© Universiti Tun Hussein Onn Malaysia Publisher's Office

IJIE

Journal homepage: http://penerbit.uthm.edu.my/ojs/index.php/ijie
ISSN : 2229-838X e-ISSN : 2600-7916

The International Journal of Integrated Engineering

The Tendency of Nature towards Hexagon Shape Formation due to Minimizing Surface Energy

Ab Aziz bin Mohd Yusof[1]*, Mohd Al-Fatihhi Mohd Szali Januddi[2], Kamariah Md Isa[1], Muhamad Noor Harun[3]

[1]Faculty of Mechanical Engineering,
Universiti Teknologi MARA (UiTM) Cawangan Johor, MALAYSIA

[2]Facilities Maintenance Engineering Section, Malaysian Institute of Industrial Technology,
Universiti Kuala Lumpur, MALAYSIA

[3]Schoolof Mechanical Engineering, Faculty of Engineering,
Universiti Teknologi Malaysia, MALAYSIA

*Corresponding Author

DOI: https://doi.org/10.30880/ijie.2022.14.01.004
Received 21 September 2020; Accepted 15 February 2021; Available online 07 March 2022

Abstract: Nature, for instance, bubble and honeycomb, tend to form a hexagon shape naturally. The array of bubbles and honeycomb is formed by merging and sharing the common wall with the adjacent unit. Even though each of the unit shapes size may vary, the noticeable elements that built up the array are hexagons. There are many regular and irregular shapes available in nature, but the shape formation still leads to hexagon at the end of the shape evolving due to surface tension. Based on the phenomenon, this study was carried out to investigate the effect of surface tension, energy, and geometry features, which affect the tendency of hexagon formation. The study was carried out by comparing hexagon with triangle, trapezium, and square. From the result, it is found that the reduction of surface energy ranged from 10-23 percent from the initial shape. As expected, the hexagon shape is packed with the lowest surface parameter and very stable in single unit or array form by showing the lowest energy reduction. The energy content is a reflection to structure equilibrium and its stability for nature tendency.

Keywords: Nature, hexagon, surface tension, minimizing energy

1. Introduction

Our nature is built out with many shapes, patterns, and geometry designs, either human-made or natural. The geometries can be identified as circles, squares, stars, diamonds, and many others [1-4]. Among the shapes, hexagon abounds in nature as one of the unique shapes that packed many advantages in the design [1, 2]. The tendency of hexagon shape formation becomes more apparent in the microfluid scale due to the surface tension properties [5, 6]. One question arises of how the surface tension contributes to hexagon formation and how it affects the natural tendency shown on bubble array and honeycomb [4, 7, 8].

The spectacular shape structure forms in nature have not yet rigorously explored either using experimental or numerical procedure. It is interesting to understand the causes of surface energy's shape formation especially when it involves a thin wall such as a bubble or honeycomb. It is expected that the structure always seeks the lowest energy state for a given geometry [9]. One significant contribution from this knowledge is developing the advanced lightweight structure such as hexagon metal foam for porous structure application [10].

**Corresponding author: abaziz86@uitm.edu.my*

Generally, the shape can be classified into regular, which all the sides and inside angles are equal. In contrast, irregular does not have equal sides and internal angles [11]. Triangle is the basic form that can be found everywhere which the total of internal angle is 180 degrees and is known to be the smallest total angle compared to the other shapes. Meanwhile, the square's formation can be easily found in nature as a cube's crystalline or present in the typical salt crystal structure [12]. However, even though the possibility for trapezium to form naturally is small, but this shape is considered important as it is built up from the combination of the triangle and square shape.

Meanwhile, the hexagon is a regular shape with a six-straight-sided polygon. The tendency of hexagon form naturally is higher than other mentioned shapes and interesting to be explored [3, 13, 14]. The hexagon contains the internal angle between the adjacent side of 120 degrees [2, 3]. One of the examples is the study done by Qiancheng Zhang et al. (2015). He mentioned that the design principle of honeycomb structure-property is related to hexagon shape, and it poses higher structural integrity under different types of loading conditions, making this porous type of structure exceptional [2].

