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A certification method for the milling process of free-form surfaces using a test part

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It is generally admitted that the manufacturing of free-form surfaces requires the use of a CAD-CAM system. The toolpath accuracy and the dimensional quality of the final shape have to be in accordance with the geometrical specifications. But most of the time, the final parts present deviations from the expected shape. These deviations may be due to either the toolpath calculation (CAM system) or the cutting process itself. In the paper, we propose an analysis of the whole milling process to point out the possible sources of errors. These errors generally lead to geometrical deviations and the final part does not meet the required specifications. As the errors can be linked to geometrical particularities of the shape, we propose a test part associated with check means to bring out problems. The milling of this part using two different techniques of toolpath generation shows that obviously both toolpaths are not error-free and that errors result from different geometrical particularities of the part surfaces.

1. Introduction

Nowadays, high speed milling allows the realization of parts with no rectification thanks to the high quality of metal cutting (Konig, 1992), (Paro J et al, 1995). The envelope of the toolpath essentially defines the final part shape. Thus, the toolpath has to be as accurate as possible since no polishing is made on the part.

The precision of the whole machining process has to lead to a low roughness, to a nice aspect of the part and of course, to the respect of geometrical specifications (form deviation, roughness, ...). The part dimensions are otherwise given by the geometrical model. But as the final quality is linked to the precision of all the steps of the machining process, the control of each of them yields the global control. As a result, the toolpath generation has to be good enough to ensure the competitiveness of the process.

An analysis of the milling process shows the different areas where problems may appear.

The milling process can be divided as follows (figure 1) :

- geometrical description,
- surface modeling with a CAD system,
- toolpath calculation with a CAM system,
- program translation to be understood by the numerical controller via post-processor,
- milling of the part on a machine tool.

The model resulting from the surface description by the CAD system is taken as the reference. As we are only interested in the three-axes milling with a hemispherical tool, the program translation via a post-processor is considered as perfect.

Thus, the precision of the milling process results from the accuracy of the toolpath generation and the cutting of the part on a machine tool.

In the paper, we propose a test part that allows to evaluate the accuracy of the calculated toolpath and that of the cutting process separately.

figure 1 : the milling process of a free-form surface

We essentially present the approach that leads us to elaborate the test part. A list of the different sources of errors or inaccuracies is established, separating errors coming from unexpected behaviors of the toolpath generation with a CAM system from those coming from the machining process, e.g., C^1 discontinuities on the part may provide errors in the toolpath calculation. To each kind of errors corresponds one or more surfaces on the test part. In other words, each surface of the test part is elaborated so as to make either the toolpath calculation or the part cutting potentially fail.

As our objective is to control the milling of various kinds of parts on the basis of the milling of the test part, checking means are set up that allow to evaluate the errors or the inaccuracies. In particular, we have based our approach on the evaluation of the geometrical deviation between the machined surface and the nominal surface at each point. This allows to verify if the machined surface meets the main used geometrical specification for sculptured surfaces : the form deviation.

2. The sources of error in the global process

The aim of manufacturing with a CAD system is to produce a part that meets the required specifications. The most relevant tolerance specification for sculptured surface is the form deviation (profile tolerance) (Austin et al, 1997). The part is said within tolerance if the machined surface is inside two envelope surfaces given by sweeping a sphere of radius t along the nominal surface (ISO 1101, 1983) (figure 2). In addition to the form specification, one can specify the roughness of the finished surface too.

figure 2 : tolerance of form deviation

Commonly, in 3-axis milling with a ball-end cutter tool, the calculated toolpath with a CAM system is a polyhedral line which represents the centers or the extremities of the hemispherical tool. Obviously this sampling of the shape induces dimensional deviations between the final shape and its geometrical representation. But, as we will see next, only a few toolpath generation techniques rely on the geometrical specifications.

