Algorithm Symmetric 2-DLDA for Recognizing Handwritten Capital Letters

Ismail Husein[1], Rina Widyasari[2]
Universitas Islam Negeri Sumatera Utara Medan, Indonesia

Article Info

Article history:

Received July 21, 2021
Revised January 12, 2022
Accepted March 05, 2022

Keywords:

Capital Letter
Recognition Using
DLDA

ABSTRACT

Statistical pattern recognition is the process of using statistical techniques to obtain information and make informed decisions based on data measurements. It is possible to solve the doubt inherent in the objective function of the 2-Dimension Linear Discriminant Analysis by employing the symmetrical 2-Dimension Linear Discriminant Analysis approach. Symmetrical 2-dimensional linear discriminant analysis has found widespread use as a method of introducing handwritten capital letters. Symmetric 2-DLDA, according to Symmetric 2-DLDA, produces better and more accurate results than Symmetric 2-DLDA. So far, pattern recognition has been based solely on computer knowledge, with no connection to statistical measurements, such as data variation and Euclidean distance, particularly in symmetrical images. As a result, the aim of this research is to create algorithms for recognizing capital letter patterns in a wide range of handwriting. The ADL2-D symmetric method is used in this study as the development of the ADL2-D method. The research results in an algorithm that considers the left and right sides of the image matrix, as opposed to ADL2-D, which does not consider the left and right sides of the image matrix. In pattern recognition, the results with symmetric ADL2-D are more accurate.

Corresponding Author:

Ismail Husein,
Department of Mathematics,
UIN Sumatera Utara Medan,Indonesia
Email: husein_ismail@uinsu.ac.id

1. INTRODUCTION

The Pattern recognition is the process of recognizing patterns regularities in data by using automated algorithm. The process in pattern recognition systems starts from the selection of patterns as sensors, then the patterns are entered into processing techniques, representation charts, and finally, the process of modelling decision making [1].

In 2-Dimensional When it comes to pattern identification in this study, statistical classification is the most effective method [2]. Pattern recognition can be done using a variety of algorithms. Writing with LDA is a successful application of Linear Discriminant Analysis (LDA). Latent Dirichlet Allocation (LDA) is a subspace investigation method for studying the low-dimension structure of high-dimension data. In the vector space character projection, this method simultaneously minimizes the distribution within class (Sw) and maximizes the distribution between classes (Sb).[3, 4].

Matrices such as images, and in general the image is not symmetrical Xi XiT, then the distribution matrix between classes and distribution matrices in a class is defined not single. So there are a number of possible choices for determining the appropriate objective function of LDA [5, 6]. The previous researches about LDA such as [7] were using LDA for face recognition which divide images to same class and different class. Then [8] improved the present 2DLDA algorithm by its modification to more precise, class-dependent estimations repeated separately for each class and evaluated several scenarios using the TIMIT corpus in phoneme-based continuous speech recognition task. The next work be done by [9, 10] that improve the recognition accuracy of Local Binary Pattern (LBP) on low-resolution face recognition. In 2017, [4] investigated new methods to address the problem of plant species identification/classification using 2D digital leaves images and the approach used two features extraction methods based on one-dimensional (1D) and two-dimensional (2D) and the Bagging classifier. Next research same with [11, 12], that [13] employed new approach that matched the low resolution images with high resolution based on 2D-LDA. Based on recent researches, this paper employs a novel approach to recognition handwritten of capital letter use a develop method from [14]. We know that everyone write a capital letter such A letter in different style, and how to recognize it, so this paper will show that to recognize a capital letter as a symmetric image by using symmetrical 2-dimension linear discriminant analysis. [15] The symmetric ADL2D method is more accurate than other symmetrical methods because the recognition process partitions and then forms a left and right matrix. The level of accuracy is presented in the form of a graph obtained from the value of the objective function of the two functions [16, 17].

This manuscript's future writing will be organized as follows: Section 2 discusses Research Methodology and Materials; and third section discusses and explains the research findings and discusses computational illustration and is followed by the fourth section, which summarizes the study's findings and makes recommendations for future research.

2. RESEARCH METHOD

2.1. Explaining Linear Discriminant Analysis

In the classification process, first measure the observational characteristics of the sample. Extract all the information contained in the sample to calculate the sample-time value for a curve-shaped pattern, and the level of blackness of the pixels for a figure, as shown in Figure 1 [4] .