Honeycomb and bubble array are the best examples to explain hexagon shape in nature [5, 7, 15]. Both cases are heavily depending on surface tension and the interfacial energy's stability to form the final shape [5, 15]. When the magnitude of surface tension increased due to external factors such as temperature causes the material to melt [15], the material condition will significantly affect the interfacial force between the shape unit as the interfacial force is the function of the contact parameters and surface tension property of the fluid [16].

According to all the mentioned factors, this study was initiated to investigate nature’s tendency to form a hexagon shape compared to other regular or irregular shapes which the objective then extended to structure array. The initial shape revolves to achieve the minimum the surface area, consequently minimise the contained energy. Each of the exciting shapes has its features, classified as parameters of contact edges, surface area, and the internal angle between the adjacent walls. At the end of the study, a comprehensive understanding will be obtained.

2. Method

The study was conducted by considering the vital parameter at the microfluidic scale where the surface tension property is significant compared to the fluid’s weight. The model was considered as a wall film. The mathematical models for the related physics are considered before proceeding to an interactive program of Surface Evolver. The final surface structure was obtained using a simplicial triangle element. The final result after evolving the input file is energy contain, area, and final geometrical shape.

2.1 Mathematical Model

Surface tension is the physical properties of the fluid. The tension happens due to the Van Der Waals interaction between the fluid interface molecule. Let ϒ represent surface tension where the unit is dyne/cm, L is the length or parameter of the line where force act. Then the force magnitude is approximate as:

$$F = \Upsilon \, x \, L \tag{1}$$

For the homogenous fluid, surface tension’s energy is described as equation 2 [5]. The reduction of the stored energy shows the capability of minimising the area based on equation 2.

$$E = \Upsilon \, x \, A \tag{2}$$

While evolving the shape, fluid alters the area, and work is done by the surface tension force. Work is negative as the area was reduced and stored in the atom/molecule as potential energy. The reduction ΔL in a minimal amount alters the energy about dW at the specific width, w [17]. The energy from surface tension is defined as the following:

$$dW = F\,dL = 2\gamma w\,dL \quad \text{Where w x dL} = \text{Area} \tag{3}$$

For the bubble and honeycomb cases, the negative work is considered when the contact parameter's length reduces when minimising the area. Based on the equation, the final shape of the interested geometries under the specified constraints is obtained using the Newton-Raphson method, in which the vertex coordinates, edges, and surfaces iterated to reach the minimum surface energy according to the local minimum [17, 18]. To fit with the current research interest, Plateau laws’ internal form structure was used the describe the model [5]. The fluid film always meets in threes at 120 degrees, and film means curvature is constant for the whole structure.

2.2 Computational Study

The model was set as a hollow column of wall fluid film before assigning to Surface Evolver [19]. The program used to study the fluid’s geometrical shape based on surface tension and energy using an input file [17]. In the input file, the initial geometry was defined using the vertices, edges, and faces elements. The surface tension of 72 dynes/cm was assigned to the model to represent water surface tension [20]. The study simplified the material properties to water to represent the bubble and elucidated the common thing between bubble and honeycomb structure formation.

The study was carried out with a few assumptions. First, all the faces were assigned as constant tension of wall film. Second, the film is assumed so dry until the structure degrades into face structure and can be managed as a two-dimensional structure [5, 21]. The third gravitational acceleration act to the structure is not included.

This study started with an investigation of 6 types of hollow column shape. It included the regular and irregular shape of a triangle, trapezoid, square, and hexagon, as shown in Fig 1. All the shapes were set to fit inside 1.2 cm x 1.2 cm x 1.2 cm of three dimensions the surface working area by projecting the two-dimensional area upward. The surface geometry represents using triangular tessellation. This triangular tessellation was refined progressively, stage by stage to achieve an acceptable level of accuracy. The refinement process, r was started with 1 stage and combined with two stages of gradient descent step, g (r; g2). The process of (r;g2) was repeated for three times before increasing the number of stages for 1 stage of refinement, r and 15 stages of gradient descent step, g (r; g 15) and The process was ended with 1 stage of refinement, r and 20 stages of gradient descent step, g (r; g 20). The same series of refinements and gradient step also applied to the other five input files of different hollow column shapes.

