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Essays on earnings dynamics

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Abstract

In this thesis, I analyze workers' earnings dynamics through different angles. In the first chapter, I estimate explicit age-varying distributions of idiosyncratic persistent and transitory earnings shocks over workers' life-cycles using a German administrative data set. Large positive shocks, both transitory and persistent, are characteristic for the first eight years of the working life. After the age of 50, large negative shocks become a major source of earnings risk. Between the ages of 30 and 50, most shocks are small and transitory. Large persistent positive shocks that occur early in the working life help to rationalize large wealth and consumption shares of the top one percent in an incomplete markets model. In the second chapter, I estimate an earnings and job mobility model before and after the Hartz reforms in Germany. After the Hartz reforms, full-time work fell and part-time, marginal employment and concurrent employment rose. Wage inequality increases at the bottom of the distribution. This comes as a result of lower full-time wages for males and the rise of part-time work. With the empirical model I then simulate employment and earnings trajectories and obtain lifetime values of earnings. This estimation shows that there is an increase in the inequality of lifetime values stemming at the bottom of the distribution. Generally, both lower wages and a larger hazard of falling and remaining in part-time employment explain the lower lifetime earnings. However, for males, lower wages has twice the impact in the decrease of earnings than the new employment transitions after the Hartz reforms. In the third chapter, I study the process of human capital accumulation on the job. This has important implications for life-cycle inequality and cross-country differences in earnings. Using a novel dataset from Chile I document a significant gap in wage growth measured using job ad information versus the one observed in surveys of workers. I develop a lifecycle structural model where workers face frictional labour markets, uncertainty with respect to match quality with firms and uncertainty about human capital accumulation on the job. I quantify how much of the gap in job ad wage growth vs workers' survey wage growth is due to job market frictions (time not working) versus learning frictions. I find that failure to learn is the main reason behind the gap.

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Author's declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

SIGNED: Manuel Antonio Sanchez Garcia.......... DATE: 17 January 2022

Table of Contents

List of Tables

Table | **Page**

List of Figures

Figure **Page**

CHAPTER 1

MODELING LIFE-CYCLE EARNINGS RISK WITH POSITIVE AND NEGATIVE SHOCKS

with Felix Wellschmied

Statement on coauthorship: this chapter is joint work with Felix Wellschmied. I have made significant contributions to all aspects of the work. Data cleaning, econometric model specification, estimation, counterfactuals, identification analyses and engagement in conferences were done by myself alone. Felix Wellschmied contributed to the development of the consumption and savings model, the interpretation of results and placement in the literature.

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1.1 Introduction

Individual earnings risk changes over the life-cycle. During the early stage of the working life, finding all-year-round employment and moving up the job-ladder are associated with large individual earnings fluctuations.[1] During prime-age (ages 30-50), workers settle into more stable employment and large earnings changes become less frequent. Once closer to retirement, periods of non-employment and losing a high-tenured job become major risks.[2] Karahan and Ozkan (2013), Blundell et al. (2015), and Lopez-Daneri (2016) study this age variation in terms of changing variances of idiosyncratic transitory and persistent earnings shocks. This paper also decomposes male earnings changes into transitory and persistent earnings shocks, but it goes beyond age-variations in the second moment of these shocks. In particular, it estimates explicitly age-varying distributions of positive and negative earnings shocks. We find that the probabilities to experience large positive and large negative earnings shocks vary substantially with age, and this age-variation furthers our understanding of households' consumption and savings decisions.

Using German administrative individual earnings data, we first document that moments of positive and negative residual earnings growth behave very differently from each other over the life-cycle. Positive residual earnings growth is relatively rare before the age of 30, but growth rates are large on average and highly dispersed.[3] The relatively frequent negative residual earning growth is small on average leading to a positively skewed distribution of residual earnings growth. The average size and the dispersion of positive residual earning growth fall throughout the life-cycle, and the average size and the dispersion of negative residual earnings growth grow throughout the life-cycle. Brought by less frequent large positive residual earnings growth and more frequent large negative residual earnings growth, the distribution of residual earnings growth becomes negatively skewed from age 40 onwards. After the age of 50, negative residual earnings growth is 10% larger on average and its variance is 60% larger than at age 25. In contrast, positive residual earnings growth is 70% smaller on average and its variance is 70% smaller than at age 25. Finally, the first autocovariance of positive growth is small relative to the first autocovariance of negative growth throughout the life-cycle.

Using simulated methods of moments, we estimate a parametric model that maps the distribution of residual earnings growth into age-varying distributions of transitory and persistent earnings shocks. We obtain these distributions explicitly by modeling shocks as a mixture of specified parametric distributions, similar to Geweke and Keane (2000), Bonhomme and Robin (2010), and Guvenen et al. (2016). To be specific, we parametrize residual log earnings as a mixture of three components that, given our decomposition of the data, have a natural interpretation: a positive, a negative, and a mean-zero component. The latter is a transitory normally distributed shock. In addition to this shock, with age-varying probabilities, workers draw either

[1]See Topel and Ward (1992).

[2]See Jung and Kuhn (2015).

[3]On average, earnings rise when young and decline when old. We study deviations from this predictable age pattern.

an innovation to their positive component, an innovation to their negative component, or no further shock. An innovation to the positive (negative) component is a combination of a transitory and a persistent log-normally distributed shock. Hence, persistent and transitory shocks are partially correlated in our model which deviates from the more standard zero-correlation assumption in the literature. A positive correlation allows the model to be consistent with the earnings dynamics occurring around two prominent (observable) persistent labor market shocks: unemployment and job-to-job transitions. That is, earnings are lowest on average in the year of an unemployment spell but return partially to their former level afterward (see also Jacobson et al. (1993)). Similarly, average earnings are highest in the year of a job-to-job transition but reverse somewhat thereafter. To capture the age-varying frequency and severity of these and other earnings shocks, we allow the means and variances of the parametric shock distributions to vary with age. These age variations in the shocks underlying the three components, together with the age-varying sampling probabilities of the three components, allow the model to generate rich age-variations in the overall distributions of transitory and persistent earnings shocks.

We find that at prime-age, most workers experience only small transitory shocks. At age 40, only 33% of workers experience any persistent earnings shocks during a given year. These probabilities are much higher, around 58 percent, at ages 25 and 55. Turning to the properties of these persistent shocks, we find that the autocorrelations of persistent positive and persistent negative shocks are above 0.97, i.e., these shocks are close to permanent. The probability to draw a positive persistent shock increases from 11% at age 25 to 44% at age 55. Nevertheless, experiencing a positive persistent increase in log earnings of more than 0.2 is 7 times more likely at age 25 than at age 55. The reason is that the mean and the variance of persistent positive shocks are about 5 times larger at age 25 than at age 55. Persistent negative shocks show the exact opposite life-cycle behavior of positive shocks. They are small, have little dispersion, and occur with relatively high frequency early in life, and become rare, large on average, and more dispersed late in life. To put these findings in perspective to the U-shaped variance of persistent shocks over the life-cycle found by Karahan and Ozkan (2013), our results imply that the initial decrease is entirely driven by positive persistent shocks becoming less dispersed and the later increase is entirely driven by negative persistent shocks becoming more dispersed.

