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Nikolai Nikolaevich Luzin at the crossroads of the dramatic events of the European history of the first half of the 20th century

Abstract

Nikolai Nikolaevich Luzin's life (1883–1950) and work of this outstanding Russian mathematician, member of the USSR Academy of Sciences and foreign member of the Polish Academy of Arts and Sciences, coincides with a very difficult period in Russian history: two World Wars, the 1917 revolution in Russia, the coming to power of the Bolsheviks, the civil war of 1917–1922, and finally, the construction of a new type of state, the Union of Soviet Socialist Republics. This included collectivization in the agriculture and industrialization of the industry, accompanied by the mass terror that without exception affected all the strata of the Soviet society. Against the background of these dramatic events took place the process

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of formation and flourishing of Luzin the scientist, the creator of one of the leading mathematical schools of the 20th century, the Moscow school of function theory, which became one of the cornerstones in the foundation of the Soviet mathematical school. Luzin's work could be divided into two periods: the first one comprises the problems regarding the metric theory of functions, culminating in his famous dissertation *Integral and Trigonometric Series* (1915), and the second one that is mainly devoted to the development of problems arising from the theory of analytic sets. The underlying idea of Luzin's research was the problem of the structure of the arithmetic continuum, which became the super task of his work.

The destiny favored the master: the complex turns of history in which he was involved did not prevent, and sometimes even favored the successful development of his research. And even the catastrophe that broke out over him in 1936 – "the case of Academician Luzin" – ended successfully for him.

Keywords: *D. Egorov, set theory, theory of functions of a real variable, Moscow school of function theory, W. Sierpinski, axiom of choice, continuum hypothesis, M. Suslin, effectivism, Borel set, analytic set*

Nikołaj Nikołajewicz Łuzin na skrzyżowaniu dramatycznych wydarzeń w historii Europy pierwszej połowy XX wieku

Abstrakt

Życie Mikołaja Nikołajewicza Łuzina (1883–1950) i twórczość wybitnego rosyjskiego matematyka, członka Akademii Nauk ZSRR i zagranicznego członka Polskiej Akademii Umiejętności, przypadają na bardzo trudny okres w historii Rosji: dwie wojny światowe, rewolucja 1917 w Rosji, dojście do władzy bolszewików, wojna domowa 1917–1922, wreszcie budowa nowego typu państwa – Związku Socjalistycznych Republik Radzieckich, obejmująca kolektywizację w rolnictwie i industrializację przemysłu, czemu towarzyszył masowy terror, który bez wyjątku dotknął wszystkie warstwy społeczeństwa radzieckiego. Na tle

tych dramatycznych wydarzeń przebiegał proces powstawania i rozkwitu naukowca Łuzina, twórcy jednej z głównych szkół matematycznych XX wieku – moskiewskiej szkoły teorii funkcji, która stała się jednym z kamieni węgielnych radzieckiej szkoły matematycznej. Twórczość Łuzina można podzielić na dwa okresy: pierwszy obejmuje zagadnienia dotyczące metrycznej teorii funkcji, których kulminacją jest jego słynna rozprawa *Całka i szeregi trygonometryczne* (1915), a drugi, poświęcony głównie rozwojowi problemów wynikających z teorii zbiorów analitycznych. Ideą leżącą u podstaw badań Łuzina był problem struktury kontinuum arytmetycznego, który stał się nadrzędnym zadaniem jego pracy.

Przeznaczenie sprzyjało mistrzowi: złożone zwroty historii, w które był wplątany, nie przeszkadzały, a czasem nawet sprzyjały pomyślnemu rozwojowi jego badań. I nawet katastrofa, która wybuchła w 1936 roku – „przypadek akademika Łuzina" – zakończyła się dla niego pomyślnie.

