VCS Experiment - Radiative corrections

L. Correa

VCS Experiment
Radiative corrections

Loup Correa
Institut für Kernphysik - Johannes Gutenberg-Universität Mainz
and
Laboratoire de Physique Corpusculaire de Clermont-Fd

January 20th, 2014

In this note we summarise the set of radiative corrections in photon-electroproduction process experiments. Then we describe and compare the different ways to determine and apply those corrections in a particular kinematical setting of our VCS experiment.

PCCF RI 14-04

1 Introduction & Motivation

The cross section of the photon electroproduction process ($ep \rightarrow ep\gamma$) is the sum of three contributions :

-Bethe Heitler

-Born part of Virtual Compton Scattering (VCS)

-Non Born part of VCS

This process is described through those 5 diagrams, figure 1

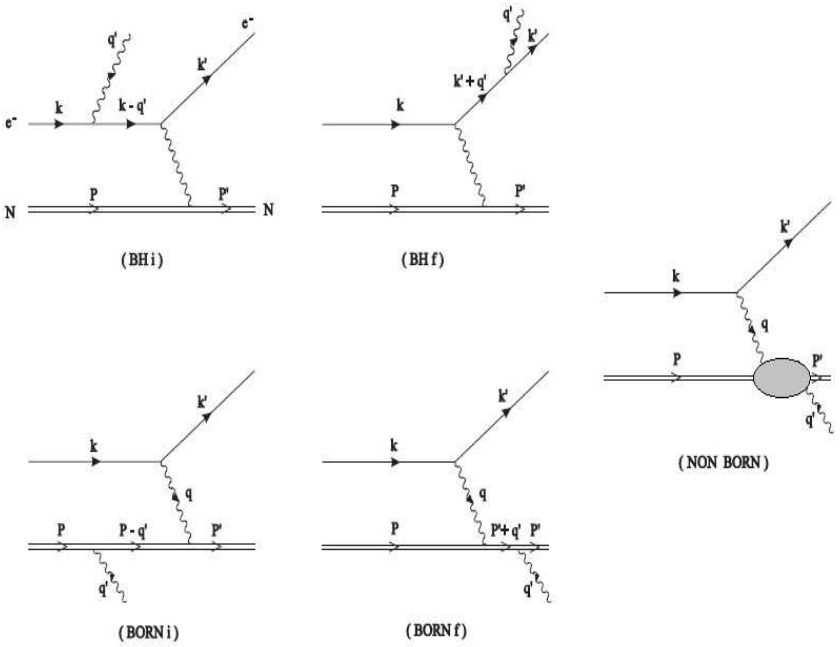

FIG. 1 – photon electroproduction diagrams, Bethe Heitler diagrams, VCS Born and Non Born diagrams

In our VCS experiment the final electron and proton are detected, but the final photon is identified by the missing mass method. So higher-order processes can occur (for instance the emission of another real photon) and will be an unwanted part of the experimental cross section. We must determine this contribution in order to correct the experimental cross section from it. So :

$$\sigma_{final} = \sigma_{exp} * F \tag{1}$$

Where σ_{final} is the cross section corresponding to the sum of the five diagrams of photon electroproduction process, figure 1.

σ_{exp} is our VCS experimental cross section at the raw level, i.e. before applying the radiative corrections, it contains the five 1^{st} order diagrams that we need but also higher-order processes that we want to remove.

F is the radiative correction term to determine. It is a sum of several contributions, expressed through δ correction terms, that will be detailed in the following sections.

Expected precision

We created and used a program which determines F, and δ correction terms, by following the theoretical calculations from [1]. The theorists assert that the approximations made will generate a 1% incompressible-theoretical error on the F factor. So this 1% error on the final cross section will be our guideline in every conclusions.

VCS phase space

For our VCS analysis the phase space is defined by five quantities, two which are fixed :

- The virtual photon four-momentum vector q_{cm}.
- The polarisation parameter ϵ.

And three variable quantities :

- The real photon four-momentum vector q'_{cm}.
- θ_{cm} the angle between the emitted real photon and the virtual photon.
- ϕ_{cm} the angle between the leptonic and the hadronic planes.

Those three variable quantities are decomposed into three-dimensional bins, constituting the phase space. Our program will give us a bin-per-bin F value, but it may not be necessary to correct the cross section bin-per-bin. In order to answer that question we will check the F phase-space dependence.

Study aims

The purpose of this study is summarized into four objectives.

1 - Identify components of F and their order of magnitude.

2 - Study the phase-space dependence of F and determine if one can apply an overall factor, or if one must apply a bin-per-bin F.

3 - Compare the different systems, see **(2.1.1)**, where F is calculated.

4 - Estimate the impact of higher-order correction terms on the final cross section.

Part **(2)** is a description of F factor components, the δ correction terms. Part **(3)** contains our results, the first two objectives are investigated in the first subsection **(3.1)**, the two others in their own subsection **(3.2)** and **(3.3)**.

2 Diagrams, approximation and δ correction terms

There are two types of diagrams affecting the experimental cross section ; the ones with real supplementary photons, and those with supplementary virtual photons. Initially we will consider only processes having the higher contribution to the final cross section, that is to say those with a cross section proportional to

the elementary charge power six e^6. That leads to treat diagrams with only one supplementary virtual or real photon.

