Evolution of Asteroid Orbits in a Restricted Three-Body Simulation

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Abstract

We study the evolution of asteroid orbits in a restricted three-body problem formulation in two dimensions, consisting of the Sun, the planet Jupiter and an unspecified asteroid of negligible mass. It was discovered by Kirkwood [l] that the distribution of asteroid orbits contains gaps for orbits whose period is commensurate with that of Jupiter. Detailed computations in three-dimensional, many-body formulations found that test bodies initially placed in a forbidden orbit did not develop large eccentricities or leave the gap even after the passage of 10^5 years [2]. While previous two-dimensional, three-body simulations, an extension of earlier work [3], showed significant departure of asteroids placed in forbidden orbits in fewer than 10 revolutions, our present work shows such orbits to be stable for at least 25,000 years. The results suggest two things: One, the two-dimensional, three-body, reduced problem (modeled in figure 3) is consistent with more detailed three-dimensional, many-body models, contrary to previous work. Secondly, the numerical integrations of the coupled equations of motion for Jupiter and the asteroid are highly sensitive to the precision in the method of computation. The demand for precise calculations for accurate predictions brings forth a problem of time complexity. With the resources available, simulations longer than 25,000 years were not practically possible.

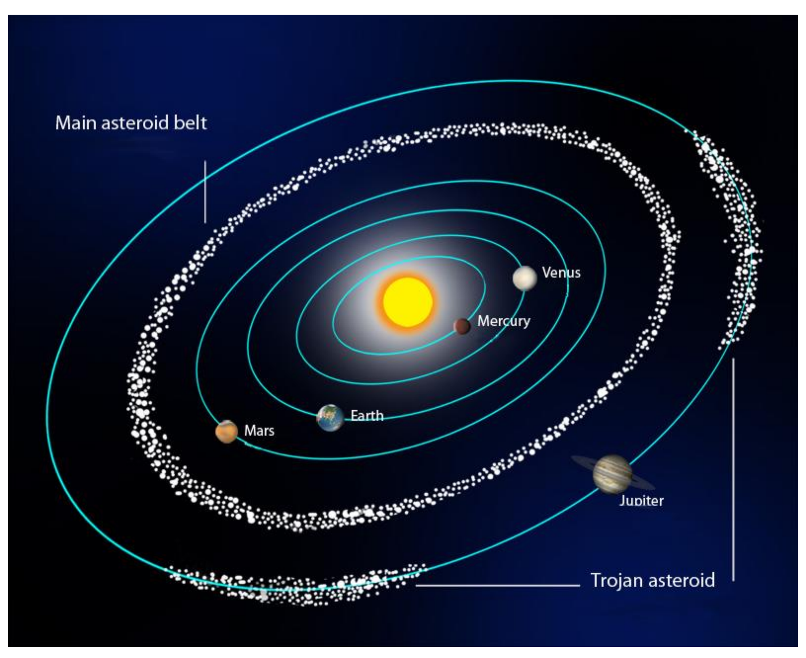

Fig.1

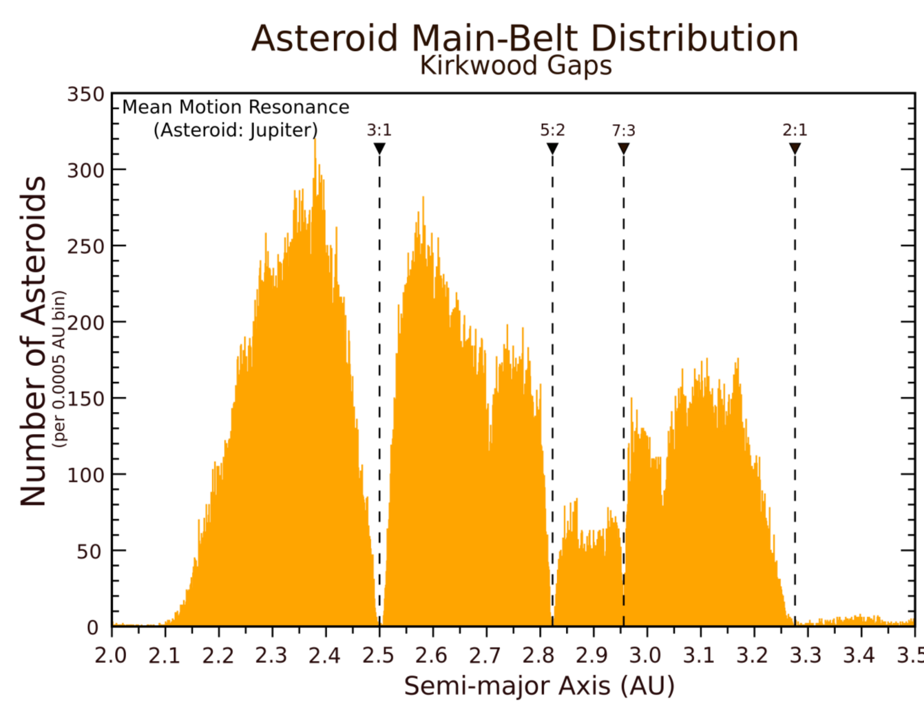

Fig. 2

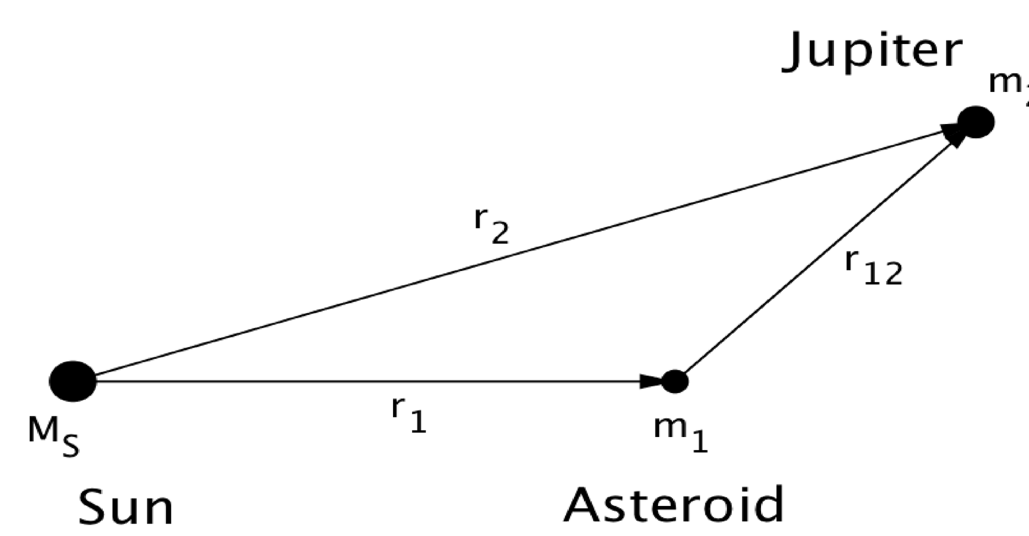

Fig. 3

Introduction