Słowa kluczowe: *D. Egorov, teoria zbiorów, teoria funkcji zmiennej rzeczywistej, moskiewska szkoła teorii funkcji, W. Sierpiński, aksjomat wyboru, hipoteza kontinuum, M. Suslin, efektywizm, zbiór Borela, zbiór analityczny*

1. Introduction

This paper concerns the work of the outstanding Russian mathematician, member of the USSR Academy of Sciences and foreign member of the Polish Academy of Arts and Sciences Nikolai Nikolaevich Luzin (1883–1950).[1] I will begin my story with the events of the Russian revolution of 1905, which opened a new period in Russian history, followed in 1914 by the battles of the First World War and the revolutions that unfolded during it: this of February and the Great October 1917 revolutions. The revolution grew into a grandiose civil war (1917–1922), after which the Bolsheviks, headed by the first chairman of the Council of People's Commissars of the RSFSR V.I. Lenin, launched a gigantic

[1] There are no generally accepted rules for writing Russian surnames in the Latin alphabet. In this paper we use the rules adopted by the American Mathematical Society. In accordance with them, the surname "Лузин" is transcribed as "Luzin". Although in the list of references to this article, the reader will come across another spelling of the same surname – "Lusin".

social experiment – the construction of "the first classless society in history". After his death in 1924, I.V. Stalin, whose task was the construction of a new state – the collectivization and industrialization of the country – finally faced a terrible war, which in the Russian historiographic tradition received the name of the Second World War.

2. Lift to the mathematics

All these events fell on the period of adult life of Luzin, born in a family of a small businessman, a pupil of the Tomsk *gymnasium*. His parents dreamed that their only child would become an engineer. At the time, this profession was very prestigious in the country, especially in Siberia, because in those years the Great Siberian Route – the legendary Transsib – had been constructed. His parents chose for him the newly opened Imperial St. Petersburg Polytechnic Institute – the parental ambitions did not allow them to be content with the newly opened (in 1900 (!)) Tomsk Polytechnic Institute. However, the *gymnasium* certificate was far from brilliant and in order to enter the capital's polytechnic, it was necessary to confront serious competition. Uncertain in his skills, the young candidate was forced to choose a roundabout route. A certificate on the completion of two years at a faculty of physics and mathematics of any Russian university gave him the permission to enter and follow without any competition senior courses of the capital Polytechnic.

In the fall of 1901, Luzin already became its student. The lectures of N.V. Bugaev, N.E. Zhukovsky, B.K. Mlodzeevsky, D.F. Egorov, as well as all the atmosphere that pervaded the Polytechnic, amazed and captivated the impressionable provincial.

> As N.N. said, the very first lecture on mathematics solved the problem. The years of mediocre teaching in the *gymnasium* could not manifest the absolute qualifications and tastes for doing mathematics, but they were shown by the very first talented lecture of a university professor. After listening to it, N.N. was firmly convinced that he did not want to be any engineer and would not be one – he *would be a mathematician: the question of a career was decided firmly and once and for all* (it is a slightly modifited translation from Tiulina 2006, pp. 274–275; author's italics – S.D.).

3. In the search of one's own road in mathematics

The next important step in Luzin's creative biography was his entry into the circle of the young professor D.F. Egorov (1869–1931), who immediately was able to appreciate his mathematical talent. At the same time he was in a position to detect the weaknesses of Luzin's creative nature: the lack of firmness in character, the extreme impressionability, as well as his suspiciousness and the lack of self assurance. In combination with youthful maximalism, all these led to nervous breakdowns, often ending in periods of prolonged depressions. Egorov used all the strength of his character and his outstanding pedagogical gift to direct the progress of his student in the right direction, carefully monitoring his studies and mood swings. He did this with the highest degree of tactfulness, correcting rather than directing. His letters to Luzin (see Egorov 1980), which were miraculously preserved, allow us to evaluate this achievement.

The main reason for Luzin's first scientific trip in 1905 to Paris (then he was still to be a student!) was the revolutionary events in Moscow in 1905–1906. When Egorov was informed that revolutionary-minded student friends turned Luzin's room, which he rented in Arbat's street, into a warehouse of bombs and proclamations, he was afraid that his talented student would be involved in revolutionary activities. He did all his best in order to send him to then peaceful Paris. In Paris, Luzin read a lot, attended the lectures of famous French mathematicians and took advantage of the meetings of the Mathematical Society, and thus gradually discovered his own guidelines in the world of mathematics of that time. This scientific trip contributed to the formation of his talent and paved the way to the circle of ideas of the set theory and the theory of functions of a real variable along the paths laid by the French mathematicians, E. Borel, R. Baire and H. Lebesgue. It is important to note that it was then that *he paid special attention to Borel's point of view on actual infinity, expressed by him in the famous dispute about the axiom of choice that had just erupted. This dispute and Borel's position in it will subsequently play an important role in the formation of his ideology.* In Paris, he also pondered over the problem concerning the theory of ordinary differential equations, which he chose as the topic of his future essay to obtain a 1st degree diploma, which gave him the right to remain at the university in order "to prepare for professorship", that is, in today's terms, to remain in a postgraduate school.