Transitory shocks to the positive and negative components are large and highly dispersed. On a life-time average basis, the variance of transitory negative shocks is 2.6 times larger than the variance of transitory positive shocks. Moreover, it is 11 times larger than the variance of persistent negative shocks. As a consequence, most large negative shocks are transitory. A negative change in log earnings of more than 0.2 is in 71% of the cases due to a transitory shock. The corresponding number for positive shocks is only 55%. The difference is even more pronounced early in life. At age 25, 94% of all negative changes in log earnings of more than 0.2 are the result of a transitory shock. In contrast, 50% of all positive changes in log earnings of more than 0.2 result from persistent shocks at age 25.

Next, we introduce this estimated earnings risk into an Aiyagari (1994) type model to study the implications of age-varying, non-normally distributed risk for consumption and savings decisions. We contrast the results to the widely used age-invariant risk model (*AIRM*) with mean-zero normally distributed transitory and persistent shocks. Compared to this latter model, the large but rare persistent positive shocks early in life imply, as in the data, a relatively high dispersion in the right tail of the cross-sectional earnings distribution. A few lucky workers, therefore, accumulate large wealth holdings for life-cycle purposes, particularly to finance consumption during retirement, and hold a relatively large share of the overall wealth. This channel has a strong amplification mechanism for cross-sectional wealth inequality because these shocks occur early in life; thus, they imply large cross-sectional differences in lifetime earnings. Compared to the *AIRM*, the share of wealth holdings by the top 1% more than doubles, bringing the model closer to the data.

Similar to wealth inequality, consumption inequality is more pronounced in the right tail of the cross-sectional distribution in our model than in the *AIRM*. That is, the ratio of consumption of the top 1% relative to the median worker is relatively high and it grows relatively rapidly over the life-cycle. This shift of resources away from the median and towards the highest lifetime consumption workers reduces welfare in our model relative to the *AIRM*. Counteracting this effect, consumption inequality at the bottom of the distribution is somewhat lower in our age-varying risk model. Measuring welfare in terms of the consumption an unborn household is willing to pay to insure against idiosyncratic earnings heterogeneity, we find that the former effect dominates, that is, the welfare costs of incomplete insurance markets are higher in the age-varying risk model.

Age-varying non-normally distributed risk also helps to explain the dynamics of cross-sectional consumption inequality over the life-cycle. In specific, large negative tail shocks late in life increase the desired stock of workers' precautionary savings. We show that more precautionary savings and a shift towards more persistent and positive shocks increase the speed at which consumption dispersion increases late in life. As a result, the cross-sectional variance of log consumption grows close to linear in age, which is consistent with the German data analyzed by Fuchs-Schúndeln et al. (2010).

Our findings contribute to the recent macroeconomic literature that studies the implications of non-normally distributed shocks for individuals' savings and consumption. Civale et al. (2017) show that wealth inequality decreases when earnings shocks become more negatively skewed. Castañeda et al. (2003) calibrate the earnings process such that it matches the observed right tail of the wealth distribution which implies a "superstar" earnings state. The large and persistent positive shocks we find early in the life-cycle have qualitatively the same effect. De Nardi et al. (2019) use a two-step approach to study higher-order earnings risk. First, they estimate the model proposed by Arellano et al. (2015) and, thereafter, estimate Markov processes on simulated data resulting from step one. Importantly, this approach allows for non-linear log earnings dynamics that imply shocks being less persistent; therefore, less costly in terms of welfare. Our finding that a shift of resources towards the right tail of the earnings distribution increases the welfare costs relative to an age-invariant risk model is complementary to theirs.

The rest of the paper is organized as follows. Section 1.2 describes the German data set. Section 1.3 presents the moments of residual earnings growth over the life-cycle. Section 1.4 describes the econometric model. Finally, Section 1.5 introduces our earnings process into a life-cycle savings model.

1.2 Data and Sample Construction

1.2.1 Data Description

Our data source is the *Sample of Integrated Labour Market Biographies* (*SIAB*) for the years 1975-2010. The data originates from the German notification procedure for social security. This requires employers to report their employees' working spells, earnings, and some socioeconomic information. The data covers the population of German employment except for civil servants, the self-employed, and regular students (about 20% of the employment-population). From this population, the German employment agency draws a 2% random sample of individuals' careers. In total, the data has information on 1,594,466 individuals and 41,390,318 unique person-year records. Thus, *SIAB* provides a large number of career-long earnings profiles with little measurement error.

We focus on the earnings risk of workers with a high attachment to the labor force and abstract from any employment decisions resulting from earnings shocks. We drop workers in an apprenticeship, partial retirement, marginal part-time workers (*geringfügig Beschäftigte*), and part-time workers not eligible for unemployment benefits. Moreover, we only consider German male workers to avoid female decisions over maternity leave.[4] We define a worker as employed within a year when he is contracted for at least 90 days of that year. Hence, our analysis abstracts from earnings shocks arising from long-term unemployment. Following the literature that focuses on workers with a high attachment to the labor market, we keep for each individual the longest spell of earnings with at least 7 years of observations (see Meghir and Pistaferri (2004), Guvenen (2009) and Hryshko (2012)).

To avoid misinterpreting predictable earnings changes as shocks, the age range under consideration is of some importance. For the time period of our sample, a high school degree takes up to 13 years of schooling and male workers are obliged to perform 1 year of military service. Most workers enter professional training (2-3 years) thereafter. Hence, we expect workers to have made a full transition to the labor market by the age of 24. The intended retirement age in Germany used to be 65. Yet, Arnds and Bonin (2002) show that early retirement schemes lead to an average retirement age around the age of 60. Moreover, generous unemployment benefits for high tenured

[4]The moments for females are available from the authors upon request.

workers often lead to an effective retirement age of 55. To avoid these endogenous decisions, we restrict the panel to workers aged 24 to 55. Finally, we discard workers in East-Germany as those observations are only available after 1991. Our final sample contains information for 251,352 individuals with a total of 3,566,212 person-year observations.

