual supply functions, taking into account the total number of farms in the area under study represented by the farms used in the analysis. Similar methodology has been used by Gómez-Limón and Riesgo (2004) for the estimation of the demand for irrigation water in Andalusia and by Sourie (2002) and Kazakçi et al. (2007) for the estimation of the supply of energy biomass in the French arable sector.

3. Case study

3.1. Data

In this analysis we estimate the milk supply function in the Prefecture of Etoloakarnania, located in Western Greece. The Prefecture of Etoloakarnania produces 7% of the total sheep milk in Greece and includes almost 9% of the total number of Greek sheep farms (N.S.S.G., 2006). Sheep farming is a common and traditional activity in the area. The majority of farms have a small flock, which indicate that sheep farming is often a part-time or side activity. Specifically, 42% of the farms have less than 50 sheep, while less than 9% of the farms have a number of sheep larger than 200.

Thus, the estimation of the milk supply function of the area is achieved through the use of technico-economic data from three sheep farms with different flock size and milk production. Other differences amongst the selected farms (which are more or less linked to the flock size) are the amount of farm produced fodder, the labour requirements and the breeding system (extensive or intensive). The selection of farms with different sizes means that our analysis will be laid out in groups of farmers, leading to a more precise estimation of milk supply. This is essential in a multi-criteria analysis since previous studies indicate that the goals of farmers can differ between large and small farms (Gasson, 1973; Wallace & Moss, 2002). In the case of sheep farming in Greece, where 63% of the farms have a small number of livestock, studying these farms along with the larger farms and stressing any differences between them is crucial.

For the above reasons, the first selected farm is a large and commercial example. It produces part of the fodder it uses and has an annual milk yield of 135 kg/ewe. This farm represents 764 farmers in the area under study, according to the flock size (National Payment Agency of Greece – N.P.A.G. (O.P.E.K.E.P.E.), personal communication, May 2008). The second farm has a middle size flock (80 ewes), it is located in lowland area, has a lower milk yield and produces alfalfa and maize not only to cover the needs of the livestock activity but also for cash. Although this farm is a commercial farm, and the owner is a full-time farmer, it has a different production orientation than the large farm, since it aims at the production of feedstock and not only in the production of milk. According to the N.P.A.G. (personal communication, May 2008), there are about 4379 farmers in the area with a flock size of 50–200 sheep. The third farm is a small-scale farm, representing only a part-time activity for the owner. The part-time farmer produces no feedstock and receives only a supplementary income from sheep farming. This farm represents 3750 farmers in the area under study (less than 50 sheep). The main characteristics of the farms used in the analysis are summarised in Table I. It should be mentioned that the gathered data refer to the agricultural year 2004–2005 (annual data).

3.2. Model specification

The estimation of the individual supply functions supposes the construction of a linear programming model that can reflect the characteristics and constraints of each of the three farms accurately. The model used in the analysis has also been used in previous work (Sintori et al., 2009) and has undergone a slight modification. This change involves an extra constraint on the percentage of energy requirements satisfied from concentrates, which varies among farms. The model is adjusted according to the specific characteristics of each farm. The main difference of the multi-criteria model among the three farms is the different objective function (utility function). The other parts of the model (decision variables and constraints) are adapted to the specific farm features. In its basic form the model consists of 108 decision variables and 95 constraints that cover both animal and crop activities of the farms (see Appendix 1).

There are three sets of decision variables included in the model. The first set involves the production of fodder (mainly alfalfa and maize), the use of pastureland (area of different kinds of pastureland engaged by the farm) and the monthly consumption of in-farm produced or purchased fodder. The second set involves monthly family and hired labour engaged in crop and animal activities. The last set of decision variables involves the animal activities of the farm and the area engaged in the production of cash crops. It should be noted that there are four animal activities incorporated in the model, namely the production of lambs that are sold after weaning or three months after birth (rearing) and ewes that are premium eligible or not (previous CAP regime).

