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Consciousness, time and science epistemology: an existentialist approach

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Abstract

In this work, the author presents an updated state-of-the-art study about the fundamental concept of time, integrating approaches coming from all branches of human cognitive disciplines. The author points out that there is a rational relation for the nature of time (*arché)* coming from humanistic disciplines and scientific ones, thus proposing an overall vision of it for the first time. Implications of this proposal are shown providing an existentialist approach to the meaning of "time" concept.

Keywords: Time, Science, Consciousness, Existentialism, Metaphysics†.

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Tiziano (1565 approx.)

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1. Introduction

Time, so close and prolifically used, but at the same time so diffuse for humanity its ultimate nature. Already in the fifth century, "Agustín de Hipona" expressed his feeling about the real knowledge of the nature of time in the following sentence, which remains still valid generically:

"What´s time? If no one asks me, I know indeed. If I wanted to explain it to someone asking me, I feel myself unable"

(Confessions)

The concept of time is highly important in all human cognitive disciplines, is very intricate as it involves a series of facets, each of them demanding a particular special multidisciplinary analysis. Already from the beginning of our intellectual cognitive scheme (VI century BC), time appears as the "judge" of the events in Nature. Anaximander tells us, according to the few direct testimonies of his work, something like:

"The principle (arjé) of all things is the indeterminate (ápeiron). Now, where a genesis for things happens, also destruction accomplishes, according to necessity; indeed, they pay the blame to each other and the reparation of injustice, according to the order of time" [1].

Allegory of time, by Tiziano (1490-1576). London National Gallery (cf. Wikipedia)

In fact, we could affirm that since the beginning of our structural system of scientific knowledge time appears in the concept later developed and systematized by Leibniz as an expression of cause-effect relationships, where this parameter plays a fundamental role in the explanation of nature. Already in the XIX-XX century with the advent of Relativity this relation of order in principle consistent with the human feeling is broken by Einstein´s definition of time as "imaginary" magnitude, basically to make

mathematically consistent his novel concept of space-time with Riemannian geometries, according to Minkowski introduction of the mathematical framework for Relativity in his famous 1908 Cologne lecture[‡] on "space time" [2]. This is the beginning of a major cognitive misunderstanding in the human knowledge structure with respect to time. All the XXth century and first quarter of XXIst this incomprehension has predominated the fruitless multidisciplinary dialogues about nature of time. As an example, P.Yourgrau supports Gödel's thesis of the inconsistency between the human feeling of the temporal flow and the epistemological definition of Einstein's Relativity [3], thus suggesting the disappearance of time as a cognitive entity. A statement certainly meaningless, but showing the degree of bewilderment in this matter of "imaginary" time introduced by Relativity.

In Epistemology of Physics, the problem of time is a conceptual conflict between "general relativity" and "quantum mechanics" theories. In the last, flow of time is regarded as universal and absolute, whereas general relativity regards the flow of time as malleable and relative [4]. This problem raises the question of what time really is in a physical sense and whether it is truly a real, distinct phenomenon. It also involves the related question of why time seems to flow in a single direction [5], [6], [7]. Though it is recognized, for macroscopic systems, the directionality of time is directly linked to "first principles" such as the Second law of Thermodynamics, thus Universe concerned [8]. This is the so called "Thermal Time Hypothesis".

Very recently, the author of this paper proposes a topological definition of time making naturally a one-to-one map between the mathematical ontology of time and human feeling of time [13]. What is more, he shows mathematically the univoque correspondence between "Time of relativity", "human feeling of time" and "ontology of time". Thus, the misunderstanding between science and other cognitive disciplines for this fundamental parameter is almost over.

The objective of the following sections is to present an existentialist approach to time essence, after clearly stablished the mathematical nature of it in my previous work. This approach is possible once the above definition of its ontology is integrated with the other characteristics identified to complete the overall description of "time".