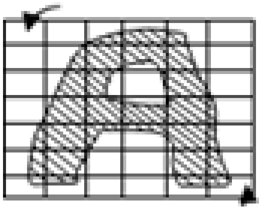


Figure 1. Example measurement of letter patterns

If a data matrix is given $H \in R^{N\times n}$, LDA method intents to encounter a transformation $K \in R^{N\times 1}$ that assigns each a_i column from matrix H, for $1 \leq i \leq p$, in the P dimension space to the b_i vector in the dimension l space. Namely $K : h_i \in R^{N\times n} \rightarrow b_i = K^T h_i \in R^l (l < P)$. In other words, LDA intents to encounter a vector space K spanned by where $K = [k_1, k_2, \ldots, k_l]$, so that each h_i is projected to K by [18].

$$(k_1^T.h_i, \ldots, k_l^T.h_i)^T \in R^l \tag{1}$$

Consider that the initial data in H is subdivided into k classes so that $H = \{\prod_1, \prod_2, \ldots, \prod_k\}$, where $\prod_i$ loads n_i the data points of class$i$ and $\sum_{i=1}^{k} p_i = p$. The classic LDA intents to encounter the optimal transformation of G so that the class structure of the initial high-dimensional space data is converted into a low-dimensional space. [18, 19].

The transformation to a lower dimension subspace is used in LDA is

$$y_i = K^T x_i \tag{2}$$

where K is the transformation to a subspace. Usually also written with $(y_1, \ldots, y_p) = K^T(x_1, \ldots, x_p)$ atau $Y = K^T X$. The main purpose of LDA is to find the value of K so that classes can be more detached in the transformation space and can conveniently be identified from the others. In the Linear Discriminant Analysis method, there are two distribution matrices, namely the in-class distribution matrix symbolized by S_w, and the inter-class distribution matrix symbolized by S_b, each construed as follows:

$$S_w = \sum_{i=1}^{c} \sum_{x_k \in \prod_i} [x_k - m_i][x_k - m_i]^T \tag{3}$$

$$S_b = \sum_{i=1}^{c} n_i [m_i - m][m_i - m]^T \tag{4}$$

Where p_i is the number of fragment in the class x_i , and m_i is the average image of-i and m is the overall average. The class average formula and the overall average is as follows: $m_i = \frac{1}{n_i} \sum_{x \in \prod_i} x$ is the average of the i-class, and $m = \frac{1}{n} \sum_{i=1}^{k} \sum_{x \in \prod_i} x$ is the overall average. General optimizations in Linear Discriminant Analysis:

$$max_k J(k) = tr \frac{S_b(Y)}{S_w(Y)} = tr \frac{K^T S_b(X) K}{K^T S_w(X) K} \tag{5}$$

This research are applied research that apply symmetric 2-D linear Discriminant Analysis to recognize handwritten of capital letter that has a symmetric form. We use testing data and training data. The training data are capital letter from computer based, whereas the testing data are handwritten capital letter. The procedure of this research shown in flow chart below

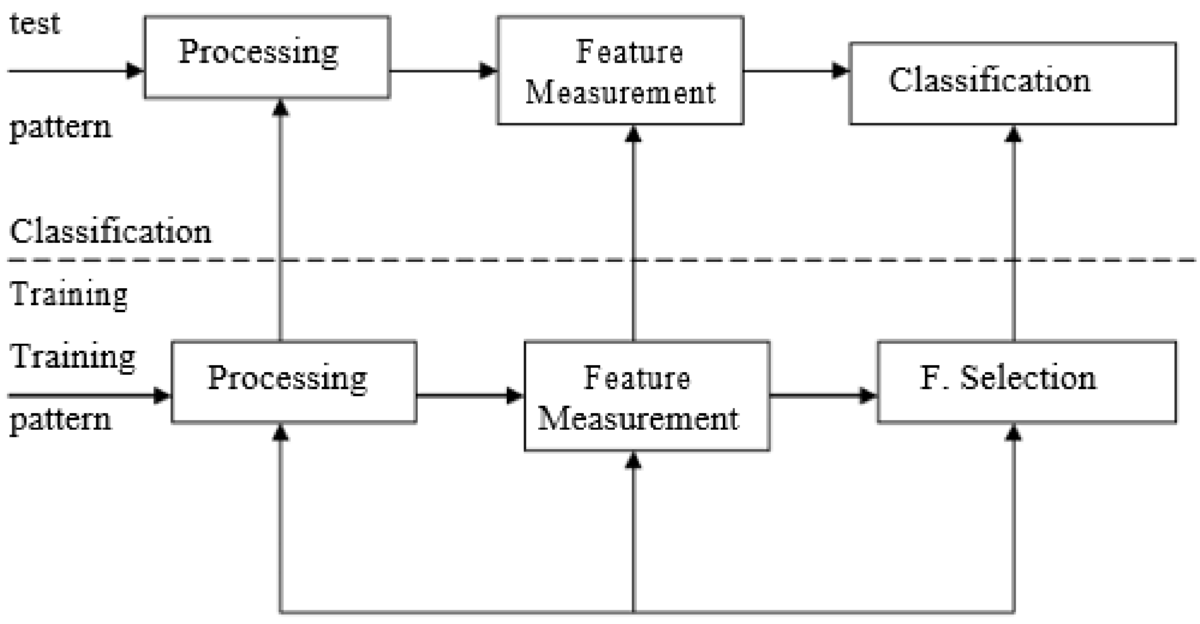


Figure 2. Statistical pattern recognition model

Set of figure $\{X_i\}_{i=1}^{n}$ and label of each class, testing data is handwritten capital letter
L_0, R_0 inisialization
Frequency c for orthogonalisazion
Inisialization

a) $L \leftarrow L_0, R \leftarrow R_0$
b) Compute M_k, $k = 1, 2, \ldots K$ and M
   M_k is average of each class, and M is overall average
c) $t \leftarrow 0$