The structure was modelled as in Fig. 2 to consider the effect of the Hexagon array. Each of the edges was connected to an adjacent wall. Surface tension property was assigned to each of the walls. The model allowed to move in a horizontal direction for both of top and bottom side. However, the attached wall's vertical edges deal as symmetrical due to tension that cancels each other. Thus, to simplify the model, the vertical edges then assigned symmetrical fixed boundary conditions.

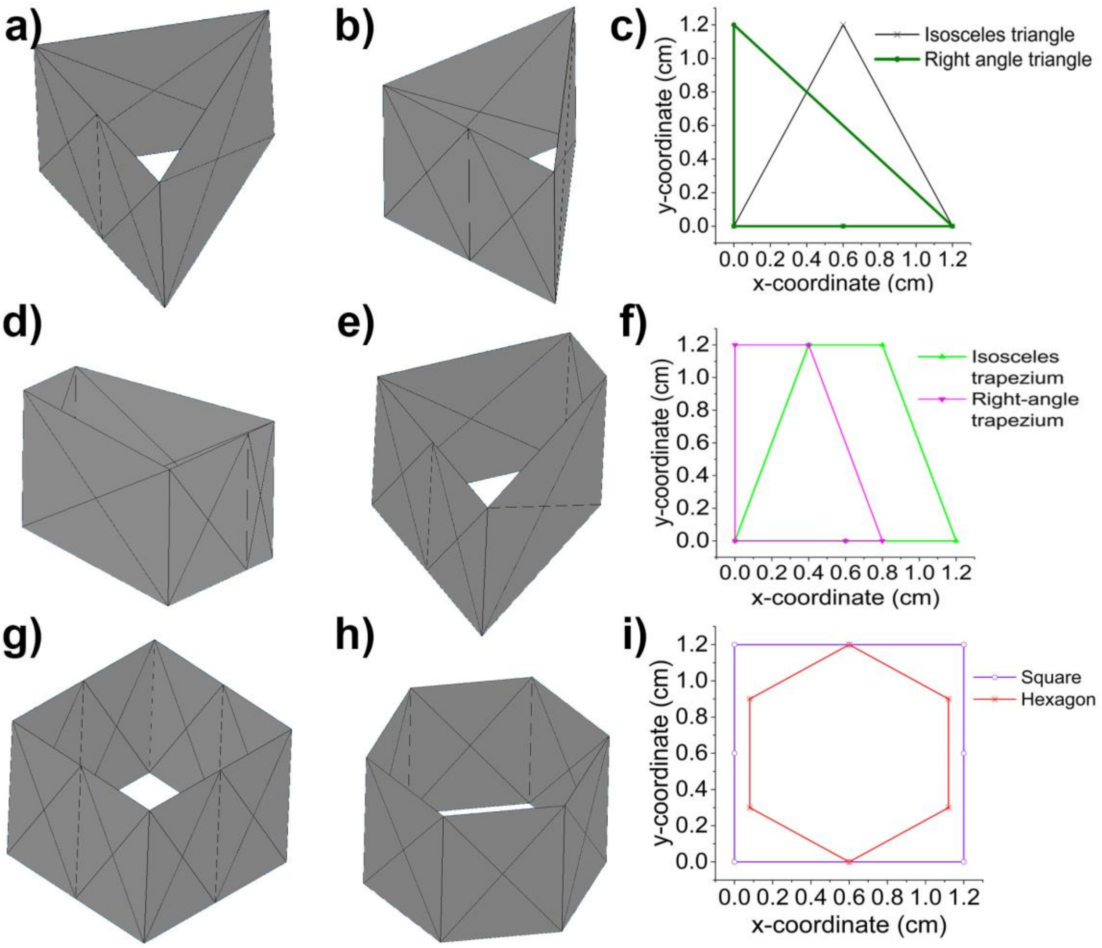

Fig. 1 - Geometry shape and its dimension used in the study a) Isosceles triangle shape; b) Right-angle triangle shape; c) Dimension of isosceles and right-angle triangle shape; d) Isosceles trapezium shape; e) Right-angle trapezium shape; f) Dimension of isosceles and right-angle trapezium shape; g) Square shape; h) Hexagon shape; i) Dimensions of Square and Hexagon shape

For comparison purposes, hexagon without attached wall faces was run firstly. The final shape and percentage of energy reduction after the model evolved was recorded. Then proceed with the hexagon array, as in Figure 2(b). It was expected that minimal energy reduction to happen as the array model is very stable.