Table I. Main characteristics of the farms used in the analysis.

	Large farm	Medium farm	Small farm
Size			
Gross margin (€)	36,986	20,798	3263
Number of ewes	262	80	20
Total land (including pastureland) (Stremmas)	885	90	26
Total irrigated land (Stremmas)	85	75	3
Intensity			
Milk yield/ewe (Kg)	135	85	128
Labour/ewe (Hr)	33	9	13
Energy requirements/ewe (MJ)	3723	2768	3115
Alfalfa yield (Kg/Stremma)	1320	2000	–
Maize yield (Kg/Stremma)	910	1290	–
Production orientation			
Alfalfa for cash (Stremmas)	0	25	0
Maize for cash (Stremmas)	0	22	0
Gross margin from sheep farming to total gross margin (%)	97%	36%	80%

The constraint matrix includes land constraints (total own land, irrigated land, available pastureland, etc.), the monthly distribution of produced fodder, monthly nutrient requirements (dry matter, Net Energy of Lactation – NEL (MJ), digestible nitrogen), monthly labour requirements of all activities and policy constraints (number of premium eligible ewes). For the estimation of the nutrient requirements of the flock the methodology described by Zerbas et al. (2000) has been used. The mathematical expression of the constraint matrix and the decision variables are presented in Appendix 1.

3.3. Initial set of goals

Five tentative goals are used in this analysis. The first goal is the maximisation of the total gross margin which is considered the main economic goal of farmers and therefore is widely used in decision-making models (e.g. Piech & Rehman, 1993). But Greek farmers often place more value on keeping their expenses (mainly variable costs) low, than on making maximum profit. For this reason we have also included the minimisation of variable cost at the initial set of goals, following a number of studies (e.g. Piech & Rehman, 1993). The third goal refers to the minimisation of family labour. This goal is strongly linked to the farmer's attempt to increase his leisure time. The importance of this goal is stressed in a number of studies of farmers' goals (e.g. Barnett et al., 1982).

The fourth goal refers to the minimisation of all purchased feed and is linked mainly with the increasing concern about the quality and hygiene of fodder and rather secondly to maintaining expenses at a low level. Farmers often prefer to feed their livestock with fodder produced on the farm. This attempt is evident in farmers that consume part of their products, or wish to produce and promote quality products. The last goal is the minimisation of the cost of foreign labour (e.g. Piech & Rehman, 1993; Berbel & Rodriguez-Ocana, 1998). This is a major concern of farms that attempt to utilise family labour to increase farm income. But this is not the only reason, since hired labour is not always abundant. Consequently, farmers may need to restrict the amount of the livestock so as to depend only on family labour. The five goals used in this analysis and their mathematical expressions are given below (see Appendix 1 for the indices, parameters and decision variable notation):

1. Maximisation of gross margin (in €):

$$f(1) = \text{Max}\ [\sum_{ti} \text{gr\_marc}_{ti,\ \text{sales}} \cdot \text{crop}_{ti,\ \text{sales}} + \sum_{r}\sum_{a} \text{gr\_mara}_{a,r} \cdot \text{anim}_{a,r} - \sum_{g} \text{rqwc}_{g} \cdot \text{gland}_{g} - \sum_{t}\sum_{fi} \text{rqwc}_{fi,t} \cdot \text{feed}_{fi,t} - \sum_{t}\sum_{fs} \text{rqwc}_{fs,t} \cdot \text{feed}_{fs,t} - \sum_{r}\sum_{a} \text{rqwc}_{a,t} \cdot \text{anim}_{a,r} - \sum_{ti} \text{rqwc}_{ti} \cdot \text{crop}_{ti,\ \text{con,sales}} - \sum_{t}\sum_{l} \text{lab}_{l,\text{hire},t} \cdot w_{l,\text{hire}}]. \quad (6)$$