Do
Compute $S_W^R, S_W^L, S_b^R, S_b^L$
$R \leftarrow R + \delta \frac{\partial J}{\partial R}$
$l \leftarrow L + \delta \frac{\partial J}{\partial L}$
$t \leftarrow t + 1$
if (t mod c)=0
$R \leftarrow$ eigenvector from $(S_W^L)^{-1} S_b^L$
$L \leftarrow$ eigenvector from $(S_W^R)^{-1} S_b^R$
End if
Output L, R.

3. RESULT AND ANALYSIS

3.1. In 2-Dimensional Linear Discriminant Analysis

The preeminent discrepancy between the classic LDA and 2-DLDA that the researchers propose in this study is about data representation. Classical LDA uses vector representations, whereas 2-DLDA works with data in matrix representations. In using the 2-DLDA method, it will be seen that the representation leads to eigen-decomposition of the matrix with a smaller size. More specifically, 2-DLDA involves eigen-decomposition matrices of $r \times r$ and $c \times c$ sizes, which are much smaller than the classical LDA matrices. It has been agreed in 2-DLDA that a set of images is symbolized. by $X = (X_1, X_2, \ldots, X_n)$, $X_i \in \Re^{r \times c}$. With the same clairvoyance as the classical LDA, 2-DLDA tries to find a linear transformation

$$Y_i = L^T X_i R \tag{6}$$

so the different classes are detached. In other words, the image or image X must be transformed into the form $Y_i = L^T X_i R$

$$M_i = \frac{1}{n_i} \sum_{x \in \prod_i} X, M = \frac{1}{n} \sum_{i=1}^{k} \sum_{x \in \prod_i} X \tag{7}$$

M_i is here to measure the average in each column of the X matrix, because the calculation of the distance between the matrices and the distance between the matrices and the distance between the matrices (based on discriminant statistics in statistics) will be carried out in formula 8.

Suppose that is the average of the i-th class, $1 \leq i \leq k$, and means the overall average. In 2-DLDA, researchers regard images as two-dimensional signals and intend to detect two transformation matrices $L \in$ and $R \in$ and assigns each H_i member for $1 \leq x \leq n$, to a B_i matrix so that $B_i = L^T H_i R$.

Similarly to classic LDA, 2-DLDA intends to encounter optimal L and R transformations (projections) so the class structure of the initial high-dimensional space is converted to a low-dimensional space. An innate metric affinity between matrices is the Frobenius[22]. The square of the distance from within-class and between classes can be calculated as below:

$$D_W = \sum_{i=1}^{k} \sum_{x \in \prod_i} ||X - M_i||_F^2, D_b = \sum_{i=1}^{k} n_i ||M_i - M||_F^2 \tag{8}$$

This formula is used to measure the distance between classes and the distance within the class with M_i as in formula $trace(MM^T) = ||M||_F^2$, for a matrix M, so obtained :

$$D_W = trace\left(\sum_{i=1}^{k}\sum_{x\in\prod_i}||X - M_i||_F^2\right) \tag{9}$$

$$D_b = trace\left(\sum_{i=1}^{k} n_i||M_i - M||_F^2\right) \tag{10}$$

So trace here is the same as counting (summing) the values on the main diagonal of the matrix so that the form 8 becomes 9 and 10.

In low-dimensional space, the result of linear transformations L and R, the distance between classes and between-classes becomes:

$$\bar{D}_W = trace\left(\sum_{i=1}^{k}\sum_{x\in\prod_i} L^T(X - M_i)RR^T(X - M_i)^T L\right) \tag{11}$$

$$\bar{D}_b = trace\left(\sum_{i=1}^{k} n_i L^T(X - M_i)RR^T(X - M_i)^T L\right) \tag{12}$$

Because the image X must be linearly transformed so that the distance between the matrices and the distance in the matrix must also be transformed so that the form 9 and 10 become the form 11 and 12.

The optimum of transformation of L and R will maximize $\bar{D}_b$ and minimize $\bar{D}_W$, because of the difficulty of calculating the optimum of L and R concurrently, the following is the algorithm for 2-DLDA. More particularly, for a settled R, we can calculate the optimum of L by determining the same optimization problem with equation 12. By calculating L, we can then amend R by determining another optimization problem as the only solution in equation 6.