2. Time idea through time: a brief[§]

Roughly speaking, we could say time is one of the dimensions of the "spatio temporale" reference system human beings use to describe reality: the dimension allowing us a comprehension of the dynamics of things. In a static world, time is a superfluous

[‡] "*Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality. . .*"

[§] For a detailed and multidisciplinary exposition on the item, [12]. Our aim here is a basic historical introduction to the problematic of the misunderstanding in XXth century concerning "time definition" recently solved by J.J. Sánchez in [13].

concept. We introduce time into our conceptual apparatus to be able to talk about changes and movements. By the way, it's also worth pointing out that time is also meaningless in an eternity framework.

As stated before, Ionic school, with Anaximander (VI BC), provides a first somewhat poetic identification for the meaning of "time": the "judge" who imposes order on natural events.

Aristotle (IV BC) begins his analysis of time by noting that there is no time without movement, without any one-to-one identification between them. Time is an aspect, a dimension of movement; enabling us to order events (according to the "before" and "after") within the movement, [14]. If time is the measure of motion, motion is in turn (for example, in clocks) the measure of time. In fact, time and movement measure each other. Newton (1642–1727) basically agrees with this hypothesis, developing the concept of "absolute time" splitting, thus differentiating, the nature of time from the measurement of it. (*Principia Mathematica*, Escolio 1st). For Kant (1744–1804), following Newton's arguments, time and space are homogeneous. But above all they are conditions "*a priori*" for all our experience and knowledge.

On the other hand, Leibniz (1646-1716) was the one introducing the idea of "order" related to the notion of time. Concerning this first appearance of the concept of causality in relation to time, the German mathematician and philosopher wrote: "I have pointed out many times that I consider space ,as well as time, as something purely relative: space describes the order of existence, and time describes the order of a sequence". In other words, "space" from the point of view of possibility describes the order of things existing simultaneously. Thus, Leibniz considered space and time in association with the changes of material objects, being time the framework for causality and space the framework for simultaneity. This last concept supposed to be absolute..... for time being. We'll come back to this. Just remarking at this point that this idea of complete order "in time" is the way for a first mathematical topological assessment of

time: Leibniz associated this order in time with the Real Line, which mathematically has a total order.

XIXth century, as a conceptual prologue to the development of modern physics, pushes the XXth to a disruptive and -in principle- tragic leap in relation to the temporal concept. Indeed, the appearance of Einstein's theories and their mathematical framework as Minkowski spaces has conceptually three major effects:

1 Disappearance of absolute simultaneity. "Absolute motion" concept is already rid away, at the beginning of the scientific method age (Galilean dynamics transformations related to cinematics).
2 Space-time interrelated (Lorentz-Einstein-Poincaré transformations).

3 Postulation of nature of time magnitude as "imaginary number", which brings as an immediate consequence -by mathematical properties of this number- the disappearance of the total order relations inside this mathematical set: origin of the cognitive disagreement between cognitive disciplines in relation to the meaning of time, due to ignorance of the basic algebraical properties of “imaginary numbers” for non-mathematicians.

A relevant aspect that arises additionally in the XXth century is the development of the justification of the irreversible temporal arrow as a consequence of the Second Fundamental Principle of Thermodynamics, established and justified by Reichenbach [15]. Without going into deep detail, to my view, the fundamental contribution of this great work is the reasoned justification of the irreversibility of time due to fundamental laws of maximum entropy, explicitly pointing out the

difference between mathematical process description and its observational real feasibility, thus excluding mechanical considerations in any justification of temporal orientability.

Paraphrasing Reichenbach "For whatever process observed, a reverse mathematical description is always possible, but an inverse process may be unfeasible". In this way, it is worth saying that from the microscopical point of view (*statistical mechanics)*, time definition coming from the non commutativity of the quantum microscopical states possible for a given macroscopic configuration provides identical result for macroscopic entropy-based time definition initially proposed by Carnot and Boltzmann from the late XIXth century. This is the thermodynamic approach to irreversibility of time.

