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# Wingtip Vortices of a Biomimetic Micro Air Vehicle 

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#### Abstract

Wingtip vortices are generated behind a wing that produces lift. They exhibit a circular pattern of spinning air that generates an additional drag force, the induced drag, reducing the aerodynamic performance of an aircraft. Moreover, the wingtip vortices can pose a hazard to airplane maneuvers, mainly in take-off and landing operations. This chapter describes a review of the lifting-line vortex theory applied to a biomimetic Micro Aerial Vehicle (MAV) with Zimmerman planform. Therefore, the horseshoe vortex model is deeply explained and the estimations of vortex velocity distribution, lift, and induced drag are obtained with this simple model. These results are compared with experimental data obtained from wind tunnel testing by using Particle Image Velocimetry (PIV). Finally, the vorticity maps in the wake of this MAV are obtained from PIV measurements.


Keywords: tip vortices, biomimetic, micro aerial vehicle, induced drag, vorticity

## 1. Introduction

The aeronautic industry has developed a growing interest in Unmanned Aerial Vehicles (UAVs). These vehicles have been designed for multiple missions where the human factor is not required. Therefore, in dangerous missions, unhealthy environments, or inaccessible areas, human accidents can be avoided. The UAVs can be distinguished into different categories according to their performance characteristics. In this context, the relevant design parameters are weight, manufacturing costs, and size. Mainly, the manufacturing costs have been the key point for that engineers and researchers could be focused on developing smaller vehicles in order to perform unmanned activities. This group of smaller vehicles is known as Micro Aerial Vehicles (MAVs) [1-3]. Their main features are low aspect ratio (AR) and low range operation. Research centers and universities have been able to investigate new designs of MAVs due to their low manufacturing costs and small size. This is the case of aerodynamic challenges posed in the work of Moschetta [4]. The MAV designs have taken into account the fixed-wing, coaxial, biplane, and tilt-body concept. Marek [5] performed experimental tests to determine the aerodynamic coefficients in six different types of platforms. The Zimmerman and elliptical planforms resulted in having the highest lift coefficient. Hence, Hassannalian and Abdelkefi [6] designed and manufactured a fixed-wing MAV based on the Zimmerman planform. Also, other authors designed the Dragonfly MAV using Zimmerman planform [7, 8].

The chapter will begin with a description of the biomimetic Micro Air Vehicle (MAV) $[9,10]$, then the horseshoe theory will be explained and applied to the
studied vehicle. Consequently, the experimental facility, the Particle Image Velocimetry technique, and the description of the experimental tests will be defined. Then, the vorticity and several vortex models will be defined and applied to the experimental data obtained from the Wind Tunnel. At the end of the chapter, the formulation which relates the axial vorticity and the circulation will be presented and finally, the lift coefficient will be obtained.

## 2. Micro air vehicle geometry

The geometry of the studied Micro Air Vehicle (MAV) is based on Zimmerman planform and Eppler 61 airfoils for the wing configuration and Whitcomb II airfoils for the fuselage (see Figure 1).


Figure 1.
Biomimetic MAV model (dimensions in mm).


Figure 2.
Zimmerman planform.

Wingtip Vortices of a Biomimetic Micro Air Vehicle
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| Parameter | Value |
| :--- | :---: |
| Wing tip Chord $\boldsymbol{c}_{\boldsymbol{t}}$ | 0.025 m |
| Wing root Chord $\boldsymbol{c}_{r}$ | 0.200 m |
| Wing taper ratio, $\boldsymbol{\lambda}$ | 0.124 |
| Aspect ratio, $\boldsymbol{A R}$ | 2.500 |
| Wingspan, $\boldsymbol{b}$ | 0.320 m |
| Mean aerodynamic chord, $C M A$ | 0.141 m |
| Mean geometry chord, $C M G$ | 0.127 m |
| Wing reference area, $\boldsymbol{S}$ | $0.040 \mathrm{~m}^{2}$ |
| Dihedral angle, $D_{h}$ | $10^{\circ}$ |
| Fuselage length, $l$ | 0.300 m |
| Fuselage width, $d$ | 0.060 m |

Table 1.
MAV features.

The Zimmerman wing consists of two halves of ellipses connected at one point of reference which corresponds to $1 / 4$ of the maximum wing root chord ( $c_{r}=200 \mathrm{~mm}$ ) for one half of the ellipse and $3 / 4$ of the $c_{r}$ for the other half of the ellipse. Figure 2 shows the planform of the micro air vehicle and their dimensions. The rest of the geometrical features are shown in Table 1.