In the summer of 1906, he returned to Moscow and, as he later wrote in his autobiography:

> in 1906 he submitted a test essay on the topic "On one method of integrating differential equations". In the same year, he graduated from the University with a 1st degree diploma and remained at the Department of Pure Mathematics led by Professor Dmitry Fedorovich Egorov (…) (cited from Volkov 2005, pp. 14–15).

4. Continuum hypothesis

The interval from the beginning of 1907 to the end of the spring semester of 1910 allowed Luzin to "prepare his professorship". So, as it was expected, he was obliged to pass the master's exams, to choose a topic for further research (that is, the topic of a future master's thesis) and to prepare the first mock lectures (*Probevorlesung*). Apparently, for a long time he could not decide on the choice of the topic, and only at the spring of 1909 that it began to transpire, the set theory. In April of this year, in a letter to his university friend P.A. Florensky, who later became an outstanding theologian and philosopher, he wrote:

> In the summer I am thinking to prepare for the mock lectures and to develop several topics: "Possibility of projective geometry of transcendental curves" and the "Continuum-problem" (see Luzin 1989, p. 158).

Luzin's choice of the continuum hypothesis as the topic of his *Probevorlesung* indicates the adoption of set theory in his sphere of special interests. *The very continuum hypothesis became the subject of his constant reflections*.

Apparently, already during his *gymnasium*'s studies in Tomsk was Luzin's interest for philosophy manifest. He called it a "poisoning" with philosophy. It must also be stressed that regarding mathematics, he was not at all attracted by difficult problems to solve or by the search to design their solution, ingenious *kunststück*, allowing to bypass the arising difficulties – unlike mathematicians, for whom their science is a kind of sport, he was not a "problem solver". As a matter of fact, *he had the qualification to be a mathematician-philosopher*. This feature of his creative talent distinguished him in the mathematical community and found its expression in various forms. This is where his friendship with P.A. Florensky (1882–1937), and his election in 1929 as a member of the USSR Academy of Sciences in the class of "philosophy". This feature of his creative nature was especially stressed by the astute H. Lebesgue in his preface to Luzin's lectures on analytic sets[2]. This feature made him subsequently a convenient target for attacks by philosophizing ideologists on the Soviet "mathematical front". It also lies at the basis of the difficulties arising from his choice of this topic of research. He was not entirely happy with the topic of his "candidate" essay, regarding the methods of integrating differential equations. He was not attracted by the prospect of tackling problems of differential geometry in the direction developed by his respected teacher, as his classmate S.S. Byushgens did. For his studies, he was looking for a topic with a philosophical context and the arithmetic continuum became such a topic for him, a resonance of Pythagoras's secret teaching. Hence his interest in the Cantor continuum hypothesis.

The concentration of his efforts to solve it fell on the period from mid-winter to the beginning of the summer of 1910. Yet, the hypothesis did not succumb to his efforts and having postponed it for future times (postponed not abandoned), he decided to tackle the problems of the theory of functions of a real variable. He was even going to announce a course on this topic at the university for the fall semester (after all he had in mind to become a privat-docent of Moscow University!). However, the circumstances of his life seemed to be against his plans, although in reality all these events ended up being for his good: his supervisor managed to secure a scientific trip for him to Göttingen and Paris.

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[2] I will cite only two fragments from Lebesgue's preface: "(...) in short, the qualities of a philosopher are needed. Mr. Luzin explores issues from a philosophical point of view and arrives at mathematical results: unprecedented originality!" or "(…) he has philosophical interests, and he confesses to them. Mathematical requirements and philosophical requirements are constantly combined, even, one might say, fused. Although his book is an essay on mathematics written by a mathematician for mathematicians, almost every page of it clearly shows this close connection between philosophical and mathematical thoughts, which makes the monograph extremely significant and absolutely extraordinarily attractive" (Lusin 1930, pp. IX and XI, respectively; translated by S.D.).