The description of the calculation of L (bottom triangular matrix) and R (upper triangular matrix).

3.2. L Calculation

For a settled R, $\bar{D}_W$ and $\bar{D}_b$ can be rephrased as

$$\bar{D}_W = trace(L^T S_W^R L), \bar{D}_b = trace(L^T S_b^R L) \tag{13}$$

where

$$S_W^R = \sum_{i=1}^{k}\sum_{x\in\prod_i}(X - M_i)RR^T(X - M_i)^T, S_b^R = \sum_{i=1}^{k} n_i(X - M_i)RR^T(X - M_i)^T \tag{14}$$

Optimal L can be calculated by figuring out the succeeding optimization problem: $maxLtrace((L^T S_W^R L)^{-1}(L^T S_b^R L)$. The solution can be procured by clarifying the problem of generalizing the following eigenvalues: $S_W^R x = \lambda S_b^R x$. Because S_W^R in general it is nonsingular, the optimum L can be obtained by calculating an eigen-decomposition on $(S_W^R)^{-1} S_b^R$. Remark that the size of the matrices S_W^R and S_b^R are $r \times r$ (square matrix), which is smaller than the size of the matrices S_W and S_b in the classic LDA [13].

3.3. R Calculation

Then calculate R for a settled L. $\bar{D}_W$ and $\bar{D}_b$ can be written back as

$$\bar{D}_W = trace(R^T S_W^L R), \bar{D}_b = trace(R^T S_b^L R) \tag{15}$$

where

$$S_W^L = \sum_{i=1}^{k} \sum_{x \in \prod_i} (X - M_i) L L^T (X - M_i)^T, S_b^L = \sum_{i=1}^{k} n_i (X - M_i) L L^T (X - M_i)^T \tag{16}$$

Optimal R can be calculated by figuring out the succeeding optimization problem: $maxRtrace((R^T S_W^L R)^{-1}(R^T S_b^L R)$. The solution can be procured by clarifying the problem of generalizing the following eigenvalues: $S_W^L x = \lambda S_b^L x$. Because S_W^R in general it is nonsingular, the optimum R can be obtained by calculating an eigen-decomposition on $(S_W^L)^{-1} S_b^L$. Remark that the size of the matrices S_W^L and S_b^L are $r \times r$ (square matrix).

3.4. Symmetrical 2-Dimension Linear Discriminant Analysis (Symmetrical 2-DLDA)

It has been stated in the previous chapter that the classification approach with 2-Dimension Linear Discriminant Analysis (2 DLDA) raises a fundamental problem of doubt: There are two ways to delineate in-class distribution matrices S_W.

$$\begin{aligned} S_W(XX^T) &= \sum_{j=1}^{k} \sum_{x_i \in \pi_i} (X_i - M_j)(X_i - M_j)^T \\ S_W(X^T X) &= \sum_{j=1}^{k} \sum_{x_i \in \pi_i} (X_i - M_j)^T (X_i - M_j) \end{aligned}$$

and there are two ways to delineate the distribution matrix between classes S_b.

$$\begin{aligned} S_b(XX^T) &= \sum_{j=1}^{k} n_j (M_j - M)(M_j - M)^T \\ S_b(X^T X) &= \sum_{j=1}^{k} n_j (M_j - M)^T (M_j - M) \end{aligned}$$

Consequently, in the space of transformation can be written

$$S_b(YY^T), S_b(Y^T Y), S_W(YY^T), S_W(Y^T Y)$$

In general, images are not symmetrical $X_i \neq X_i^T$, then

$$\begin{aligned} S_b(YY^T) &\neq S_b(Y^T Y), \\ S_w(YY^T) &\neq S_w(Y^T Y) \end{aligned}$$

For this argument, the objective function of LDA is dubious, which raises a number of choices:

$$\begin{aligned} J_1 &= tr \frac{S_b(YY^T)}{S_w(YY^T)} \\ J_2 &= tr \frac{S_b(Y^T Y)}{S_w(Y^T Y)} \\ J_3 &= tr \left[\frac{S_b(YY^T)}{S_w(YY^T)} + \frac{S_b(Y^T Y)}{S_w(Y^T Y)} \right] \\ J_4 &= tr \left[\frac{S_b(Y^T Y)}{S_w(Y^T Y)} \frac{S_b(YY^T)}{S_w(YY^T)} \right] \\ J_5 &= tr \left[\frac{S_b(YY^T) + S_b(Y^T Y)}{S_w(YY^T) + S_w(Y^T Y)} \right] \end{aligned}$$

[14]

Symmetrical 2-Dimensional Linear Discriminant Analysis in solving the ambiguous problem above inspired by a key observation: if the picture is symmetrical, namely $X_i = X_i^T$, then

$$\begin{aligned} S_w(XX^T) &= S_W(X^TX), \\ S_b(XX^T) &= S_b(X^TX) \end{aligned}$$

