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Chapter

$(\alpha, \beta)-$Pythagorean Fuzzy Numbers Descriptor Systems

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Abstract

By using pythagorean fuzzy sets and T-S fuzzy descriptor systems, the new (α, β)-pythagorean fuzzy descriptor systems are proposed in this paper. Their definition is given firstly, and the stability of this kind of systems is studied, the relation of (α, β)-pythagorean fuzzy descriptor systems and T-S fuzzy descriptor systems is discussed. The (α, β)-pythagorean fuzzy controller and the stability of (α, β)-pythagorean fuzzy descriptor systems are deeply researched. The (α, β)-pythagorean fuzzy descriptor systems can be better used to solve the problems of actual nonlinear control. The (α, β)-pythagorean fuzzy descriptor systems will be a new research direction, and will become a universal method to solve practical problems. Finally, an example is given to illustrate effectiveness of the proposed method.

Keywords: Pythagorean fuzzy sets, T-S fuzzy descriptor systems, stability

1. Introduction

Pythagorean fuzzy sets [1–4] were proposed by Yager in 2013, are a new tool to deal with vagueness. Pythagorean fuzzy sets maintain the advantages of both membership and non-membership, but the value range of membership function and non-membership function is expanded from triangle to quarter circle. The expansion of the value area makes the amount of information of pythagorean fuzzy sets expand 1.57 times that of the intuitionistic fuzzy sets, and ensures that intuitionistic fuzzy sets are all pythagorean fuzzy sets. They can be used to characterize the uncertain information more sufficiently and accurately than intuitionistic fuzzy sets. Pythagorean fuzzy sets have attracted great attention of a great many scholars that have been extended to new fields and these extensions have been used in many areas such as decision making, aggregation operators, and information measures. Due to theirs wide scope of description cases are very common in diverse real-life issue, pythagorean fuzzy sets have given a boost to the management of vagueness caused by fuzzy scope. Pythagorean fuzzy sets have provided two novel algorithms in decision making problems under Pythagorean fuzzy environment.

Takagi-Sugeno (T-S) fuzzy systems [5–9] has been applied on intelligent computing research and complex nonlinear systems. T-S fuzzy systems have also been extended to new fields and these extensions have been used in many areas by a great many scholars. However, the membership functions of T-S fuzzy systems cannot make full use of the all uncertain message in the premise conditions. So we decide to study the new (α,β)-pythagorean fuzzy descriptor systems in order to solve practical control problems more easily and feasible.

The advantages of (α, β)-pythagorean fuzzy descriptor systems are the following:

1. Pythagorean fuzzy sets maintain the advantages of both membership and non-membership, but the value range of membership function and non-membership function is expanded from triangle to quarter circle. The expansion of the value area makes the amount of information of pythagorean fuzzy sets expand 1.57 times that of the intuitionistic fuzzy sets. They can be used to characterize the uncertain information more sufficiently and accurately than intuitionistic fuzzy sets.

2. The membership function and non-membership function of pythagorean fuzzy sets can be easy to be defined. The value ranges of membership function and non-membership function are also more consistent with objective reality and many hesitant problems and people's thinking.

3. Pythagorean fuzzy sets can ensure that intuitionistic fuzzy sets are all pythagorean fuzzy sets, i.e. intuitionistic fuzzy sets are the special examples of pythagorean fuzzy sets. So intuitionistic fuzzy control systems can be changed into (0,1)-pythagorean fuzzy control systems.

4. (α,β)-pythagorean fuzzy descriptor systems are a broader generalization of T-S fuzzy descriptor systems i.e. T-S fuzzy descriptor systems are the special examples of (α, β)-pythagorean fuzzy descriptor systems.

5. We can judge the degree of weight in the control process according to the value of membership function and non-membership function of the rules. By setting the values of α and β, we decide whether the rules will participate in the final calculation, thereby reducing the calculation process and improving the control efficiency and effectiveness.

6. In fact, (α, β)-pythagorean fuzzy descriptor systems are consistent with the control methods of human being. This method is to imitate the control process of people and also solves the most difficult problem for humans.

The rest of this paper is organized as follows: In Section 1, the basic concepts of T-S fuzzy descriptor systems are introduced. In Section 2, (α,β)-pythagorean fuzzy descriptor systems are firstly proposed. Then the relationship of T-S fuzzy descriptor systems and (α,β)-pythagorean fuzzy descriptor systems are discussed in Section 3. (α,β)-pythagorean fuzzy controller and the stability of (α,β)-pythagorean fuzzy descriptor systemsare deeply researched in Section 4. In Section 5, a numbers examples is given to show the corollaries are corrected. We discussed in detail the effects of controls in several cases. Through this practical example, we find that the selection of pythagorean fuzzy membership functions in the premise conditions of the rules has a great influence on the control effect. Therefore, the choice of pythagorean fuzzy membership functions must be determined after more tests, and we can not completely believe the original given functions. Finally, the conclusion is given in Section 6.