In this section the correction terms, δ, are detailed but not the cross section calculations, those are available in the reference [1]. Each δ is calculated with the approximation that $Q^2 >> m^2$, where Q^2 is the opposite of the square of the virtual photon four-momentum vector and m the electron mass (this approximation is a part of the 1% incompressible-theoretical error).

2.1 Supplementary real photon

Supplementary real photons, produced by bremsstrahlung and having a small energy, are called soft photons. The production diagrams are listed figure 2.

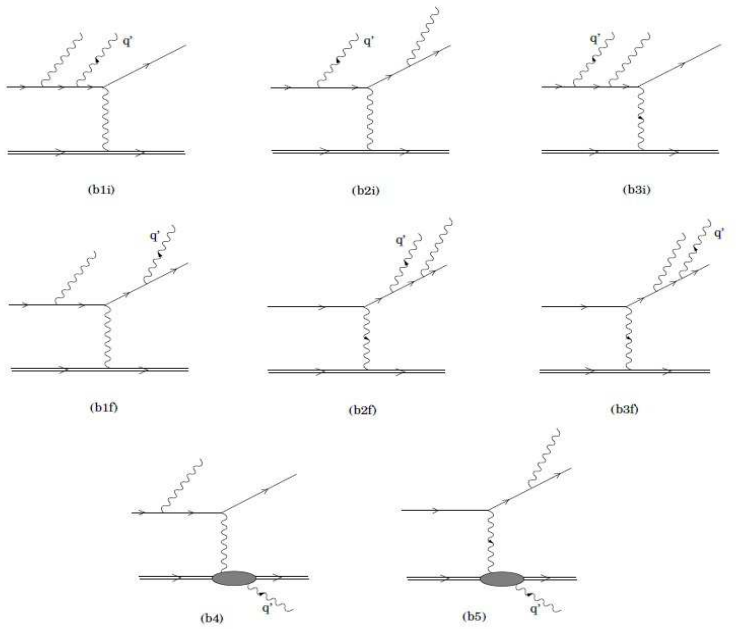

FIG. 2 – Supplementary real photon diagrams (the VCS photon is the one next to the q' symbol)

For these processes, the theoretical cross-section calculation leads to the correction term δ_r. It can be expressed as a sum of two terms : $\delta_r = \delta_{r1} + \delta_{r2}$.

$$\begin{aligned} \delta_{r1} &= \frac{\alpha_{em}}{\pi} ln(\frac{(\Delta E_s)^2}{\widetilde{E}_e \widetilde{E}'_e})[ln(\frac{Q^2}{m^2}) - 1] \\ \delta_{r2} &= \frac{\alpha_{em}}{\pi}(-\frac{1}{2}ln^2(\frac{\widetilde{E}_e}{\widetilde{E}'_e}) + \frac{1}{2}ln^2(\frac{Q^2}{m^2}) - \frac{\pi^2}{3} + Sp(cos^2(\frac{\widetilde{\theta}_e}{2}))) \end{aligned} \quad (2)$$

Where :

α_{em} is the electromagnetism fine structure constant.

ΔE_s the maximum energy that the soft photon can reach. It depends on the analysis's upper cut on the missing mass squared as described in the result section.

All the tilde quantities are expressed in a system different from the lab frame or the center of mass frame. The choice of this system depends on the $(\gamma^* p)$ experiment.

$\widetilde{E}_e$ is the energy of the incoming electron.

$\widetilde{E}'_e$ is the energy of the outgoing electron.

$\widetilde{\theta}_e$ is the polar angle of the outgoing electron wrt the incoming one.

2.1.1 System choice

The δ_r calculations contain an integration over the soft photon phase-space. This phase space has a elaborated shape in the lab system, complicating the solving. In order to get a spherical shape, one has to perform the calculation in an other system. Finding the right frame will depend on the experiment.

Detection of outgoing electron and real photon

In such experiments, if one measures the outgoing electron and photon (mainly used for DVCS experiments), the integration volume will be spherical in the system S_1 of the recoiling nucleon + soft photon. this system is defined by : $\vec{p'} + \vec{l} = \vec{p} + \vec{q} - \vec{q'} = 0$ where $\vec{l}$ is the soft photon momentum. In this system the tilde quantities will be expressed as :

$$\begin{array}{lcl} \widetilde{E}_e & = & \frac{M_N}{M_{m1}}(E_e - \frac{Q^2}{2M_N} - \frac{k.q'}{M_N}) \\ \widetilde{E}'_e & = & \frac{M_N}{M_{m1}}(E'_e + \frac{Q^2}{2M_N} - \frac{k'.q'}{M_N}) \\ sin^2(\widetilde{\theta}_e/2) & = & \frac{E_e E'_e}{\widetilde{E}_e \widetilde{E}'_e} \end{array} \quad (3)$$

with M_{m1} the missing mass of the system S_1, defined as : $M_{m1}^2 = (p' + l)^2$ and calculated, by knowing ΔE_s, with :

$$\Delta E_s = \frac{M_{m1}^2 - M_N^2}{2M_{m1}} \quad (4)$$

The kinematical variables are the ones defined in the figure 1.