2. Minimisation of the variable cost (in €):

$$f(2) = \text{Min}\ [\sum_{g} \text{rqwc}_{t,g} \cdot \text{gland}_{g} + \sum_{t}\sum_{fi} \text{rqwc}_{fi.t} \cdot \text{feed}_{fi,t} + \sum_{t}\sum_{fs} \text{rqwc}_{fs,t} \cdot \text{feed}_{fs.t} + \sum_{r}\sum_{a} \text{rqwc}_{a,t} \cdot \text{anim}_{a,r} + \sum_{ti} \text{rqwc}_{ti} \cdot \text{crop}_{ti,\ \text{“con,sales”}} + \sum_{t}\sum_{l} \text{lab}_{l,\text{hire},t} \cdot w_{l,\text{hire}}]. \quad (7)$$

2. Minimisation of the family labour (in hours):

$$f(3) = \text{Min} \sum_{l}\sum_{t} \text{lab}_{l,\text{own}.t}. \quad (8)$$

3. Minimisation of the amount of purchased fodder (in MJ)[2]:

$$f(4) = \text{Min} \sum_{fs}\sum_{t} y_{fs,\text{energy}} \text{feed}_{fs,t}. \quad (9)$$

4. Minimisation of hired labour (in hours):

$$f(5) = \text{Min} \sum_{l}\sum_{t} \text{lab}_{l,\text{hire},t}. \quad (10)$$

4. Results of the analysis

4.1. Utility functions

In order to build the multi-criteria model for each of the farms we use the methodology described in a previous section for the elicitation of the individual utility function. The first step of the analysis is to obtain the pay-off matrix for each of the farms and apply the L_1 criterion. This way we estimate the weights attached to each of the initial goals. For the large farm, the analysis indicates that the farmer aims at maximising gross margin with a weight of 37%. But the farmer mainly aims at minimising hired labour (52%), since the farm actually has high labour requirements, especially for grazing. The weight of the minimisation of purchased fodder is low but non-negligible (11%). The other two of the initial goals receives zero weight, as far as the large farm is concerned. Using these weights and Equation (4), we can estimate the utility function of the farmer:

$$U_1 = 0.37{*}f_1/15,682 - 0.11{*}f_4/1,446,487 - 0.52{*}f_5/41,630. \quad (11)$$

For a medium size farm, which is also commercial, the main attribute of the utility function is the maximisation of the gross margin, since the weight attached to this objective is 55%. Another important attribute in the utility function of this farm is the minimisation of purchased fodder, since one of the farm's main activities is the production of alfalfa and maize, not only for consumption but also for cash. The weight of this attribute is 39%. A smaller weight is given at the minimisation of variable costs (6%). According to the estimated weights, the utility function for this farmer is shown below:

$$U_2 = 0.55{*}f_1/4799 - 0.06{*}f_2/3643 - 0.39{*}f_4/4539. \quad (12)$$

Finally, as far as the small farm is concerned, the analysis indicates that the farmer intends not only to maximise gross margin but mainly to minimise family labour. The weights attached to these objectives are 23% and 77%, respectively. The weight attached to the gross margin maximisation is smaller than in the case of larger farms. On the other hand, the minimisation of family labour is only included in the utility function of the owner of the small farm, where it receives the highest weight. The reason for this is that the owner of the third farm is only a part-time farmer. This pluriactive farmer probably needs to save on labour inputs so that he can invest time and effort in his off-farm activities. The estimated weights derive the utility function shown below:

$$U_3 = 0.23{*}f_1/2209 - 0.77{*}f_3/682. \quad (13)$$

4.2. Model validation

The utility functions estimated above are then optimised (to the existing price level), subject to the model constraints to approximate farmers' behaviour. It should be noted that, because of the small weight attached to the gross margin maximisation objective, an additional constraint has been used in the case of the small farm that does not allow the estimated gross margin to be less than 70% of the observed one. To allow for comparison, the traditional gross margin maximisation objective function is also optimised. First, the predicted values of all objectives, according to both the traditional and the multi-criteria model, are compared (Amador et al., 1998). But in order to decide on the ability of the multi-criteria model to reproduce farmers' behaviour, the decision variable of space has to be taken into account as well. Tables II–IV summarise the predicted values of the objectives and the decision variables for the farms. The observed values are also included in the tables. The last two columns contain the absolute deviations of the predicted values from the observed values, in the case of gross margin maximisation and the maximisation of the estimated utility function. The total deviation from the observed behaviour is also presented and the last row contains the ratio of the deviations (total