2.1. CAM system errors

Each calculation technique uses its own approximation criteria to generate the toolpath. This involves different calculated toolpaths for a unique CAD model and hence various machined shapes. Moreover, the discretization techniques may lead to a final shape that does not meet the required specifications. Calculation techniques rely on the evaluation of the successive positions of the tool which are tangent to the original surface. At each point of the surface, the tangent location of the tool verifies (figure 3):

$$\overrightarrow{CL} = \overrightarrow{CP} + R\,\overrightarrow{n} \tag{1}$$

where CP is the contact point, CL the cutter location and $\overrightarrow{n}$ the local normal. The whole toolpath is then defined by the set of the CL points for which two successive CL points are linked together by a straight line.

figure 3 : definition of the cutter location, *CL data*

To achieve this calculation, authors first evaluate the offset surface of the original surface with the tool radius for offset value. The offset surface may be determined in an explicit manner or an implicit. For that last case, the problem is solved for a discrete number of points.

Classically, toolpath generation techniques can be divided into four main types.

The first one consists in generating an offset of the part whose sampling gives the toolpath (Kim K.I, Kim K, 1995), (Kim C.B et al, 1995), (Lai J.Y, Wang D.J, 1994), (Tang K et al, 1995).

If $\vec{Q}(u,v)$ is the surface equation, the offset is given by :

$$\vec{Q}^{off}(u,v) = \vec{Q}(u,v) + R\ \vec{n}(u,v) \tag{2}$$

where R is the tool radius and $\vec{n}(u,v)$ the normal to the surface at each point.

In that case, the main difficulty is that the offset surface is completely defined only if the normal is known at each point. We can notice different sources of errors presented in figure 4 :

- when the tool radius is locally greater than the curvature radius of the surface, self intersections generally appear,

- shapes such as "butterfly" provide an inversion of the normal that makes the calculations fail,

- C^1 discontinuities on the surface, for example sharp edges, lead to C0 disontinuities on the offset surface.

figure 4 : geometrical singularities for the normal evaluation

Such calculation techniques must be combined with surface analysis to define areas that require corrections but this is not yet completely achieved.

The second consists in determining the tool contact positions and in deducting their center locations (Bobrow J.E, 1985), (Choi B.K, Jun C.S, 1989). In this case, the obtention of the offset surface is implicit. The problem is defined by (1) and has to be solved after the discretization of the original surface in contact points. Of course, a wrong sampling of the tool contact locations may lead to the risk of missing out small dimension surfaces. In addition to the errors linked to a bad definition of the normal at each point, a possible source of errors is the interferences between the tool and the surface (figure 5).

figure 5 : interference tool-surface

The third type consists in machining a polyhedral approximation of the surface (Hwang J.S, 1992). The determination of the cutter locations is then easier for realized from elementary facets. The precision of the toolpath generation is given by the sampling step of the polyhedral surface and if the step is too large small details may be missed out (figure 6).

figure 6 : offset evaluation from polyhedral approximation

The last technique is the determination of the toolpath with the method of the inverse offset (Suzuki H et al, 1991). The main interest of such a technique is that the offset calculation is very simple and avoids undercuts since the calculation takes the orientation of the tool axis into account, as well as the whole working part of the tool. The main difficulty is for the milling of vertical walls that are parallel to the tool axis.

Different approaches can be taken to specify the range of the authorized deviations. Some CAM systems allow the machined surface to be included between two offsets of the model shape. More commonly, others allow the specification of criteria to determine the pass computation. These criteria may be the chordal deviation in the driving tool direction and the path interval in the perpendicular direction (Kim K.I, Kim K, 1995). Generally, to compute the distance between two CL data in the driving direction, authors locally approximate the surface by a constant curvature surface, most often a sphere (Choi B.K et al, 1988). This assumption is widely violated when a sharp edge is treated. The path interval is generally used to compute the discretization step between two successive paths. Lin and Koren (1996) present the models used by most of the authors. Most of the time, the path interval is calculated from the maximum scallop height allowed. Works rely on the following assumptions : tool locations are tangent to the surface ; the surface is continuous and can be approximated by a circle in a perpendicular plane to the driving direction. These hypotheses are not always verified and, due to the chordal deviation, the actual scallop height is greater than the authorized one (Tournier 1996).

The toolpath will be considered as error-free if the whole set of CL data allows the respect of the geometrical specifications. At the end of this analysis, we can conclude that techniques of toolpath generation may introduce errors due to the different approximations that have to be done; such errors are generally not easy to evaluate or to correct. Moreover, discretization parameters are rarely linked to the geometrical specifications. As a result the final surface may not meet the required specifications. Austin et al propose a discretization technique that takes the geometrical specification into account : the edge length of the triangular facets is calculated so as to comply with the profile tolerance.