5. Metric theory of functions. Göttingen – Paris

His arrival in Göttingen (Fig. 1) at the end of 1910 marked the beginning of a period of exceptional creativity for him.

Fig.1: N.N. Luzin with his wife Nadezhda Mikhailovna (right) with the Lavrentiev family - from left to right: Misha Lavrentyev, future vice-president of the USSR Academy of Sciences, organizer of the Siberian branch of the USSR Academy of Sciences, his father Alexei Lavrentievich, future professor of Kazan University, then Moscow University, his mother Anisia Mikhailovna. Göttingen 1911.

He plunged into a wonderful creative atmosphere: working in the local libraries, making acquaintances with first-class mathematicians (E. Landau, D. Hilbert, etc.), and finally ensuring friendly contacts in the circle of Russian mathematicians who studied there. This helped him not only to overcome the consequences of a difficult mental state caused by his failure to prove the continuum hypothesis, but also to find the strength to make a leap into the theory of trigonometric series, into the field of the metric theory of functions, which became for him, and later for his students, a field of successful research. The *first period of his glorious creative biography* thus began. The first visible success on this path appeared in his first publication "Über eine Potenzreihe" in *Rendiconti del Circolo Matematico di Palermo* (1911, v. 32), a journal that was becoming fashionable in those years. It should be noted that this article was sent to the editorial office of the journal at the insistence of E. Landau, as for a long time Luzin hesitated to submit it. In addition to the outstanding results it presented, this publication constituted the foundation for his research regarding the theory of functions of a complex variable.

Luzin's creative tension in 1911 (that is, in the spring semester in Göttingen and in the fall semester in Paris) was very fruitful. Its visible expression, in addition to the article just named, was the extraordinary publishing activity that followed already in 1912: 7 papers published in *Matematicheskii Sbornik* (3 articles) and in *Comptes Rendus de l'Académie des Sciences Paris* (4 notes). In particular, they contain Luzin's well-known C-property theorem (see Luzin 1912a; Lusin 1912b). The origins of these results lie, of course, in Göttingen. The continuation was in Paris during the years 1912–1914. Nikolai Nikolaevich, although not immediately, was remarkably blended into Borel's – Lebesgue's – Baire's circle. Excellent relationships were built with each of them. He became very close friends with A. Denjoy (1884–1973). In this milieu, he worked well. His results were immediately published in the Parisian *Comptes Rendus* (notes 1912, 1913, 1914). The kernel of his future dissertation gradually grew, which in its main outlines had developed by the summer of 1914. The odds were in his favor as he managed to freely cross Germany by train just before the outbreak of hostilities on the German-Russian front, which unfolded in August 1914.

6. Metric theory of functions. Moscow

The war, although it made serious adjustments to the life of the Russian scientific community (reduction of funds, which led to a slowdown in the release of scientific publications, as for example, *Matematicheskii Sbornik*, etc.), did not disrupt the progressive nature of its course. In Moscow, Luzin was waiting for a group of students (and what sudents (!): we can quote the names of D.E. Menshov, A.Ya. Khinchin, P.S. Aleksandrov), prepared for him by Egorov, i.e. his future students. According to Russian laws, students were released from conscription. (This also extended to persons "remaining at the university to prepare for professorship" and to privat-docents.) Luzin was able to accomplish the editing and in 1915 publish his thesis (see Luzin 1915) which was defended on April 27, 1916 having as official opponents D.F. Egorov and L.K. Lakhtin. The defense turned into a real triumph for Nikolai Nikolaevich: on Egorov's recommendation, due to the special scientific merits of the monograph, the Academic Council understood the significant context thereof and accepted to bypass the Master's degree and the thesis was accepted as a doctoral dissertation in pure mathematics.

7. Arithmetic continuum structure

In 1914–1916 saw the first generation of Luzin's disciples appear: the legendary Luzitania was born. The first disciples were the mentioned Menshov, Khinchin and Aleksandrov. Then also M.Ya. Suslin (Fig. 2), who was a genius find from among the peasants of the Saratov province.

Fig. 2: N.N. Luzin and W. Sierpinski at the apartment of prof. D.F. Egorov. Luzin is sitting, Sierpinski and Egorov are standing. Moscow 1915.