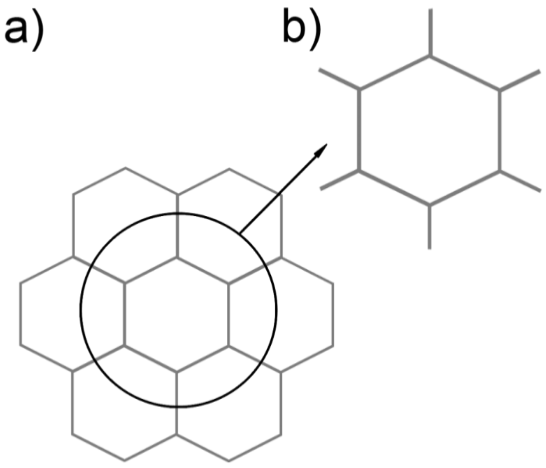

Fig. 2 - Array of hexagon resemble bubble and honeycomb shape formation (a) Hexagon attached to the neighbor unit and sharing the same wall; (b) Simplification model of hexagon array

3. Result

The initial area and interfacial energy of the column structure are shown in Fig. 3. The higher area contains higher energy in agreement with equation 2. Square is the highest surface area and interfacial energy, followed by Isosceles trapezium at 14 per cent lower than square, Right-angle triangle, 15 per cent lower than square, Isosceles triangle 20 per cent lower to the square. Meanwhile, Right angle-triangle and hexagon have almost the same area and interfacial energy, respectively, at 24 and 25 per cent lower than square.

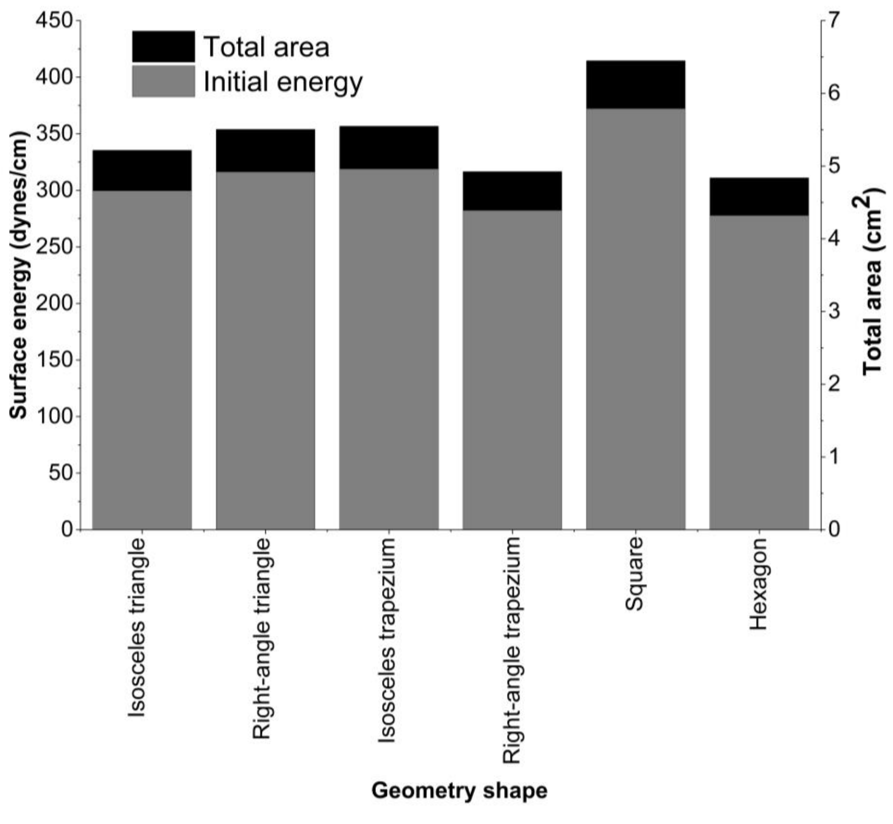

Fig. 3 - Total area of the geometry and energy contains surface tension for each geometry shape