## 3. The horseshoe vortex: Biot-Savart law

In this section, a previous formulation of the wingtip vortex will be presented. The 3D wings can be modeled by vortex filaments. The horseshoe is the simplest mathematical model of potential flow to represent the aerodynamics of a wing aircraft. That consists of the bound vortex (vortex filament of the wing) and the trailing vortices formed by the semi-infinite filament vortex that represents the wingtips.

The horseshoe is a 3-D vortex that can be represented with an arbitrary shape according to the Helmholtz vortex theorems:

- The circulation $\Gamma$ is constant along the vortex length.
- The vortex has to be extended to $\pm \infty$, form a closed-loop, or end at a solid boundary.

In this context, the velocity field of a 3-D vortex by applying the Biot-Savart Law is defined by the following expression Eq. (1), [11].

$$
\begin{equation*}
\vec{V}(x, y, z)=\frac{\Gamma}{4 \pi} \int_{-\infty}^{+\infty} \frac{d \vec{l} \times \vec{r}}{|\vec{r}|^{3}} \tag{1}
\end{equation*}
$$

where $\vec{r}$ is extended from the integration point to the field point and the arc length element $d \vec{l}$ points follow the direction of positive circulation.


Taking into account the straight vortex of Figure 3, $h$ is defined as the nearest perpendicular distance from the vortex line and $\theta$ is the angle between the radius vector $\vec{r}$ and the vortex line (are defined in Eq. (4) and (5)).

$$
\begin{gather*}
r \equiv|\vec{r}|=\frac{h}{\sin \theta}  \tag{2}\\
l=-\frac{h}{\tan \theta}  \tag{3}\\
d l=\frac{h}{\sin ^{2} \theta}  \tag{4}\\
d \vec{l} \times \vec{r}=(d l r \sin \theta) \hat{\theta} \tag{5}
\end{gather*}
$$

Now, the velocity field can be recalculated as Eq. (6):

$$
\begin{equation*}
\vec{V}=\frac{\Gamma}{4 \pi h} \hat{\theta} \int_{0}^{\pi} \sin \theta d \theta=\frac{\Gamma}{2 \pi h} \hat{\theta} \tag{6}
\end{equation*}
$$

To reproduce the wingtip vortices of the studied MAV, a simple model based on the superposition of the freestream flow ( $U_{\infty}$ ) and a horseshoe vortex is described. The horseshoe vortex can be defined as the sum of three segments that can be seen in Figure 4: two free-trailing vortices at each tip of the wing (segment $A B$ and segment CD ) that are connected by a bound vortex spanning the wing (segment BC ). As explained previously, the circulation $\Gamma$ along the entire vortex line is constant, and the vortex lines have to extend downstream to infinity (see Figure 3). This potential solution is not very effective since the local lift to span is constant over the wingspan and in the real MAV model, the local lift is zero at the tip of the wings. A scheme of the horseshoe vortex model is defined in Figure 4.

The velocity field downstream of the wing in $x=$ constant planes is similar to the potential solution generated by a horseshoe vortex except near the vortex axes. Now, to obtain the vertical velocity distribution of the potential vortex in our MAV, it is necessary to know the wing chord ( $c=0.2 \mathrm{~m}$ ), wingspan ( $b=1.6 c$ ), and chord distance downstream of the trailing edge of the wing $(x=3 c)$. Therefore, the following two non-dimensional variables ( $\eta$ and $\zeta$ ) need to be defined (Eq. (7)):

$$
\begin{equation*}
\eta=\frac{x}{a}=3.75 ; \zeta=\frac{y}{a} \tag{7}
\end{equation*}
$$

where $a$ is the semi-wingspan, $a=\frac{b}{2}$ (see Figure 2), $\eta$ and $\zeta$ are the nondimensional coordinates according to the x -axis and y -axis, respectively.


Figure 4.
Scheme of the horseshoe vortex model.

Then, the non-dimensional vertical velocity $\psi(\zeta)$ can be defined as Eq. (8):

$$
\begin{equation*}
\psi(\eta)=\frac{w(\zeta)}{\frac{\Gamma}{4 \pi a}} \tag{8}
\end{equation*}
$$

which presents a different formulation depending on the vortices defined in each of the segments (see Figure 3):

$$
\begin{gather*}
\psi_{A B}(\eta)=\frac{-1}{(\zeta+1)}\left[1+\frac{\eta}{\sqrt{(\zeta+1)^{2}+(\eta)^{2}}}\right]  \tag{9}\\
\psi_{C D}(\eta)=\frac{1}{(\zeta-1)}\left[1+\frac{\eta}{\sqrt{(\zeta-1)^{2}+(\eta)^{2}}}\right]  \tag{10}\\
\psi_{B C}(\eta)=\frac{-1}{\eta}\left[\left[\frac{\eta+1}{\sqrt{(\zeta+1)^{2}+(\eta)^{2}}}\right]-\left[\frac{\eta-1}{\sqrt{(\zeta-1)^{2}+(\eta)^{2}}}\right]\right] \tag{11}
\end{gather*}
$$