Asteroids, also known as minor planets or planetoids, are rocky bodies ranging in size from dust particles to 1,000 km across. While most are contained in stable orbits in a belt located between the orbits of Jupiter and Mars, some have highly eccentric orbits and can cross the plane of Earth's orbit. Figure 1 depicts the location of the Asteroid Main Belt in the Solar System. Figure 2 shows the distribution of asteroid orbits, specifically the distribution of semi-major axes of the orbits. Gaps in the distribution occur at positions where an orbit semi-major axis is an integer fraction of that of Jupiter. These are known as the Kirkwood gaps and the absence of asteroids in such orbits is attributed to resonant phenomena associated with the gravitational influence of Jupiter.

Present Work

In a two-dimensional, three-body formulation we study the evolution of orbits of test asteroids with an initial location at forbidden values of the semi-major axis. The exact problem is a three-dimensional many-body problem requiring the solution of many second-order non-linear coupled differential equations. With only the Sun, Jupiter, and an asteroid in a planar configuration, the number of coupled differential equations is reduced to four. They are further simplified by neglecting the effect of asteroid mass upon the motion of Jupiter. The simplified equations of motion for the asteroids and graphs of the computed orbital evolution for an asteroid placed in the 2:1 gap are shown below. Earlier simulations (figure 5) showed large deviations from this orbit in less than 10 revolutions. Our current work, considering the same model with more precise numerical integration, suggests a more stable evolution of the asteroid. Figure 6 shows the orbit for its first 250 years, where the z-axis in the left plot is of time.