Connes et al. [16] showed that elemental quantum transitions in particles forming a macroscopic state induce a natural self-order among them. Thus, coming from the non commutative mathematical characteristics of their quantum description, a natural flux of time naturally appears. The mathematical foundation detail is out of scope of this article, just pointing out it comes from “Tomita-Takesaki theorem” application in the “Von Neumann noncommutative algebra” set of the physical variables in a system [17]. In other words, this contribution shows that physical time has a thermodynamical origin (thermal state of the system), thus firmly demonstrating the irreversibility feature for the time-flow due to quantum considerations in the microscopic possible states for a certain macroscopic system.

Finally, C. Rovelli shows us in [18] the rational, and therefore objective, properties that completely determine the nature of time:

1 Concept of simultaneity is not absolute; at most, it has local validity.
2 Flow of time is asymmetrical for thermodynamic reasons: Reality runs from the past (determined) to the future (indeterminate) through the "now".

3 Measurement of evolution of time is relative, being the relation of the measurement between two different observers provided by the so called “Lorentz-Einstein-Poincaré transformations”.
4 Quantum features previously pointed out have an immediate additional consequence intrinsic to the nature of the theory: time does not run continuously, but at discrete intervals whose magnitude is within the order of the so-called *"Planck time"*.
5 The same author acknowledges that the similarity between the subjective time of human experience and the objective nature of the characteristics of time indicated remains unresolved. Recently, the author of this work identified and proposed a rational definition of the nature of time [13], stablishing the relationship between both cognitive levels –the scientific and the empirical from human experience; thus proposing to culminate the human misunderstanding in relation to time definition. Let's proceed to summarize the proposal and add some self comments below.

3. The conceptual unification for the rational definition of time as a mathematical manifold parameterized by the subjective human experience of time flow

This author –in a former work about the subject [9] - using a topological construction has initially proposed the following definition for the nature of time (or essence):

“Ontology of magnitude of time is represented as a 1-dimensional manifold (with boundary) in the $\mathbb{R}^2$ plane (or equivalent $\mathbb{C}$ body set), oriented and embedded with respect to natural human parameterization".

Briefing his argument developed, the key points justifying the definition rely on:

1 Distinguish between nature of time and natural parameterization of it as a mathematical -object-curve- due to human feeling of time flow, thus ordering completely the parameter set. Within this parameter set, mathematically named as covering space, measurements of time flow (or increments of time) occur.
2 Compactification of the above covering space using Alexandroff Theorem [19] and subsequent identification of the quotient topological space under the following equivalence relation: "nothing can be said of either at the beginning of Universe or at the end".
3 Identification of the equivalence class set induced by the above equivalence relation and proposal of one canonical element, as S^1 variety. Formally, one can point out the consistency with the fundamental topological *theorem of classification for 1-d manifolds* [20].

$$S^1(s): I \to \exp\left(i\frac{2\pi}{a}s\right) \subset \mathbb{C}, I \subset \mathbb{R}, interval\ [0, a)$$

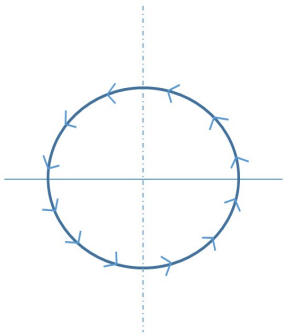

Fig.1: Im(S^1)⊂$\mathbb{C}$ geometrical representation, as defined as 1-d oriented variety.

The author points out that this definition is not equivalent as the cyclic S^1 circumference. One topological argument -not unique- behind: covering space of circumference is the $\mathbb{R}$, while covering space for the "time manifold" proposed is [0,a) in the quotient topology described. This difference has induced major misunderstandings in science. As an example, and without pretending to be exhaustive, Nielsen et al. [8] had already rejected 1-d manifold as definition of time confusing the manifold with circumference. In addition, the authors show a mathematical inconsistence in the arguments provided by its definition of time as the whole set $\mathbb{R}$ defined as (-∞, 0) ∪ [0, ∞). Topologically, this argumentation is redundant since once stablished an oriented parameterization the above indicated subsets are equivalent. We agree in their second conclusion about Second Law of Thermodynamics with a more simple (and realistic) argument: the natural feeling for humans of flow time justifying the "Thermal time hypothesis". Thus, we demonstrate time and its flow is unique (up to homeomorphisms); in consequence, their allusion about the lack of anthropic principle in Cosmology is unfounded.