Finally, the total non-dimensional vertical velocity is defined as the sum of the three velocities of the vortices (Eq. (12)):

$$
\begin{equation*}
\psi(\zeta)=\psi_{A B}(\zeta)+\psi_{B C}(\zeta)+\psi_{C D}(\zeta) \tag{12}
\end{equation*}
$$

In the following Figure 5, the total non-dimensional vertical velocity distribution of this MAV is presented only for the section located at 3c downstream of the trailing edge of the wing and for the angle of attack of $10^{\circ}$.

To obtain a better understanding of the flow behavior of these vortices and how they interact between them, in Figure 6 the non-dimensional vertical velocity only of the AB free-trailing vortex region is presented. The blue line shows the velocity distribution of the $A B$ free-trailing vortex while the dashed red and black lines correspond to the velocity of the bound vortex (BC in Figure 4) and the CD freetrailing vortex, respectively. It is clearly noted that both vortices, bound vortex, and CD free-trailing vortex are not affecting the AB free trailing vortex since their


Figure 5.
The non-dimensional vertical velocity at $3 c$ downstream of the trailing edge of the $M A V$ wing.


Figure 6.
The non-dimensional vertical velocity at $3 c$ downstream of the trailing edge of the MAV wing.
velocity values are very small. As a consequence, in that region only the flow presence from the AB free-trailing vortex itself.

## 4. The experimental set-up

In this section, the experimental setup will be presented. All experimental tests were carried out in a Low-Speed Wind Tunnel at the Instituto Nacional de Técnica Aeroespacial (INTA) in Madrid (Spain). This wind tunnel has a closed circuit with an elliptical open test section of $6 \mathrm{~m}^{2}$. The DC engine, which is situated at the opposite side of the test section, works at 420 V , allowing a maximum airflow speed of $60 \mathrm{~m} / \mathrm{s}$ with a turbulence intensity lower than $0.5 \%$. Figure 7 shows the Low-Speed Wind Tunnel of INTA.


Figure 7.
Components of the low-speed wind tunnel of INTA.
The MAV model was tested with a freestream velocity of the wind tunnel of $10 \mathrm{~m} / \mathrm{s}\left(U_{\infty}=10 \mathrm{~m} / \mathrm{s}\right)$, which results in a Reynolds number of $1.3 \times 10^{5}$ based on the wing root chord $\left(c_{r}=0.20 \mathrm{~m}\right)$. This analysis was performed for the cruise configuration (with an angle of attack equal to $0^{\circ}$ ). The experimental tests consisted in obtaining various transversal planes of the flow field at different sections downstream of the trailing edge of the wing.

The test experiments were carried out by using a full-scale model made in plastic material (PLA) by means of additive manufacturing and attached to a wood board by means of a streamlined support strut (see Figure 8). Only half of the model was studied due to its symmetry. Moreover, the MAV model and the wood board had to be painted in black in order to avoid reflections of the laser plane. The CCD camera was located behind the model (Figure 8), parallel to the flow stream of the wind tunnel.

The flow field velocity was determined by Particle Image Velocimetry (PIV). PIV is an advanced experimental technique that has the advantage of measuring the velocity field in a non-intrusive manner. This technique measures the velocity of the flow by analyzing flow images pairs [12]. For achieving this, PIV requires tracer particles that have to be seeded in the airflow. Olive oil was chosen for the


Figure 8.
Experimental setup.
generation of the tracer particles. A laser sheet has to be generated in order to go through the tracer particles and illuminate them. Two Neodymium-Yttrium Aluminum Garnet (Nd:YAG) lasers and an optical system were used for achieving this. The two lasers Nd:YAG has a pulse energy of 190 mJ within a time gap of $22 \mu \mathrm{~s}$. The location of the tracer particles has to be recorded by a high-resolution camera with $2048 \times 2048$ pixels equipped with a lens Nikon Nikkor 50 mm 1:1.4D. A crosscorrelation implemented via Fast Fourier Transform (FFT) is carried out over small image regions in order to obtain the averaged displacement of the tracer particles. The field of view (FOV) of the camera was $190 \times 190 \mathrm{~mm}^{2}$. The flow images are divided into interrogation window of $32 \times 32$ pixels. By using the Nyquist Sampling Criteria, these interrogation windows are overlapped by $50 \%$. Moreover, the peak of correlation is adjusted to the subpixel accuracy by Gaussian approximation. A final post-processing task to remove spurious data and fill the empty vectors is needed. Therefore, a local mean filter based on a neighbor kernel window of $3 \times 3$ vectors was applied.