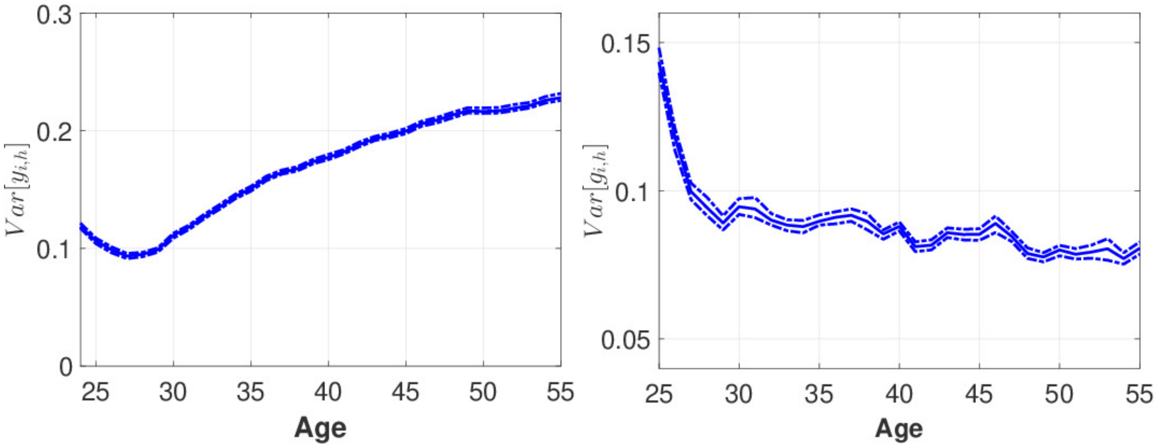
For each calendar year, we aggregate an individual's earnings across all job spells. We deflate earnings using the German consumer price index of 2010.[5] Changes in real earnings may arise from inflation, a change in working hours, a change in employer, an unemployment spell, bonuses, promotions, etc. Workers entering the sample for the first time are statistically expected to enter in the middle of the year. Daly et al. (2016) show that this may lead to a bias in the estimates of permanent shocks. To avoid this bias, we assume that earnings in the months preceding the first employment spell are the same as the observed months in that year. Similarly, we assume that earnings in the months following the last employment spell are the same as the observed months in that year. Following Dustmann et al. (2009), we drop real daily-earnings that are below 5 euros. Daily-earnings are top-coded by the limit liable to social security. On average, this affects around 6% of observations per year. We follow Daly et al. (2016) and impute daily earnings from an extrapolated Pareto density fitted to the non-top-coded upper-end of the observed distribution for each year. Alternatively, we could drop workers affected by top-coding. The moments of residual earnings growth are almost identical for the two approaches. We opt for the former because it allows us to infer the entire cross-sectional earnings distribution of the German employment-population.[6]

Our interest is in annual earnings changes that are idiosyncratic to the individual. To this end, we remove predictable changes from earnings growth by running cross-sectional regressions for each workers' age. The regressions control for an education dummy, year dummies, region of residence, and 14 major industries. Next, we assign each individual to a birth cohort defined as being born in a seven-year interval starting in 1923. Figure 1.A2 in the Appendix shows, using as an example the variance of residual earnings over the life-cycle, that the data features both a calendar time and a cohort effect. The latter may partially arise from the data not reporting one time payments before 1984. Following Blundell et al. (2015), we average all data moments across cohorts to eliminate these types of time effects, assigning equal weight to all cohorts. Therefore, our results can be interpreted as the risk a typical cohort is facing. To compute the cross-sectional earnings inequality over the life-cycle, we follow Deaton and Paxson (1994) and regress the cross-sectional variance of log earnings on a full set of age and cohort dummies. We compute the cross-sectional variance at age 24 as the mean of the cohorts' intercepts.

Figure 1.A3 in the Appendix compares the resulting life-cycle moments of the variance, skewness, and kurtosis of earnings growth to those reported in Guvenen et al. (2016) for the US. The life-cycle behavior of these moments is remarkably similar across the two countries, yet,

[5] We obtain the consumer price index from OECD data; `https://data.oecd.org/price/inflation-cpi.htm`.

[6] By doing so, we assume earnings growth behaves similarly for the top decile of the German earnings distribution relative to the rest of the distribution.



(a) Variance of log earnings

(b) Variance of earnings growth

Notes: Panel (a) displays the cross-sectional variance of log earnings by age. It displays the age coefficients of a regression of the variance of log earnings on a cohort and age dummies. Panel (b) displays the variance of residual earnings growth across ages. The dashed lines display block-bootstrapped 95% confidence intervals.

Figure 1.1: Variance of Residual Earnings and Earnings Growth

there are some differences in their levels. The age-averaged variance of earnings growth is two to three times larger in the US. For one, Guvenen et al. (2016) impose a \$1500 lower income limit to enter into their sample which is less stringent than the limit implied by our restrictions on minimum wages and working days.[7] Moreover, the US data includes income from operating a business that is more volatile than employees' earnings. Yet, there are also some institutional differences between the countries worth highlighting. For many German sectors, wage floors are centrally bargained implying more nominal wage rigidity. Moreover, Germany has strong employment protection for high tenured workers that leads to a lower probability of becoming unemployed but also to a lower probability of finding a new job. Bachmann et al. (2013) show that both the German accession and separation rate of workers within establishments are only 60% of the US level, yet such switches are a major source of earnings volatility. This latter fact also contributes to skewness being less negative on average in Germany. We find that skewness becomes more negative as we loosen the requirement for the number of days worked to enter into the sample, i.e., negative skewness is strongly driven by workers reducing their amount of working days from one year to the next. In Germany, there are fewer of such non-employment events.

1.3 Moments of Residual Earnings Growth

This section highlights the salient features of residual earnings dynamics over the life-cycle. Figure 1.1a displays the cross-sectional variance of residual log earnings across ages, $Var(y_{i,h})$,

[7] Lowering the work requirement to 65 days increases the variance of residual earnings growth from 0.089 to 0.098.

where i denotes the individual and h denotes age. The variance is falling for the first three years and reaches a low of 0.09. Inequality accelerates up to age 40 when its growth slows down somewhat. In total, between the ages of 27 and 55, residual earnings inequality more than doubles. Guvenen et al. (2016) show that cross-sectional inequality also doubles over the life-cycle in the US. However, the cross-sectional variance of earnings at labor market entry is substantially higher in the US (0.47 at age 27).

We now turn to the dynamics in residual earnings that create this life-cycle pattern in inequality. A common way to identify earnings shocks is to study the covariance structure of residual earnings growth (we use interchangeably the terms growth/innovations/changes), $g_{i,h}$. Figure 1.1b plots its cross-sectional variance over the life-cycle. The variance declines by almost 43% between the age of 24 and age 55 with most of the decline, close to 80%, occurring before the age of 30.