Main implications and consequences achieved and derived from the time definition are:

1 Consistent with all cosmological theories where there is a final for the Universe either in a finite or infinite steps. In any case, there is a mathematically equivalence of time as a physical fundamental magnitude under an identified quotient metric space, subset of the topological space ($\mathbb{C}$, T_u), where "T_u" is the Euclidean typical topology induced by Euclidean distances. Being $\mathbb{C}$ isomorphic to $\mathbb{R}^2$ as Euclidean metric spaces. It is worth pointing out that even Conformal Cyclic Cosmology approach is compatible with this time definition [21],[22],[23].

2 The definition is unequivocally unique, taken into account the equivalency between the identified homeomorphisms, and the subsequence quotient space identified. The canonical element of this second quotient space identified provides us with the mathematical model of the magnitude "time" valid for all epistemological approaches in Science in general. And what is more, it proposes the formal relationship between the human feeling of ordered time-flow and its objective (thus, scientific) essence. Topologically, supported by the *theorem of classification for 1-d manifolds*.

3 The thermal time hypothesis is supported by the human feeling of flow time parametrization, thus completely ordering events in the covering space set of parameterization of "time curve" (the interval $I \subset \mathbb{R}$). In principle, there is no a preferred sense for time flow but thermodynamics considerations briefed by C. Rovelli indicates that once the sense chosen, this is irreversible [16], [18].

4. Time and consciousness: the existentialist cause-effect relation

Our purpose here to assess the question why, how and when our human feeling of flow time runs parallel to 2nd Law of Thermodynamics (increased Entropy, $\Delta S>0$). Rovelli [18] briefs a deep analysis about this, establishing a consistent link between neuroscience and thermodynamics to propose a consistent answer: human feeling time is a consequence of our perception of existence, through the foundation of our self identity. Three steps for this:

1 Identification of our being with a point of view about the world, from the individual interpretation –perception- of the information collected in its interaction with "what exists". To this author's opinion, not restricted to purely empirical experiences.

2 Classification of our aforementioned perception into disjoint classes of entities physiologically reflected in dynamic and flexible neural network models [24]. According to this, our "concepts" could be formally expressed as a neural "steady-state" fixed point induced by recurrent structures resulting from the information processing.

3 The memory: matching perception with the above stored "concepts", we can extrapolate potential events arriving with our experience, thus providing a series of possible scenarios allowing us to prepare our behavior for the best adaptation of our being. Two immediate consequences:

a) Flow time appears as a consequence of the extrapolation: a monotone and continuous link from the present to the future thanks to my past experience for my optimal adaptation to the environment based on individual decision-making. It is straightforward to conclude this complete ordered set is aligned with Entropy Law, since in future always $\Delta S>0$.

b) Existence as a final result of a continuous process: self identification/information assessment and classification/ evolutive decision making process.

For details about dynamics -non linear- of neural networks and memory processes in brain as an evolutionary capability of human beings [25], [26].

Summarizing, the topological nature of time is completed by an existentialist relation with a self-determination of our identity process, providing the parameterization of the recover space of the "time manifold": the flow time. Mathematically, in S^1 there is no *a priori* preference for the sense of the loop, but once formally established, it remains, by the above considerations.

5. Epistemological implications in Cosmology: towards a revision of the space-time manifold for Universe?

The above definition of nature of time leads us to check implications in mathematical structure of Cosmos as 4d manifold: the so called "space time" from Einstein's Relativity. Effectively, from Hawking and Ellis [27], and even a little before Penrose [21], [22], [28], it is generally accepted that the Universe is defined mathematically as a 4d manifold, with a Lorentzian metric and an "associated affine connection". Starting from here, Let's take into account the algebraic axiom – the so called "classification axiom"- to define a class or set, as a collection of mathematical objects satisfying: "a" belongs to the class {x:F(x)} if and only if F(a) is true and besides, a is a set", accordingly to Von Neumann–Bernays–Gödel set theory (NBG). It is straightforward that both "space"-like objects and "time"-like ones -the ones coming from the proposed manifold- form the class (or set) "Universe". To check the consistency of the argument, it is straightforward to see that the rest of axioms of N-B-G theory for class/set theory are fulfilled [29].

