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João Ferreira do Amaral[1]

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Abstract

This paper develops concepts and theoretical models that can prove useful for the study of technological revolutions both from the point of view of economic growth theory and of economic history. The basic concepts are innovative capital, technological wave and technological revolution and a comparison is made with other concepts such as industrial revolution and social revolution in the Marxian sense.

Keywords: economic growth; digital revolution; technological progress; innovation

JEL codes : E10, E11,E22,N10,O30

Introduction

This paper develops concepts and theoretical models that hopefully can prove useful for the study of technological revolutions both from the point of view of economic growth theory and of economic history.

We are now at the beginning of the so called digital revolution. In such a context the way we look at technological progress is crucial for the study of the questions related to the future of the world economy that in large measure will certainly be determined by the digital revolution.

For six decades (and even more if we consider the pioneer work of Schumpeter) growth theory considers technological progress the determinant factor of growth.

However we must recognize that it is a factor difficult to measure and even more difficult to introduce in a macroeconomic growth model.

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Growth theory has followed two basic ways of modelling technological progress (or to use a more neutral term, technological innovation):

a) The first assumes that technological innovation is exogenous, that is something not related to relevant economic variables. There are two alternatives:

- as manna falling from Heaven

- as something associated with another production factor (mainly capital or labour)

The association with capital may in turn take two alternatives: one alternative considers technological innovation as something that increases the productivity of equipment; the other as something that results from the increasing proportion of the value of the equipment considered by its characteristics to be innovative (that we call *innovative capital*) on the total value of capital

b) The second way assumes that technological innovation is endogenous that is, a good that is produced in the economy and tries to determine the respective conditions of production.

Excluding the manna case, usually introduced in the models as a real function of time - that is little more than a confession of ignorance - there is no incompatibility between the other ways of formalizing technological innovation. All depends on the scope of the analysis.

If we use a simpler model it is not necessary to explain how technological innovation is produced. The same happens for many simple, aggregate models of economic growth where for example, no explanation concerning the production process of capital goods is provided. However this absence of explanation it is not incompatible with the consideration of equations that describe the accumulation of productive factors (including technological innovation), although no reason is given why the accumulation takes the forms considered. There is a *description* and not an *explanation*.

If we use a more complex model then it is necessary to formalise an explanation for the production of technological innovation as it is also necessary to explain for example the reasons for the accumulation of capital goods or for the growth of labour force.

It is not always the case that a more complex model gives us more useful results to understand growth phenomena.

This is so because the theories that have been developed so far concerning the production of technological innovation are far from satisfactory mainly because it is not possible to quantify technological innovation in a satisfactory way - even in principle - since it is a radically qualitative factor. However it is possible to quantify the *consequences* of technological innovation.

That is why we prefer a simpler model where technological innovation is considered exogenous but associated with another productive factor (capital).

In this paper no endogenous explanation is provided for technological innovation so that the models used are more descriptive than explanative ones but we think that they may help to understand the way that technological waves and technological revolutions behave or happen in time.

The paper is structured as follows:

In section 1 the concept of technological wave (TW) is introduced.

In section 2 the time path of an isolated TW is obtained when the wave follows an upward path, the *ascending phase*.

In section 3 we determine the conditions necessary for the development of a TW when there is a limited, although increasing productive capacity in the economy.

In section 4 the existence of several simultaneous TW and the occurrence and character of technological revolutions are discussed and formalised.

In the very short section 5 we discuss some possibilities regarding empirical studies using this type of models.

Finally we conclude with a comparison of the concept of technological revolution with other concepts including Marx's concept of social revolution.

1. Technological waves (TW)

1.1 General concept

We assume that the technological innovation acting on a given economy at each time period or moment is the resultant of several basic technological innovations, each of which we call a *technological wave* (TW). A TW is characterized by a sustained increase in the share of total capital of a certain type of capital (with new characteristics or new uses) that we call *innovative capital.*

This means that each TW is characterized by a technological innovation that is incorporated in a growing part of the total equipment of the economy.

Perhaps the main question that can be asked regarding this definition is what criteria we should use to qualify a specific kind of equipment as innovative.

There are at least two possibilities of looking at innovative capital: one emphasises a *substantial* characteristic of the equipment; the other emphasises a *functional* characteristic of the equipment.