2.2. Errors of the machining process

The machining process is also a source of the dimensional difference between the final shape and its geometrical definition. The errors may come from the treatment by the numerical controller, the rigidity of the milling machine and the cutting process (Makino H, 1988), (Weck M, Ye G, 1990). High speed milling requires high speed cutting and high feed-rates, thus a high speed treatment of the toolpath.

If the cutting process is not the same on the whole shape, it may induce differences of roughness on the finished surface. It is the case with a three-axes milling because the orientations of the tool axis relative to the shape surface involve a non-constant cutting process.

An abrupt variation of the feed-rate causes marks on the shape due to the inertia of the machine tool. This variation of the feed-rate comes from the numerical controller for different reasons:

- the transmission speed of the data is less than the treatment speed, so the machine slows down while the numerical controller is waiting for information (Schultz H, Moriwaki T, 1992), (Yeung M.K, Walton D.J, 1994),
- the milling time of an elementary toolpath is smaller than that allowed by the numerical controller, so the machine slows down (Yeung M.K, Walton D.J, 1994),
- the rapid variation of the normal orientation on the shape requires changes of the tool movement, hence some accelerations on the different axes.

Unfortunately, these kinds of deviations can only be prevented by a look ahead function of the numerical controller, and we can only notice their effect at the end of the cutting process.

3. The test part

For an efficient checking of the process, the test part must be representative of the CAM problems and the machining process problems. The main problems are, as seen previously, the treatment of the small curvature radius, the normal singularities on the shape, the length of an elementary toolpath, the rapid evolution of the normal orientation and the changes in the process cutting. We have to consider the details with small dimensions and the undercut shapes too.

These different sources of deviations can be shown using surfaces which present geometrical particularities. They are classified in seven types:

1 - small curvature radius on concave surface,

2 - geometrical discontinuities of the surfaces: gap, hole and overlapping,

3 - singularities in the evaluation of the normal : undefined normal, normal inversion,

4 - undercut shape,

5 - surface with great curvature radius,

6 - small dimensions,

7 - sharp edge between surfaces in convex relation and vertex.

A solution to analyze the machining process is to get a geometrically defined shape which contains all the necessary elements to bring out the behavior of the whole process. Thus, the test part has to be geometrically defined and easy to check. As the existing test parts do not take all of these specifications into account, we propose one that considers these issues (Thiebaut F, 1995).

The part is mostly composed of canonical but representative surfaces in order to minimize the potential deviations of the CAD system (assumed to be perfect in our work). Of course, the surfaces could be modelled by Nurbs surfaces or other models without limiting the proposed approach. As far as possible, the choice of the surfaces is realized to show only one of the problems for each surface. For most cases, the size of the geometrical characteristics is defined relatively to the used tool radius (R=5 mm). A representation of the resulting part is given with the recapitulation of the different surfaces that compose it figure 7.

figure 7: the test part

Type 1 particularities are analysed by the means of three different kinds of *grooves* in order to essentially determine the behavior of the toolpath generator. The *groove* located near the details presents a radius that is smaller than the tool radius and allows to know if some interferences occur between the shape and the tool. The width of the second is exactly equal to the diameter of the tool, so the tool is tangent on both sides. The evolutional width of the third one can provide the minimum required curvature radius of a machinable shape (figure 8).

figure 8: a specifical shape, *the grooves*

Type 2 singularities are directly brought out with *the crossing faces*, *the hole*, *the gap and the overlapping*. These discontinuities are generated with planar surfaces in order to minimize the other deviations that may occur during the machining of biparametric surfaces.

Type 3 particularities require surfaces whose the normal is not defined, inversed or not continuous. We take these mathematical degeneration into account on triangular patches where the normal is not defined, on one vertex, by the use of *the tetrahedron*. *The "butterfly" surface* is a patch defined by two parametric segments whose parametric orientations are opposed. The result is a surface presenting an inversion of its normal. The last differentiation concerns the curvature continuity between linking surfaces. The most representative associated surfaces are the sequences from *plane with torus and torus with sphere.*

Type 4 particularity is analyzed by the means of *the undercut shape*.