Table II. Observed and predicted values of the objectives and decision variables for the large farm.

	Traditional model	Multi-criteria model	Observed values	Absolute deviation (multi-criteria model)	Absolute deviation (traditional model)
Values of objectives					
Gross margin (€)	41572	39057	36986	0.06	0.12
Variable cost (€)	60949	32068	31680	0.01	0.92
Family labour (h)	4843	4570	4843	0.06	0.00
Purchased fodder (MJ)	786048	250753	324844	0.23	1.42
Hired labour (€)	19680	9011	7958	0.13	1.47
Total deviation				0.49	3.93
Relative fit					0.12
Decision variables					
Number of ewes	380	237	262	0.10	0.45
Alfalfa produced[a]	72	50	40	0.25	0.80
Maize produced[a]	8	32	40	0.20	0.80
Total pastureland[a]	800	800	800	0.00	0.00
Other crops[a]	5	3	5	0.40	0.00
Total deviation				0.95	2.05
Relative fit					0.46

[a]Stremmas.

Table III. Predicted and observed values of the objectives and decision variables for the middle farm.

	Traditional model	Multi-criteria model	Observed values	Absolute deviation (multi-criteria model)	Absolute deviation (traditional model)
Values of objectives					
Gross margin (€)	21,438	20,398	20,798	0.02	0.03
Variable cost (€)	7798	7504	8153	0.08	0.04
Family labour (h)	2756	2657	2274	0.17	0.21
Purchased fodder (MJ)	0	0	0	0.00	0.00
Hired labour (€)	438	401	350	0.15	0.25
Total deviation				0.42	0.53
Relative fit					0.79
Decision variables					
Number of ewes	157	105	80	0.31	0.96
Alfalfa produced[a]	37	41	35	0.17	0.06
Maize produced[a]	29	25	31	0.19	0.07
Total pastureland[a]	15	15	15	0.00	0.00
Other crops[a]	9	9	9		
Total deviation				0.67	1.09
Relative fit					0.61

[a]Stremmas.

deviation in the case of the multi-criteria model/total deviation in the case of the traditional model) (André & Riesgo, 2007). The estimated utility function yields better results in all three farms. This means that the multi-criteria model can represent the behaviour of farmers more accurately than the traditional gross margin maximisation model.

Specifically, in the case of the first farm the suitability of the multi-criteria model compared to the traditional model is clear, especially when examining the values of objectives, where the relative fit index is 0.12 (Table II). The traditional model fails to simulate the actual behaviour, especially in the case of the purchased fodder and cost of hired labour.

As far as the basic decision variables are concerned, the number of ewes is better simulated in the multi-criteria model. Furthermore, the produced alfalfa and maize is better simulated using the multi-criteria model. As for the middle farm, the multi-criteria model has an increased ability to reproduce farmers' behaviour, compared to the traditional model as well, especially in the case of the number of ewes (Table III).

Finally, as far as the small farm is concerned, the superiority of the multi-criteria model compared to the traditional model is clear in both the objective and the decision variable space (Table IV).

4.3. Milk supply functions

After validating the utility function for each farm we can move on to estimating the individual supply

Table IV. Predicted and observed values of the objectives and decision variables for the small farm.