For instance during the first industrial revolution the equipment incorporating steam engines was an innovative equipment because of its own *substantial* characteristics.

During the same period, traditional tools conjugated with new equipment helped to increase the proportion of industrial goods in total production and that means that even traditional equipment may be considered as innovative not because of its characteristics but because of the purpose of the use of its *functional* capabilities.

The substantial character of innovative equipment is found in new capital goods. The functional character is found in capital goods that are necessary to produce new, innovative goods.

The way we look at innovative capital depends on the questions we ask in our investigation of technological innovation.

Our main focus here will be the technological waves that are propelled by intelligent capital goods so that our point of view stands more on the substantial side. But we give in subsection 1.2 an example of the combination of the two points of view, although we don't develop this more refined analysis in the present paper.

By definition a TW is a phenomenon limited in time. On the other hand, after an innovative phase, other innovations will replace the up to that moment innovative capital that stabilizes as a proportion of total capital and subsequently declines. This happens for both cases that is, for innovative capital in the substantial sense but also in the functional sense.

A TW may originate a *technological revolution* if it causes important changes in the way that production is effectuated especially if it coincides with other TW. In section 4 we look at technological revolutions and its consequences.

Summing up, a TW is characterized by:

- a growing proportion of innovative on the total value of capital of the economy. The value of the innovative capital at moment *t* is represented by *C(t)*

- a value of the stock of capital at moment *t* designated by *K(t)*

- a real function of time, $a(t)$, $0 \le a(t) \le 1$, monotonically increasing, assumed to be continuous and differentiable, defined by the relation $a(t) \equiv C(t)/K(t)$

Remark 1. Note that from the methodological point of view there is a difference between the present formulation and the traditional way of looking to innovation as a function of time. In the present formulation the function with values *a(t)* although mathematically a function of time is not *caused* by the passing of time. It reflects decisions of economic policy and firm policies that can in principle be made explicit. We don´t introduce such an explanation because as we have mentioned above as far as we can see is not yet available any solid theory that could help us to make explicit the variables that determine the values of the function *a*. From our point of view in the absence of such a theory is better to keep the model simple. However there is a cost: the relation $a(t) \equiv C(t)/K(t)$ is not a causal relation but only a description of the evolution of the proportion of innovative capital.

Remark 2. The function *a(t)* is considered monotonically increasing because we focus only on the ascending phase of the TW.

1.2 Innovative capital and digital revolution

In what concerns the so called digital revolution we can elaborate a little more on the characteristics of the respective innovative capital.

The development of the digital economy can be seen from two points of view. One measures the progress of the digital economy as the growing weight of the activities more directly linked to digital economy on the value of GDP (this is the case for instance of the IMF paper, 2018, on the measurement of the digital economy).

The other point of view - not necessarily incompatible with the first - is the one that informs the present paper that is that focus on the increase of innovative digital capital.

To be more concrete let us give an example of a possible approach.

Start by introducing two dichotomous classifications one related to substantial characteristics and the other with functional characteristics.

The first classification, related to substantial characteristics distinguishes between intelligent and non-intelligent equipment. The first kind includes those capital goods that use a program to perform tasks of production/transmission of information (computers) or of production of goods and services (robots, according to the ISO definition[2]).

The second classification is of a functional type and distinguishes between equipment that produces innovative products and equipment that produces traditional products:

If we indicate with the indexes *ij (i=1,2; j=1,2)* respectively intelligent and non-intelligent capital goods (index *i*) and respectively capital that produces innovative and capital that produces non-innovative products (index j) we can write for total capital existent at moment *t*

$$K(t) = K_{11}(t)+ K_{12}(t)+ K_{21}(t)+ K_{22}(t)$$

The choice of the more adequate indicators to represent innovative capital depends of the purpose of the research. From our point of view an important indicator – that we use in the following sections -is the sum

$$C(t) \equiv K_{11}(t)+ K_{12}(t)$$

that is, considering total intelligent equipment as innovative capital, and the associated proportion of capital on total capital $C(t)/K(t)$.