Suslin discovered the existence of a new type of sets, called A-sets or analytic sets, which made a great impression among the mathematicians. Before Suslin's paper, it was believed that in all the diverse practice the mathematicians confine themselves to the so-called B-sets, Borel sets. Suslin modified this generally accepted concept: he constructed an example of a flat Borel set, the projection of which on the line was not a Borel set[3]. The new class of sets became the main object of research

[3] There is an eyewitness testimony about how the student Suslin reported his discovery to N.N. Luzin. The outstanding Polish mathematician W. Sierpinski, who lived and worked in Moscow at the time, acted as a witness. He recounted (quoted

of the Luzin's school, which allowed the young master and his disciples to take the leading positions in descriptive set theory.

Here are a few remarks about this event in Luzin's work. Dealing with questions of the metric theory of functions (dissertation!), Luzin did not abandon his ideas regarding continuum, i.e. the continuum hypothesis. So, already in 1914 he presented a note "Sur un problème de M. Baire" in *Comptes Rendus de l'Académie des Sciences Paris* (see Lusin 1914) on which Egorov, in his review of Luzin's report on a scientific trip abroad, noted the following:

> I think that on this path N.N. Luzin will introduce something new into the fundamental problem concerning the power of the continuum (cited from Golubev, Bari 1951, p. 19).

Accomplishing his dissertation, Luzin, now a doctor of pure mathematics and an extraordinary professor at Moscow University (his title was approved in December 1916), concentrated his own efforts as well as those of his disciples on the problems of a descriptive set theory, and in particular, focused on the problem of the nature of the arithmetic continuum.

In the summer of 1915, his student Aleksandrov proved that every uncountable Borel set has the power of a continuum. This result, published in 1916 in the *Comptes Rendus* of the Paris Academy of Sciences, gave the solution to Cantor's problem for Borel sets – the very ones that were believed at that time to exhaust the supply of sets used in it. Suslin's discovery put an end to such ideas and returned the continuum hypothesis to its former attractivness. Both Luzin himself and his disciples took up the new prospects. *This is how a new period began in his creative biography,* although a difficult period started for Russia, one of revolutions and a civil war.

This epoch, hard for the Russian society, did not interrupt the life of educational institutions. New young people came to these

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from Igoshin 1996, p. 45): "I happened to witness how Suslin conveyed his remark to Luzin and handed him the manuscript of his first work. Luzin took the young student's message very seriously and confirmed that he had indeed found a mistake in the work of the famous scientist (H. Lebesgue – S.D.). I also read Suslin's manuscript immediately after Luzin and I know how Luzin helped his student and how he guided his work."

institutions, in particular those for whom just the day before enrollment at a university was a difficult task. For example, for the immigrants from Jewish townships. Despite the external unfavorable circumstances, Luzin's school amazes with its creative activity at that period.It was extremely important for its successful functioning that France was an ally in this war: the French mathematicians (they were the recognized leaders in the topic that Luzin's school was engaged in) were open to Russians in order to present their communications on the pages of French mathematical journals including the *Comptes Rendus* of the French Academy of Sciences and were now at the service of Muscovites. As soon as the possibilities for trips abroad began to open up again, Luzin and his disciples started to travel a lot and had the opportunity to visit principal European mathematical centers, including, of course, the French ones. To what extent the French mathematical community showed its concern for the problems of Muscovites appeared in its involvement in the events regarding the "case of the academician N.N. Luzin" (Dugac 2000).

Fig. 3: Mikhail Yakovlevich Suslin (1894–1919).

An important moment in Luzin's creative biography and in the history of research on the theory of sets and functions of a real variable in Moscow was his collaboration with W. Sierpinski (1882–1969) and his entourage. Let us note that their very acquaintance was a matter of chance, an accident, provoked by the events of the First World War. Its beginning found Sierpinski on the territory of Russia. As a citizen of Austria-Hungary, he was interned in Vyatka. Thanks to the intervention of Moscow University professors D.F. Egorov and B.K. Mlodzeevsky, they managed to obtain the permission to modify his place of residence and to fix him in Moscow. Here he became close friends with Luzin (Fig. 3).