## 5. The vorticity in the wingtip wake

The vorticity is defined as the curl of the flow velocity, by the following expressions (Eq. (13) and Eq. (14)),

$$
\begin{gather*}
\vec{\omega}=\nabla \times \vec{V}  \tag{13}\\
\vec{\omega}=\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right) \vec{i}+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right) \vec{j}+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \vec{k} \tag{14}
\end{gather*}
$$

The two-dimensional (2D $y$-z plane) streamwise vorticity $\omega_{x}=\xi$ can be determined from measured velocities by solving Eq. (15), which depends on the velocity spatial derivatives, as follows,

$$
\begin{equation*}
\xi=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=(\nabla \times \vec{V}) \cdot \vec{i} \tag{15}
\end{equation*}
$$



Figure 9.
Non-dimensional axial vorticity measured by PIV at 1.4 c downstream of the trailing edge of the model $\left(U_{\infty}=10 \mathrm{~m} / \mathrm{s}\right.$, cruise : $\alpha=\beta=0^{\circ}$ ).

The axial vorticity had to be obtained with central differencing in crossflow velocities. The non-dimensional form of axial vorticity component $\tilde{\xi}$ is given by the following expression (Eq. (16)):

$$
\begin{equation*}
\tilde{\xi}=\frac{\left(\frac{b}{2}\right) \cdot \xi}{U_{\infty}} \tag{16}
\end{equation*}
$$

where $b$ is the whole spanwise of the model and $U_{\infty}$ is the frestream velocity.
Figure 9 shows the non-dimensional axial vorticity after taking PIV measurements in the wake downstream when the vehicle was flying in a cruise condition. It can be seen that the peak of maximum axial vorticity (red region) takes place at the wingtip, and from there it starts to decrease.

## 6. Circulation and vorticity

By analyzing the flow downstream of the aircraft model, this flow can be studied as the 2 D wingtip wake and the vorticity is related to the velocity circulation from Stokes theorem by the following expression (Eq. (17)), [11].

$$
\begin{equation*}
\Gamma=\oint_{C} \vec{V} \cdot d \vec{l}=\iint(\nabla \times \vec{V}) \cdot \vec{n} \cdot d A \tag{17}
\end{equation*}
$$

where $C$ is a closed curve, $\vec{V}$ is the flow velocity on a small element defined on the closed curve, and $d l$ is the differential length of that small element. As the plane streamwise is the 2D-yz plane, we have $\vec{\omega}=\xi \vec{i}$, and the unit normal vector $\vec{n}=\vec{i}$, then (Eq. (18)),

$$
\begin{equation*}
\Gamma=\oint_{C} \vec{V} \cdot d \vec{l}=\iint \vec{\omega} \cdot \vec{n} \cdot d A=\iint \xi \cdot d A \tag{18}
\end{equation*}
$$

## 7. Evolution of the vorticity

The Navier-Stokes equations in vector form for an incompressible flow are given by,

$$
\begin{gather*}
\nabla \cdot \vec{V}=0  \tag{19}\\
\frac{\partial \vec{V}}{\partial t}+\vec{V} \cdot \nabla \vec{V}=-\nabla\left(\frac{p}{\rho}+g z\right)+\nu \nabla^{2} \vec{V} \tag{20}
\end{gather*}
$$

The vorticity equation (Eq. (13)) is obtained by taking the curl of the NavierStokes equation, as follows,

$$
\begin{gather*}
\nabla \times(\nabla \cdot \vec{V})=0  \tag{21}\\
\nabla \times\left(\frac{\partial \vec{V}}{\partial t}+(\vec{V} \cdot \nabla) \vec{V}=-\nabla\left(\frac{p}{\rho}+g z\right)+\nu \nabla^{2} \vec{V}\right) \tag{22}
\end{gather*}
$$

By calculating each term, where the conservation of vorticity is Eq. (23),

$$
\begin{gather*}
\nabla \times\left(\frac{\partial \vec{V}}{\partial t}\right)=\frac{\partial \vec{\omega}}{\partial t}  \tag{23}\\
\nabla \times((\vec{V} \cdot \nabla) \vec{V})=(\vec{V} \cdot \nabla) \vec{\omega}-(\vec{\omega} \cdot \nabla) \vec{V}  \tag{24}\\
\nabla \times\left(-\nabla\left(\frac{p}{\rho}+g z\right)\right)=0  \tag{25}\\
\nabla \times\left(\nu \nabla^{2} \vec{V}\right)=\nu \nabla^{2} \vec{\omega} \tag{26}
\end{gather*}
$$

and finally, the vorticity equation is,

$$
\begin{equation*}
\frac{\partial \vec{\omega}}{\partial t}+(\vec{V} \cdot \nabla) \vec{\omega}=(\vec{\omega} \cdot \nabla) \vec{V}+\nu \nabla^{2} \vec{\omega} \tag{27}
\end{equation*}
$$