The smooth surface combines all the type 5 particularities. The association of two revolution surfaces with a ruled surface, allows to know the differences of the behavior of both the toolpath generator and the cutting process in function of the curvature and the concavity.

Type 6 particularities are created with cuboids of small dimensions. These cuboids are located either on vertical or horizontal faces, thus it is possible to know if *details* are either missed by the extremity of the tool or by its side.

The main objective of the type 7 surfaces is to analyze the process cutting and more specifically the behavior of the numerical controller. Indeed, the easiest way to determine the pursuit deviation (characteristic of the numerical controller) is to produce it between planar surfaces: *the right cylinder and the parallelepiped.* For such surfaces, we expect the CAM system to generate a toolpath without deviation. Thus, if there are deviations on the machined surface, they entirely result from the pursuit deviation of the numerical controller.

The main characteristics of the test part are now defined. The surfaces can also be used to study the influence of the tool axis orientation relative to the surface on the quality of the cutting process, for example: *the right cylinder* presents some associated surfaces with the same orientation relative to the tool axis but that are milled "upwards" and "downwards". Moreover, an analysis can be made in five-axes milling to determine the capability of the toolpath generator to propose a toolpath with no useless rotation of the spindle or to know the influence of a low and permanent rotation of the spindle on the final part.

Let us now see the checking means that are associated with this test part and allow to evaluate the different kinds of deviations.

4. Checking means to assess the manufacturing process

4.1. CAM system

The checking of the toolpath is based on the fact that it is known and on the knowledge of its geometrical shape. The existing methods which allow to compare the geometrical part and the toolpath are either approximation methods that use a sampling surface: the Z-buffer method (Kim C.B et al, 1995), the point vector method (Chappel I.T, 1983), (Jerard R.B et al, 1989a), (Jerard R.B et al, 1989b) and the visual method (Suzuki H et al, 1991) or exact methods : CSG representation of the toolpath (Jerard R.B et al, 1989a), (Hanada T et al, 1994). This method is interesting but the computations lead to great model sizes, and this approach is only used to analyze local deviations.

In the paper, the assessment of the manufacturing process is realized relative to the form deviation : the machined surface has to lie inside the two envelope surfaces defined by sweeping a sphere of radius t along the nominal surface. The machined surface is only known through a set of points and the assessment is realized from the evaluation of the geometrical deviation at each point, which is the distance between a point on the machined surface and the nominal one.

Relative to the CAM system and the toolpath calculation, the geometrical deviation is calculated in two steps. First the point on the machined surface is determined by the use of the Z-buffer method. From a point on a cartesian grid defined on the nominal surface, the machined point is determined by the intersection between the vertical line and the envelope toolpath (figure 9).

figure 9: calculation of the geometrical deviation

For an elementary toolpath (straight line between two CL data), the enveloppe toolpath is materialised by a cylinder limited by two half-spheres. The intersection gives the Z_M value of the machined point for which (X_M, Y_M) are defined by the grid. M belongs to the vertical line and M belongs to the cylinder, so M verifies :

$$\frac{\left|\overrightarrow{C_m M} \wedge \overrightarrow{C_M M}\right|}{\left|\overrightarrow{C_m C_M}\right|} = R \tag{3}$$

with M such as :

$$\begin{cases} \overrightarrow{C_m C_M} \bullet \overrightarrow{C_m M} > 0 \\ \overrightarrow{C_m C_M} \bullet \overrightarrow{C_M M} < 0 \end{cases} \tag{4}$$

If (4) is not verified we have to compute the intersection with the half spheres.

The step of the grid is obviously an essential element since it characterizes the number of assessment points. In the second step, the geometrical deviation is calculated, i.e. the distance of the machined point to the nominal one.

The distance between each point M which belongs to the toolpath and the machining face (which is a re-limitation of the surface) is given by (5), where P is the projection of the M point on the face :

$e = \overrightarrow{PM} \bullet \vec{n}$ where $\vec{n}$ is the surface normal (5)

The toolpath is said error-free if the geometrical deviations are all in the authorized range.

