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# Optical propagation of partially coherent light through anisotropic biological tissues by Green's functions

J. L. Ganoza-Quintana, F. Fanjul-Vélez\*, J. L. Arce-Diego\*

Biomedical Engineering Group, TEISA Department, University of Cantabria, Av de los Castros s/n, 39005, Santander (Spain)

## ABSTRACT

Biomedical optical techniques of treatment, characterization and surgery depend on light propagation in biological tissues. As biological tissues are turbid media it is necessary to adequately analyze its influence on optical propagation parameters, such as coherence. The influence of a scatterers distribution can be analyzed using Green's functions. Green's functions are sets of impulse responses of inverse operators of differential linear operators with homogeneous boundary conditions. Optical random beams, mainly Gaussian-based, are employed to model light propagation in turbid biological tissues by Green's functions. Enhanced contrast by coherence could distinguish malignant from healthy tissues or provide diagnostic interpretation.

**Keywords:** Propagation of light, Scattering in biological tissues, Green's functions in optics, Optical properties, Numerical approaches for light-tissue interactions

## 1. INTRODUCTION

Biomedical optical techniques of treatment, characterization and surgery are strongly dependent on light propagation in tissues. Light propagation accuracy and a priori estimations are particularly significant in diagnostic techniques. Optical diagnostic techniques usually rely on intensity measurements, such as microscopy [1], fluorescence [2,3] or diffuse reflectance spectroscopy [4]. One way of further increasing contrast is the addition of polarization parameters [5,6], for instance in Optical Coherence Tomography [7]. Although, strictly speaking, reflection and refraction are also a result of scattering (absorption and reemission of electromagnetic energy by material oscillators), in practice the term scattering is used in a more restricted sense for processes that change the propagation of light from an ordered way to a random one. As biological tissues are turbid media it is necessary to adequately analyze the influence of scattering on optical propagation parameters, such as coherence. The notion of coherence is defined more generally by the correlation properties between quantities of an optical field. The influence of scatterers distribution can be analyzed using Green's functions, sets of impulse responses of inverse operators of differential linear operators with homogeneous boundary conditions. Optical random beams, mainly Gaussian-based, are employed to model light propagation in turbid biological tissues by Green's functions. For biological tissues with geometric anisotropy, the 3D power spectrum depends on parameters of the optical source and also on optical parameters of biological tissues, such as the outer and the inner scale, the refractive index variance and the anisotropic coefficients in each direction. Both the outer scale and the inner scale can be found by calculating the fractal dimension of a biological tissue using the box counting technique. The variance of the refractive index of a biological tissue can be calculated using phase contrast microscopy. Enhanced contrast by coherence could distinguish malignant from healthy tissues or provide diagnostic interpretation. The complexity of the problem increases as biological tissues present usually high scattering. As biological tissue are turbid media, with a consequent great scattering influence, it is necessary to adequately analyze its influence on optical propagation parameters, such as coherence. The influence of a particular distribution of scatterers can be analyzed by Green's functions [8]. Electromagnetic propagation could be then considered, including coherence phenomena.

Biomedical optical techniques of treatment, characterization and surgery depend on the propagation of light in biological tissues. As the biological tissue is a turbid media, with great influence of scattering, it is necessary to adequately analyze its influence on optical propagation parameters, such as coherence. The influence of a scatter distribution can be analyzed using Green's functions.

\*[fanjul@unican.es](mailto:fanjul@unican.es); [arcedj@unican.es](mailto:arcedj@unican.es); phone +34 942 20 67 30; fax +34 942 20 18 73; [www.teisa.unican.es/toa](http://www.teisa.unican.es/toa)

Next section 2 contains the theoretical model to analyze the light propagation in highly scattering biological tissues by Green's functions. Section 3 shows some of the results obtained and its discussion. Finally, section 4 includes the conclusions of the work.