	Traditional model	Multi-criteria model	Observed values	Absolute deviation (multi-criteria model)	Absolute deviation (traditional model)
Values of objectives					
Gross margin (€)	4494	2292	3263	0.30	0.38
Variable cost (€)	5096	2055	3108	0.34	0.64
Family labour (h)	952	270	671	0.60	0.42
Purchased fodder (MJ)	141,594	53,158	73,567	0.28	0.92
Hired labour (€)	24	0	6	1.00	3.00
Total deviation				2.52	5.36
Relative fit					0.47
Decision variables					
Number of ewes	45	21	20	0.05	1.25
Total pastureland[a]	23	26	23	0.13	0.00
Other crops[a]	3	0	3	1.00	0.00
Total deviation				1.18	1.25
Relative fit					0.94

[a]Stremmas.

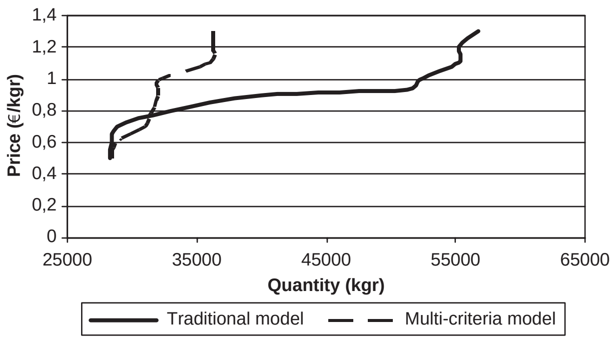

Figure 1. Milk supply of the large farm.

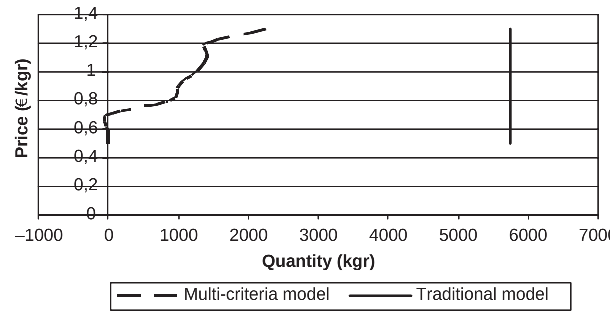

Figure 3. Milk supply of the small farm.

functions, by parametrising the price of milk. The supply for the large farm is presented in Figure 1. The supply function estimated through the use of the traditional gross margin maximisation model is also presented in the same figure. As can be observed, the supply function is less steep when the traditional model is used, which implies a higher elasticity, especially in the area of the current price level (0.8–1 €/kg). But if price falls lower than this level, then the response of the farmer is higher than that estimated using the traditional model.

In Figure 2, supply functions of the medium farm under the assumption of gross margin maximisation and under the estimated utility function maximisation are presented. As can be seen the two functions look similar. This resemblance can be explained by the fact that gross margin maximisation receives a high weight in the utility function of the farmer.

Nevertheless, as in the case of the first farm, the use of the utility function restricts the milk supply in lower levels and the supply shifts to the left. As mentioned in the case of the large farm, the elasticity of the alternative supply function is higher than that of the supply function estimated by the traditional model, in low price levels (in the range of 0.4–0.6 €/kg).

Finally, Figure 3 presents the individual supply functions for the small farm. The results indicate that the use of the traditional single objective model provides an inelastic supply function at the milk price range examined. Under the assumption of gross margin maximisation, the farm produces a large quantity of milk at all price levels. This result is rather unrealistic, since the actual milk produced is less than 20% of what the traditional model suggests. On the other hand, the multi-criteria model provides a different form of the supply function, which has a high elasticity, especially in the low price levels. In fact the farmer is willing to produce milk only if the price of milk is higher than 0.75 €/kg.