4.2. Machining process

At first, a visual analysis allows to detect the potential presence of marks on the final part. In order to identify the errors due to the machining process. The final part has to be measured by use of a Coordinate Measuring Machine. In order to elaborate a collision-free probe path, the movements of the probe can be built from the toolpath. The position of the measuring points are selected among the CL data and the measuring direction is given by the normal to the surface at each point. At this stage, the comparison with the nominal model provides the errors due to the CAM system and the machining process. The specific CAM system errors may be assessed by the means previously exposed. Thus, the machining process error can be determined by difference. The knowledge of the failing of each chain link during the entire process obviously allows the checking of the whole process.

5. Validation of the certification method

In order to validate the certification method, we have carried out the entire milling process for the test part. Two different toolpath generation techniques have been tested. The CAM parameters, i.e, the machining direction, the maximum chordal deviation, Mt, and the maximum scallop height, hc, are given in table 1.

Machining direction	Maximum chordal deviation	Maximum scallop height
Parallel to the plane xOz	0.01 mm	0.01 mm

table 1: CAM parameters

These parameters are not directly linked to the geometrical specifications. But a previous analysis (Tournier, 1996) has shown that the respect of such parameters implies that the machined surface is included between two offset surfaces of the nominal one, for which the offset values are given by -Mt and Mt + hc. So, the toolpath generation technique is assessed if the geometrical deviation at each machined point belongs to the range [-Mt, Mt + hc].

Figure 10 presents a map of the geometrical deviations that means the points on the part which are out of the authorized range for two different generation techniques.

figure 10: map of the geometrical deviations

Technique a) is a good one for the only points for which the geometrical deviations are out of tolerance are located in areas where the minimum curvature radius is less than the tool radius. This essentially corresponds to the connection zones between the surfaces. These error points obviously exist with technique b). In addition, technique b) generates other error zones:

- on the *butterfly* shape the normal inversion is not correctly treated : the adjoined plane is machined at the same altitude and as a result the geometrical deviations are equal to 0.5mm,

- in the case of concave surfaces machined by the tool side we find numerous overcut points essentially in areas where the curvature radius is slightly greater than the tool radius; the geometrical deviations reach 0.09mm,

- on the sides defined by X= Xmin and X = Xmax, the geometrical deviations reach -0.12; this provides undercut points on the part boundaries without apparent reason.

We can notice that for the technique b) the right cylinder presents a CAD modeling error which implies a systematic geometrical deviation of 0.1mm. To conclude, the treatment of the grooves and the undercut shapes is different from the two techniques but both are error-free.

The second step of the validation of the certification method is the milling of the test part and its visual analysis. The realization is carried out from the technique a) toolpath which presents less calculation errors. The visual analysis of the part brings out local marks and systematic marks wherever the curvature radius is less than the tool radius. Such kinds of marks, probably due to the cutting process, should disappear for a high speed cutting. On the other hand, most details are missed out.

6. Conclusion

The certification method for the milling process of free-form surfaces we propose is efficient. It is based on a test part composed by a set of geometrically defined surfaces and on checking means. The function of the test part is to bring out areas that can pose problems to both the toolpath generator and the machining process. The checking means allow to essentially evaluate the toolpath generation techniques with the calculation of the geometrical deviations. Through two cases, the interest of the test part is seen for we notice that errors effectively exist. It shows for example the difficulty of the technique b) to manage concave surfaces. The machining of the test part allows to complete the first analysis by a global view of the whole process. The visual analysis shows unacceptable behaviors that lead to marks on the part and that decrease the quality of the machined surface.

A measurement of the test part associated with the determination of the toolpath generator errors should permit to deduce the machining process behavior. This point is on the way of completion.

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Machining direction	Maximum chordal deviation	Maximum scallop height
Parallel to the plane xOz	0.01 mm	0.01 mm

table 1:

Captions

table 1: CAM parameters

Captions

figure 1 : the milling process of a free-form surface

figure 2 : tolerance of form deviation

figure 3 : definition of the cutter location, *CL data*

figure 4 : geometrical singularities for the normal evaluation

figure 5 : interference tool-surface

figure 6 : offset evaluation from polyhedral approximation

figure 7: the test part

figure 8: a specifical shape, *the grooves*

figure 9: calculation of the geometrical deviation

figure 10: map of the geometrical deviations