Another indicator, based on functional considerations probably useful for other theoretical contexts is

$$[K_{11}(t)+K_{21}(t)] / K(t)$$

that is the proportion of intelligent and non-intelligent equipment that produces innovative products (both consumption and capital goods) on total capital.

In the next section we consider a TW represented by an increase in the proportion of substantial innovative capital (computers and robots) that is $C(t) \equiv K_{11}(t)+ K_{12}(t)$ on total

[2] An "actuated mechanism programmable in two or more tasks with a degree of autonomy, moving within its environment, to perform intended tasks".

capital. As already mentioned we concentrate our analysis on the ascending phase of the TW when the total stock of capital increases with time.

2. A simple model of TW: the evolution of GDP and of the stock of capital

We base the analysis on the following assumptions:

a) The net investment proportion of innovative capital on GDP is determined by exogenous factors that are not analysed is this paper and it is described by a real function of t,

$u(t) \equiv C'(t)/Y(t)$ (for any function with values $x(t)$, $x'(t)$ stands for the derivative of x at point t).

b) The net investment proportion of total capital on GDP, is a function v that takes the values

$v(t) \equiv K'(t)/Y(t)$

Remark. Obviously, $u(t) \leq v(t)$ and more realistically, $u(t) < v(t)$.

c) The evolution of the ratio of the two investment ratios $u(t)/v(t)$ (represented by $s(t)$) verifies the following relation:

1) $s(t) = [1-a(t)]S_0 + a(t)S_1$

where S_0 and S_1 are positive constant numbers, with $1 \geq S_1 \geq S_0$ and where $a(t)$ is the ratio $C(t)/K(t)$.

The relation 1) is based on the idea that the larger is the stock of innovative capital relatively to the stock of total capital, the higher is the ratio of the net investment ratio of innovative capital to the net investment ratio of total capital.

Note also that this assumption makes sense only for an ascending phase of the TW. In the descending phase $s(t)$ declines for high values of $a(t)$.

These three assumptions allow us to describe the TW without entering in the discussion about causality relations.

Assuming $a(t)$ continuous and differentiable we have

$a(t) = C(t)/K(t)$ so that

2) $a'(t) = [u(t)/v(t) – a(t)]K'(t)/K(t)$

Therefore

3) $a'(t) = [S_0 - (1- S_1 + S_0)a(t)]K'(t)/K(t)$

As we concentrate in the ascending phase of the TW, *a(t)* is a monotonous increasing function and the stock of capital also increases so that

$S_0 - (1- S_1 + S_0)a(t) > 0$

As we assumed $1 > S_1 - S_0$

we get

4) $a(t) < S_0/(1- S_1 + S_0)$

That gives us an upper limit (≤ 1) for the value of *a(t)* in the ascending phase.

Using these relations we can determine the time-path for the stock of capital and for GDP

Dividing both members of 3) by $[S_0 - (1- S_1 + S_0)a(t)]$ and integrating we get

$[S_0 - (1- S_1 + S_0)a(t)] = c^*K(t)^{-(1- S_1 + S_0)}$

where *c** is the constant number

$c^* = K(0)^{(1- S_1 + S_0)} [S_0 - (1- S_1 + S_0)a(0)]$

so that

5) $K(t) = K(0)\{[S_0 - (1- S_1 + S_0)a(0)]/ [S_0 - (1- S_1 + S_0)a(t)]\}^{1/(1- S_1 + S_0)}$

and we obtain a path for the evolution of the stock of total capital.

Based on this path we get the path for GDP after obtaining the derivative of the expression of *K(t)*

6) $K'(t) = B(0)\ a'(t)/ [S_0 - (1- S_1 + S_0)a(t)]^{(2- S_1 + S_0)/(1- S_1 + S_0)}$

where $B(0) \equiv K(0) [S_0 - (1- S_1 + S_0)a(0)]^{1/(1- S_1 + S_0)}$

Dividing by *v(t)* we get

7) $Y(t) = K'(t)/v(t) = B(0)\ a'(t)/\{v(t)[S_0 - (1- S_1 + S_0)a(t)]^{(2- S_1 + S_0)/(1- S_1 + S_0)}\}$

Equation 7) should not be interpreted as expressing a *causality* relation that is, as if the value *Y(t)* was determined by the values of *v(t)* and *a(t).* What the equation 7) expresses is a *compatibility* relation between the values *v(t)*, *a(t)* and *Y(t)*. This relation will be used again in section 3.