The law of vorticity evolution is a convective vector diffusion equation given by the following expression,

$$
\begin{equation*}
\frac{D \vec{\omega}}{D t}=(\vec{\omega} \cdot \nabla) \vec{V}+\nu \nabla^{2} \vec{\omega} \tag{28}
\end{equation*}
$$

The viscous term $\left(\nu \nabla^{2} \vec{\omega}\right)$ causes the vortex to diffuse through the fluid flow. By using index notation, the vorticity equation for 3D flow is given by,

$$
\begin{equation*}
\frac{\partial \omega_{i}}{\partial t}+u_{j} \frac{\partial \omega_{i}}{\partial x_{j}}=\omega_{j} \frac{\partial u_{i}}{\partial x_{j}}+\nu \frac{\partial^{2} \omega_{i}}{\partial x_{k} \partial x_{k}} \tag{29}
\end{equation*}
$$

For a 2D flow, the stretching term is absent, and the corresponding equation is,

$$
\begin{equation*}
\frac{\partial \omega_{i}}{\partial t}+u_{j} \frac{\partial \omega_{i}}{\partial x_{j}}=\nu \frac{\partial^{2} \omega_{i}}{\partial x_{k} \partial x_{k}} \tag{30}
\end{equation*}
$$

Equivalently, in vector form, for a 2D flow we have the velocity is perpendicular to the vorticity, so $\vec{V} \cdot \vec{\omega}=0$. The velocity is $\vec{V}=(0, V, W)$ and vorticity $\vec{\omega}=$ $\left(\omega_{x}, 0,0\right)$, so that,

$$
\begin{gather*}
\vec{\omega} \cdot \nabla \vec{V}=0  \tag{31}\\
\frac{D \vec{\omega}}{D t}=\frac{\partial \vec{\omega}}{\partial t}+(\vec{V} \cdot \nabla) \vec{\omega}=\nu \nabla^{2} \vec{\omega} \tag{32}
\end{gather*}
$$

where the operator $\frac{D}{D t}=\frac{\partial}{\partial t}+(\vec{V} \cdot \nabla)$ is the material derivative and it describes the evolution along the flow lines.

## 8. Decay of wingtip vortices

The study of the decay of wingtip vortices under the assumption of 2D flow with $\omega_{y}=\omega_{z}=0$, velocity $V_{x}=0$ and $\partial / \partial x=0$, can be performed by the 2D viscous diffusion of vorticity equation, given by,

$$
\begin{align*}
& \frac{\partial \vec{\omega}}{\partial t}=\nu \nabla^{2} \vec{\omega}  \tag{33}\\
& \frac{\partial \omega}{\partial t}=\nu \cdot \Delta \omega \tag{34}
\end{align*}
$$

Where $\omega=\omega_{x}$ and $\Delta$ is the Laplacian operator.
Assuming axisymmetric flow, in cylindrical coordinates,

$$
\begin{equation*}
\frac{\partial \omega}{\partial t}=\frac{\nu}{r} \cdot \frac{\partial}{\partial \mathbf{r}}\left(r \frac{\partial \omega}{\partial r}\right) \tag{35}
\end{equation*}
$$

The initial vorticity for the study of decay point vortex in an unbounded domain is given by a 2 D delta function in the plane $y z$,

$$
\begin{equation*}
\omega(\vec{x}, t=0)=\Gamma_{0} \delta(y) \delta(z) \tag{36}
\end{equation*}
$$

Introducing the dimensionless similarity variable [13],

$$
\begin{equation*}
\epsilon=\frac{r}{\sqrt{\nu t}} \tag{37}
\end{equation*}
$$

and the nondimensional combination $\omega \nu t / \Gamma_{0}$ can be expressed as an unknown function $g$ of the variable $\epsilon$, defined as

$$
\begin{equation*}
\omega \nu t / \Gamma_{0}=g(\epsilon) \tag{38}
\end{equation*}
$$

So that,

$$
\begin{equation*}
\omega=\frac{\Gamma_{0}}{\nu t} g(\epsilon)=f(t) g(\epsilon) \tag{39}
\end{equation*}
$$