Notations: Throughout this paper, R^n and $R^{n \times m}$ denote respectively the n dimensional Euclidean space and $n \times m$ dimensional Euclidean space. PFS denotes pythagorean fuzzy set.

2. Preliminaries

This section will briefly introduce some baisc definitions and theorems on pythagorean fuzzy sets and T-S fuzzy descriptor systems.

Definition 1.1 [1–4] Let X be a universe of discourse. A PFS P in X is given by.

$$P = \{ <x, \mu_P(x), \nu_P(x)> \mid x \in X \},$$

where $\mu_P: X \to [0,1]$ denotes the degree of membership and $\nu_P: X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set P, respectively, with the condition that $0 \leq (\mu_P(x))^2 + (\nu_P(x))^2 \leq 1$. The degree of indeterminacy $\pi_P(x) = 1 - (\mu_P(x))^2 - (\nu_P(x))^2$.

For convenience, a pythagorean fuzzy number $(\mu_P(x), \nu_P(x))$ denoted by $p = (\mu_P, \nu_P)$.

Definition 1.2 [10, 11] T-S fuzzy descriptor systems are as follows:

Rule i: if $x_1(t)$ is F_1^i and...and $x_n(t)$ is F_n^i, then.

$$E\dot{x}(t) = A_i x(t) + B_i \mu(t)$$
$$y(t) = C_i x(t) + D_i \mu(t)$$

Where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in R^n$ and $\mu(t) \in R^m$ are the state and control input, respectively; A_i, B_i, C_i and D_i are known real constant matrices with appropriate dimension;

E is a singular matrix; $F_1^i, F_2^i, \cdots, F_n^i (i = 1, 2, \ldots, r)$ are the fuzzy sets.

By fuzzy blending, the overall fuzzy model is inferred as follows.

$$E\dot{x}(t) = A(t)x(t) + B(t)\mu(t)$$
$$y(t) = C(t)x(t) + D(t)\mu(t)$$

where

$$A(t) = \sum_{i=1}^{r} h_i(x(t))A_i, B(t) = \sum_{i=1}^{r} h_i(x(t))$$
$$B_i, C(t) = \sum_{i=1}^{r} h_i(x(t))C_i, D(t) = \sum_{i=1}^{r} h_i(x(t))D_i,$$

and $h_i(x(t))$ is the normalized grade of membership, given as.

$$h_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{r} \omega_i(x(t))}, \omega_i(x(t)) = \Pi_{i=1}^{n} \mu_{ij}(x_j(t)),$$

which is satisfying

$$0 \leq h_i(x(t)) \leq 1, \sum_{i=1}^{r} h_i(x(t)) = 1,$$

$\mu_{ij}(x_j(t))$ is the grade of membership function of $x_j(t)$ in F_j^i.

3. $(\alpha, \beta)−$pythagorean fuzzy descriptor systems

As T-S fuzzy descriptor systems are very familiar to us, and pythagorean fuzzy sets are a new tool to deal with vagueness. So we decide to study the new $(\alpha, \beta)-$

pythagorean fuzzy descriptor systems in order to solve practical control problems more easily and feasible. Next, the related definitions of $(\alpha, \beta)-$pythagorean fuzzy descriptor systems are gradually given.

Definition 2.1 $(\alpha, \beta)-$pythagorean fuzzy descriptor systems are as follows:

Rule i: if $x_1(t)$ is P_1^i and...and $x_n(t)$ is P_n^i, then.

$$E\dot{x}(t) = A_i x(t) + B_i \mu(t) \qquad (1)$$

$$y(t) = C_i x(t) + D_i \mu(t) \qquad (2)$$

where $x(t) = [x_1(t)), x_2(t), \cdots, x_n(t)]^T \in R^n$ and $\mu(t) \in R^m$ are the state vector and the control input vector, respectively; $y(t)$ is the measurable output vector; A_i, B_i, C_i and D_i are known real constant matrices with appropriate dimension; E is a singular matrix; $P_1^i, P_2^i, \ldots, P_n^i (i = 1, 2, \ldots, r)$ are all pythagorean fuzzy sets.