## 2. SCATTERING AND GREEN'S FUNCTIONS THEORETICAL MODEL

One of the fundamental problems of field theory is the construction of solutions to linear differential equations when there is a specified source and the differential equation must satisfy certain boundary conditions. Green's functions provide a method for obtaining these solutions. Let  $F_{Ld}[f(x)] = g(x)$  be a linear differential operator plus certain boundary conditions for  $f(x)$ ,  $BC[f(x)]$  and represented in the usual Signals and Systems scheme (considering one-dimensional signals and derivative operators). In a physical problem, usually  $g(x)$  is known — source of some physical magnitude — while  $f(x)$  is the magnitude generated by  $g(x)$ . This leads directly to the definition of the Green's function as the set of functions (distributions in a rigorous way) which are the set of impulse responses of the inverse system  $F_{Ld,BC}^{-1}[g(x)] = f(x)$ . These functions, denoted by  $G(x; x')$ , have to satisfy the Green's function problem,

$$\left\{ \begin{array}{l} F_{Ld,BC}[G(x; x')] = \delta(x - x') \\ BC[G(x; x')] \end{array} \right\} \rightarrow f(x) = \int_{x'} g(x')G(x; x')dx' \quad (1)$$

Scattering in biological media represents one of the most limiting effects for optical diagnostic imaging. Along with light intensity, coherence is particularly affected by the scattering mechanism [9]. The scattered wave  $U_s(r, \omega)$  from a monochromatic incident wave  $U_i(r, \omega)$  can be expressed as a total  $U(r, \omega)$  wave, assuming the first-order Born approximation, as:

$$U(r, \omega) \sim U_i(r, \omega) + \int_D F(r', \omega)U_i(r', \omega)G(|r - r'|, \omega)d^3r' \quad (2)$$

In equation (2),  $F(r, \omega)$  is the scattering potential of the medium, and  $G(|r - r'|, \omega)$  is the outgoing free-space Green function. If incident light is partially coherent, as it is a more general case, with a cross-spectral density function  $W_i(r_1, r_2, \omega)$ , the cross-spectral density of the scattered wave can be expressed as:

$$W_s(r_1, r_2, \omega) = \iint_D W_i(r'_1, r'_2, \omega)F^*(r'_1, \omega)F(r'_2, \omega)G^*(|r_1 - r'_1|, \omega)G(|r_2 - r'_2|, \omega)d^3r'_1d^3r'_2 \quad (3)$$

The cross-spectral density can be expressed as a function of the spectral densities  $S_i(r'_1, \omega)$  and  $S_i(r'_2, \omega)$ , and the spectral degree of coherence  $\mu_i(r'_1, r'_2, \omega)$ :

$$W_s(r_1, r_2, \omega) = \iint_D \sqrt{S_i(r'_1, \omega)}\sqrt{S_i(r'_2, \omega)}\mu_i(r'_1, r'_2, \omega)F^*(r'_1, \omega)F(r'_2, \omega)G^*(|r_1 - r'_1|, \omega)G(|r_2 - r'_2|, \omega)d^3r'_1d^3r'_2 \quad (4)$$

The coherence properties can be also expressed in the space-time domain by the mutual coherence function:

$$\Gamma_s(r_{s1}, r_{s2}, \tau) = \frac{1}{r^2} \int_0^\infty S_i(\omega) \tilde{F}^*[k(s_1 - s_0), \omega] \tilde{F}[k(s_2 - s_0), \omega] e^{-i\omega\tau} d\omega \quad (5)$$

The Gaussian Schell-model (GSM) is based in the cross-spectral density function has the structure,

$$W^{(G)}(\vec{\rho}_1, \vec{\rho}_2; \omega) = A_0^2(\omega) e^{-\frac{\rho_1^2 + \rho_2^2}{4\sigma^2(\omega)}} e^{-\frac{(\vec{\rho}_1 - \vec{\rho}_2)^2}{2\delta^2(\omega)}}, \quad (6)$$

where superscript (G) stands for the Gaussian Schell-model,  $A_0^2$  is the maximum value of the spectral density (attained on the axis) and the root-mean-square (r.m.s.) widths  $\sigma^2$  and  $\delta^2$  are independent of position but generally depend on frequency.