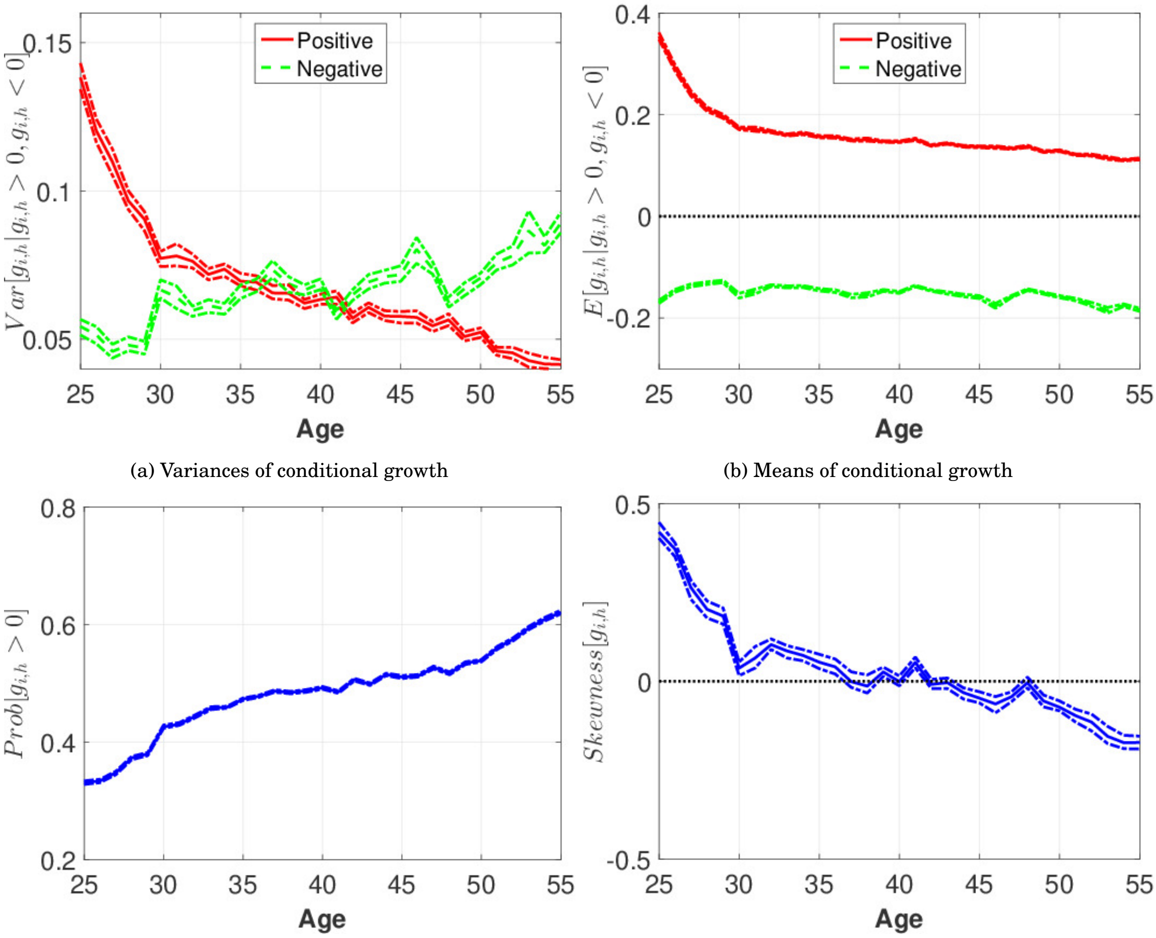
To better understand the changes in the distribution of residual earnings growth that lead to the decreasing variance, we study separately positive, $g^+_{i,h}$, and negative, $g^-_{i,h}$, residual earnings growth. Figure 1.2a displays the conditional variances of these innovations, $Var(g_{i,h}|g_{i,h,} \lessgtr 0)$. The figure shows that the decline in the variance of residual earnings growth up to age 30 results from positive changes becoming less dispersed. In contrast, the variance of negative residual earnings growth slightly increases during these years. Afterward, the variance of positive growth continues to decline and the variance of negative growth continues to increase. The latter is more than 60% larger at age 55 than at age 25.

Figure 1.2b shows that the average sizes of conditional residual earnings growths closely track their variances. Positive residual earnings growth is large on average early in life, and it becomes smaller throughout the life-cycle. Mean negative residual earnings growth is almost constant until the age of 50 and becomes larger in absolute size thereafter. Figure 1.2c plots the probability to observe a positive innovation at each age, $Prob(g_{i,h} > 0)$. Its behavior over the life-cycle reconciles the different means of conditional growths with the mean-zero unconditional growth. Early in life, close to 70% of innovations are negative, but the probability of a positive change is increasing throughout the working life and reaches 62% at the age of 55. Average positive growth becoming more likely with age implies that the distribution of earnings growth becomes more negatively skewed as workers age. Figure 1.2d shows that the distribution is initially positively skewed, and skewness turns negative around the age of 40.[8] Note that this decline in skewness is driven by simultaneous changes in both tails of the distribution over the life-cycle. That is, we observe a simultaneous decline in the occurrence of large positive residual earnings growth and a rise in the occurrence of large negative residual earnings growth.

Guvenen et al. (2016) highlight that US earnings growth features fat tail behavior. We find a similar magnitude of kurtosis in the German data.[9] What is more, Figure 1.3a shows that kurtosis increases in a concave fashion throughout the life-cycle. At its peak, it is 5 times larger

[8]To avoid outliers affecting the skewness, we use Kelly's measure of skewness.

[9]To avoid outliers affecting the kurtosis, we use Crow-Siddiqui's measure of kurtosis (Crow and Siddiqui (1967)).



(a) Variances of conditional growth

(b) Means of conditional growth

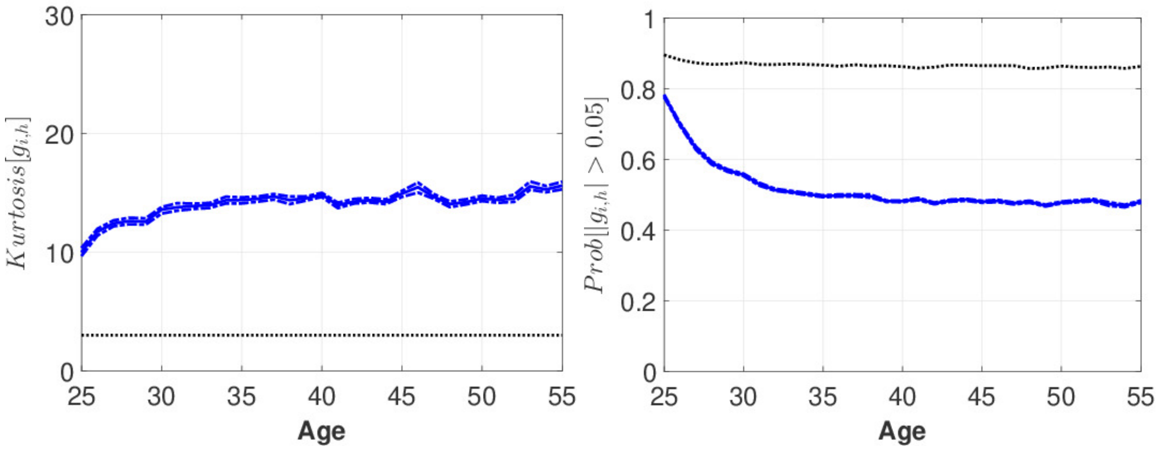
(c) Fraction of positive growth

(d) Kelly's skewness of growth

Notes: Panel (a) displays the variance of residual earnings growth across ages conditional on residual earnings growth being positive (negative). Panel (b) displays the corresponding means of residual earnings growth. Panel (c) depicts the fraction of residual earnings growth that is positive at each age. Panel (d) displays Kelly's skewness measure of residual earnings growth across ages. The dashed lines display block-bootstrapped 95% confidence intervals.