By fuzzy blending, the overall fuzzy model is inferred as follows.

$$E\dot{x}(t) = A(t)x(t) + B(t)\mu(t)$$

$$y(t) = C(t)x(t) + D(t)\mu(t)$$

where

$$A(t) = \sum_{i=1}^r h_i(x(t))A_i, B(t) = \sum_{i=1}^r h_i(x(t))B_i,$$

$$C(t) = \sum_{i=1}^r h_i(x(t))C_i, D(t) = \sum_{i=1}^r h_i(x(t))D_i,$$

and $h_i(x(t))$ is the normalized grade of membership, given as.

$$h_i(x(t)) = \frac{h_{i_{(\alpha,\beta)}}(x(t))}{\sum_{i=1}^r h_{i_{(\alpha,\beta)}}(x(t))}, i = 1, 2, 3, \cdots, r;$$

where

$$h_{i_{(\alpha,\beta)}}(x(t)) = \begin{cases} h_i^1(x(t)) & \text{when } h_i^1(x(t)) \geq \alpha \text{ or } h_i^2(x(t)) \leq \beta \\ 0 & \text{else} \end{cases}, \alpha + \beta \leq 1, i = 1, 2, 3, \cdots, r;$$

$$h_i^1(x(t)) = \frac{\mu_{P^i}(x(t))}{\sum_{i=1}^r \mu_{P^i}(x(t))}, h_i^2(x(t)) = \frac{\nu_{P^i}(x(t))}{\sum_{i=1}^r \nu_{P^i}(x(t))},$$

where $h_i^1(x(t))$ and $h_i^2(x(t))$ are respectively positive and negative membership functions.

$$\sum_{i=1}^r h_{i1}(x(t)) = 1, \sum_{i=1}^r h_{i2}(x(t)) = 1;$$

$$\mu_{P^i}(x_j(t)) = \prod_{j=1}^r \mu_{P_j^i}(x_j(t)), \nu_{P^i}(x_j(t)) = \prod_{j=1}^r \nu_{P_j^i}(x_j(t));$$

$\mu_{P_j^i}(x_j(t))$ and $\nu_{P_j^i}(x_j(t))$ is the membership function value of $x_j(t)$ that belongs and does not belong to the intuitionistic fuzzy numbers set P_j^i.

Remark 2.1:

1. We can judge the degree of weight in the control process according to the value of the positive and negative membership functions of the rules. By setting the values of α and β, we decide whether the rules will participate in the final calculation, thereby reducing the calculation process and improving the control efficiency and effectiveness.

2. In fact, (α,β)-pythagorean fuzzy descriptor systems are consistent with the control methods of human being. People generally proceed appropriate control at one point by the past experience, i.e. people's decisions are decided and implemented at roughly one point. This method is to imitate the control process of people

3. The relations between (α,β)-pythagorean fuzzy descriptor systems and T-S fuzzy descriptor systems

Firstly, the relation of T-S fuzzy descriptor systems and (α,β)-pythagorean fuzzy descriptor systems is studied through an example.
When $\alpha = 0, \beta = 1,$ then

$$h_i(x(t)) = h_{i_{(\alpha,\beta)}}(x(t)) = h_{i_1}(x(t)) = \frac{\mu_i^M(x(t))}{\sum_{i=1}^r \mu_i^M(x(t))}, h_{i_2}(x(t)) = 0, \mu_i^M(x(t))$$
$$= \prod_{j=1}^n \mu_{ij}^M(x_j(t)).$$

Then the special (0,1)-pythagorean fuzzy descriptor systems are T-S fuzzy descriptor systems. In other words, T-S fuzzy descriptor systems are all the special (0,1)-pythagorean fuzzy descriptor systems. Therefore, it is easy to get the following Theorem 3.1.

Theorem 3.1 T-S fuzzy descriptor systems are all the (α,β)-pythagorean fuzzy descriptor systems.

Proof:It is so easy, so omit.

4. (α, β)−pythagorean fuzzy numbers controller

Now we continue to study the feedback control and stability of pythagorean fuzzy descriptor systems according to the traditional research path of the control systems.

Suppose.

Rule i: if $x_1(t)$ is $P_1^i(x_1(t))$ and ... and $x_n(t)$ is $P_n^i(x_n(t))$, then.

$$u(x(t)) = \sum_{i=1}^r h_i(x(t)))G_i x(t) \qquad (3)$$

where $G_i(i = 1,2,..., r)$ are the state feedback-gains matrices.

$$h_i(x(t)) = \frac{h_{i_{(\alpha,\beta)}}(x(t))}{\sum_{i=1}^r h_{i_{(\alpha,\beta)}}(x(t))}, i = 1, 2, 3, \cdots, r;$$

where

$$h_{i_{(\alpha,\beta)}}(x(t)) = \begin{cases} h_i^1(x(t)) & when\ h_i^1(x(t)) \geq \alpha\ or\ h_i^2(x(t)) \leq \beta \\ 0 & else \end{cases}, \alpha + \beta \leq 1, i = 1, 2, 3, \cdots, r;$$

$$h_i^1(x(t)) = \frac{\mu_{P^i}(x(t))}{\sum_{i=1}^{r}\mu_{P^i}(x(t))}, h_i^2(x(t)) = \frac{\nu_{P^i}(x(t))}{\sum_{i=1}^{r}\nu_{P^i}(x(t))},$$

where $h_i^1(x(t))$ and $h_i^2(x(t))$ are respectively positive and negative membership functions.