The solution of this problem uses a new data representation that is symmetric linear transformation.

$$\begin{pmatrix} 0 & Y_i^T \\ Y_i & 0 \end{pmatrix} = \varGamma^T \begin{pmatrix} 0 & X_i \\ X_i^T & 0 \end{pmatrix} \varGamma, \varGamma = \begin{pmatrix} 0 & L \\ R & 0 \end{pmatrix}$$

In Fukunaga (1990), matrix is defined as:

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \ddots & \vdots \\ 0 & \dots & \sigma_n \end{pmatrix}$$

that is, the diagonal matrix is mainly the value of the variance of the data and other elements 0. The linear transformation above is equivalent to the linear transformation $Y_i = L^T X_i R$ and $Y_i^T = R^T X_i^T L$. We also have

$$\left\| \begin{pmatrix} 0 & X_i^T \\ X_i & 0 \end{pmatrix} - \varGamma \begin{pmatrix} 0 & Y_i \\ Y_i^T & 0 \end{pmatrix} \varGamma^T \right\|^2 = 2||X_i - LY_iR^T||^2$$

Therefore, optimization using (L, R) is equivalent to optimization using $\varGamma$.

Other than that, by using symmetric linear transformations produced a theorem:

Theorem 1: The single objective function of LDA for2-DLDA is

$$J_{ADL2-D} = tr\frac{S_b(YY^T)}{S_w(YY^T)} = tr\frac{S_b(Y^TY)}{S_w(Y^TY)} \quad (17)$$

$$J_{ADL2-D} = tr\left(\frac{R^TS_b^LR}{R^TS_W^LR} + \frac{L^TS_b^RL}{L^TS_W^RL}\right) \quad (18)$$

Using theorem 1, in the case non symmetrics matrices which cause S_w and S_b in X space to be doubly defined it also causes S_w and S_b in Y space to be doubly defined. So

$$\begin{aligned} J_1' &= tr\frac{S_b(YY^T)}{S_w(YY^T)} = tr\frac{R^TS_b^LR}{R^TS_w^LR} \\ J_2' &= tr\frac{S_b(Y^TY)}{S_w(Y^TY)} = tr\frac{L^TS_b^RL}{L^TS_w^RL} \end{aligned}$$

In an independent optimization approach, to get R can be done by maximizing J_1' (reject J_2') and then obtaining L by maximizing J_2' (rejecting J_1'). This is not consistent in optimizing the objective function, which is when maximizing J_1', J_2' has decreased and vice versa. This problem can be solved by two techniques namely first, when maximizing J_1', must calculate J_2'. But, on the other hand also need to know how to combine J_1' and J_2'. The simple combination that can be done is $J = J_1' + J_2'$, i.e.:

$$J = tr\frac{R^TS_W^RR + L^TS_b^LL}{R^TS_w^RR + L^TS_W^LL} \quad (19)$$

Secondly is how to optimize the objective function. The result to maximizing $max_R J$ can be simply done by calculating eigenvectors from, the same calculations as the Linear Discriminant Analysis method.

However, the objective function described in equation 19 cannot be used to determine the trace of a single ratio of the two distribution matrices. This happens because the objective function cannot be solved in the same direction through eigenvector calculations (same as standard LDA). However, this can be overcome by developing an efficient algorithm using the gradient-up approach. This approach reduces objective functions. The derivative of the matrix function is done by using the basic matrix algebra

contained in Fukunaga's book (1990). The results are shown in the following Lemmas:
Lemma 2: Let $P_L = L^T S_b RL$, $Q_L = L^T S_W^R L$, $P_R = R^T S_b^L R$, and $Q_R = R^T S_b LR$
Dervative of objective function J_{ADL2-D} in equation (20) as follow
For $\frac{\partial J}{\partial R}$ obtained

$$\begin{aligned} \frac{\partial}{\partial R} tr \frac{R^T S_b^L R}{R^T S_w^L R} &= 2S_b^L RQ_R^{-1} - 2S_W^L RQ_R^{-1} P_R Q_R^{-1} \text{ and} \\ \frac{\partial}{\partial R} tr \frac{L^T S_b^R L}{L^T S_w^R L} &= 2\sum_{k=1}^{K} \sum_{A_i \in \pi_k} (H_i - M_k)^T LQ_L^{-1} L^T (H_i - M_k) R \\ &\quad -2\sum_{k=1}^{K} (M_k - M)^T LQ_L^{-1} P_L Q_L^{-1} L^T (M_k - M) R \end{aligned} \tag{20}$$