Figure 1.2: Variances and Means of Conditional Residual Earnings Growth

than what is suggested by a normal distribution. The large kurtosis implies that a substantial fraction of workers experiences very small residual earnings changes. To put this into perspective, Figure 1.3b displays the fraction of residual earnings growth by age that is above 5 percent (in absolute value). In the cross-section, 48 percent of workers experience a residual log earnings change of less than 5 percent. Figure 1.3b also displays the fraction of workers with an earnings change of more than five percent that is implied by a normal distribution with the same variance as the data. In that case, only 13 percent of workers would experience such a small change. What is more, in the data, the share of workers experiencing small residual earnings changes has a strong age dimension. At age 40, 50 percent of workers experience residual earnings changes of magnitude smaller than 5 percent. In contrast, between the ages of 25 and 30, only 21 percent



(a) Kurtosis of growth

(b) Share of growth > 5%

Notes: Panel (a) displays the Crow-Siquiddi's measure of kurtosis of residual earnings growth across ages. The gray dotted line is the kurtosis of a normal distribution. Panel (b) displays the fraction of residual earnings growth that is larger than 5% in absolute value. The gray dotted line shows the fraction that would result from a normal distribution with the same variance as the data at each age. The dashed lines display block-bootstrapped 95% confidence intervals.

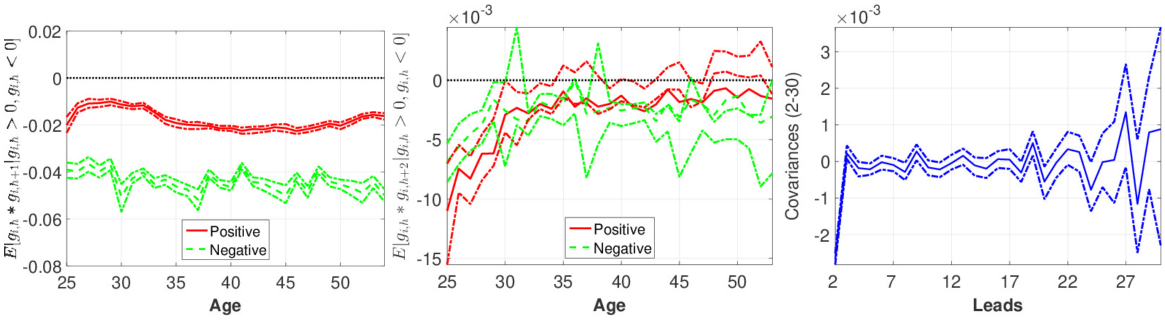
Figure 1.3: Kurtosis of Residual Earnings Growth

of innovations are smaller than 5 percent, a change of almost 30 percentage points. Under the assumption of normally distributed earnings growth, the change in the fraction would only be 3.9 percentage points.

So far, we have not addressed the persistence of earnings changes. The literature commonly differentiates between persistent (e.g., promotions, large health shocks) and transitory (e.g., bonuses, temporary sickness) changes. To understand the persistence of earnings changes, we study the first and second conditional autocovariances. A negative first autocovariance of residual earnings growth implies that part of the contemporaneous growth is offset the following year, i.e., it provides information regarding the amount of mean reversion. The second autocovariance identifies whether this mean reversion lasts longer than one year. Figures 1.4a and 1.4b display the conditional first and second autocovariances of residual earnings growth, respectively. The first autocovariance of positive growth is small relative to the first autocovariance of negative growth. Neither shows a pronounced life-cycle pattern. The second autocovariance is negative for both types of earnings changes, but it is small in size after the age of 30 in either case. Figure 1.4c displays the age-averaged (unconditional) autocovariance at longer leads. All autocovariances after lead two oscillate around zero suggesting that the mean reversion of earnings changes takes place during the first two years.

1.3.1 Sources of Earnings Innovations

Taken together, the data suggest that positive (negative) residual earnings fluctuations are particularly large before the age of 30 (after the age of 50). We end this section by briefly showing



(a) First autocovariance (b) Second autocovariance (c) Unconditional autocovariances

Notes: Panel (a) displays the first autocovariance of residual earnings growth by age conditional on residual earnings growth at the current period being positive (negative). Panel (b) displays the second-order autocovariance. Panel (c) shows the unconditional autocovariance of residual earnings growth beyond the first lead. The dashed lines display block-bootstrapped 95% confidence intervals.

Figure 1.4: Autocovariances of Residual Earnings Growth

that two observable labor market events, job-to-job transitions and non-employment spells are quantitatively important to understand these large labor earnings changes. Appendix 1.7.1 displays in more detail the earnings changes accompanying these labor market events.

First, we consider workers younger than age 30. We define a large positive innovation as a positive change in residual log earnings of at least 0.2 (or approximately 22%). Consistent with the job-ladder effects documented by Topel and Ward (1992), we find that in 32% of cases where we observe a large positive earnings change early in life, the individual changes his establishment. Topel and Ward (1992) also show that young workers' careers are characterized by repeated non-employment spells between jobs. In this vein, we ask how many of the large positive innovations in the data coincide with workers increasing the number of working days during a year. We define a "substantial" increase in working days as an increase in contracted days by more than 30 days from one year to the next. Around 29% of large positive earnings innovations early in life are associated with such an increase in working days.

Turning to workers older than age 50, we define a large negative innovation as a negative change in residual log earnings of at least -0.2 (or approximately -19%). Jacobson et al. (1993) show that reemployment earnings are substantially lower after losing a highly tenured job. To understand the importance of this effect for elderly workers in Germany, we calculate the share of large negative earnings changes associated with the worker changing his establishment. We find that the worker changes establishments in only 7% of cases where we observe a large negative innovation. Put differently, losing a high paying job and reentering with a lower-paying job is not a common phenomenon for elderly German workers. Instead, large negative residual earnings changes are predominantly associated with a reduction in working days. Workers reduce their amount of working days by at least 30 per year in 57% of the cases where we observe a large negative earnings change.

1.4 A Time Series Model of Earnings Dynamics

1.4.1 Model

We model residual log earnings as the sum of permanent initial inequality and a stochastic component:

$$y_{i,h} = \underbrace{\alpha_i}_{\text{initial heterogeneity}} + \underbrace{u_{i,h}}_{\text{stochastic component}}, \tag{1.1}$$

where $\alpha_i \sim N(0,\sigma^2_\alpha)$. α_i is the only source of deterministic unobserved inequality between workers in our model. Appendix 1.7.5 shows that our results are mostly invariant when including deterministic heterogeneity in individual earnings growth.