Their cooperation, which continued even during the years when the relations between Poland and the USSR were almost hostile, ended only with Luzin's death. Luzin and his students actively collaborated with Polish colleagues, primarily with mathematicians from Sierpinski's circle, they corresponded with them, published in *Fundamenta Mathematicae*, and even from time to time visited each other. When the events unfolded into "the case of the academician N.N. Luzin", Sierpinski took an active part attempting to help his Moscow friend (Dugac 2000, pp. 120–126).

8. At the summit of the creative way

In 1922, Luzin returned from Ivanovo-Voznesensk, where he worked with a group of his disciples at the local Polytechnic Institute[4] since 1918, and the regular meetings of his seminar were re-established. It was in those years that the last generation of Luzitania was formed. Even then, its disintegration was already outlined[5] as was the formation of a group of students around Luzin (P.S. Novikov, L.V. Keldysh and others), with whom he continued his studies on descriptive set theory. These were the years of the highest upsurge of the *second period of his work*, marked by the 1930 Paris edition of his "Lectures on Analytic Sets and Their Applications" (with a foreword by H. Lebesgue and a note by W. Sierpinski – see Lusin 1930). This book summed up the results of the research conducted by him and his students on the theory of analytic and projective sets, and outlined a program for further work on a detailed study regarding the structure of the arithmetic continuum.

[4] The civil war that engulfed Russia after the 1917 revolutions made living conditions in Moscow unbearable. Fleeing from hunger and cold, university teachers went to the provinces, where it was much easier to survive.

[5] This "disintegration" was the beginning of a chain reaction in expanding the research topics of Luzin's disciples, which in turn laid the foundation for the future "Soviet mathematical school" with its all-encompassing nature.

Here the destiny, which seemingly still favored Luzin, turned its back on him. In 1936, the notorious "case of the academician Luzin" was launched (see Demidov, Levshin 2019). And although he came out of this "trial" with minimal losses (another manifestation of his luck in a collision with the historical vicissitudes of the twentieth century), he came out broken both morally and physically. Always unsure of himself, impressionable and suspicious, Luzin took the betrayal of his disciples especially hard. Being of weak health, he was ill a lot and died four and a half years after the end of the war.

Until the end of his days, he pondered the secrets of the arithmetic continuum, in particular the continuum hypothesis. Luzin did not consider sufficient its solution within the framework of the axiomatic set theory in the perspective outlined by Hilbert, although he recognized its importance and desirability. By the way, he foresaw the possibility of proving its independence from the axioms of the set theory long before K. Gödel's results. He did not consider sufficient the solution that could be reached (and was achieved by P. Cohen in 1963) along this path. He did not consider the situation in connection with the fifth postulate in geometry to be acceptable in the case of the arithmetic continuum, i.e. the presence of different geometries. The arithmetic continuum must be unique. This is what he said about it back in 1927 in his talk at the 1st All-Russian Congress of Mathematicians (Fig. 4):

Fig. 4: The inaugural session of the All-Russian Congress of Mathematicians. At the presidium table (from left to right): N.N. Luzin (reading), D.F. Egorov, S.N. Bernstein, V.V. Stepanov. Moscow. April 27, 1927.

> The first thing that comes to mind is that establishing the power of the continuum is a matter of a free axiom, like the axiom about parallels for geometry. But while all the other axioms of Euclid's geometry are invariable and the axiom of parallels is varied, the very meaning of the spoken or written words changes: a point, a straight line, etc. – should the meaning of these terms change if we make the power of the continuum movable on the alephic scale, all the time proving the consistency of this movement? The power of the continuum, if only to think of it as a multitude of points, constitutes a single kind of reality and it must be on the alephic scale where it is, and it is natural if defining this position is difficult or, as Hadamard would have added, even impossible for us human beings (Luzin 1958, pp. 515–516).

9. The end of the road

Slowly recovering after the events of the "case", Luzin apparently did not feel sufficiently strong to continue working at the same level on the study concerning the structure of the arithmetic continuum (Fig. 5).