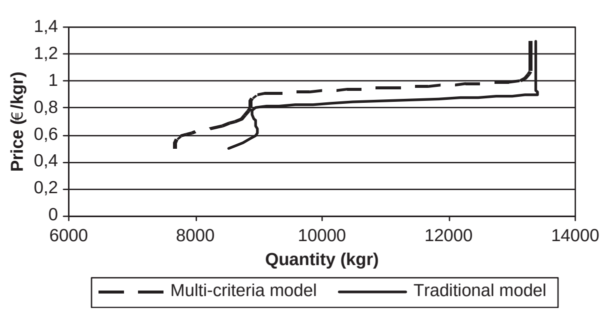

Figure 2. Milk supply of the middle farm.

The above analysis indicates that price changes affect the smaller farms more than the larger ones, especially at low price levels. Part-time farmers will engage in the activity only if the price of milk is high enough. This means that ensuring the milk price level leads to the retention of the part-time sheep farming activity.

Before estimating the total milk supply of the area, it should be mentioned that the structure of the model we have used in this analysis allows farmers to fine-tune their milk supply by adjusting the number of sheep and not the adjustment of milk yield per ewe. As described in Appendix 1, this happens because the number of ewes is included as an endogenous variable in the model, while the milk yield is an exogenous variable. Although in practice the farmer can adjust both the number of sheep and milk yield per ewe, evidence from other studies indicates that the elasticity of milk supply is explained mainly from the flock size elasticity (see, e.g. Rayner, 1975).

4.4. Aggregate milk supply

In the previous section, we have used the farm-specific utility functions to estimate the milk supply for each decision-making unit. The next step in our analysis involves the aggregation of the individual supply to estimate the total milk supply for the area of Etoloakarnania. This is estimated by the weighted addition of the individual supply functions (Gómez-Limón & Riesgo, 2004). The supply function estimated is presented in Figure 4, which also presents the aggregate supply function that corresponds to the traditional, gross margin maximisation

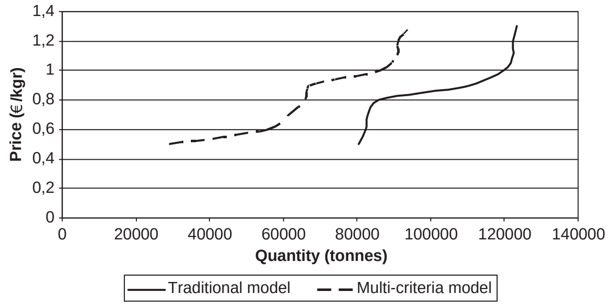

Figure 4. Aggregate milk supply.

model. The alternative supply function indicates a lower milk supply at all price levels. Using the traditional model to estimate the regional supply would lead to a serious and unrealistic overestimation of this supply. Furthermore, the alternative supply function is less elastic than the traditional one in the prevailing price range (0.8–1 €/kg), but more elastic at low price levels. This means that the inclusion of multiple goals in our model smoothens the reaction of farmers to price changes since their behaviour is also influenced by other motives (some among them may be irrational, from the *homo economicus* point of view).

5. Concluding remarks

In this analysis a multi-criteria model is used to evaluate the supply function of sheep milk in the Prefecture of Etoloakarnania. First a detailed whole farm model adapted to livestock is built to incorporate decision variables and constraints for all animal and crop activities. Then the individual utility functions are obtained through a non-interactive methodology, so that the drawbacks of the interactive methods can be limited. The weights attached to the objectives of the farmers are estimated using the actual values of the objectives, and the multi-attribute utility function is then used to reproduce their behaviour. By parametrising the milk price the individual supply functions are derived and finally the total supply function is estimated as the weighted addition of the individual functions.

The first outcome of the analysis is that sheep farmers aim to achieve multiple goals, one of which is the maximisation of gross margin. This objective is a more important attribute of the utility function of the larger and more commercial farms under study but the weight assigned to this objective is small in the cases of the less commercial part-time farmer. This farmer aims mainly at the minimisation of family labour since he has other non-farm activities to attend to.