The final form of the investigated shapes is shown in Fig.4. Due to minimising surface area to achieve the minimum surface energy and form the stable structures, all the vertical edges become curved. However, the curved edges were not happened on the horizontal sides due to fixed constrained boundary conditions.

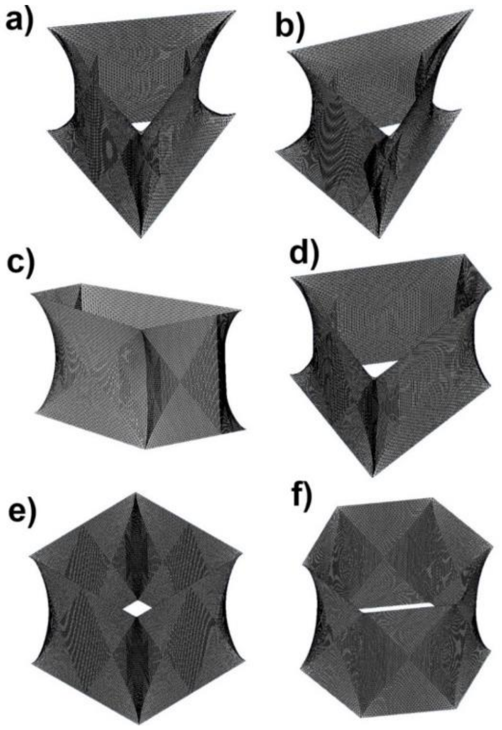

Figure 4 - Geometry shape due to surface tension a) Isosceles triangle shape; b) Right-angle triangle shape; c) Isosceles trapezium shape; d) Right-angle trapezium shape; e) Square shape; f) Hexagon shape

The percentage of energy reduction when the initial shape revolves to the final was obtained in Fig. 5. The reduction happened due to the surface tension effect. The graph shows a decreasing pattern in the percentage of energy reduction from the triangle to the hexagon. Isosceles triangle experiences a higher energy reduction compared to the Right-angle triangle and followed by Isosceles trapezium. The constant reduction was indicated from the Isosceles trapezium to the Right-angle trapezium and square. The smallest reduction happens to hexagon when compared to other shapes. The percentage of reduction ranged from 10-23 per cent.

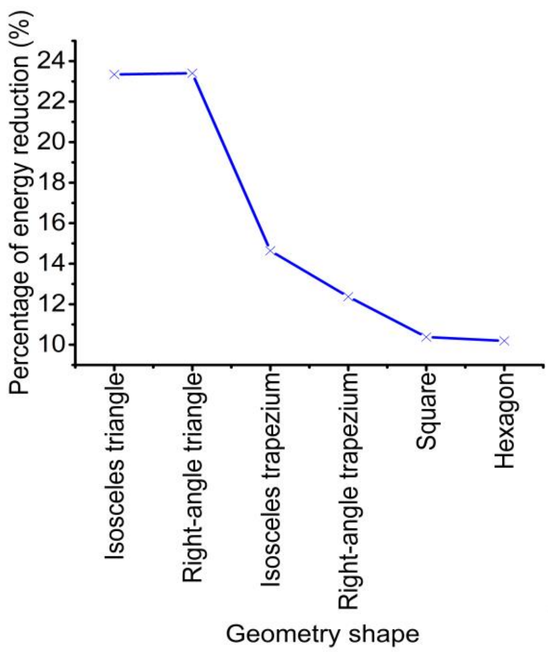

Fig. 5 - Reduction of surface energy for each of the shapes as the geometry achieved minimal surface area