Thereafter we will use the notations $\delta_{r1,photon}$ and $\delta_{r2,photon}$ to designate the bremsstrahlung correction terms calculated in the S_1 system.

Detection of outgoing electron and proton

In VCS experiments, such as ours, one measures the outgoing electron and proton. For these experiments the integration volume will be spherical if one considers the system S_2 of the VCS photon + soft photon, defined by : $\vec{q'} + \vec{l} = \vec{p}_N + \vec{q} - \vec{p'}_N = 0$. This leads to :

$$\begin{array}{lcl} \widetilde{E}_e & = & \frac{M_N}{M_{m2}}(E_e - \frac{Q^2}{2M_N} - \frac{k.p'}{M_N}) \\ \widetilde{E}'_e & = & \frac{M_N}{M_{m2}}(E'_e + \frac{Q^2}{2M_N} - \frac{k'.p'}{M_N}) \\ sin^2(\widetilde{\theta}_e/2) & = & \frac{E_e E'_e}{\widetilde{E}_e \widetilde{E}'_e} \end{array} \quad (5)$$

M_{m2} is the missing mass of the system S_2 defined as : $M_{m2}^2 = (q'+l)^2$ and calculated with :

$$\Delta E_s = \frac{M_{m2}}{2} \tag{6}$$

Thereafter we will use the notations $\delta_{r1,proton}$ and $\delta_{r2,proton}$ to designate the bremsstrahlung correction terms calculated in the S_2 system.

Elastic case

In (ep) elastic scattering experiments the integration volume is spherical in the lab frame, so the tilde quantities will be simply defined as :

$$\begin{array}{rcl} \widetilde{E}_e & = & E'_e \\ \widetilde{E}'_e & = & E_e \\ \widetilde{\theta}_e & = & \theta_e \end{array} \tag{7}$$

Using this system will lead to $\delta_{r1,EL}$ and $\delta_{r2,EL}$ notations.

2.2 Supplementary virtual photon

A supplementary virtual photon is produced by three kinds of processes, the vacuum polarisation process, the electron self-energy process and processes with an additional vertex between lines.

The correction terms presented here are the ones from the (ep) elastic case. We made this choice because the VCS case terms, detailed in [2], contain integral calculation which are extremely difficult to calculate. Anyway, according to [2], using the elastic case calculation will generate a less than 1% error on the final cross section. In [2], for $Q^2 = 0.3\ GeV^2$, the elastic-case calculation increases the correction term by 0.8%. So we will use the elastic case calculation in our VCS program and assume that our final result will have about a 1% precision, which is consistent with our theoretical error.

2.2.1 The vacuum polarisation

The vacuum polarisation process, figure 3, leads to the following correction term :

$$\delta_{vac} = \frac{\alpha_{em}}{\pi}\frac{2}{3}(-\frac{5}{3} + ln(\frac{Q^2}{m^2})) \tag{8}$$

For our VCS experiment where Q^2 is fixed, this term does not depend on the three varying phase-space variables : q'_{cm}, θ_{cm} and ϕ_{cm}, defined in the first section, so it's a constant.

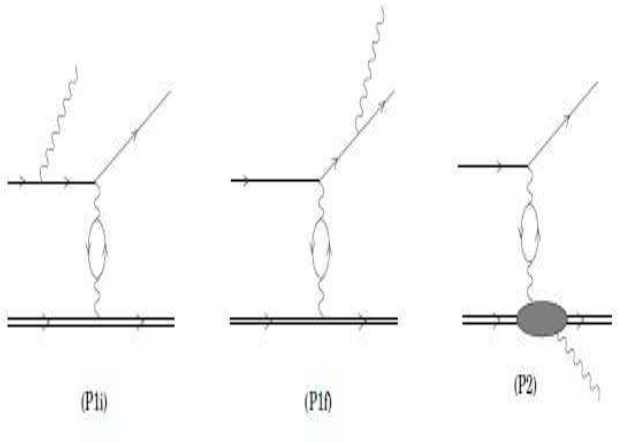

FIG. 3 – Vacuum polarisation diagrams

2.2.2 The supplementary vertex

The vertex correction figure 4, also depends only on Q^2 and is described through :

$$\delta_{vertex} = \frac{\alpha_{em}}{\pi}(\frac{3}{2}ln(\frac{Q^2}{m^2}) - 2 - \frac{1}{2}ln^2(\frac{Q^2}{m^2}) + \frac{\pi^2}{6}) \quad (9)$$

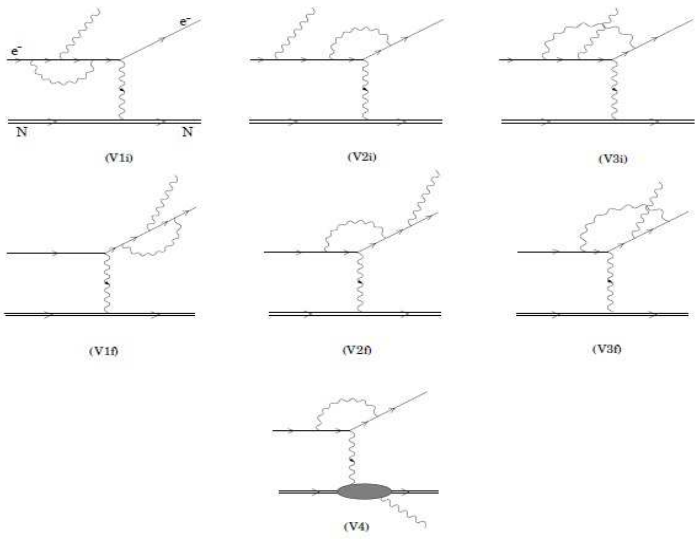

FIG. 4 – Vertex correction diagrams

2.2.3 The electron self-energy process

The electron self energy process leads to a correction term :$\widetilde{\sum}(k)$, available in [1]. For an on-shell electron ($k = m$) this terms is exactly zero. Consequently, in the VCS case, this correction has only to be applied for internal lepton lines, as presented on figure 5.