The analysis indicates that the performance of the mathematical model built to optimise the operation of a crop-livestock farm can improve through the use of multiple objectives. In this study this has proven very useful since it leads to a more robust estimation of the milk supply function. The estimated supply function reveals that farmers are less responsive to price changes than the traditional gross margin maximisation model suggests. Individual supply functions can also be used to predict the reaction to price changes for different groups of farms, helping policy makers to design more effective and targeted measures. For example, when milk price is lower than 0.75 €/kg, policy makers have to adopt a support scheme, if they wish to preserve the part-time sheep farming activity or if they wish to maintain the current milk supply. This is because the milk supply curve is elastic at low price levels, as indicated by the multi-criteria model (Figure 3). The need for this support scheme cannot be predicted by the traditional, single objective model, neither in the case of the small farm nor in the case of the entire sector (Figure 4). Similarly, the proposed methodology can be used to predict the impact of alternative policy measures on different farm types.

Finally, it should be noted that in this analysis we have used the additive form of the utility function, but the use and applicability of other forms of the utility function can also be investigated. This study is a first attempt to build a multi-criteria model to study the behaviour of livestock farmers, and estimate milk supply; therefore, further research is required. The existence of other objectives, such as minimisation of risk, is another concept for future research.

Notes

1. The family of L metrics is a series of measures used to estimate the distance between two points x^1 and x^2. The notion of L metrics can be explained using the following expression, which is a generalisation of the Euclidean distance:

$$L_p = [\sum_{j=1}^{n} |x_j^1 - x_j^2|^p]^{1/p}.$$

When $p = 2$, the expression reduces to the Euclidean distance and when $p = 1$, the expression reduces to the L_1 metric. For values of $p > 2$, it is not possible to give a geometrical interpretation of the distance measure but for some dimensions these distances can be computed. Also as p increases more weight is given to the largest deviation and therefore when $p = \infty$ the L_∞ metric, that is given exclusively by the largest deviation, is formed. A more detailed presentation of the L metrics and their usefulness to multi-criteria analysis is included in Romero and Rehman (1989, pp. 86–89).

2. The variable $feed_{fi,t}$ refers to kilograms of purchased fodder of various types, with different nutritional and energy value. Therefore minimising the sum of all purchased fodder would lead to the substitution of low nutritional value crops (used in larger amount) with high nutritional value crops (used in smaller amount). To avoid this error we use the parameter

$y_{fs,\ energy}$ as a normalising factor. This means that the fourth goal expresses the "purchased energy" measured in MJ.

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Appendix 1

Mathematical expression of the constraints and decision variables of the LP model:

Indices	
ti	cultivated crops (P = {maize, alfalfa, other})
fi	cultivated fodder (T = {maize, alfalfa})
fs	purchased fodder (N = {maize, alfalfa})
a	animal activities (A = {sheep3, sheep-3})
r	animal premiums (C = {elig, nelig})
m	destination of produced fodder (M = {con, sale})
l	Destination of labour (L = {crops, flock})
s	origin of labour (S = {own, hire})
t	month
g	type of pastureland (G = {rent, own, com})
u	nutritional value (U = {dry matter, nitrogen, energy})
Model parameters	
Yield_{ti}	crop yield (kg)
$y\_gz_{t,u}$	nutritional value of pastureland per month (kg)
$y_{fi,u}$	nutritional value of produced fodder (kg)
$y_{fs,u}$	nutritional value of purchased fodder (kg)
$n_{a,t,u}$	monthly feed requirements (kg)
$n_{at,u}$	annual feed requirements (kg)
$w_{l,s}$	wage (€/hr)
$\text{rclab}_{ti,t}$	monthly labour requirements for crops (hr)
$\text{ralab}_{ti,t}$	monthly labour requirements for animal activities (hr)
$\text{avail}_{l,t}$	available family labour per month (hr)
own_land	available owned land (stremma[a])
rent_land	available pastureland for rent (stremma)
irr_land	irrigated land (stremma)
graz_mun	available communal pastureland (stremma)
land	total land (stremma)
num_elig	number of premium eligible ewes (number)
gr_marc_{ti}	gross margin of crops (gross revenue minus variable costs except labour) (€)
$\text{gr\_mara}_{a,r}$	gross margin of animal activities (gross revenue minus all variable costs except labour and feed costs) (€)
rqwcg	variable costs required for pastureland (€/stremma)
rqwc_{ti}	variable costs required for crops (€/stremma)
rqwc_{a}	variable costs required for animal activities (€/ewe)
rqwc_{fi}	monthly cost of produced fodder (€/kg)
rqwc_{fs}	cost of purchased fodder (€/kg)
percent_energy	percent of energy covered from concentrates
Decision variables	
$\text{crop}_{fi,\text{con}}$	produced fodder for consumption (kg)
$\text{crop}_{ti,\text{sasles}}$	cash crops (stremma)
$\text{feed}_{fs,t}$	monthly purchased fodder (kg)
$\text{feed}_{fi,t}$	consumption of produced fodder/month (kg)
$\text{lab}_{l,s,t}$	labour per month, destination and origin (hr)
gland_{g}	pastureland (stremma)
$\text{anim}_{a,r}$	ewe (number)