We want our model to capture the rich dynamics of positive and negative residual earnings growth over the life-cycle. We achieve this by modeling shocks to the stochastic component of residual earnings as an age-varying mixture of several specified parametric distributions. To be specific, we let $u_{i,h}$ consist of a mean zero component and, following our analysis above, a positive and a negative component that all have age-varying properties:

$$u_{i,h} = \underbrace{W^+_{i,h}}_{\text{positive}} + \underbrace{W^-_{i,h}}_{\text{negative}} + \underbrace{\iota^n_{i,h}}_{\text{mean zero}}, \tag{1.2}$$

where $\iota^n_{i,h} \sim N(0,\sigma^2_{\iota^n})$ is a transitory shock to earnings that realizes for each individual at every age. The positive component, $W^+_{i,h}$, and the negative component, $W^-_{i,h}$, contain both a persistent and a transitory part:

$$W^+_{i,h} = \underbrace{w^+_{i,h}}_{\text{persistent}} + \underbrace{\tau^+_{i,h}}_{\text{transitory}} \qquad W^-_{i,h} = \underbrace{w^-_{i,h}}_{\text{persistent}} + \underbrace{\tau^-_{i,h}}_{\text{transitory}} \tag{1.3}$$

$$w^j_{i,h} = \rho^j w^j_{i,h-1} + \xi^j_{i,h} \text{ for } j = -,+ \qquad \tau^j = \iota^j_{i,h} + \theta^j \iota^j_{i,h-1} \text{ for } j = -,+ \tag{1.4}$$

Thus, innovations to the positive and the negative components are a combination of a persistent, $\xi^j_{i,h}$, and a transitory, $\iota^j_{i,h}$ shock. These shocks have by assumption the same sign which deviates from the independence assumption common in the literature. This structure captures a wide range of economic phenomena. For example, consider the case of workers losing their job shown in Appendix 1.7.1. Average residual earnings are lowest, probably resulting from a reduction in the number of days worked, in the year of displacement, recuperate somewhat afterward, but they stay persistently lower than before the displacement. The model will identify this as an innovation to the negative component of log earnings. The initial reduction in working days will be identified as the transitory shock. The longer-lasting earnings loss will be identified as the persistent shock. The appendix also shows that job-to-job transitions display a similar pattern. Earnings are highest in the year of a job-to-job transition, possibly due to signing bonuses, but reverse on average towards their old level thereafter. Such a move up the job ladder will be identified as a shock to the positive component. The initial overshooting of earnings will be identified as the transitory shock and the longer-lasting earnings increase will be identified as the persistent shock. Note that the model does, nevertheless, not impose a perfect correlation between persistent and transitory shocks because the mean zero shocks, $\iota^n_{i,h}$, realize additionally at each age.

We let the probability to receive innovations to the positive and negative components vary with age. Mutually exclusive, and at each age, an individual draws with probability p^-_h an innovation to his negative component, (both $\xi^-_{i,h}, \iota^-_{i,h}$), and with probability p^+_h an innovation to his positive component, (both $\xi^+_{i,h}, \iota^+_{i,h}$). With probability $1-p^+_h-p^-_h$ he draws neither.[10] We specify second order polynomials in age for these probabilities:[11]

$$p^j_h = \delta^j_I + \delta^j_{II}h + \delta^j_{III}h^2 \text{ for } j = -,+; \;\; h = 0 \text{ at age 24.} \tag{1.5}$$

Different from most of the literature on earnings dynamics, we explicitly specify the shock distributions. The persistent and the transitory shocks to the positive and negative components follow age-varying log-normal distributions:[12]

$$\xi^+_{i,h} \sim exp(N(\mu^+_h, \sigma^2_{\xi^+,h})), \;\; \xi^-_{i,h} \sim -exp(N(\mu^-_h, \sigma^2_{\xi^-,h})) \tag{1.6}$$

$$\iota^+_{i,h} \sim exp(N(\mu^+_h, \sigma^2_{\iota^+,h})), \;\; \iota^-_{i,h} \sim -exp(N(\mu^-_h, \sigma^2_{\iota^-,h})) \tag{1.7}$$

The log-normal specification allows the model to match the fat tails of the residual earnings growth distribution. To provide intuition for this, Figure 1.A4 plots the density function of earnings growth. We do not impose it, but it is natural to think of the mean zero component as mostly representing small changes in real earnings that are close to zero (inflation, small changes in hours, etc...), thus, capturing in part the many earnings changes close to zero. In contrast, the positive and negative components mostly allow the model to match the fat tails of the distribution.[13]

To accommodate for the age-variation in the variances of positive and negative residual earnings growth, the dispersion parameters in equations (1.6) and (1.7) vary with age in a linear fashion:

$$\sigma_{k^j,h} = \gamma_{a,k^j} + \gamma_{b,k^j}h \text{ for } j = -,+ \text{ and } k = \xi,\iota; \;\; h = 0 \text{ at age 24.} \tag{1.8}$$

[10] In particular, we obtain a draw from a uniform distribution, $s_{i,h} \sim U(0,1)$, for each worker at each age, and assign the innovation to the negative component of that worker if $s_{i,h} \in [0, p^-_h]$. Similarly, we assign an innovation to the positive component of that worker if $s_{i,h} \in (p^-_h, p^+_h + p^-_h]$. Finally, we assign no innovation to these components if $s_{i,h} \in (p^+_h + p^-_h, 1]$.

[11] We find that moving to a third order polynomial provides little improvement in the model fit to the data.

[12] To keep the number of parameters manageable, we impose the same location parameters for transitory and persistent shocks.

[13] The log-normal assumption is also more convenient for the estimation of the model than a symmetric distribution. With the log-normal specification, the tail of the positive (negative) shock distribution does not cross into the negative (positive) domain, providing stability in the implied moments of the process, particularly the conditional autocovariances.

Also, to allow for age-varying conditional means, the location parameters of these shocks are age-varying:

$$\mu_h^j = \lambda_a^j + \lambda_b^j h \text{ for } j = -,+ \;\; ; h = 0 \text{ at age 24.} \tag{1.9}$$

Different from Karahan and Ozkan (2013) and Blundell et al. (2015), we do not allow the variances of shocks to change arbitrarily with age but, to keep the number of parameters manageable, restrict the age variations to be linear. In our framework, age variations in the unconditional distributions of transitory and persistent shocks arise from the age-variations in the parametric shock distributions (equations (1.8) and (1.9)) together with the age-varying sampling probabilities of the three components of log earnings (equation (1.5)). Figure 1.A5 in the Appendix shows that, as a result, the model generates non-linear moments, among them the variance of residual earnings growth, that are very similar to the data.

As workers accumulate different shocks over their life-cycles, the process implies that the variance of log residual earnings is increasing over the life-cycle. However, Figure 1.1a shows that residual earnings inequality is decreasing during the initial years. We interpret this initial decline as resulting from heterogeneity in the initial transitory components:

$$\iota_{i,0}^j \sim exp(N(\mu_0^j, \sigma_0^j)), \text{ for } j = -,+. \tag{1.10}$$

1.4.2 Identification

We estimate the model by the method of simulated moments (MSM) and use the block bootstrapping procedure suggested by Horowitz (2003) to obtain standard errors that we report in Table 1.A4. We target three main sets of empirical moments over the life-cycle: (i) moments of unconditional residual earnings growth: the mean, skewness, kurtosis, fraction of shocks above 5%, and the autocovariance function; (ii) moments of conditional positive and negative residual earnings growth: the means, variances, share of positive changes, and the first and second autocovariances; and (iii) the variance of residual log earnings. In our main specification, we estimate 28 parameters using 461 moments. Sections 1.7.2 and 1.7.4 in the Appendix describe further details about the estimation procedure and the set of moments.[14]

The matrix of first derivatives (evaluated at the minimum) of the moment conditions with respect to the parameter vector has full rank suggesting that our selected data moments do identify the model. The Online Appendix provides a visualization of this test. It displays the partial impact of each parameter on each moment evaluated at the minimum. Most parameters

[14] To estimate earnings shocks from residual earnings growth, we require that the information set of the econometrician is the same as that of the worker. Quite likely, it is impossible for the worker to predict wage changes conditional on all the observables that we use in our regressions; therefore, we may underestimate earnings risk. However, our moments are almost unchanged when excluding some of the observables. At the same time, a worker may have more information than the econometrician about the path of his earnings, thus, leading to an overstatement of risk.

affect all moments simultaneously. To gain some intuition for the identification, we briefly discuss here which moments are the most affected by the different parameters.