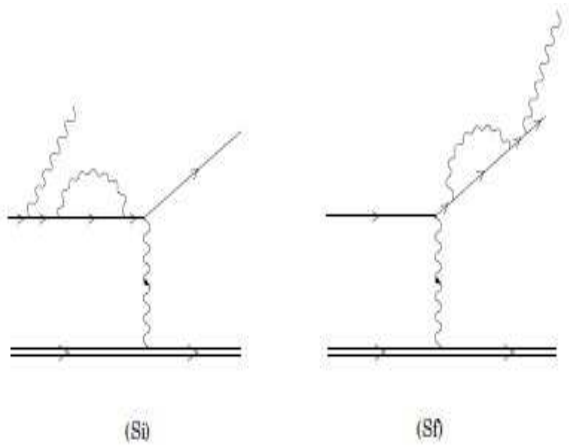

FIG. 5 – Electron self energy diagrams

Therefore, for the VCS case the complete-virtual correction can be written as a sum of the three processes detailed above :

$$\delta_{virtual,VCS} = \delta_{vertex,VCS} + \delta_{vac,VCS} + \widetilde{\sum}(k)$$

But we chose to use the elastic-case calculations for the virtual corrections and, in the elastic case, such diagrams (figure 5) are higher order processes that can be ignored. So, the complete-virtual correction for the elastic case can be written as :

$$\delta_{virtual,elastic} = \delta_{vertex} + \delta_{vac}$$

Finally the error made by ignoring the elastic self-energy process is a part of this difference between $\delta_{virtual,VCS}$ and $\delta_{virtual,elastic}$ given above(section **(2.2)**), and is estimated in [2] as 0.8% of the final cross-section (at $Q^2 = 0.3\ GeV^2$).

2.3 Supplementary virtual and real photon from the proton side

A supplementary photon can also be produced from the proton side, by rare proton bremsstrahlung and vertex corrections. Unlike corrections from the electron side, which can be calculated model independently from QED, the proton side

corrections will depend on a model for proton structure. This model dependence will become important if one wants a experimental precision at the 1% level [1].

This correction is described through two terms which also take into account the two-photon exchange process (direct and crossed) :

$$\delta_1 = \frac{2\alpha_{em}}{\pi}(ln(\frac{4*(\Delta E_s)^2}{Q^2 x})ln(\eta) + Sp(1-\frac{\eta}{x}) - Sp(1-\frac{1}{\eta x}) \tag{10}$$

$$\begin{aligned}\delta_2^{(0)} = & \frac{\alpha_{em}}{\pi}(ln(\frac{4*(\Delta E_s)^2}{M_N^2})(\frac{E'_N}{|\vec{P}'_N|}ln(x)-1)+1 \\ & +\frac{E'_N}{|\vec{P}'_N|}(-\frac{1}{2}ln^2(x) - ln(x)ln(\frac{\rho}{M_N^2}) + lnx - 2Sp(1-\frac{1}{x^2}) + 2Sp(-\frac{1}{x}) + \frac{\pi^2}{6}))\end{aligned} \tag{11}$$

With :
M_N the mass of the baryon (here proton)
$\rho^2 = Q^2 + 4M_N^2$
$x = \frac{(Q+\rho)^2}{4M_N^2}$
$\eta = E_e/E'_{el} = 1 + E_e(1 - cos(\theta_e)/M_N$. Where E_e, E'_{el} and θ_e are respectively in the lab frame : the energy of the incoming electron, the energy of the elastic outgoing electron and the angle between the outgoing and incoming electrons.

One goal of this study is to find the impact of those correction terms on the total correction.

A third term $\delta_2^{(1)}$ also describes the proton side process. It depends on the proton form factors and appears to be less than 1% of the total correction term F, reference [3]. That is why this term will not be calculated in this study.

2.4 F exponentiated form

The radiative correction term F, for a +1 charged hadron, is simply a sum of all the process contributions and so is defined as :

$$\frac{1}{F} = 1 + \delta_{vac} + \delta_{vertex} + \delta_r + \delta_1 + \delta_2^{(0)} \tag{12}$$

But in order to approximately take into account the higher-order radiative-corrections we can use an exponential form for elastic and non-elastic cases. This was demonstrated in references [4] [5] and leads to a exponentiated form only for the vertex and the soft emission parts of the correction.

$$\frac{1}{F_{EXP}} = \frac{e^{\delta_{vertex}+\delta_r}}{(1-\frac{\delta_{vac}+\delta_1+\delta_2^{(0)}}{2})^2} \tag{13}$$

In the last part we will compare the results obtained with those two formulas in order to approximate the higher-order corrections.