[a]1 Stremma = 0.1 Ha.

The mathematical expression of the constraint matrix is the following:
Distribution of produced feed crops:

$$\text{yield}_{fi} \cdot \text{crop}_{fi,con} = \sum_t \text{feed}_{fi,t} \qquad \forall fi \in \text{FI}$$

Feed requirements

$$\sum_g y\_\text{gz}_{t,u} \cdot \text{gland}_g + \sum_{fi} y_{fi,u} \cdot \text{feed}_{fi,t} + \sum_{fs} y_{fs,u} \cdot \text{feed}_{fs,t} \geq \sum_r \sum_a n_{a,t,u} \cdot \text{anim}_{a,r} \;\; \forall t \in T, \forall u \in U$$

Minimum annual energy requirements satisfied from concentrates:

$$y_{fi,\text{energy}} \cdot \text{yield}_{fi} \cdot \text{crop}_{fi,\text{con}} + \sum_t y_{fs,\text{energy}} \cdot \text{feed}_{fs,t} \geq \text{percent\_energy} \cdot \sum_a \sum_r n_{\text{at,energy}} \cdot \text{anim}_{a,r}$$

$$fs == \text{maize}, \; fi == \text{maize} \quad .$$

Labour requirements for crops:

$$\sum_{ti} \text{rclab}_{ti,t}(\text{crop}_{ti,\text{sales}} + \text{crop}_{fi,\text{con}}) \leq \sum_s \text{lab}_{\text{crops},s,t} \qquad \forall t \in T$$

Available family labour:

$$\text{lab}_{l,\text{own},t} \leq \text{avail}_{l,t} \qquad \forall t \in T$$

Labour requirements of the flock:

$$\sum_{\text{a}} \text{ralab}_{a,t} \text{anim}_{a,r} \leq \sum_s \text{lab}_{flock,s,t} \frac{8}{9} \qquad \forall t \in T$$

Available irrigated land:

$$\sum_{ti} (\text{crop}_{ti,\text{sales}} + \text{crop}_{fi,\text{con}}) \leq \text{irr\_land}$$

Available own land:

$$\sum_{ti} (\text{crop}_{ti,\text{sales}} + \text{crop}_{fi,\text{con}}) + \text{gland}_{\text{own}} \leq \text{land}$$

Communal pasture land (pastureland, property of the municipality, distributed among livestock farms according to their ewe rights. In exchange, livestock farms pay a small fee to the municipality):

$$\text{gland}_{\text{mun}} \leq \text{graz\_mun}$$

Available land for rental:

$$\text{gland}_{\text{rent}} \leq \text{rent\_land}$$

Number of ewe rights:

$$\sum_a \text{anim}_{a,\text{``elig''}} \leq \text{num\_elig}$$