$$\sum_{i=1}^{r} h_{i_1}(x(t)) = 1, \sum_{i=1}^{r} h_{i_2}(x(t)) = 1;$$

$$\mu_{P^i}(x_j(t)) = \prod_{j=1}^{r} \mu_{P_j^i}(x_j(t)), \nu_{P^i}(x_j(t)) = \prod_{j=1}^{r} \nu_{P_j^i}(x_j(t));$$

$\mu_{P_i^j}(x_j(t))$ and $\nu_{P_j^i}(x_j(t))$ is the membership function value of $x_j(t)$ that belongs and does not belong to the intuitionistic fuzzy numbers set P_j^i.

If we take (3) into (1, 2), we can get.

$$E\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x(t))h_j(x(t))(A_i + B_iG_j)x(t) \tag{4}$$

$$y(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x(t))h_j(x(t))(C_i + D_iG_j)x(t) \tag{5}$$

The system stability is guaranteed by determining the feedback gains G_j.

Basic LMI-based stability conditions guaranteeing the stability of the above control system in the form of (4, 5) are given in the following theorem.

Theorem 4.1 The system (3) is asymptotically stable, if there exist matrices $N_j \in R^{m \times n}$ (j = 1,2,3,..., r) and $K = K^T \in R^{n \times n}$ such that the following LMIs are satisfied:

$$K > 0 \tag{6}$$

$$E^TK = K^TE \geq 0 \tag{7}$$

$$Q_{ij} = A_iK^{-1} + K^{-1}A_i^T + B_iN_j + N_j^TB_i^T < 0 \forall i,j \tag{8}$$

where the feedback gains are defined as $G_j = N_jK$ for all j.

Proof: Considering the quadratic Lyapunov function.

$$V(x(t)) = x^T(t)E^TKx(t),$$

where $0 < K = K^T \in R^{n \times n}$.

then

$$\dot{V}(x(t)) = \dot{x}^T(t)E^TKx(t) + x^T(t)E^TK\dot{x}(t) = (E\dot{x}(t))^TKx(t) + x^T(t)K^T(E\dot{x}(t))$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_jx^T(t)KK^{-1}\{A_i^TK + K^TN_j^TB_i^TK + K^TA_i + K^TB_iN_jK\}\}K^{-1}Kx(t),$$

let $Z = Kx(t)$, then

$$\dot{V}(x(t)) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j x^T(t) K K^{-1} \left\{ A^T{}_i K + K N_j^T B_i^T K + K A_i + K B_i N_j K \right\} K^{-1} K x(t)$$

$$= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j Z \left(K^{-1} A^T{}_i + N_j^T B_i^T + A_i K^{-1} + B_i N_j \right) Z.$$

As $Q_{ij} = A_i K^{-1} + K^{-1} A_i{}^T + B_i N_j + N_j{}^T B_i{}^T < 0$, so the system (3) is asymptotically stable.

5. Simulation example

Example 5.1: Considering an inverted pendulum, subject to parameter uncertainties [12–15] as the nonlinear plant to be controlled. The dynamic equation for the inverted pendulum is given by.

$$\ddot{\theta}(t) = \frac{g \sin(\theta(t)) - a m_p L \dot{\theta}(t)^2 \sin(2\theta(t))/2 - a \cos(\theta(t)) \mu(t)}{4L/3 - a m_p L \cos^2(\theta(t))}$$

Where $\theta(t)$ is the angular displacement of the pendulum, $g = 9.8$ m/s^2 is the acceleration due to gravity, $m_p \in [m_{p_{\min}}, m_{p_{\max}}] = [2,3] kg$ is the mass of the pendulum, $M_c \in [M_{\min}, M_{\max}] = [8, 12]$.

Kg is the mass of the cart, $a = 1/(m_p + M_c)$, $2L = 1$ m is the length of the pendulum, and $u(t)$ is the force (in newtons) applied to the cart. The inverted pendulum is considered working in the operating domain characterized by $x_1 = \theta(t) \in [-5\pi/12, 5\pi/12]$ and $x_2 = \dot{\theta}(t) \in [-5,5]$.