3 Results

Thereafter δ_1, $\delta_2^{(0)}$, δ_{vertex}, δ_{vac}, $\delta_{r1,proton}$, $\delta_{r1,photon}$, $\delta_{r1,EL}$, $\delta_{r2,proton}$, $\delta_{r2,photon}$, $\delta_{r2,EL}$, F and F_{EXP} are, for the whole phase space, calculated by our program according to their expression described above. All these results are obtained for the VCS q2-02-inp kinematics, with :

$q_{cm} = 458\ MeV/c$
$q'_{cm} = 62.5,\ 87.5,\ 112.5\ or\ 137.5\ MeV/c$
$\epsilon = 0.85$
$\phi_{cm} \in [-175; 175]\ (degree)$
$cos(\theta_{cm}) \in [-0.975; 0.975]$

To perform the calculation we need to set a value for the maximal energy of the emitted soft photon ΔE_s. It has to be related to the analysis's upper cut on the missing mass squared as : $cut = 4 * \Delta E_s^2$.

For us $\Delta E_s = 25MeV$ and $cut = 2500MeV^2$.

Note about the VCSSIM user

In VCSSIM, which is our simulation code (playing the same role as Simul++ in A1), the δ_{r1} correction is already implemented by a Monte Carlo method and should not be added to the final F result. Later all conclusions will be made with and without implementing the analytical δ_{r1} calculation of section **(2.1)** to our program. We will also refer someone who use a Monte Carlo determination of δ_{r1} as a VCSSIM user (even if one use another simulation code like Simul++).

3.1 Phase space dependence and correction terms averaged values

The goal here is to determine the need to apply a final correction averaged on the whole phase space or to apply a bin-per-bin correction. Of course a phase space dependence is expected only for the correction terms which depend on ϕ_{cm}, θ_{cm} or q'_{cm}.

3.1.1 $\delta_{r,EL}$, δ_{vertex} and δ_{vac} : the corrections with no phase space dependence

For those three corrections there is no phase space dependence, as expected from formulas. We obtained for our kinematics :

$$\begin{aligned} \delta_{r1,EL} &= -0.207 \\ \delta_{r2,EL} &= 0.210 \\ \delta_{vertex} &= -0.168 \\ \delta_{vac} &= 0.021 \end{aligned}$$

It appears that the vertex correction is higher than the others since the two parts of the bremsstrahlung correction compensate each other.

3.1.2 δ_1 correction : the small phase space dependence

The first proton side correction δ_1 shows a very weak dependence on q'_{cm} only. We obtained :

$$\overline{\delta_1} \quad = \quad -0.0039907 \pm 0.0000758$$

Here, the uncertainty is simply the gap between the mean (obtained by averaging over the whole phase space) and an extremum (the extremum that δ_1 reaches in a particular bin of the phase space).

So for $q'_{cm} \in [62.5; 137.5] MeV/c$ it seems that if one applies an averaged correction of this term one will make an error on the final cross-section well under 1%.

3.1.3 $\delta_2^{(0)}$ correction : the θ_{cm} dependence

The second proton side correction $\delta_2^{(0)}$ depends on θ_{cm} and q'_{cm},figures 6 and 7. Figure 7 shows that $\delta_2^{(0)}$ varies approximately from -0.001 to 0.002 for our setting.

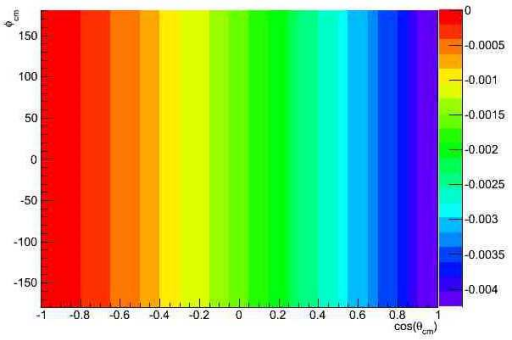

FIG. 6 – δ_2 as a function of ϕ_{cm} and $cos(\theta_{cm})$ for $q'_{cm} = 62.5 MeV/c$

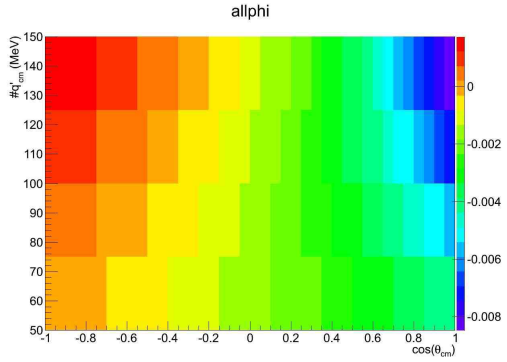

FIG. 7 – δ_2 as a function of q'_{cm} and $cos(\theta_{cm})$ for $\phi_{cm} = 55°$

We obtained for the whole studied phase space $\overline{\delta_2} = -0.0018 \pm 0.0067$, here again the uncertainty is the gap between the mean and an extremum. So it seems that if one applies an averaged correction of this term one will make an error on the final cross-section under 1%.