The optical turbulence is well explained by the presence of irregularities in the refractive index or, so-called, "turbulent eddies," appearing due to fluctuations in various physical properties of matter, such as temperature, pressure and concentration of inhomogeneous chemical content. Such eddies are created in different types of matter through certain physical/chemical/biological mechanisms. Shearing and mixing of different parts of the irregular structures under influence of winds in atmosphere, currents in the ocean, cell growth and fluid transfer in bio-tissues lead to a mechanism of energy transfer among eddies of different sizes. The largest possible size of an eddy in the turbulent process is taken as the

definition of the outer scale  $L_0$  of turbulence. Larger eddies break down further into smaller ones with energy until the size of the eddy reaches the lower limit when the energy dissipates. The size of the smallest eddy before dissipation defines the inner scale  $l_0$ .

The most important statistical characteristics of the refractive index in the three-dimensional space are the first two moments: the mean value of a field [10],

$$n_0(\vec{r}) = \langle n(\vec{r}) \rangle_M, \quad (7)$$

and its covariance function,

$$B_n(\vec{r}_1, \vec{r}_2) = \langle [n(\vec{r}_1) - n_0(\vec{r}_1)][n(\vec{r}_2) - n_0(\vec{r}_2)] \rangle_M \quad (8)$$

where the angular brackets with subscript  $M$  denote the ensemble average over the realizations of the medium.

The interaction of the electromagnetic fields with the turbulent media is a very complex process in the general case when the latter are anisotropic and inhomogeneous. In the inertial range of scales the random media are often assumed to be homogeneous (statistical moments of the field are translationinvariant). Under such circumstances the relation between the spatial covariance function  $B_n(\vec{r})$  and the power spectrum  $\Phi_n(\vec{\kappa})$ , which determines the distribution of energy among the eddies of different sizes, has the form of the three-dimensional Fourier transform pair,

$$B_n(\vec{r}) = \iiint_{-\infty}^{\infty} e^{i\vec{\kappa}\vec{r}} \Phi_n(\vec{\kappa}) d^3\kappa, \quad (9)$$

$$\Phi_n(\vec{\kappa}) = \left(\frac{1}{2\pi}\right)^3 \iiint_{-\infty}^{\infty} e^{i\vec{\kappa}\vec{r}} B_n(\vec{r}) d^3r, \quad (10)$$

Here  $\vec{\kappa} = (\kappa_x, \kappa_y, \kappa_z)$  is the three-dimensional vector, whose components have the units  $m^{-1}$ , representing spatial frequencies.

A scalar GSM light beam is incident on a soft anisotropic biological tissue with anisotropy factors  $\mu_x = \mu_z \neq \mu_y$  in the plane  $z = 0$ , termed source plane or plane of incidence and propagates through it in the positive half-space  $z > 0$ .

For biological tissues with geometrical anisotropy the three-dimensional power spectrum can be written in the following form, in similarity with other anisotropic random media, [11],

$$\Phi_n(\kappa_x, \kappa_y, 0) = \frac{(2\pi)^3 \sigma_n^2 \mu_x \mu_y \mu_z e^{-\frac{\kappa_x^2}{\kappa_{m0}^2} - \frac{\kappa_y^2}{\kappa_{m0}^2}}}{\kappa_0^{3-\alpha} [\kappa_0^2 + 4\pi^2 (\mu_x^2 \kappa_x^2 + \mu_y^2 \kappa_y^2)]^{\frac{\alpha}{2}}}, \quad 3 < \alpha < 4. \quad (11)$$