Calculating the derivatives quantities from the earlier equation,

$$
\begin{array}{r}
\frac{\partial \omega}{\partial t}=\frac{\partial f(t)}{\partial t} g(\epsilon)+f(t) \frac{\partial g(\epsilon)}{\partial t}=-\frac{\Gamma_{0}}{\nu t} \frac{1}{t} g(\epsilon)+f(t) \frac{d g(\epsilon)}{d \epsilon} \frac{\partial \epsilon}{\partial t} \\
\frac{\partial \omega}{\partial t}=-f(t) \frac{1}{t} g(\epsilon)-f(t) \frac{\epsilon}{2 t} \frac{d g(\epsilon)}{d \epsilon}=-f(t) \frac{1}{t}\left(g+\epsilon g^{\prime} / 2\right) \tag{41}
\end{array}
$$

On the other hand,

$$
\begin{array}{r}
\frac{\partial \omega}{\partial r}=f(t) \frac{\partial g(\epsilon)}{\partial r}=f(t)\left(\frac{\partial \epsilon}{\partial r} \frac{d g(\epsilon)}{d \epsilon}\right)=f(t)\left(\frac{\epsilon}{r} \frac{d g(\epsilon)}{d \epsilon}\right) \\
\frac{\partial \omega}{\partial r}=f(t) \frac{\partial g(\epsilon)}{\partial r}=f(t)\left(\frac{\partial \epsilon}{\partial r} \frac{d g(\epsilon)}{d \epsilon}\right)=f(t)\left(\frac{\epsilon}{r} \frac{d g(\epsilon)}{d \epsilon}\right) \\
\frac{\partial}{\partial r}\left(r \frac{\partial \omega}{\partial r}\right)=\frac{\partial \epsilon}{\partial r} \frac{d}{d \epsilon}\left(f(t) \epsilon \frac{d g(\epsilon)}{d \epsilon}\right)=\frac{\epsilon}{r} f(t) \frac{d}{d \epsilon}\left(\epsilon \frac{d g(\epsilon)}{d \epsilon}\right) \tag{44}
\end{array}
$$

And substituting in (35), the following expression is derived,

$$
\begin{equation*}
2\left(\epsilon g^{\prime}\right)^{\prime}+\epsilon^{2} g^{\prime}+2 g \epsilon=0 \tag{45}
\end{equation*}
$$

Where ' denotes a derivative respect to, and finally, the equation is integrated

$$
\begin{equation*}
g(\epsilon)=A \exp \left(\frac{-\epsilon^{2}}{4}\right) \tag{46}
\end{equation*}
$$

The condition of the flow circulation is equal to $\Gamma_{0}$ at any time, gives,

$$
\begin{equation*}
\Gamma_{0}=\int_{0}^{\infty} \omega 2 \pi r d r=4 \pi A \Gamma_{0} \tag{47}
\end{equation*}
$$

so that $A=1 / 4 \pi$, and the solution of the $g(\epsilon)$ function is,

$$
\begin{equation*}
g(\epsilon)=\frac{1}{4 \pi} \exp \left(\frac{-r^{2}}{4 \nu t}\right) \tag{48}
\end{equation*}
$$

Finally, the solution of vorticity is given by the axisymmetric Lamb-Osteen vortex by,

$$
\begin{equation*}
\omega=\frac{\Gamma_{0}}{4 \pi \nu t} \exp \left(\frac{-r^{2}}{4 \nu t}\right) \tag{49}
\end{equation*}
$$

The swirl velocity is,

$$
\begin{equation*}
V_{\theta}=\frac{\Gamma_{0}}{2 \pi r}\left(1-\exp \frac{-r^{2}}{4 \nu t}\right) \tag{50}
\end{equation*}
$$

and the circulation is,

$$
\begin{equation*}
\Gamma=\Gamma_{0}\left(1-\exp \frac{-r^{2}}{4 \nu t}\right) \tag{51}
\end{equation*}
$$