3.1.4 The phase-space dependence of the δ_r correction for a photon-detection experiment

Here we studied the dependence of the bremsstrahlung correction δ_r over the whole phase space, a small dependence is observed on figures 10 and 11.

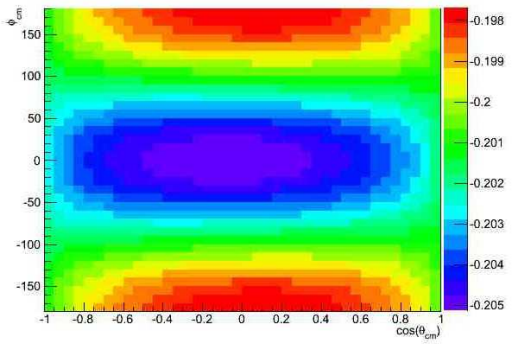

FIG. 8 – $\delta_{r1,photon}$ as a function of ϕ_{cm} and $cos(\theta_{cm})$ for $q'_{cm} = 62.5 MeV/c$

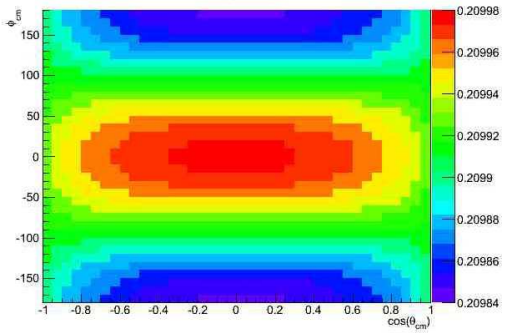

FIG. 9 – $\delta_{r2,photon}$ as a function of ϕ_{cm} and $cos(\theta_{cm})$ for $q'_{cm} = 62.5 MeV/c$

As previously we made an average over the whole phase space including the full range in q' (figures 8 and 9 are only an axemple of $\delta_{r,photon}$ values at a given q'). The result is that the dependence appears to be small :

$$\begin{aligned} \overline{\delta_{r1,photon}} &= -0.2017 \pm 0.0084 \\ \overline{\delta_{r2,photon}} &= 0.2100 \pm 0.0002 \end{aligned}$$

So if one does a VCS experiment with a photon detector there is no need to take into account the phase space dependence for the δ_r term. This is an interesting result because one may want to use the $S1$ system calculation, which is simpler, for a proton detection experiment. A specific study will be done further to know if the error generated by this "wrong" system choice will overpass 1% error on the final cross-section. Also we have to check if the $\delta_{r,proton}$ correction term, calculated in the $S2$ system, is as dependent on the phase-space as this $\delta_{r,photon}$ term.

For a photon-detection experiment all the correction terms are now calculated. No phase-space dependence higher than 1% is observed. So we can determine an averaged value of F on the whole phase space for our kinematics :

$$\frac{1}{\overline{F}} = 0.855 \pm 0.010$$

Where the uncertainty is essentially due to the $\delta_{r1,photon}$ dependence.

3.1.5 The δ_r correction phase-space dependence for a proton-detection experiment

In such experiments one has to carefully treat the area around the Bethe-Heitler peaks. Those are phase-space areas where the cross section increase tremendously

due to the Bethe Heitler process. Therefore no VCS experiment is done near the peaks and we are not interested in this area's results. As we can see on figures 10 and 11 the phase space dependence of δ_r cannot be neglected around the peaks (when $\phi_{cm} = 0°and\ cos(\theta_{cm}) \approx -0.35\ or\ 0.2$). So we use a cut ($|\phi_{cm}| > 30°$), which will remove the Bethe-Heitler-peaks area, and see if we can still apply or not an averaged value of δ_r.

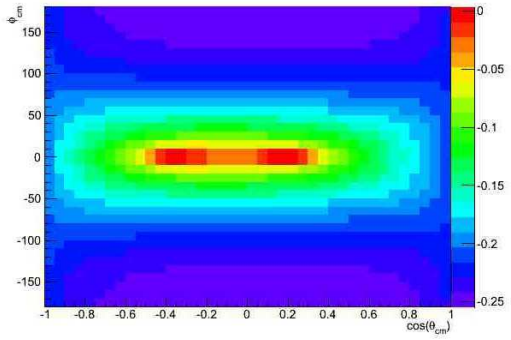

FIG. 10 – $\delta_{r1,proton}$ as a function of ϕ_{cm} and $cos(\theta_{cm})$ for $q'_{cm} = 62.5MeV/c$

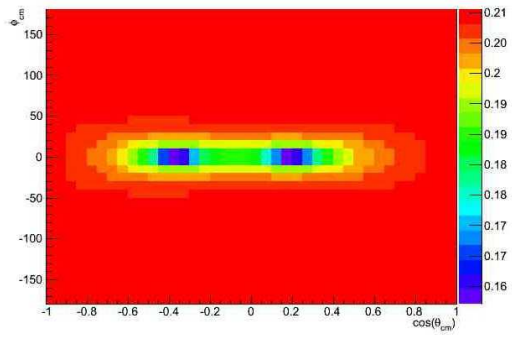

FIG. 11 – $\delta_{r2,proton}$ as a function of ϕ_{cm} and $cos(\theta_{cm})$ for $q'_{cm} = 62.5MeV/c$

With the cut, we obtained :

$$\begin{aligned} \overline{\delta_{r1,proton}} &= -0.2409 \pm 0.2434 \\ \overline{\delta_{r2,proton}} &= 0.2101 \pm 0.0002 \end{aligned}$$

The phase-space dependency does exist only for $\delta_{r1,proton}$ and is tremendous. But a VCSSIM user, who does not have to calculate δ_{r1} analytically, only considers $\delta_{r2,proton}$ which is almost constant outside the Bethe-Heitler peaks. So, the radiative correction doesn't need to be done bin-per-bin.