Rule 1: If $x_1(t)$ is M_1^1, $x_2(t)$ is M_2^1, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 10.0078 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.1765 \end{pmatrix} \mu(t);$$

Rule 2: If $x_1(t)$ is M_1^2, $x_2(t)$ is M_2^2, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 10.0078 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.0261 \end{pmatrix} \mu(t);$$

Rule 3: If $x_1(t)$ is M_1^3, $x_2(t)$ is M_2^3, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 18.4800 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.1765 \end{pmatrix} \mu(t);$$

Rule 4: If $x_1(t)$ is M_1^4, $x_2(t)$ is M_2^4, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 18.4800 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.0261 \end{pmatrix} \mu(t);$$

Next, according to the ideas based on the principles of interpolation and interval coverage, we firstly change the interval-valued T-S fuzzy model of inverted

pendulum into the special (α,β)- pythagorean fuzzy descriptor systems of inverted pendulum as follows.

Rule 1: If $x_1(t)$ is $P_1^1(x_1(t))$, $x_2(t)$ is $P_2^1(x_2(t))$, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 10.0078 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.1765 \end{pmatrix} \mu(t);$$

Rule 2: If $x_1(k)$ is $P_1^2(x_1(t))$, $x_2(t)$ is $P_2^2(x_2(t))$, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 10.0078 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.0261 \end{pmatrix} \mu(t);$$

Rule 3: If $x_1(t)$ is $P_1^3(x_1(k))$, $x_2(t)$ is $P_2^3(x_2(t))$, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 18.4800 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.1765 \end{pmatrix} \mu(t);$$

Rule 4: If $x_1(t)$ is $P_1^4(x_1(k))$, $x_2(t)$ is $P_2^4(x_2(t))$, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 18.4800 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.0261 \end{pmatrix} \mu(t);$$

According to the theorem 4.1, we can get.

$$K = \begin{pmatrix} 1/7 & -1/7 \\ -1/7 & 8/7 \end{pmatrix}, K^{-1} = \begin{pmatrix} 8 & 1 \\ 1 & 1 \end{pmatrix}, N_1 = N_3 = (100 \quad 100), N_2 = N_4 = (1000 \quad 1000),$$

So the above (α,β)-pythagorean fuzzy descriptor systems of inverted pendulum is asymptotically stable, and the state feedback-gains matrices $G_1 = G_3 = (0 \quad 100)$, $G_2 = G_4 = (0 \quad 1000)$.

The first case, suppose $x_1(0) = -\frac{11\pi}{29}, x_2(0) = -0.88, \alpha = 0.3, \beta = 0.25$, then take the variable $x_1(t)$ as the main factor of the control, and according to **Table 2** we can control in three steps, i.e. $x_1(0) = -\frac{11\pi}{29} \to x_1(t_1) \to x_1(t_2) \approx 0$ and $0 < t_1 \leq t_2$.

When $x_1(0) = -\frac{11\pi}{29}, x_2(0) = -0.88$, and $\alpha = 0.30, \beta = 0.25$, according to **Table 2** we can get $\mu_{P_1^1}(x_1(0)) = \mu_{P_1^2}(x_1(0)) = 0.69$, $\nu_{P_1^1}(x_1(0)) = \nu_{P_1^2}(x_1(0)) = 0.72, \mu_{P_1^3}(x_1(0)) = \mu_{P_1^4}(x_1(0)) = 0$, $\nu_{P_1^3}(x_1(0)) = \nu_{P_1^4}(x_1(0)) = 1$, $\mu_{P_2^1}(x_2(0)) = \mu_{P_2^2}(x_2(0)) = 0.02, \nu_{P_2^1}(x_2(0)) = \nu_{P_2^2}(x_2(0)) = 1$, $\mu_{P_2^3}(x_2(0)) = \mu_{P_2^4}(x_2(0)) = 0.40$, $\nu_{P_2^3}(x_2(0)) = \nu_{P_2^4}(x_2(0)) = 0.92$, noteworthy, $\mu_{P_1^1}(x_1(0)) + \nu_{P_1^1}(x_1(0)) = \mu_{P_1^2}(x_1(0)) + \nu_{P_1^2}(x_1(0)) = 1.41 > 1$, $\mu_{P_2^3}(x_2(0)) + \nu_{P_2^3}(x_2(0)) = \mu_{P_2^4}(x_2(0)) + \nu_{P_2^4}(x_2(0)) = 1.32 > 1$. Then according to Definition 2.1, taking it one step further, we can get $h_1^1 = 0.49, h_1^2 = 0.22, h_2^1 = 0.49, h_2^2 = 0.22, h_3^1 = 0.01, h_3^2 = 0.28, h_4^1 = 0.01, h_4^2 = 0.28$, so $h_1 = 0.5, h_2 = 0.5, h_3 = 0, h_4 = 0$, according to 4.2, so the overall fuzzy model of the $(0.30, 0.25)$- pythagorean fuzzy descriptor systems is.