The swirl velocity can be rewritten as,

$$
\begin{equation*}
V_{\theta}=\frac{\Gamma_{0}}{2 \pi r}\left(1-\exp \left(-1.2526\left(r / r_{c}\right)^{2}\right)\right) \tag{52}
\end{equation*}
$$

where $r_{c}$ is the core radius, defined as the distance from the vortex center to the point with the higher swirl velocity, and given by,

$$
\begin{equation*}
r_{c}=2.24 \sqrt{\nu t} \tag{53}
\end{equation*}
$$

## 9. Analysis of vortex models and experimental data

The velocity components which define a 2-D vortex are typically the swirl velocity $V_{\theta}$, the axial velocity $V_{z}$ and the radial $V_{r}$ velocity. The last two components usually are neglected in many applications as they are very small compared to swirl velocity, and are defined as follows,

$$
\begin{gather*}
V_{\theta}=\frac{\Gamma}{2 \pi r}  \tag{54}\\
V_{r}=0 \tag{55}
\end{gather*}
$$

$$
\begin{equation*}
V_{z}=0 \tag{56}
\end{equation*}
$$

Several tip vortex models are usually studied, but this chapter is only focused on some of them, displayed in Figure 10. The first method is the Rankine vortex model, being the simplest formulation with a finite core. Therefore, the vortex is divided into two parts: the viscous core and the potential vortex. The viscous core is rotating as a solid body near the vortex center while the potential vortex remains away from the vortex center. The velocity in the potential vortex is decreasing hyperbolically with the radial coordinate [14, 15]. Therefore, the following expressions represent the swirl velocity distribution $V_{\theta}$ in the tip vortex,

$$
\begin{gather*}
V_{\theta}(\tilde{r})=\left(\frac{\Gamma}{2 \pi r_{c}}\right) \cdot \tilde{r} 0 \leq \tilde{r} \leq 1  \tag{57}\\
V_{\theta}=\frac{\Gamma}{2 \pi r} \tilde{r}>1 \tag{58}
\end{gather*}
$$

Where $r_{c}$ is the viscous core radius and $\tilde{r}=\frac{r}{r_{c}}$ is the non-dimensional radial coordinate.

The second vortex model is the Lamb-Oseen vortex which is a simplified solution of one-dimensional Navier-Stokes equations for laminar flow which is defined by the following expression,

$$
\begin{equation*}
V_{\theta}(\tilde{r})=\left(\frac{\Gamma}{2 \pi r}\right) \cdot\left[1-e^{-\alpha(\tilde{r})^{2}}\right] \tag{59}
\end{equation*}
$$

where $\alpha=1.2526$ is the Oseen parameter.
An alternative tip vortex formulation is given by Vatistas in Ref. [15]. This method is based on a group of desingularized algebraic swirl velocity profiles for vortices which present continuous distributions of flow quantities. The swirl velocity is defined by,

$$
\begin{equation*}
V_{\theta}(\tilde{r})=\frac{\Gamma}{2 \pi r_{c}} \cdot \frac{\tilde{r}}{\left(1+\tilde{r}^{2 n}\right)^{1 / n}} \tag{60}
\end{equation*}
$$

where $n$ is an integer.


Figure 10.
Swirl velocity distribution inside a tip vortex was obtained by several tip vortex models.

The Scully vortex model is the previous formulation when the integer is $n=1$, and it is defined as,

$$
\begin{equation*}
V_{\theta}(\tilde{r})=\left(\frac{\Gamma}{2 \pi r_{c}}\right) \cdot \frac{\tilde{r}}{\left(1+\tilde{r}^{2}\right)} \tag{61}
\end{equation*}
$$

when the integer is $n=2$, the swirl velocity of the vortex formulation is,

$$
\begin{equation*}
V_{\theta}(\tilde{r})=\left(\frac{\Gamma}{2 \pi r_{c}}\right) \cdot \frac{\tilde{r}}{\sqrt{1+\tilde{r}^{4}}} \tag{62}
\end{equation*}
$$

It is important to notice that when the integer $n \rightarrow \infty$, the swirl velocity distribution corresponds to the Rankine method.

Figure 11 shows the flow field velocity in a normal section to the flow located at 3 chords downstream of the MAV model. The 2d vortex can be observed clearly and the color scale indicates that the velocity is increasing near the center of the vortex.

It is possible to obtain a better visualization of the flow field distribution by looking at Figure 12. This PIV map is obtained for the angle of attack of $10^{\circ}$. The plotted streamlines reveal the location of the vortex center (places at $\mathrm{Y}=\mathrm{Z}=0 \mathrm{~mm}$ ), the region of the vortex core (yellow region), and the external region (green area).

Extracting the data value of the swirl velocity as measured by the PIV technique we can obtain Figure 13 when the experimental data are plotted with curves of theoretical vortex models. The blue scatter dots which its trend is approached by a 6th-degree polynomial (red continue line).

Also, the distributions of the swirl velocity obtained by the theoretical vortex models as Rankine, Lamb-Oseen, and Scully are represented in Figure 11.

The analysis of this graph shows the wingtip vortex method which presents the most accurate fit to the MAV is obtained with the tip vortex model of Scully. Subsequently, there is a deviation between the two approaches (experimental data and Scully) which depends on the distance from the vortex center. The ratio between both of them is assessed by the parameter $k(r)$ defined as


Figure 11.
Wingtip vortex in the MAV wake at $3 c$.