Where  $\alpha$  is the power spectrum slope,  $\sigma_n^2$  is the variance of the refractive index of the bio-tissue,  $\mu_x, \mu_y$  and  $\mu_z$  are the anisotropic strength coefficients in each direction. Further,  $\vec{\kappa}_0$  is the large-scale cut-off frequency vector with magnitude  $\kappa_0 = \sqrt{\kappa_{x0}^2 + \kappa_{y0}^2 + \kappa_{z0}^2}$  and components  $\kappa_{x0} = \frac{2\pi}{L_x}, \kappa_{y0} = \frac{2\pi}{L_y}, \kappa_{z0} = \frac{2\pi}{L_z}$ , with  $L_x, L_y$  and  $L_z$  being the outer scales along  $x, y$  and  $z$ , directions, respectively:  $L_x = \mu_x L_0, L_y = \mu_y L_0$  and  $L_z = \mu_z L_0$ . Small-scale cut-off frequency vector  $\vec{\kappa}_m$  has magnitude  $\kappa_m = \sqrt{\kappa_{xm}^2 + \kappa_{ym}^2 + \kappa_{zm}^2}$  and components  $\kappa_{xm} = \frac{2\pi}{l_x}, \kappa_{ym} = \frac{2\pi}{l_y}, \kappa_{zm} = \frac{2\pi}{l_z}$  with  $l_x = \mu_x l_0, l_y = \mu_y l_0$  and  $l_z = \mu_z l_0$ .

For an anisotropic turbulent medium and the vector with Cartesian coordinates  $\vec{\rho} = (\xi, \eta)$ , the spectral density is given by,

$$S(\vec{\rho}, z) = \frac{1}{\sqrt{\Delta_x(z)\Delta_y(z)}} e^{-\frac{\xi^2}{2\sigma_0^2\Delta_x(z)}} e^{-\frac{\eta^2}{2\sigma_0^2\Delta_y(z)}} \quad (12)$$

### 3. RESULTS AND DISCUSSION

Figure 1 is plotted from Eq. (11) and it shows the changes in the spectral density of the beam along the propagation path. It is observed that increasing the propagation distance produces an elliptical beam profile.

### Spectral density of the coherent GSM beam propagating in the bio-tissue

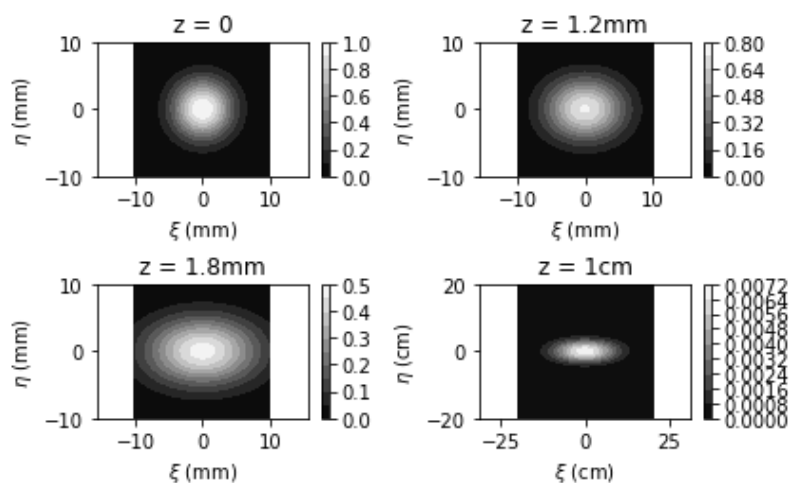


Figure 1. Spectral density of the coherent GSM beam propagating in the bio-tissue at different distances  $z$ .

## 4. CONCLUSIONS

Biomedical optical techniques of treatment, characterization and surgery are strongly dependent on light propagation. Further parameters based on polarization or coherence can provide increased contrast in diagnostic techniques. Light propagation in static highly scattering biological tissues can be analyzed by Green's functions, including coherence phenomena. Results for dermatological tumoral tissues were obtained, and the potential diagnostic contrast of coherence parameters was analyzed.

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