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 0.25 \\ 2.5019 & -13.929 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},$$

The solution of the systems is $x(t) = \begin{pmatrix} -\dfrac{11}{29}\pi + 0.25t \\ 0.2172 - 0.045t - 1.0972e^{-13.93t} \end{pmatrix}$

When $x_1(4) \approx -0.19103, x_2(4) \approx 0.0372$, and $\alpha = 0.30, \beta = 0.25$, according to **Table 2**, we can get $\mu_{P_1^1}(x_1(4)) = \mu_{P_1^2}(x_1(4)) = 0.03$, $\nu_{P_1^1}(x_1(4)) = \nu_{P_1^2}(x_1(4)) = 1, \mu_{P_1^3}(x_1(4)) = \mu_{P_1^4}(x_1(4)) = 0.20$, $\nu_{P_1^3}(x_1(4)) = \nu_{P_1^4}(x_1(4)) = 0.98$, $\mu_{P_2^1}(x_2(4)) = \mu_{P_2^2}(x_2(4)) = 0.50$, $\nu_{P_2^1}(x_2(4)) = \nu_{P_2^2}(x_2(4)) = 0.87$, $\mu_{P_2^3}(x_2(4)) = \mu_{P_2^4}(x_2(4)) = 0$, $\nu_{P_2^3}(x_2(4)) = \nu_{P_2^4}(x_2(4)) = 1$, noteworthy, $\mu_{P_1^1}(x_1(4)) + \nu_{P_1^1}(x_1(4)) = \mu_{P_1^2}(x_1(4)) + \nu_{P_1^2}(x_1(4)) = 1.03 > 1$, $\mu_{P_2^1}(x_2(4)) + \nu_{P_2^1}(x_2(4)) = \mu_{P_2^2}(x_2(4)) + \nu_{P_2^2}(x_2(4)) = 1.37 > 1$. Then according to Definition 2.1, taking it one step further, we can get $h_1^1 = 0.49$, $h_1^2 = 0.23$, $h_2^1 = 0.49$, $h_2^2 = 0.23$, $h_3^1 = 0.01$, $h_3^2 = 0.27$, $h_4^1 = 0.01$, $h_4^2 = 0.27$, so $h_1 = 0.50$, $h_2 = 0.50$, $h_3 = 0$, $h_4 = 0$, then according to 4.2, so the overall fuzzy model of the (0.3,0.3)- pythagorean fuzzy descriptor systems is.

$$\begin{pmatrix} \dot{x}_1(t-4) \\ \dot{x}_2(t-4) \end{pmatrix} = \begin{pmatrix} 0 & 0.25 \\ 2.5019 & -13.929 \end{pmatrix} \begin{pmatrix} x_1(t-4) \\ x_2(t-4) \end{pmatrix},$$

The solution of the systems is $x(t) = \begin{pmatrix} -0.19103 + 0.25t \\ 0.03754 - 0.045t - 0.00034e^{-13.93t} \end{pmatrix}$

When $x_1(4.764) \approx -0.0000344$, $x_2(4.764) \approx 0.00323$, so the overall fuzzy model of the (0.30, 0.25)-pythagorean fuzzy descriptor systems is E-asymptotic stability. But it takes a shorter time (**Figure 1**).

The second case(interval-valued T-S fuzzy model of inverted pendulum), suppose $x_1(0) = -\frac{11\pi}{29}, x_2(0) = -0.88$, then take the variable $x_1(t)$ as the main factor of the control, and according to **Table 1** we can control in three steps, i.e. $x_1(0) = -\frac{11\pi}{29} \to x_1(1) \to x_1(t) \approx 0$.

When $x_1(0) = -\frac{11\pi}{29} \in \left[-\frac{11\pi}{29}, 0\right)$, $x_2(0) = -0.88 \in [-0.88, 0)$, and $\lambda_1 = \lambda_2 = \frac{1}{2}$, according to **Table 1**, we can get $h_1 = 0.26$, $h_2 = 0.58$, $h_3 = 0.05$, $h_4 = 0.11$, according to theorem 4.1, so the overall interval-valued fuzzy model of the interval-valued fuzzy descriptor systems is

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -48.561 & -59.925 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},$$

The solution of the systems is $x(t) = \begin{pmatrix} -1.2229e^{-0.82t} + 0.0319e^{-59.1t} \\ 1.0048e^{-0.82t} - 1.8848e^{-59.1t} \end{pmatrix}$;