The previous result can be made more useful if we introduce an additional assumption on the evolution of the proportion *a(t)* that reflects empirical findings for several phenomena related to technological progress :

d) The path *a(t)* in its ascending phase of the TW is well represented by the logistic function

$a(t) = Q/(1+Pe^{-nt})$

where $n > 0$, $Q > 0$, $P = Q/a(0) - 1$

As we will see in section 3 we have necessarily $P > 1$, so that $Q/a(0) > 2$ and according to 4) above $a(0) < \min [Q/2, \; S_0/(1- S_1 + S_0)]$.

Substituting this expression for *a(t)* in 7) we get the following path for the evolution of GDP:

$$8)\ Y(t) = B(0)\ [nPQe^{-nt}/(1+Pe^{-nt})^2]/\{v(t)[S_0 - Q\ (1- S_1 + S_0)/(1+Pe^{-nt})]^{(2- S_1 + S_0)/(1- S_1 + S_0)}\}$$

We have determined a path for GDP that is consistent with the assumptions. However we still don't know if the path is compatible with the evolution of the productive capacity of the economy, because so far there is no mention of a production function.

In the next section we deal with this problem.

3. TW and the production function of the economy. The concept of innovative capital and technological progress

3.1 Continuous time

Obviously the time path given by 8) is feasible only if there is sufficient productive capacity, which we represent by an aggregate production function.

We assume the following two characteristics of the production function:

a) The only limitative primary factor is capital, that is human capital and natural resources are not limitative. It is a simplification that does not affect the main results of the analysis (the possible existence of technological restrictions when we include human capital in a different context is discussed in Amaral, 2019).

b) The impact of technological innovation on the productive capacity of the economy is described by an increasing function of a, so that the real impact of technological innovation is to increase the average productivity of capital.

With these assumptions at moment t the maximum value of GDP that can be produced is given by $Y^*(t) = f(a(t))K(t)$, where f is an increasing function of a (and also of t since we focus only at the ascending phase of the TW).

Note however that the interpretation of the equality $Y^*(t) = f[a(t)]K(t)$ is not the same as the equality $Y^*(t) = MA(t)K(t)$, used in some growth models where $A(t)$ is a factor representing technological progress.

In the this case $A(t)$ is a somewhat mysterious undefined factor, because it represents a time dependent average productivity of capital with no explanation of the reason why this productivity growths.

According with our formulation $f[a(t)]$ increases because the proportion of innovative capital on total capital increases and there is no mystery about this.

On the other hand – and this is a crucial difference – in our formulation the variation of $a(t)$ and consequently of $f[a(t)]$ is bounded (because $a(t) < 1$), something that does not necessarily happen using the other formulation.

Some of other possible properties of f can also be of interest.

3.2 Properties of function f

First note that all the influence of time on the productivity of capital is mediated by the function $a(t)$ since we reject the fall of the manna . We have a composite function $f[a(t)]$ and not $f[a(t),t]$.

Additionally we assume that the properties of function f reflect the spreading process of the innovation that leads to the accumulation of innovative capital. One way of interpreting this process is to look at possible concavity/convexity properties. When f is concave in an interval $[a^*, a^{**}]$ this means that increases in the value of f due to increases in the value of a decline when a increases. This may be interpreted as representing a phase of the spread of the innovation in its beginnings when network effects are still small. If later on, for higher values of a, $a > a^{**}$ f is convex this means that the network effects in that phase became important.

This interpretation can be used to obtain the behaviour of f as a composite function of t in the special case where $a(t)$ is monotonous increasing as illustrated by the following example.

First note that we may write

$df/dt = (df/da)a'(t)$

As df/da is positive (an increase in the proportion of innovative capital increases the productivity of capital) and as we consider only the ascending phase of the wave (so that $a'(t)$ is positive for all t), df/dt is positive and f is a monotonous increasing function of t.

Taking the second derivative we get

$d^2f/dt^2 = (d^2f/da^2)\, a'(t)^2 + (df/da)a''(t)$

The behaviour of function $f(a)$ is conditioned by the existence or non-existence of network effects associated to the innovative capital. If those network effects become significant in due time a possible situation as we mentioned above is one such that f is a concave function in the period where network effects are not significant (that is for a less than a certain value a^*) and convex for values higher than a^*.