At this point, it is worth a comment on one of the most famous configuration models from Universe: "the "Gödel universe" [30], exact solution of the Hilbert-Einstein field equations for gravity. It defines two subspaces of the way: {x,t,y} and {z}. The metric of the solution is given in the form

$$ds^2 = dt^2 + dx^2 - \frac{1}{2}\exp\left(2\sqrt{2}\omega x\right)dy^2 + dz^2 - 2\exp\left(\sqrt{2}\omega x\right)dtdx$$

where $\omega > 0$ is a constant related to the Eintein's cosmological constant Λ.

That metric is directly the sum of 2 transitive actions on its manifold: first action in *(t, x, y)* and the second in the *z*-coordinate. Thus, presenting temporal vortices leading to time closed-loops that physically (for thermodynamics considerations) are meaningless, as he immediately recognized, but from more formal topological arguments. Even tough, this author points out taking also into account NBG axiomatic for sets, Gödel's sets identified as {t,x,y} and {z} are inconsistent. Effectively, if "time" is considered as a mathematical object satisfying a certain definition, be F(t), the {x,y,z} must belong to a

different set, whose F{x,y,z} is a different predicate defining another disjoint primitive concept. The question then arising is to formally identify the consistent topology of Space time manifold as union of "time set" and "space set" using the IV axiom of NBG theory, having identified the topology of "time set". This author's proposal relies on the space product topology with the conditions provided for the Lorentz-Einstein Poincaré relations, semiorthogonal group $O(3,1)$, for Restricted Relativity between different observers. Be $A \in O(3,1)$, a certain automorphism between two reference systems for Universe manifold M, moving relatively at a uniform speed $\boldsymbol{u}$.

According to Relativity, the metrics associated to each reference system is invariant under the action of A:

$$ds^2 = g_{ij} dx^i \, dx^j \quad i,j = 1, \ldots .4.$$

The above invariance provides a metrics dependence between both sets (-space and time-) for a certain observer; once these metrics are identified for each set, the related metric topology follows immediately, as the product of each topological (sub)space. So, formally we are in a position to conjecture that topologically space-time manifold, let $\mathcal{M}$, can be expressed as:

$$\mathcal{M} \sim (Y, T_d) \times S^1.$$

Being (Y, T_d) the topological set for "space set", induced by the metrics chosen in the set. S^1 is the canonical element for the oriented 1d manifold defining "nature of time" proposed in this work.

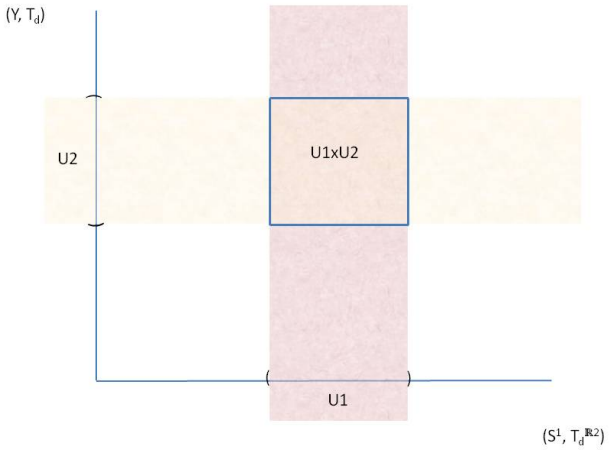

Fig.2: Conceptual schema showing the topological construction of topology proposed of "space time" 4 d manifold as a product space. U_1,U_2 are open sets in the respective original topological spaces.