When $x_1(1) = -0.5386 \in [-0.5386, 0)$, $x_2(1) = 0.4425 \in [0, 0.4425)$, and $\lambda_1 = \lambda_2 = \frac{1}{2}$, according to **Table 1**, we can get $h_1 = 0.32$, $h_2 = 0.25$, $h_3 = 0.24$, $h_4 = 0.19$, according to 4.2, the overall interval-valued fuzzy model of the interval-valued fuzzy descriptor systems is
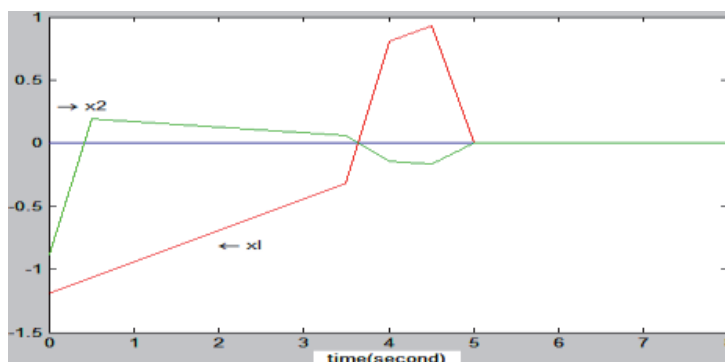
Figure 1.
$x_1(t)$ and $x_2(t)$ under the (0.30,0.25)- pythagorean fuzzy descriptor systems.

| Left membership functions	Right membership functions
$M_{M_1^1}^1(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$M_{M_1^1}^3(x_1) = 1 - 0.23e^{-\frac{x_1^2}{0.25}}$
$M_{M_1^2}^1(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$M_{M_1^2}^3(x_1) = 1 - 0.23e^{-\frac{x_1^2}{0.25}}$
$M_{M_1^3}^1(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$M_{M_1^3}^3(x_1) = e^{-\frac{x_1^2}{1.2}}$
$M_{M_1^4}^1(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$M_{M_1^4}^3(x_1) = e^{-\frac{x_1^2}{1.2}}$
$M_{M_2^1}^1(x_2) = 0.5e^{-\frac{x_2^2}{0.25}}$	$M_{M_2^1}^3(x_2) = e^{-\frac{x_2^2}{1.5}}$
$M_{M_2^2}^1(x_2) = 1 - e^{-\frac{x_2^2}{1.5}}$	$M_{M_2^2}^3(x_2) = 1 - 0.5e^{-\frac{x_2^2}{0.25}}$
$M_{M_2^3}^1(x_2) = 0.5e^{-\frac{x_2^2}{0.25}}$	$M_{M_2^3}^3(x_2) = e^{-\frac{x_2^2}{1.5}}$
$M_{M_2^4}^1(x_2) = 1 - e^{-\frac{x_2^2}{1.5}}$	$M_{M_2^4}^3(x_2) = 1 - 0.5e^{-\frac{x_2^2}{0.25}}$

Table 1.
The membership functions of the IT-2 T-S fuzzy model of inverted pendulum.

Membership functions	Non-membership functions
$\mu_{P_1^1}(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$\nu_{P_1^1}(x_1) = \sqrt{1 - \left(1 - e^{-\frac{x_1^2}{1.2}}\right)^2}$
$\mu_{P_1^2}(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$\nu_{P_1^2}(x_1) = \sqrt{1 - \left(1 - e^{-\frac{x_1^2}{1.2}}\right)^2}$
$\mu_{P_1^3}(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$\nu_{P_1^3}(x_1) = \sqrt{1 - \left(0.23e^{-\frac{x_1^2}{0.25}}\right)^2}$
$\mu_{P_1^4}(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$\nu_{P_1^4}(x_1) = \sqrt{1 - \left(0.23e^{-\frac{x_1^2}{0.25}}\right)^2}$
$\mu_{P_2^1}(x_2) = 0.5e^{-\frac{x_2^2}{0.25}}$	$\nu_{P_2^1}(x_2) = \sqrt{1 - \left(0.5e^{-\frac{x_2^2}{0.25}}\right)^2}$
$\mu_{P_2^2}(x_2) = 1 - e^{-\frac{x_2^2}{1.5}}$	$\nu_{P_2^2}(x_2) = \sqrt{1 - \left(1 - e^{-\frac{x_2^2}{1.5}}\right)^2}$
$\mu_{P_2^3}(x_2) = 0.5e^{-\frac{x_2^2}{0.25}}$	$\nu_{P_2^3}(x_2) = \sqrt{1 - \left(0.5e^{-\frac{x_2^2}{0.25}}\right)^2}$
$\mu_{P_2^4}(x_2) = 1 - e^{-\frac{x_2^2}{1.5}}$	$\nu_{P_2^4}(x_2) = \sqrt{1 - \left(1 - e^{-\frac{x_2^2}{1.5}}\right)^2}$

Table 2.
The membership functions and non-membership functions of (α,β)-pythagorean fuzzy descriptor systems of inverted pendulum.