Figure 12.
Velocity distribution at $3 c$.


Figure 13.
Experimental data and theoretical vortex models.

$$
\begin{equation*}
k(r)=\frac{\left(V_{\theta}\right)_{\text {polynomial }}}{\left(V_{\theta}\right)_{\text {Scully }}} \tag{63}
\end{equation*}
$$

where $\left(V_{\theta}\right)_{\text {polynomial }}$ and $\left(V_{\theta}\right)_{\text {Scully }}$ are the distributions of swirl velocity obtained in the test experiments and by the theoretical model proposed by Scully, respectively.

Finally, the distribution of experimental swirl velocity is fitted to the Scully model by the function called $\left(V_{\theta}\right)_{\text {experimental-Scully }}$ defined as,

$$
\begin{equation*}
\left(V_{\theta}\right)_{\text {experimental-Scully }}=k(r) \cdot\left(V_{\theta}\right)_{\text {Scully }} \tag{64}
\end{equation*}
$$

## 10. Lift coefficient

The lift of an airfoil can be determined by the Kutta-Joukowski theorem [11] relating the velocity and the circulation, as follows,

| Parameters | MAV model |
| :--- | :---: |
| Location | $\mathrm{x}=3$ chords |
| $\boldsymbol{\alpha}\left({ }^{\circ}\right)$ | 0 |
| $\boldsymbol{\beta}\left({ }^{\circ}\right)$ | 0 |
| $\boldsymbol{U}_{\infty}(\boldsymbol{m} / \boldsymbol{s})$ | 10 |
| $\boldsymbol{r}_{\boldsymbol{c}}(\boldsymbol{m m})$ | 4.70 |
| $\boldsymbol{V}_{\boldsymbol{\theta} \max }(\boldsymbol{m} / \boldsymbol{s})$ | 7.69 |
| $\boldsymbol{\Gamma}\left(\boldsymbol{m}^{2} / \boldsymbol{s}\right)$ | 0.45 |
| $\boldsymbol{C}_{\boldsymbol{L}}$ | 0.72 |

Table 2.
Results of the tip vortex analysis in the wake of MAV.

$$
\begin{equation*}
L^{\prime}=\rho U_{\infty} \Gamma \tag{65}
\end{equation*}
$$

By applying the earlier formulation, the total lift of the wing $L$ can be obtained from the following expression

$$
\begin{equation*}
L=\left(\rho U_{\infty} \Gamma\right) \cdot b \tag{66}
\end{equation*}
$$

where $b$ is the wingspan.
The lift coefficient $C_{L}$ is obtained by dividing the lift by $q_{\infty} S_{r e f}$,

$$
\begin{equation*}
C_{L}=\frac{\left(\rho U_{\infty} \Gamma\right) \bullet b}{q_{\infty} S_{r e f}} \tag{67}
\end{equation*}
$$

where $q_{\infty}$ is the dynamic pressure $\left(q_{\infty}=1 / 2 \rho U_{\infty}^{2}\right)$ and $S_{r e f}$ is the reference wing surface.

Table 2 shows the values of the main parameters obtained from the tip vortex analysis, including the lift coefficient, $C_{L}$.

## 11. Conclusions

Wingtip vortices generated behind an aircraft wing affect the aerodynamic performance of the aircraft while endangering take-off and landing maneuvers of the subsequent aircraft.

In this chapter, it is reviewed the theoretical background of the horseshoe vortex and several vortex models applied to a Biomimetic Micro Air Vehicle (MAV) with Zimmerman planform. The formulation of the vorticity in the wingtip wake of the MAV model has been presented as well as the expression which relates the axial vorticity and the circulation.

All experimental tests have been carried out in the Low-Speed Wind Tunnel of the Instituto Nacional de Técnica Aeroespacial (INTA) with a full-scale MAV model. Particle Image Velocimetry has been used to obtain the transversal flow field at 3 chords downstream of the trailing edge of the MAV model. The swirl velocity distribution according to the horseshoe vortex model and several vortex models (Rankine, Lamb-Oseen, Scully, and Vatistas) is plotted. The experimental results have shown that the Scully vortex has the most similar behavior to the MAV wing
vortex. The distribution of the transversal velocity as well as the axial vorticity for the section at 3 chords are presented by PIV maps. Finally, the lift coefficient by using the Kutta-Joukowski theorem is obtained.

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## Conflict of interest

The authors declare that they have not known existing or potential Conflicts of Interest, including financial or personal factors, as well as any relationship which could influence their scientific work.


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