Simplified equations of motion for asteroid

$$\ddot{x}_1 + GM_s\frac{x_1}{r_1^3} = Gm_2\left(\frac{x_2 - x_1}{r_{12}^3} - \frac{x_2}{r_2^3}\right)$$

$$\ddot{y}_1 + GM_s\frac{y_1}{r_1^3} = Gm_2\left(\frac{y_2 - y_1}{r_{12}^3} - \frac{y_2}{r_2^3}\right)$$

where G is the gravitational constant

Fig. 4

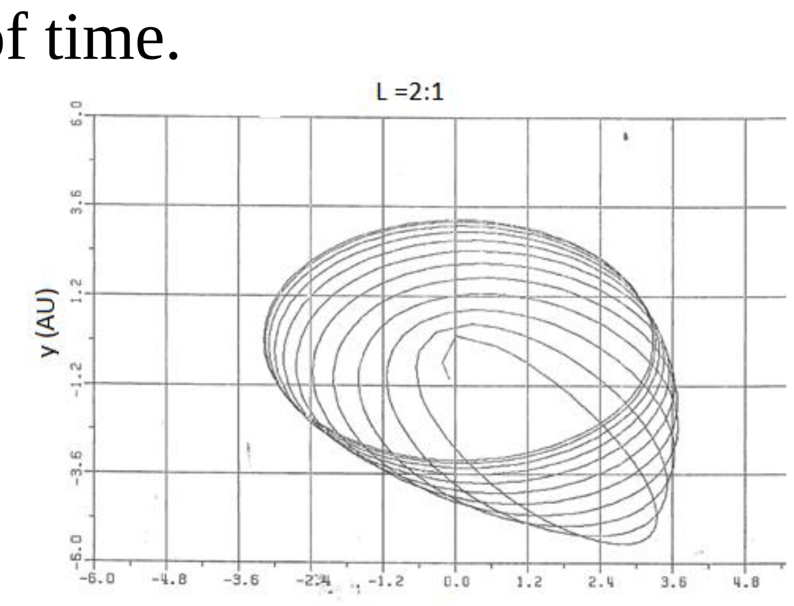

Fig. 5

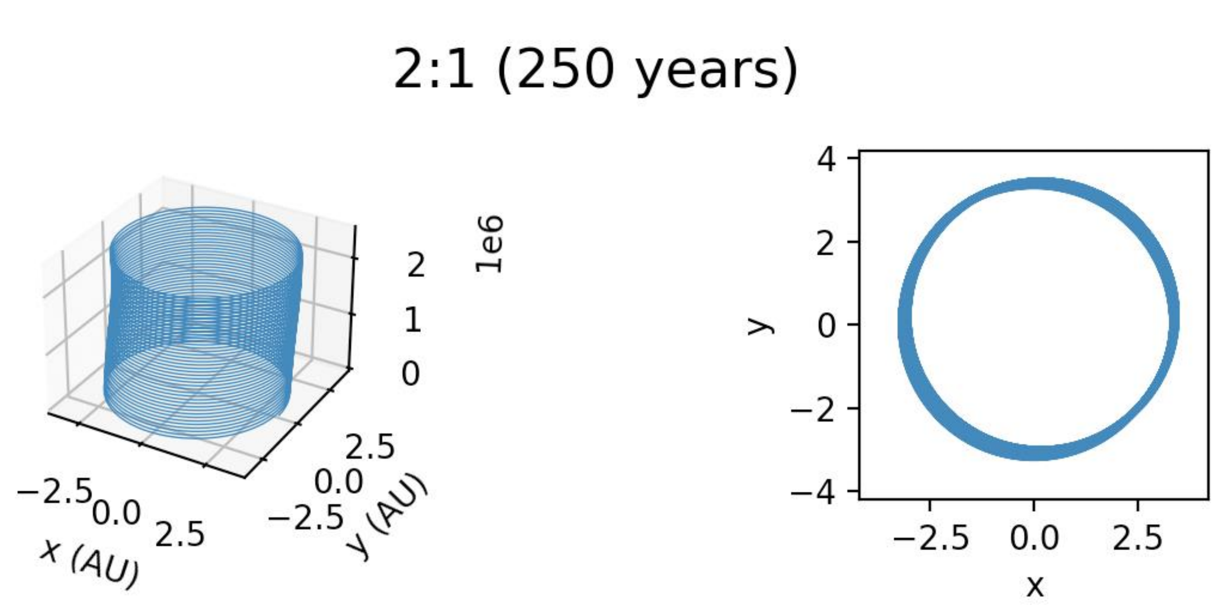

Fig. 6

Summary and Conclusions

The orbits shown are for a test body placed in the forbidden location corresponding to Jupiter-to-asteroid semi-major axis ratio of 2:1. The same behavior was observed for other theoretically forbidden orbits. Many-body calculations have shown such orbits to be stable for more than 10^5 years and our work has indicated that the simplified 2-body model agrees to a higher extent than previously thought.

References

1. D. Kirkwood, Proc. AAAS, pp.8-14 (1866).
2. See, for example, J. Wisdom, AJ, **87**, 577 (1982).
3. D. W. Kraft, Bull. Am. Phys. Soc **33**, 64 (1988).