As shown, e.g., by Hryshko (2012) the variance and first two autocovariances of earnings growth identify the variance of persistent and transitory shocks and the persistence parameter of transitory shocks in a model with one persistent and one transitory mean-zero shock. Moreover, the distant lags of the autocovariance function of earnings growth identify the autocorrelation parameter of persistent shocks. The intuition extends straightforwardly to our model with conditional shocks. The conditional variances and autocovariances, together with the unconditional autocovariance, identify the parameters $\rho^+, \rho^-, \theta^+, \theta^-, \gamma_{a,k^j}$, for $j = -, +$ and $k = \iota, \xi$. Additionally, the conditional means of these changes contain information about the location parameters λ_a^j, λ_b^j, for $j = -, +$.

Storesletten et al. (2004) show that the cross-sectional dispersion of residual log earnings across ages contains information on the model parameters in a model with one persistent and one transitory mean-zero shock. Again, the intuition carries over to our model and provides additional identification. The cross-sectional variance of residual log earnings early in life identifies initial heterogeneity. The initial changes in cross-sectional inequality identify how much of this initial inequality is permanent, σ_α, or transitory, λ_0^j and σ_0^j, for $j = -, +$. The increase in inequality over the life-cycle contains information on the size of positive and negative persistent shocks, and the shape of the increase contains information on their persistence parameters.

Finally, the fraction of positive shocks over the life-cycle, skewness, the share of shocks above 5%, and kurtosis identify the variance of the mean zero component and the sampling probabilities, $\delta_I^j, \delta_{II}^j, \delta_{III}^j$, for $j = -, +$. To see the latter point, consider an increase in the sampling probability of positive shocks. This implies a higher fraction of those and a more negatively skewed distribution of earnings growth. To understand the relationship with kurtosis, we show in the next section that the mean zero transitory shocks, ι^n, have little variance. Hence, these shocks allow the model to create a large share of shocks centered around zero, thereby, given a fixed variance, a large kurtosis in earnings growth. Put differently, lower probabilities to draw any persistent shock imply more kurtosis.

1.4.3 Description of the Empirical Results

Table 1.1 reports selected parameter estimates for the process described by Equations (1.1) to (1.10). Table 1.A3 in the Appendix reports the remaining parameters. Column (1) is the full specification of the econometric model. We estimate the autocorrelation coefficients of positive and negative persistent shocks to be close to a unit-root process. The age-averaged means of positive and negative persistent shocks are similar, but their life-cycle behaviors differ (cf. Figure 1.5a). Positive shocks decrease in size throughout the life-cycle, but negative persistent shocks are smallest early in life and become larger on average with age. Figure 1.5b shows that the variances of these two shocks differ in their size and their behavior over the life-cycle. Positive

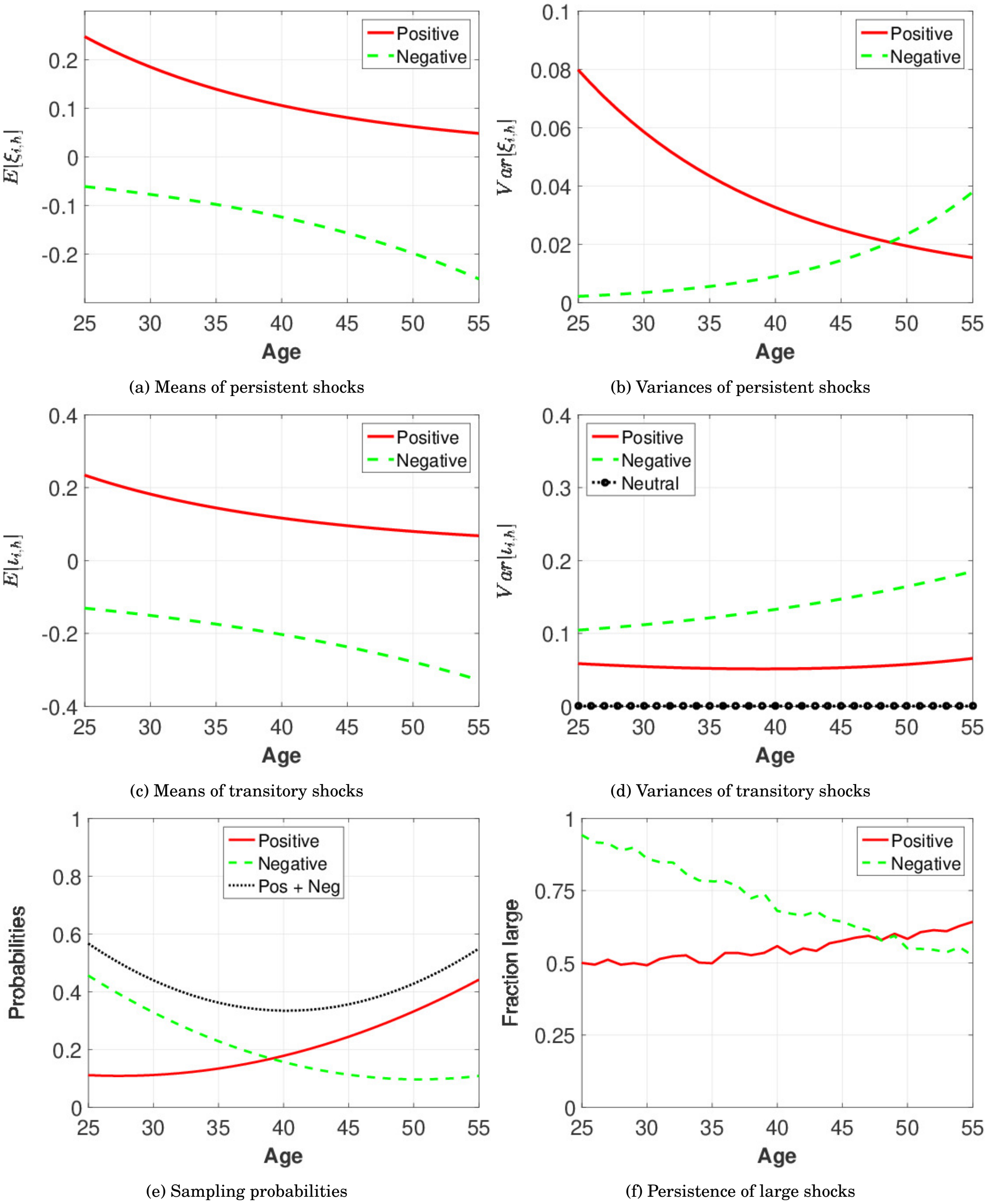
Table 1.1: Parameter Estimates of the Labor Income Process