The percentage of energy reduction happens on the investigated shapes mostly related to the wall's elastic behaviour due to surface tension and number of the corners. Isosceles found to be the highest reduction as one of its corners located at the middle between two other corners. Due to this condition, the structure's rigidity is lower a bit compared to right angle triangle at 0.1%. A 90-degree angle between the walls improves the rigidity by promoting less elasticity to the wall. Moving from 3 corners to 4 corners, the reduction of the energy content is increasingly noticeable. 15% reduction occurs to Isosceles trapezium as the number of the corner increases, preventing further reduction in surface area. 12% reduction can be seen to occurs for right-angle trapezium. 3% differences between both types of trapezium occur due to 90 degrees angle on two corners of the right angle trapezium. Less reduction of 10.4 % for square can be observed as all the corners are 90-degree angle, which further reduces wall elasticity and stiffer the shape. Hexagon shows that most minimum reduction occurs at 10.2%. In the aspect of the internal angle between the wall, most of the internal angle for hexagon increased bigger than 90 degrees.

The percentage of reduction can be divided into four categories. These categories are not directly related to each other as the scope of this study is to investigate the energy-contain in its final form as the minimum energy was achieved. The categories are based on the similarity of the features such as basic shape and the corner number. The first category is triangle shapes, as mentioned in the previous paragraph. The second category is a trapezium. The third category is square. The fourth category is the hexagon. Each of the categories contains specific features which then affects the minimising surface area and its energy reduction. The importance of these categories is to identify difficulty in achieving the most stable form through energy reduction, which the highest percentage of energy reduction cause instability on the shape.

After five series of surface evolution process based on refinement and gradient descent step using Surface Evolver software, the shape's initial form tried to fit its area and minimise the energy content by achieving the most stable form. The stable shape is defined as no more reduction in its energy content after a series of surface evolution processes. For the hexagon without the attached wall film as in Fig6 (a), the model minimises the energy by revolving the hexagon shape to circle shape as no the third wall resists the wall's movement due to surface tension effect. Due to the movement, the energy content reduces by 9 per cent. However, once the third wall is attached to the junction, it created 120 degrees of the adjacent wall's internal angle. Fig. 6(b) shows the revolution of the initial to final shape. Not much difference between these initial and final models either for the shape and energy content. From the result, the difference only 1.1x 10-6 per cent, which is close to not shape alteration at all due to stable shape and contained energy.

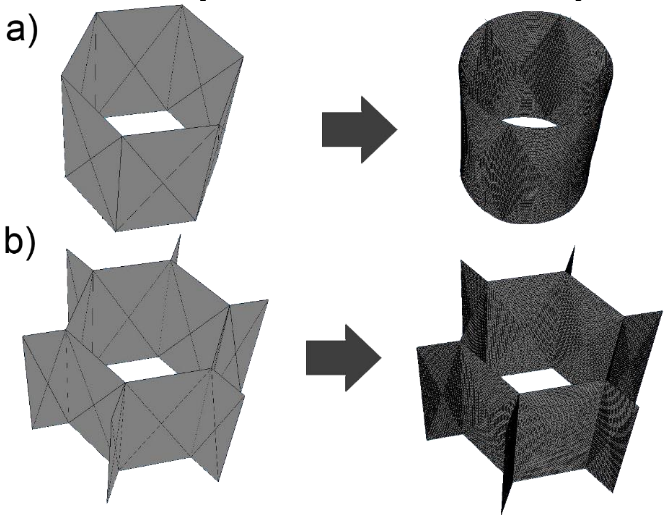

Fig. 6 - Hexagon model for the features comparison (a) Hexagon model without additional wall film attached at the junction; (b) Hexagon array model with wall film attached at the junction, which simplified the hexagon structure as in figure 2

4. Discussion

A combination of several bubbles that sharing the common wall know as a bubble raft array. The bubbles are pack close together as repeated hexagon shapes. The reason for the phenomena is going back to the packing efficiency of the single shape unit. The wall film's surface tension at an exact wall thickness tries to co-exist with the adjacent bubble by spending the minimum surface energy. Combination of the bubble re-shaping the shape and rearrange its position to fit the wall junction. It is essential to note for the bubble case, the wall film pulls its area by seeking a small area as possible to maintain the lowest total area and energy. Honeycomb structure works differently due to the wax [15, 22]. Initially, honeycomb is built by a bee as a circle structure. Over the extended period, the structure evolves as the wax then melts, reducing the degree of roundness due to increasing surface tension at the wall junction [22]. Both structures evolve by forming the equilibrium interfacial structure by providing many advantages.