On the other hand we assumed that $a(t)$ is a logistic function, so that is strictly convex for $t < t^{**}$ for a certain value t^{**} and concave for t higher than t^{**}.

If $a^* > a(t^{**})$ than there is a period (t^{**}, t^{***}) - where t^{***} is such that $a(t^{***}) = a^*$, (t^{***} is an unique value because $a(t)$ is monotonous increasing) – such that for the values that a takes in this period (t^{**}, t^{***}) we have $d^2f/da^2 < 0$ and also $a''(t) < 0$ so that $d^2f/dt^2 < 0$ and f as a function of t is consequently strictly concave in that period.

This means that in this case the positive network effects are felt only for high values of a, and this may imply that for a certain time the macroeconomic effects of the innovative capital are not sufficient to encourage the investment in this type of capital perhaps eventually originating an abortive TW.

However if $a^* < a(t^{**})$, that is if the positive network effects are felt relatively early in the process than there is at least a period (t^{***}, t^{**}) where $d^2f/dt^2 > 0$, so that f as a function of t

is convex and we can expect an acceleration in the growth of the productivity of capital. That is, a fast growing impact of the TW in that period.

From these considerations *we emphasize the importance of public policies that help to reduce the time needed for the positive network effects to be felt*.

3.3 Discrete time

All this analysis was done using continuous time functions. However this is only an approximation and may have undesirable consequences because the dimension of the relation $Y^*(t)/K(t)$ tends to infinity when $t+\delta t$ tends to t when δ tends to 0.

The dimension of K is the monetary unit (v. g. euros) and the dimension of the GDP is euros by a given temporal period (v. g. year) so that the dimension of $f(a(t)) = Y^*(t)/K(t)$ is the reciprocal of the time period. As this period tends to a moment t the dimension of $f(a(t))$ tends to infinity

To avoid this problem we consider a function with discrete time and we convert to discrete time all the results obtained for continuous time.

We consider the expression

$Y^*_t = f(a_t)\ K_t$

where t is a temporal period (not a moment) Y^*_t is the maximum value of GDP that can be obtained with K_t which is the stock of capital existing at the beginning of period t, a_t is the value of a at the beginning of the period and $f(.)$ a real function monotonous increasing with the values of a.

On the other hand using a discrete approximation of relation 7) above we get

9) $Y(t) = B(0)\ (a_{t+1} – a_t)\} / \{v_t[S_0 – (1- S_1 + S_0)a_t]^{(2- S_1 +S_0) /(1- S_1 +S_0)}\}$

where Y_t is the value of GDP produced in the period t.

Under the plausible assumption that a_t follows a path that is the discrete equivalent of the continuous logistic function $a(t) = Q/(1+Pe^{-nt})$ we have $a_t = Q/(1+Pb^t)$, $0 < b < 1$. Substituting $Q/(1+Pb^t)$ in the expression 9) we can easily obtain the discrete evolution of Y_t as a function of t and v_t.

However this result is incomplete because is still missing a feasibility condition which is the constraint that the value of Y_t cannot be higher than the value of maximum productive capacity Y^*_t, that is

$Y_t \leq Y^*_t$

Using expressions 5) and 9) we get

10) $Y(t) = B(0)\,(a_{t+1} - a_t)\} / \{v_t[S_0 - (1\text{-}\,S_1 + S_0)a_t]^{(2\text{-}\,S_1 + S_0)/(1\text{-}\,S_1 + S_0)}\}$

$\leq f(a_t)\,K(0)\,\{[S_0 - (1\text{-}\,S_1 + S_0)a(0)] / [S_0 - (1\text{-}\,S_1 + S_0)a_t]\}^{1/(1\text{-}\,S_1 + S_0)}$

which can be put in the form

11) $(a_{t+1} - a_t) / \{f(a_t)[S_0 - (1\text{-}\,S_1 + S_0)a_t]\} \leq v_t$, where $a_t = Q/(1+Pb^t)$.