For $\frac{\partial J}{\partial L}$ obtained [14]

$$\begin{aligned} \frac{\partial}{\partial L} tr \frac{L^T S_b^R L}{L^T S_w^R L} &= 2S_b^R LQ_L^{-1} - 2S_W^R LQ_L^{-1} P_L Q_L^{-1} \text{ and} \\ \frac{\partial}{\partial L} tr \frac{R^T S_b^L R}{R^T S_w^L R} &= 2\sum_{k=1}^{K} \sum_{A_i \in \pi_k} (H_i - M_k) RQ_L^{-1} R^T (H_i - M_k)^T L \\ &\quad -2\sum_{k=1}^{K} (M_k - M) RQ_R^{-1} P_R Q_R^{-1} R^T (M_k - M)^T L \end{aligned} \tag{21}$$

Using the explicit gradient formula above, an algorithm can be developed like algorithm 1 to facilitate the classification applied to computer visualization, following an efficient algorithm using the gradient-up approach.

The flow of using ADL2-D to ADL2-D Symmetrical in pattern recognition:
Enter the image that will be the object of handwriting capital letter pattern recognition
Algorithm 1 Symmetrical 2-DLDA using Gradient
Input

a) Set of figure $\{X_i\}_i = 1^n$ and label of each class
b) L_0, R_0 inisialization
b) Frequency c for orthogonalisazion

Inisialization

a) $L \leftarrow L_0, R \leftarrow R_0$
b) Compute M_k, $k = 1.2.\ldots K$ and M
   M_k is average of each class, and M is overall average
b) $t \leftarrow 0$

Do
Compute $S_W^R, S_W^L, S_b^R, S_b^L$
$R \leftarrow R + \delta \frac{\partial J}{\partial R}$
$L \leftarrow L + \delta \frac{\partial J}{\partial L}$
$t \leftarrow t + 1$
if $(tmodc) = 0$
$R \leftarrow$ eigenvector from $(S_W^L)^{-1} S_B^L$
$L \leftarrow$ eigenvector from $(S_W^R)^{-1} S_B^R$
endif
Output L, R [23]

3.5. Application of the use of Symmetrical 2-Dimension Linear Discriminant Analysis Method in an Example of Character Pattern Recognition

To facilitate understanding of Linear Discriminant Analysis (LDA) and 2-Dimension Linear Discriminant Analysis (2-DLDA), researchers present how to recognize patterns of a character using these methods and how they compare with each other. The following are examples of 2 characters A and B, with each character A and B having two patterns.

```
. . # # . .
. # . . # .
# . # # . #
# . . . . #
# . . . . #
# . . . . #
```

H1

```
# # # # . .
# . . . # .
# # # # # .
# . . . . #
# . . # # #
# # # . . .
```

H2

```
. . # # . .
. # . . # .
# # . . # #
# . # # . #
# . . . . #
# . . . . #
```

H3

```
# . # # # .
# # . . # .
# . . # . .
# # # # # .
# . . . . #
# # # # # .
```

H4

Each character pattern above is represented in a $6 \times 6 = 36$ elements matrix, then the matrix is detached into 6 classes, $H_i = \prod_1, \prod_2, \ldots, \prod 6$. $X_i = 0$ if the element represented is a dot and $X_i = 1$ if the element represented is $\sharp$, the representation matrix as follow:

$$H_1 = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In the example of the characters above, it can be seen that the characters A_1 and A_3 characters are symmetrical characters, but the matrix of character representation is not a symmetrical matrix. Since in the pattern recognition process there must be training data (training set) and test data (testing set) then in this research example, we consider H_1 and H_2 as training data, H_3 and H_4 as testing data. This aims to see whether the H_3 pattern is similar to the H_1 pattern and whether the H_4 pattern is similar to the H_2 pattern.

For the pattern recognition process, then each character representation matrix is partitioned into $k = 2$, $k = 3$, and $k = 6$ classes viz.

In high-dimensional data

The average of each class is

$$\begin{array}{llll} M_1 = 2,0553 & M_2 = 0,92912 & M_3 = 0,0726266 & M_4 = -0,18354 \\ M_5 = -0,386472 & M_6 = 0,05335 & M_7 = -0,09874 & M_8 = -0,26913 \\ M_9 = -0,07844 & & & \end{array}$$

Overall average $M = 0.2331295$.

Linear Discriminant Analysis Method
Eigenvalue matrix in-class distribution (S_w) above is

$$eigenvalue(S_W) = \begin{pmatrix} -0.0000 \\ -0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.7703 \\ 2.9668 \\ 3.9791 \\ 9.7268 \\ 13.1248 \\ 17.8440 \\ 30.8847 \\ 96.9425 \\ 603.0483 \end{pmatrix}$$

$trace(S_w) = 779.2891$. Trace is the number of eigenvalues in a square matrix of size $n \times n$ which is also the sum of the diagonal elements of the matrix.