Comparing to other shapes, hexagons required the least parameter and total length of the wall film than the other shapes in the same cross-section area. There are several advantages of the hexagon identified in the study over the other shapes: 1) Besides the required less energy with less material, the hexagon is the best shape to divide the space into the equal discrete region over the designated area [1], 2) hexagon shape could cover shape from the simplest to the most complex curved surface with the most minimal wastage [1], 3) Hexagon shape is close resemble the circle at higher roundness, 4) Stable and steady shape that involves the surface tension [1]. The lowest energy reduction shown in Fig. 4 proves the hexagon shape is stable among the others.

The total internal angle was increased from 180 up to 720 degrees, respectively, for the investigated shapes [4]. A smaller internal angle causes a higher pulling force on the wall. The pulling acting happens in both of the wall films, doubling the pulling magnitude compared to a more significant internal angle because the surface tension pulls parallel to the wall surface. The pulling force gradually reduces as the internal angle increase as the force component is directed to different directions. By adding more corners, increasing the structure's rigidity as the wall film creates many walls for unit shapes. The rigidity effect can elucidate the effect of three, four, and six walls, respectively for triangle, trapezium, square, and hexagon. Comparing Isosceles dan the right-angle shapes, the right angle model forms the most unstable shape due to the most massive energy reduction.

Hexagon shape packing with minimum energy and surface parameter because of the 120 degrees of internal angle between the wall. The arrangement causes the structure to pull the surface tension all over the surface from the steadiest pulling action, which cancel each other. The hexagon shape array was proved to form a stable structure with surface tension on the wall. The edges move at minimal displacement in agreement with packing stability criteria reported by A. Bezdek 1998 [5, 23]. Figure 6 shows how the formation happens, and each of the edges tried to adjust its position by forming a perfectly straight edge to achieve an equilibrium state.

Meanwhile, for honeycomb, it is reported that bees build the honeycomb as a circle with the curved edges initially. Over time the curved edge then slowly transforms to straight to minimise the area of the honeycomb. Bubble array, however slightly different than honeycomb due to the presence of air trapped pressure inside. This closed shape achieved the equilibrium by considering the surface tension that pulls the wall inside while pressure pushes it back outside. The pull-push action was then determined at the junction of the three bubbles.

5. Conclusion

Considering all the mentioned topics, the hexagon is most of the stable geometry shape compared to the four shapes used in the investigation due to the lowest the interfacial energy contains. Over time the initial shape tried to adjust its condition in terms of the area and minimise the energy. This condition happens with the help of surface tension that pulls all the edges. Hexagon is known to have more corners than other investigated shapes in which the corners increase the rigidity of the structure and help in achieving the structure equilibrium.

References

[1] T. C. Hales, "The honeycomb conjecture," *Discrete & Computational Geometry,* vol. 25, no. 1, pp. 1-22, 2001.

[2] Q. Zhang *et al.,* "Bioinspired engineering of honeycomb structure–Using nature to inspire human innovation," *Progress in Materials Science,* vol. 74, pp. 332-400, 2015.

[3] Q. Chen, N. Pugno, K. Zhao, and Z. Li, "Mechanical properties of a hollow-cylindrical-joint honeycomb," *Composite Structures,* vol. 109, pp. 68-74, 2014/03/01/ 2014.

[4] J. Duan, Y. Xiong, and D. Yang, "Melting Behavior of Phase Change Material in Honeycomb Structures with Different Geometrical Cores," *Energies,* vol. 12, no. 15, p. 2920, 2019.