Expression 11) gives us an important result, that is *the existence of a constraint on the total investment ratio* v_t. This constraint is determined exclusively by the proportion a_t of innovative capital on total capital and depends on time when $a_t = Q/(1+Pb^t)$. This means that, with our assumptions, for a TW to have success is necessary that the total net investment ratio of the economy (and not only the innovative net investment ratio) is not lower than the left hand side of the inequality 11).

The more productive is innovative capital (that is the higher is the value of $f(a_t)$ for a given value a_t) the lower is the value of $(a_{t+1} - a_t)/\{f(a_t)[S_0 - (1\text{-}\,S_1 + S_0)a_t]\}$. This means that the value of v_t may be lower than in the case of less productive innovative capital.

These results were obtained for a single TW. In the next section we deal with the case of a bundle of TW that is, a situation where technological progress is more intense and diversified and capable of originating a technological revolution.

4. Technological revolutions. Coherence of a technological revolution

4.1 Concept and behaviour of the variables

We define *technological revolution* generated in a given economy as a period of accelerated growth characterized by the impact of a given bundle of TW's with a certain degree of coherence. We discuss the concept of coherence at subsection 4.2.

This concept of technological revolution comes near the concept of industrial revolution adopted by Caron (1997, p. 11):

“ *radical change in the production patterns of production and consumption linked to the emergence and development of new industries*[3]” .

However our concept is more limited than Caron’s since it doesn´t consider explicitly patterns of consumption, so that it means that a technological revolution in our sense doesn´t always induce an industrial revolution in Caron’s sense.

We consider a situation where there are *m* types of innovative capital and only one type of non-innovative one.

We designate by K_i the innovative capital stock of type i $(i=1,\ldots m)$

We have obviously

$C(t) = \Sigma_{i=1}^{m} K_i(t)$

and

$K(t) = C(t) + K_{m+1}(t)$

We assume that for each stock *i* of the *m* stocks of innovative capital we have

$K_i(t) = b_i(t)K(t)$

Where $\Sigma b_i(t) < 1$ and all the $b_i(t)$ are increasing functions.

The non- innovative capital stock $K_{m+1}(t)$ is given by

$K_{m+1}(t) = [1 - \Sigma b_i(t)]K(t)$

and it is of course a decreasing proportion of $K(t)$.

Let us focus on innovative capital and define the concept of coherence of a bundle of TW’s.

4.2 Definition of coherence of a bundle of TW’s

As in the case of $a(t)$ the evolution following a logistic curve is plausible for each $b_i(t)$ that is

for each $i = 1,\ldots m$

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[3] My translation.

$b_i(t) = Q_i /(1+P_i e^{-n(i)t})$

Based on this assumption on the evolution of the b_i we define the following concept of (relative) coherence.

Definition (Coherence). Coherence in an economic revolution is higher the nearer in time are the moments of highest instantaneous velocity of the functions $b_i(t)$, $i =1,\ldots m$.

By the previous assumption

$b_i(t) = Q_i/(1+P_i e^{-n(i)t})$ with $P_i = Q_i/b_i(0) - 1$

It is easy to determine the moment t^*_i where there $b_i'(t^*_i)$ is a maximum. Making the second derivative of $b_i(t)$, equal to 0 we obtain (since the second order conditions of maximum apply)

$t(i)^* = \log P_i/n(i)$.

In order for $t(i)^*$ to be positive is necessary that $P_i > 1$ that is that $Q_i > 2b_i(0)$.

We define the degree of coherence of the bundle as the value of some expression that is a decreasing function of the dispersion of the $t(i)^*$.

To obtain a value that is always a finite number we may use the following simple expression for the degree of coherence

$N = 1/(1+M)$ where $M = \Sigma\, [t(i)^* - t^*]^2$ and $t^{**} \equiv \Sigma\, t(i)^*/m$

Of course the values of N are such that $0 < N \leqq 1$

The degree of coherence is maximum and equals 1 when all the $t(i)^*$ are equal.

High values of N indicate that the bundle of TW's is really a process that can be the basis of a technological revolution. Obviously empirical and historical research is needed for knowing what values of N can be considered as "high".

If we focus on the digital revolution we considered in section 2 a type only of innovative capital (intelligent equipment), that is $m=1$.