An additional hypothesis can be added: if *(Y, T)* is simply connected and compact, Perelmann in 2002 [31] demonstrated the Poincare´s conjecture, thus allowing us to propose the following conjecture:

$$\mathcal{M} \sim S^3 \times S^1, \textit{ being unique (up to isomorphisms).}$$

The mathematical expression of Universe is unique. Thus, conception of Universe is unique.

The 3-sphere, S^3, centered on the origin and with radius 1 is called 3-unit sphere or 3-sphere unit. It can be described as a subset of $\mathbb{R}^4$ as expressed above, or $\mathbb{C}^2$, or $\mathbb{H}$ (quaternion) [32][**].

$$S^3 = \{q \in \mathbb{H}, ||q|| = 1\}$$

Quaternions are generally represented in the form:

$$q = a + b\boldsymbol{e_1} + c\boldsymbol{e_2} + d\boldsymbol{e_3}$$

Where "*a*", "*b*", "*c*", and "*d*" are real numbers, and $\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}$ are the fundamental *quaternion units*.

* (multiplication)	1	$\boldsymbol{e_1}$	$\boldsymbol{e_2}$	$\boldsymbol{e_3}$
1	1	$\boldsymbol{e_1}$	$\boldsymbol{e_2}$	$\boldsymbol{e_3}$
$\boldsymbol{e_1}$	$\boldsymbol{e_1}$	-1	$\boldsymbol{e_3}$	$-\boldsymbol{e_2}$
$\boldsymbol{e_2}$	$\boldsymbol{e_2}$	$-\boldsymbol{e_3}$	-1	$\boldsymbol{e_1}$
$\boldsymbol{e_3}$	$\boldsymbol{e_3}$	$\boldsymbol{e_2}$	$-\boldsymbol{e_1}$	-1

Table 1: Caley's table for quaternion base elements for multiplication, showing the non commutative characteristic.

The 3-sphere is the simplest mathematical space belonging topologically to the so called family "homological spheres". J.P.Laminet has claimed in [33] that 2003-2006 WMAP data for the cosmic microwave background radiation reveals that the shape of the Universe may geometrically fit with sets belonging to these "spheres

[**] In this paper, the quaternion is inserted as an hypothesis. Here, our contribution shows that quaternion is the natural way to describe mathematically physical Universe as the simplest homological sphere, as a 3d manifold.

topological families". Further study is in progress to assess implications of this affirmation, including geometrical assessments about finite intrinsic positively curved sets in this type of 3-d manifolds.

Summarizing, our definition of nature of time leads to an unique mathematical conception of Cosmos (up to isomorphism) whose manifold can be computed and viewed globally, as opposed to most of approximations in this topic are developed up to now [34].

6. Conclusions

This work has shown a mathematical assessment, by topological analysis, of the ontological definition of the fundamental magnitude of time justifying an existentialist source of it. Some implications in Science epistemology also provided:

1 The idea of time is a subjective idea; thus a consequence related to the awareness of our existence. Without this awareness, the Universe is, as Rovelli affirms, a succession of events. The surprising thing in Science Epistemology is capable to objectivize such subjective perception through measurement, which can be performed without preferring any reference system –Relativity- and all these measures are related by the Lorentz-Einstein-Poincaré transformations.

2 Measurement of time-flow is naturally done in the topological universal recovering space $\mathbb{R}$, different to the topological space S^1, the nature of time. This confusion between recovering set and nature of time itself has been the major misunderstanding in time concept definition throughout the past XXth century,

3 since the appearance of Einstein's theories (Relativity), and now clarified by this author's contributions. Direct consequence of this definition is irreversibility of time and no "time travels" possible.

4 The flow of time has an intrinsic arrow running parallel to the fundamental law of Thermodynamics of increased entropy justified by neuronal networks dynamics in our brain (memory). This could be a complementary description from science of Heidegger existentialist theories about "Dasein" and human existence description.

5 As a final consequence, Universe mathematical description as a 4d manifold is conjecturized, offering an unique possible Universe set: no time travels nor multi universe concepts consistent with this existentialist approach to time definition is evidenced.

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