The in-class distribution matrix eigenvalue (S_w) above is

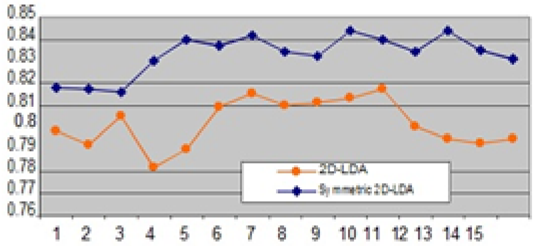
$$eigenvalue(S_W) = \begin{pmatrix} -0.0000 \\ -0.0000 \\ -0.0000 \\ -0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 73.1690 \end{pmatrix}$$

$trace(S_b)$ =number of diagonal elements $(S_b) = 73.1690$.
Optimum objective function Symmetric $LDA = \frac{tr(S_b)}{tr(S_w)} = \frac{73.1690}{779.821} = 0.093827$

Based on subsection 3.3 we show that the result that to recognize the different type of handwritten capital letter, can use 2-D LDA and Symmetric 2-D LDA. At first, we must partition it and count the average of each matrices, count the eigenvalue of those matrices to have separate L and R. At the last count the objective function to know that this method more efficient than the other LDA method.

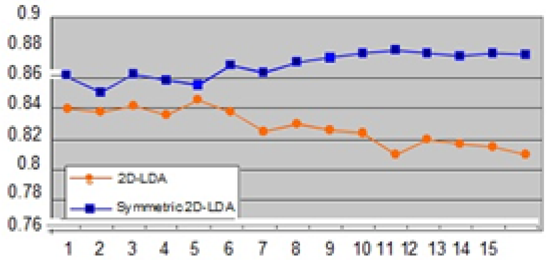
The comparation Symmetric 2-DLDA with the 2D-LDA showed in graphic below. The following graphics show the differences in pattern recognition and classification using 2-Dimensional Linear Discriminant Analysis and Symmetric 2-Dimensional Discriminant Analysis with 15 iterations. To produce this image, at first calculates the accuracy of the classification and the value of the objective function using a validity chart by dividing the data matrix into 2 parts, 1 part to be tested, namely patterns H1 and H2 as training data and patterns H3 and H4 as testing data. We partitioned the image into k = 2, k = 3, and k = 6. The objective function is calculated using the algorithm described in chapter 3. In the graphics, the blue color represents the objective function of the classification using ADL2-D Symmetrical, while the red color represents the objective function of the classification using ADL2-D.

For k=2



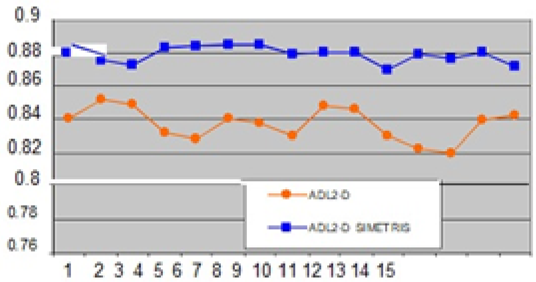
Objective Function

For k=3



Objective Function

For k=6



4. CONCLUSION

Symmetrical ADL2-D considers the symmetrical nature of the image, and transforms the image into an upper triangular matrix and a lower triangular matrix with the aim of making pattern recognition, especially handwriting patterns more accurate.The problem of doubt created by the objective function of the 2-Dimensional Linear Discriminant Analysis can be dealt with prior to implementing the symmetrical 2-Dimension Linear Discriminant Analysis approach by resolving the issue before using the symmetrical 2-Dimension Linear Discriminant Analysis approach. As a result, there is a complete objective function. By dividing the traction formulation (Sw) by the inter-class distribution matrix (Sw), the symmetrical 2-DLDA formula was obtained (Sb). The all-encompassing objective function includes everything.

In application, this method to recognize handwriting of capital letter, with testing data that handrwitten capital letter style, and training data are capital letter of computer base, we can see that the images that form a symmetric character but not symmetric matrices can separate into same class and between class, become L matrice and R matrice, at the end, based on objective function graphically show that 2D-LDA is more efficient than the other method of LDA. In addition, an efficient computational algorithm for solving objective functions in symmetric 2-DLDA is provided. 2-DLDA gives better and more accurate results than 2-DLDA when applied to high-dimensional data. It is necessary to do calculations by generating this method for further research to solve multi-linear discriminant problem.

REFERENCES

[1] Y. F. Mustafa, F. Ridho, and S. Mariyah, "Study of Handwriting Recognition Implementation in Data Entry of Survei Angkatan Kerja Nasional (SAKERNAS) Using," *International Cpnference On Data Science And Official Satistik*, vol. 1, no. 2, pp. 53–65, 2021.