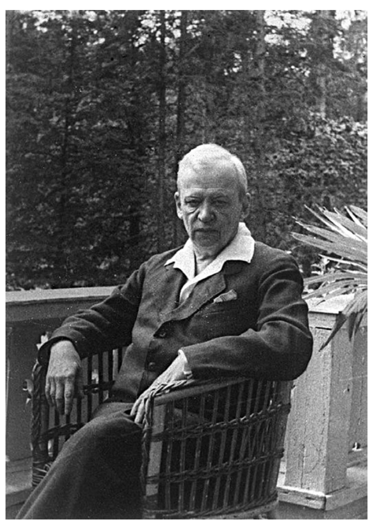

Fig. 5: N.N. Luzin in the vicinity of Moscow. The exact date remains unknown. Apparently it must be during the second half of the 1940s.

He switched to old problems, which he had pondered over in previous years, but had not brought their solution to the end – something completely different had been relevant for him at the time. So he took up the classic – from the time of K.M. Peterson – problem of deformation over a principal base and masterfully coped with it, with his results essentially covering this problem. The questions posed in his Paris lectures in 1930 were investigated and solved by the disciples or even the second generation of his students – about this; see, for example, Keldysh, Novikov 1953; Keldysh 1974; Uspenskiĭ 1985. The study of the problems arising here continues to this day exciting mathematicians – see Kanoveĭ 1985; 2008; Bogachëv 2013; Moschovakis 1980; Kanoveĭ, Liubetskiĭ 2007; 2010; 2013; Gao 2009.

The arithmetic continuum, which the already Pythagoreans began to study, continues to excite mathematicians to this day. Luzin was among them. The ideas on the continuum, awakened in him in his student years by the famous discussion about the axiom of choice, accompanied him all his life. His ideology – a version of effectivism – was formed around this topic. The most complete expression of his views on actual infinity can be found in his Paris lectures (Lusin 1930). Here he talks about the possible limits to which a mathematician, guided by the idea of actual infinity, can reach in his constructions, so that the entities introduced by him would be efficiently designed and not turn out to be virtual fictions.

Among the problems raised in these lectures is the task of reforming our ideas about the arithmetic continuum. He wrote:

> There is only a countable set of completely definable irrational numbers [...] Thus, the arithmetic continuum certainly contains indefinable points. These points, each of which has an infinite definition, are parasitic in any reasoning that can be done effectively [...] (Luzin 1958, p. 269).

So, one of the problems facing mathematics constitutes the task of “cleaning” the arithmetic continuum from parasitic solutions.

10. Instead of a conclusion

At the end of my paper, to justify its title, it is only natural to raise the question of the influence of the dramatic events of the European history of the first half of the twentieth century on the work of Nikolai

Nikolaevich Luzin. To what extent did they contribute or, on the contrary, hindered the disclosure of the creative potential of the famous mathematician?

It is clear that "our cruel age" could "offer" a wide range of possible options for such an impact on the creative path of a mathematician, who had to live and act in such a difficult time. These options were mostly negative. One does not have to look far for examples, it is enough to recall the fate of the Lvov mathematician J. Schauder (1896–1943), who died in the dungeons of the Gestapo, or the Kiev mathematician M.F. Kravchuk (1892–1942 (?)), who perished in the depths of the Gulag. Nikolai Nikolaevich was extremely lucky – the negative manifestations of the harsh century either did not harm him at all, or even served him in his favor. And even having fallen into a loud political scandal at the age of 53, he got out of it with minimal possible losses. They were even more insignificant by the standards of the harsh time: the loss of administrative positions in the academic world, a serious blow to health, which was already initially weak. Arguably, he could have done a lot to implement the scientific program outlined in the Paris lectures of 1930 (Lusin 1930) and even beyond it. However, he still managed to do something, and where he did not have the time, his students did some of the work, and everyone can get acquainted with their successes (first of all we must stress P.S. Novikov's and L.V. Keldysh's achievements, see Keldysh, Novikov 1953; Keldysh 1974; Uspenskiĭ 1985; Kanoveĭ 1985; Bogachëv 2013). Evidently, the most ambitious expectations exposed in his Paris lectures have not yet been realized: the reform of the arithmetic continuum has not yet happened. But the significance of the object itself – the arithmetic continuum – raises the stakes. Nikolai Nikolaevich still, that is 70 years after his death, remains alive in mathematics: in the students of his students, and in his ideas concerning not ephemeral concepts of mathematical thought, but its fundamental notions, such as the arithmetic continuum.

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