However we know that other types of equipment suffer the same kind of evolution. This implies that a more detailed analysis that disaggregates the intelligent equipment and non-intelligent in several types of products is needed in future work in order to study the so-called digital revolution as a real technological revolution and not just as a TW.

4.3 Improvements in innovative capital and successive TW's

It is characteristic of innovative capital that as it initiates a TW it starts also a series of improvements in itself that accelerate during the ascending phase of the TW. Unless we see these improvements as manna falling from Heaven the alternative is to consider improvements that induce major qualitative changes as the emergence of a new type of innovative capital that starts a new TW replacing the previous innovative capital when the respective TW enters in a decline phase. This is not a technological revolution but a succession of TW corresponding to the improvements of a certain kind of innovative capital that change its productivity in a significant way.

5. Thoughts on empirical analysis

This theory of TW and technological revolutions is not very exigent in what concerns data. Apart from national accounts and the data related to innovative capital which, in what concerns intelligent equipment and the respective values of a may be obtained by sampling, the two (more elusive) pieces of information needed are the function f (which again may be obtained by extrapolating from known representative cases) and the parameters S_1 and S_0. For these parameters possibly the more sensible way to proceed is to consider several pairs of values and the corresponding scenarios.

Conclusion. Technological revolution, digital revolution and social revolution

The economic and social consequences of a technological revolution may be studied using the social revolution concept of Karl Marx.

In its Preface to *A Contribution to the Critique of Political Economy*, Marx writes:

"At a certain stage of development, the material productive forces of society come into conflict with the existing relations of production or – this merely expresses the same thing in legal terms – with the property relations within the framework of which they have operated hitherto. From forms of development of the productive forces these relations turn into their fetters. Then begins an era of social revolution. The changes in the economic foundation lead sooner or later to the transformation of the whole immense superstructure".

Our concept of technological revolution is an essential component of what Marx calls the development of material productive forces of society. As we have seen in section 4 It is also a component of an industrial revolution in the concept of Caron.

In the case of a technological revolution induced among other factors by the digital revolution if there is a high degree of coherence it will probably originate a social revolution in the sense of Marx caused by the huge transformation induced by the replacement of human labour force by intelligent machines.

Present social relations namely in what concerns property and income distribution will hardly remain unchanged. If they do change a question arises that was not (and could not) be sufficiently answered by Marx's theory. The question is: in what ways (the outburst of tensions in property relations, in income distribution, in employment, in social exclusion, in network effects etc.) will be expressed the contradictions between the development of productive forces and the relations of production?

Each social revolution will no doubt have a specific way of expressing those contradictions.

But it is important to have a theoretical framework, although necessarily incomplete that explains the forms that this contradiction will take (is taking) in the case of the digital revolution as the leader of the technological revolution.

It was not the intention of this paper to develop such a framework. I limit myself to a few commentaries.

A technological revolution as any other macro-social event involves the confrontation of powers of social agents and the more so the higher is the coherence of the revolution.

Simplifying an obviously complex phenomenon the digital revolution confronts the power of those that produce intelligent machines (product innovation), the power of the entrepreneur that innovates the process of production (process innovation), the power of the AI experts that are those that really know the limits and the capacities of intelligent machines (knowledge power) and the power (opposed to all the previous ones) of those that see their jobs at risk of being replaced by machines.

The result of this confrontation will probably be that the three powers will prevail against the last one but the outcome is dependent on the capacity of reaction of those negatively affected by the digital revolution.

It is also this capacity of reaction that will determine the transformation of existing production relations and the surge or not of a social revolution. If for example the affected workers manage to mobilize their potential for social action to impose an effective control of the programmes included in the machines and new rules of income distribution or redistribution that overcomes the impact of the substitution of machines for workers then the route is open for a deep transformation of social relations and therefore to an economic and social revolution but this will probably need new objectives and processes in what concerns workers unions.

But as mentioned above it is not our intention to focus on social revolutions. Our purpose was a more limited one: to develop a type of models that are simple but that we think are apt to inform empirical research on ascending phases of technological waves and technological revolutions that in some cases (as we think is the case of the digital revolution) are coherent and powerful enough to induce an industrial revolution in the sense of Caron or even a social revolution in the Marxian sense.

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