For a proton-detection experiment all the correction terms are now calculated. We can determine an averaged value of F on the whole phase space for our kinematics. By implementing the $\delta_{r1,proton}$ calculation :

$$\frac{1}{\overline{F}} = 0.816 \pm 0.112 \qquad (\delta_{r1}\ included,\ |\phi_{cm}| > 30°)$$

The error will be higher than the 1% error expected on the final cross section. But for a VCSSIM user, without the $\delta_{r1,proton}$ calculation :

$$\frac{1}{\overline{F}} = 1.057 \pm 0.007 \qquad (\delta_{r1}\ excluded,\ |\phi_{cm}| > 30°)$$

One can apply an averaged value of F.

3.1.6 Conclusion on the corrections amplitude

We have seen that δ_{vertex}, δ_{r1} and δ_{r2} are one order of magnitude higher than δ_{vac} and two orders of magnitude higher than the proton side corrections ($\delta_2^{(0)}$, δ_1). Moreover, the proton side corrections don't reach 1% of the final cross-section. So, at the 1% level of precision, it seems to be correct to neglect the model dependency of $\delta_2^{(0)}$ and δ_1. In the same way, omitting the third proton-side correction $\delta_2^{(1)}$, which must be at least as small, will not generate high errors on F. We assume that the total error on proton side corrections is roughly of the same order as $\delta_2^{(1)}$:

$$\pm 0.002$$

However one has to carefully take into account the electron-bremsstrahlung process (δ_{r1}, δ_{r2}) and the vacuum polarisation (δ_{vac}) which are higher by one order of magnitude.

3.1.7 Conclusion on the phase-space dependence

If one wants to apply a radiative correction to a photon detection experiment, in a bin, the error made by using an averaged value of F instead of the exact bin value of F will be mainly due to the $\delta_2^{(0)}$ term. Then, in a bin, the maximum error on the final cross section will be less than 1% and will not exceed the incompressible-theoretical error.

For a proton detection experiment one can use an averaged value of F only if δ_{r1} is determined separately by the simulation (like VCSSIM or Simul++). That way the maximum error on the final cross section will be less than 1%. If one has to do the complete calculation (δ_{r1}+δ_{r2}), using an averaged value of F will cause an error much higher than 1% on several bins cross section.

Here are the two tabs obtained ; the first one is for the whole phase space, the second one is for $|\phi_{cm}| > 30°$ (the cut is applied only for the $\delta_{r,proton}$ calculation).

```
          dr1 proton | dr1 photon | dr2 proton | dr2 gamma | delta1 | delta2
   mean :   -0.22325    -0.20171     0.20883     0.21003  -0.0039907  -0.0018167
   max :   0.0025334    -0.19331     0.21079     0.21026  -0.0039149   0.0017244
   min :    -0.30133    -0.20961     0.16204     0.20975  -0.0040344  -0.0084744
   gap :     0.22578   0.0084033    0.046787  0.00027597  7.5776e-05   0.0066577
---------------------------
F (proton)= 0.83257+-0.18228
F(proton for VCSSIM user) = 1.0558+-0.045821
F (photon) = 0.85531+-0.0099876
F(photon for VCSSIM user) = 1.057+-0.0066304
---------------------------



(if|phi| >30)
          dr1 proton | dr1 photon | dr2 proton | dr2 gamma | delta1 | delta2
   mean :   -0.24094    -0.20171     0.21014     0.21003  -0.0039907  -0.0018167
```

```
   max :   0.0025334    -0.19331     0.21079     0.21026  -0.0039149   0.0017244
   min :    -0.30133    -0.20961     0.20624     0.20975  -0.0040344  -0.0084744
   gap :     0.24347   0.0084033    0.003904  0.00027597  7.5776e-05   0.0066577
---------------------------
F (proton)= 0.8162+-0.11228
F(proton for VCSSIM user) = 1.0571+-0.0068484
```

3.2 System choice

We have seen in sections **(3.1.4)** and **(3.1.5)** that the phase-space behaviour of δ_{r1} differs between a photon-detection experiment and a proton-detection experiment. $\delta_{r1,proton}$ depends on the phase-space while $\delta_{r1,photon}$ does not.

Despite this different behaviour of F_{photon} and F_{proton} with the phase space, one may want to use the system S1 calculation for a proton detection experiment, or also may want to use the elastic case calculation. Will it cause an error higher than 1% on the final cross section ?