[2] E. A. Al-Zubaidi, M. M. Mijwil, and A. S. Alsaadi, "Two-Dimensional Optical Character Recognition of Mouse Drawn in Turkish Capital Letters Using Multi-Layer Perceptron Classification," *Journal of Southwest Jiaotong University*, vol. 54, no. 4, pp. 4–9, 2019.

[3] I. M. Pegi, K. Amerta, and I. G. A. Wibawa, "Alphabet Writing Game Application Using Template Matching Cross-correlation," vol. 8, no. 4, pp. 393–399, 2020.

[4] A. Challa, "Automatic Handwritten Digit Recognition on Document Images Using Machine Learning Methods," *Blekinge Institute of Technology*, no. January, 2019.

[5] M. M. Saufi, M. A. Zamanhuri, N. Mohammad, and Z. Ibrahim, "Deep Learning for Roman Handwritten Character Recognition," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 12, no. 2, pp. 455–460, 2018.

[6] I. Raya, G. Widjaja, Z. H. Mahmood, A. J. Kadhim, K. O. Vladimirovich, Y. F. Mustafa, M. M. Kadhim, T. Mahmudiono, I. Husein, and L. Kafi-Ahmadi, "Kinetic, Isotherm, and Thermodynamic Studies on Cr(VI) Adsorption Using Cellulose Acetate/Graphene Oxide Composite Nanofibers," *Applied Physics A: Materials Science and Processing*, vol. 128, no. 2, 2022.

[7] Y. Lu, "Handwritten Capital Letter Recognition Based on OpenCV," *MATEC Web of Conferences*, vol. 277, p. 02030, 2019.

[8] M. Tokovarov, M. Kaczorowska, and M. Milosz, "Development of Extensive Polish Handwritten Characters Database for Text Recognition Research," *Advances in Science and Technology Research Journal*, vol. 14, no. 3, pp. 30–38, 2020.

[9] M. J. Al-Shamdeen, A. N. Younis, and H. A. Younis, "Metaheuristic Algorithm for Capital Letters Images Recognition," *International Journal of Mathematics and Computer Science*, vol. 16, no. 2, pp. 577–588, 2020.

[10] S. E. Husain, Muhammad Zarlis, Herman Mawengkang, "Causal Loop Diagram (CLD) Model In Planning A Sustainable Smart Sharia Tourism," *J. Phys.:Conf. Ser*, vol. 1641, no. 012099, 2020.

[11] M. Domino, M. Borowska, N. Kozłowska, A. Trojakowska, Ł. Zdrojkowski, T. Jasiński, G. Smyth, and M. Maśko, "Selection of Image Texture Analysis and Color Model in The Advanced Image Processing of Thermal Images of Horses Following Exercise," *Animals*, vol. 12, no. 4, 2022.

[12] and Liu Li, J.,Chen, Z., "Low-Resolution Face Recognation of Multi-Scale Blocking CS-LBP and Weighted PCA," *International Journal of Pattern Recognition and Artificial Intelligence*, vol. 30, no. 10, 2018.

[13] Y. Zhao, D., Chen, Z., Liu, C., Peng, "Two-Dimensional Linear Discriminant Analysis for Low-Resolution Face Recognition," *IEEE*, 2018.

[14] H. Luo, D., Ding, C., and Huang, "Symmetric Two Dimensional Linear," in *University of Texas at Arlington Press*, 2019.

[15] I. H. Gerhard Wilhelm Weber, Burcu Gurbuz, Herman Mawengkang, “Rumour Propogation : An operational Research Approach by Computional and Information Theory,” *CEJOR*, vol. 4, no. 4, 2022.

[16] I. Husein, A. Suhada, P. Chetthamrongchai, A. P. Peressypkin, A. S. Nurrohkayati, V. H. Ca, H. T. Hoi, J. W. Grimaldo Guerrero, and M. Kavitha, “Scheduling for A Container Supply Chain to Minimize Costs Using The Meta-Innovation Approach,” *Industrial Engineering and Management Systems*, vol. 20, no. 4, pp. 662–671, 2021.

[17] H. Anthony Anggrawan, Christofer Satria, “Smart Campus : Model Baru Enterprise Architecture STMIK Bumigora Mataram dalam Manajemnt Tata Kelola TIK Berbasis TOGAF ADM,” *Jurnal Mantik Penusa*, vol. 2, no. 2, pp. 127–136, 2018.

[18] A. A. Yahya, J. Tan, and M. Hu, “A Novel Handwritten Digit Classification System Based on Convolutional Neural Network Approach,” *Sensors*, vol. 21, no. 18, pp. 1–26, 2021.

[19] Y. Du, Y. Guo, M. Dou, F. Yang, I. Husein, and I. Pustokhina, “Experimental Iinvestigation of Carbonated Water Injection Incorporated with Optimum Concentration of Chemical Agents in Ultra-Low Permeable Reservoirs,” *Energy Reports*, vol. 7, pp. 3607–3613, 2021.