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -44.49 & -58.141 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

,

The solution of the systems is $x(t) = \begin{pmatrix} -0.4654e^{-0.78(t)} - 0.0732e^{-57.37(t)} \\ 0.4174e^{-0.78(t)} + 0.0251e^{-57.37(t)} \end{pmatrix}$;
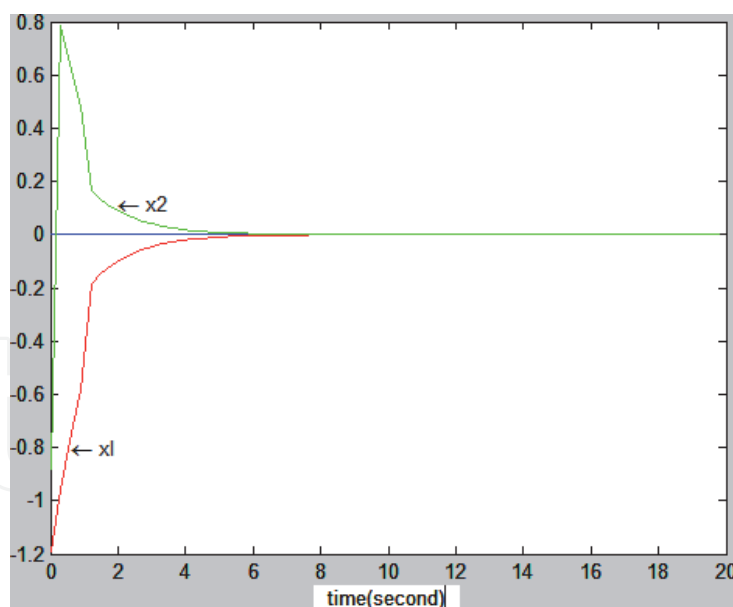
Figure 2.
$x_1(t)$ and $x_2(t)$ under the IT2 T-S fuzzy descriptor systems.

When $x_1(9.6) \approx -0.0006$, $x_2(9.6) \approx -0.0005$, so the IT2 T-S fuzzy descriptor system of the inverted pendulum will to be stable too.

Thus the stable control time of the (0.30,0.25)-pythagorean fuzzy descriptor systems of inverted pendulum is **4.836** second shorter than the interval-valued T-S fuzzy descriptor systems of the inverted pendulum (**Figure 2**).

Remark 5.1: In this way, the (0.30,0.25)-pythagorean fuzzy descriptor systems can get the better effect than the control effect of interval-valued T-S fuzzy model of inverted pendulum. It is easy to see that the (0.30,0.25)-pythagorean fuzzy descriptor systems has the best control, and can reduce the number of rules and thus reduce the amount of calculations.

In this way, it can get the better effect than the control effect of interval-valued T-S fuzzy model of inverted pendulum. Because the feedback more or less needs a little time, when the system carries out feedback instructions, but the time has gone, so the feedback that have been given are also lagging and out of date. $(α, β)−$pythagorean fuzzy descriptor systems can be closer to the actual, and easy to control the error range. The new control method is more convenient and feasible!

6. Conclusions

In this paper, the new $(α, β)−$pythagorean fuzzy descriptor systemsare firstly introduced, and more consistent with the human way of thinking and more likely to be set up and more convenient for popularization. The new$(α, β)−$pythagorean fuzzy descriptor systems is very simply and quickly. We can do not know the control principle, but we can directly achieve good control effect. The new theory can be studied in parallel to the basic framework of the original theories and easy to promote the old theories and achieve good results. In addition, we can judge the degree of weight in the control process according to the value of the positive and negative membership functions of the rules. By setting the values of$α$and$β$, we decide whether the rules will participate in the final calculation, thereby reducing the number of the rules and the calculation process, and improving the control efficiency and effectiveness. Otherwise, T-S fuzzy descriptor systems are the special examples of $(α, β)−$pythagorean fuzzy descriptor systems. $(α, β)−$pythagorean fuzzy controller and the stability of$(α, β)−$pythagorean fuzzy descriptor systems are

deeply researched. At last, a numbers example is given to show the corollaries are corrected.

But the theoretical part of the new systems need to be in-depth studied, and specific applications are also to be further developed. For example, $(\alpha, \beta)-$ pythagorean fuzzy descriptor systems can also be used as the model of autonomous learning in order to establish intelligent control, and can be used well in unmanned driving in the future. So $(\alpha, \beta)-$ pythagorean fuzzy descriptor systems is just to meet the reality requirements.

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