[5] S. Hilgenfeldt, "Bubble geometry," *Nieuw archief voor wiskunde,* vol. 3, pp. 224-230, 2002.

[6] S. Vowell, "Microfluidics Effects of Surface Tension," *Recovered from,* 2009.

[7] F. Nazzi, "The hexagonal shape of the honeycomb cells depends on the construction behavior of bees," *Scientific reports,* vol. 6, p. 28341, 2016.

[8] J. F. Harper, D. W. Moore, and J. R. A. Pearson, "The effect of the variation of surface tension with temperature on the motion of bubbles and drops," *Journal of Fluid Mechanics,* vol. 27, no. 2, pp. 361-366, 1967.

[9] V. Cardoso, "The many shapes of spinning drops," *Physics,* vol. 1, p. 38, 2008.

[10] J. Banhart, "Manufacture, characterisation and application of cellular metals and metal foams," *Progress in Materials Science,* vol. 46, no. 6, pp. 559-632, 2001/01/01/ 2001.

[11] P. Rostami, M. Sharifi, and M. Dejam, "Shape factor for regular and irregular matrix blocks in fractured porous media," *Petroleum Science,* vol. 17, no. 1, pp. 136-152, 2020/02/01 2020.

[12] N. Shahidzadeh-Bonn, S. Rafaï, D. Bonn, and G. Wegdam, "Salt Crystallization during Evaporation: Impact of Interfacial Properties," *Langmuir,* vol. 24, no. 16, pp. 8599-8605, 2008/08/01 2008.

[13] B. Wu *et al.,* "Equiangular Hexagon ‐ Shape ‐ Controlled Synthesis of Graphene on Copper Surface," *Advanced Materials,* vol. 23, no. 31, pp. 3522-3525, 2011.

[14] H. Qing and L. Mishnaevsky, "A 3D multilevel model of damage and strength of wood: Analysis of microstructural effects," *Mechanics of Materials,* vol. 43, no. 9, pp. 487-495, 2011/09/01/ 2011.

[15] C. Pirk, H. R. Hepburn, S. Radloff, and J. Tautz, "Honeybee combs: construction through a liquid equilibrium process?," *Naturwissenschaften,* vol. 91, no. 7, pp. 350-353, 2004.
[16] C. Chao, G. Xu, and X. Fan, "Effect of surface tension, viscosity, pore geometry and pore contact angle on effective pore throat," *Chemical Engineering Science,* vol. 197, pp. 269-279, 2019/04/06/ 2019.
[17] M. Rayner, G. Trägårdh, C. Trägårdh, and P. Dejmek, "Using the Surface Evolver to model droplet formation processes in membrane emulsification," *Journal of Colloid and Interface Science,* vol. 279, no. 1, pp. 175-185, 2004/11/01/ 2004.
[18] H. Chen and C. Jin, "Competition brings out the best: modelling the frustration between curvature energy and chain stretching energy of lyotropic liquid crystals in bicontinuous cubic phases," (in eng), *Interface Focus,* vol. 7, no. 4, pp. 20160114-20160114, 2017.
[19] K. A. Brakke, "The surface evolver," *Experimental mathematics,* vol. 1, no. 2, pp. 141-165, 1992.
[20] M. C. Amiri and A. A. Dadkhah, "On reduction in the surface tension of water due to magnetic treatment," *Colloids and Surfaces A: Physicochemical and Engineering Aspects,* vol. 278, no. 1, pp. 252-255, 2006/04/20/ 2006.
[21] J. Baleta, M. Vujanović, and N. Duić, "Mathematical modelling of surface tension effects in liquid wall films," *International Journal of Innovation and Sustainable Development,* vol. 11, no. 1, pp. 85-99, 2017.
[22] B. L. Karihaloo, K. Zhang, and J. Wang, "Honeybee combs: how the circular cells transform into rounded hexagons," *Journal of the Royal Society Interface,* vol. 10, no. 86, p. 20130299, 2013.
[23] A. Bezdek, K. Bezdek, and R. Connelly, "Finite and uniform stability of sphere packings," *Discrete & Computational Geometry,* vol. 20, no. 1, pp. 111-130, 1998.