We compare for δ_{r1} and for δ_{r2} the maximum gap observed for a possible VCS kinematics (when $|\phi_{cm}| > 30°$).

```
max | dr2 (proton - photon)| : 0.00372
max | dr2 (proton - EL)| : 0.00371
max | dr2 (gamma - EL)| : 0.00022

max | dr1 (proton - photon)| : 0.10802
max | dr1 (proton - EL)| : 0.09043
max | dr1 (gamma - EL)| : 0.01759
```

Here again a VCSSIM user can use any system without exceeding the 1% error on the final cross section. But the δ_{r1} calculation needs to be done properly if one uses the analytical calculation instead of the Monte Carlo method.

Finally we obtained the final F result for each system (S_1, S_2 and *elastic*) with and without the δ_{r1} analytical calculation :

```
Analytical calculation of dr1
  F (proton) = 0.78338
  F (proton) (if|phi| >30) = 0.8162
  F (photon) = 0.85531
  F (elastic) = 0.83958

No calculation of dr1 (VCSSIM user)
  F (proton)  = 1.0508
  F (proton) (if|phi| >30) = 1.0571
  F (photon) = 1.057
  F (elastic) = 1.0505
```

So one can use any system determination of the bremsstrahlung correction if one uses a Monte Carlo way to obtain δ_{r1}.

3.3 Higher-order corrections

Here we will discuss about the impact of the higher-order correction-terms which can be approximately taken into account by using the exponentiated form. Again we observe the maximum difference, in a bin, between the two kinds of calculation (linear addition, equation (12), versus the exponentiated form, equation (13)).

When we do the complete ($\delta_r = \delta_{r1} + \delta_{r2}$) calculation :

```
max | Fexp - F | proton : 0.027263
max | Fexp - F | proton (if|phi| >30): 0.027263
max | Fexp - F | photon : 0.01137
max | Fexp - F | elastic : 0.012221
```

This gives us an idea of the amplitude of some of the higher-order corrections (the exponentiated-form approximation does not describe all the higher-order processes). It appears to be higher, or about the same order, than 1% of the final cross-section. If one does the analytical calculation of δ_{r1} this will be the main source of uncertainty.

The δ_{r1} exponentiated form is already taken into account in the Monte Carlo method of VCSSIM. So, to approximate the impact of the remaining-higher-order correction-terms for a VCSSIM user, one has to use the equations (12) and (13) without δ_{r1}, i.e. $\delta_r = \delta_{r2}$.

Here also we obtain the maximum difference, in a bin, for F calculated as a VCSSIM user :

```
max | Fexp - F | proton : 0.0019899
max | Fexp - F | photon : 0.0019698
max | Fexp - F | elastic : 0.0019717
```

This difference amounts to 0.2% of the final cross-section. But, to properly analyse this result one has to carefully consider three supplementary sources of uncertainty on this 0.2% value :

1 - There is no exact calculation done of the higher-order correction-terms.

2 - The exponential form is a way to approximate only a part of the higher orders.

3 - Here we consider that a large extent of this part of the higher orders is correctly calculated through the Monte Carlo method of VCSSIM.

Anyway, if one use a Monte Carlo method to determine δ_{r1}, the uncertainty brought by omitting the higher-order correction terms is already contained in the 1% incompressible-theoretical error.

4 Conclusion

Implementing the δ_{r1} contribution by a Monte Carlo method will generate a radiative tail on an event-by-event basis, making the missing mass distribution

look like a real one. Also the Monte carlo application of δ_{r1} is well under control, as demonstrated in [1]. In this study we have shown that this way to include the δ_{r1} correction allowed us to consider the remaining radiative correction, F, as a constant over the VCS phase-space. The use of an averaged calculation of F, in any system instead of a bin-by-bin calculation, does not generate an error higher than 1%.

On the other hand, if one would calculate δ_{r1} without Monte Carlo simulation, i.e. using the analytical form, formula (2), one must apply a bin-per-bin correction and calculate δ_r with the appropriate system.

Omitting the complex $\delta_2^{(1)}$ calculation and the model dependency of the other proton-side corrections will not create an error higher than 1% on the F final value.

Anyway, in this study, an important part of the error is dominated by our choice to use the elastic case to determine δ_{vertex} and δ_{vac}.

For our kinematics we finally obtained :

$$\frac{1}{F} = 1.057 \pm 0.007_{PSA} \pm 0.01_{theo} \pm 0.008_{EC} \pm 0.002_{PSO}$$

With :

PSA, the uncertainty brought by our Phase-Space Averaging.

theo, the incompressible-theoretical error (it contains the incompressible error due to the lack of knowledge about higher order processes).

EC, the uncertainty brought by our choice to use the elastic-case determination of δ_{vertex} and δ_{vac}.

PSO, the approximate uncertainty brought by the proton-side omission of $\delta_2^{(1)}$.

We do the quadratic sum of errors :

$$\frac{1}{F} = 1.057 \pm 0.015$$

Références

[1] M. VanderHaegen, J.M. Friedrich, D. Lhuillier, D. Marchand, L. Van Hoorebeke and J. Van de Wiele, arXiv :hep-ph/0001100v1 (2000)

[2] D. Marchand, arXiv :hep-ph/0001100v1 (1998)

[3] L.C. Maximon and J.A Tjon, DAPNIA/SPHN-98-04T (2000) .

[4] F. Bloch and A Nordsieck, Phys. Rev 52, 54 (1937)

[5] D.R Yennie, S.C Frautschi and H. Suura, Ann. Phys. 13 ,379(1961)