Model:	(1)	(2)	(3)	(4)	(5)
	Full	No	No	*Macro*	*Micro*
Parameters	Model	h	ι		
ρ^-	0.9788	0.1357	0.4179	0.9685	0.9902
ρ^+	0.9767	-0.2057	0.2411		
θ^-	0.0452	-0.1014	-	-	-
θ^+	0.1530	0.9995	-		
σ_α	0.0238	0.3486	0.3249	0.0302	-
$\sigma_{\hat{\iota}}$	-	-	-	0.2364	0.1744
$\sigma_{\hat{\xi}}$	-	-	-	0.1074	0.1674
Obj. Function	**82.70**	**196.05**	**138.79**	**-**	**-**

Notes: The table displays selected parameter estimates of the earnings process described by Equations (1.1)-(1.10). The remaining parameter estimates are displayed in Table 1.A3. Table 1.A4 displays standard errors. The process is estimated by the method of simulated moments. We use the sample from *SIAB* described in Section 1.2. Column (1) is the full model. Columns (2)-(3) shut down age-dependence and transitory shocks, respectively. The last two columns display parameter estimates of the model in equation (1.11).

persistent shocks are heavily dispersed early in life. Their variance decreases from 0.08 at age 24 to 0.02 at age 55. In contrast, the variance of negative persistent shocks is close to zero early in life and reaches 0.035 at age 55. Figure 1.5e shows that early in life, about 43% of workers receive a negative persistent shock and this probability is decreasing to 13% late in life. In contrast, the probability to receive a persistent positive shock is increasing throughout life. The joint probability to receive any persistent shock during a year is U-shaped over the life-cycle and is particularly low around the age of 40 when 67% of workers receive no such shock. That is, they only receive a transitory mean-zero shock. The variance of these latter shocks is close to zero for most of the life-cycle. Put differently, during ages when individuals are unlikely to receive shocks to their positive or negative component, they face little earnings risk.

Unlike transitory mean-zero shocks, transitory shocks to the positive and negative components do present major earnings risks. Figures 1.5c and 1.5d show that particularly negative transitory shocks are large on average and highly dispersed throughout the life-cycle. In fact, Figure 1.5f shows that most large negative shocks, defined as a log earnings decrease of at least 0.2, are transitory. Early in life, almost all large negative shocks are transitory. The share declines with age and reaches close to 50% at age 55. Likewise, most large positive shocks are transitory, yet, the share of large positive shocks explained by transitory shocks is somewhat smaller than in



(a) Means of persistent shocks

(b) Variances of persistent shocks

(c) Means of transitory shocks

(d) Variances of transitory shocks

(e) Sampling probabilities

(f) Persistence of large shocks

Notes: The figures display age specific estimates from the earnings process described by Equations (1.1)-(1.10). Panel (a) displays the means of persistent shocks. Panel (b) shows the variances of persistent shocks. Panel (c) displays the means of transitory shocks. Panel (d) displays the variances of transitory shocks. Panel (e) shows the probabilities of drawing a shock to the positive and negative components. Panel (f) displays the fraction of shocks above 0.2 that are transitory.

Figure 1.5: Model Predictions

the case of large negative shocks. In contrast to large negative shocks, the share of large positive shocks explained by transitory shocks is increasing with age. It increases from 50% at age 24 to over 60% at age 55.

Figure 1.A5 in the Appendix compares the targeted moments in the model to the data. Moreover, Table 1.A2 shows the loss function with respect to the different data moments. Overall, the model fits the data moments closely. The main conceptual issue is that the model cannot rationalize (by construction) a cross-sectional inequality that is decreasing for several years at the beginning of the life-cycle.

1.4.4 Discussion of the Empirical Results

Age-varying distributions turn out to be key in fitting the moments of residual earnings growth over the life-cycle. In Column (2), we restrict the means and variances of all shocks to constants across ages. In this case, changes in the sampling probabilities of the age-invariant distributions drive all the life-cycle dynamics. Relative to our full model, the loss function more than doubles. Figure 1.A13 in the Online Appendix shows that the model generates little age variation in the moments of residual earnings growth. In particular, the model fails to match the decrease in the variance of positive shocks, the age variation in the share of positive shocks, and the resulting decrease in skewness over the life-cycle.[15]

Column (3) highlights the importance of distinguishing between persistent and transitory shocks.[16] Omitting transitory shocks provides a substantial worse model fit and raises the objective function. The estimate for the autocorrelations of persistent shocks is substantially lower without transitory shocks. The intuition is the following: when neglecting transitory shocks, the moment estimator implies $\rho << 1$ to match the negative autocovariances of earnings growth at lag one and two. Column (3) shows that particularly the estimated autocorrelation of persistent positive shocks decreases. Similarly, Guvenen et al. (2016), who also estimate a model with mixture probabilities, find that positive persistent shocks are only mildly persistent. They allow, similar to us, for a purely transitory shock, but, different from us, they model the other two components as pure $AR(1)$ processes. We find that by modeling the positive and negative components to be a combination of both transitory and persistent shocks, our model identifies persistent shocks that are close to permanent and, at the same time, identifies most large shocks as being purely transitory.

We find age variations in the variances of shocks that are similar to those in Karahan and Ozkan (2013) using PSID data. Our specification allows for a deeper understanding of these life-cycle variations. In particular, the decreasing dispersion in persistent shocks early in life is entirely driven by a decreasing dispersion of positive shocks. Similarly, the increasing dispersion

[15] The poor life-cycle behavior of the model also implies counter-intuitive parameter estimates, e.g., persistent shocks are estimated to be almost transitory.

[16] Figure 1.A14 in the Online Appendix displays all the model moments over the life-cycle.

of persistent shocks late in life is entirely driven by an increasing variance of negative shocks. Finally, the increasing variance in transitory shocks is mostly driven by an increasing variance of negative transitory shocks. Our results regarding the persistence of a typical shock early in a worker's life-cycle are somewhat different from theirs, though. They find that a typical shock is less persistent when young than at prime age. Instead, we find that the share of persistent shocks is declining until prime-age (see Figure 1.5e).

Finally, we compare the results to the earlier literature that models a single age-invariant mean zero $AR(1)$ shock process. To capture the decline in the variance of log earnings at young ages, we extend this framework and allow for an age varying variance of transitory shocks at age 24:

$$\hat{y}_{i,h} = \alpha_i + \hat{Z}_{i,h} + \hat{\iota}_{i,h}, \quad \mathbb{E}(\hat{\iota}_{i,h}) = 0,\ Var(\hat{\iota}_{i,h}) = \sigma^2_{\hat{\iota}} \tag{1.11}$$

$$\hat{Z}_{i,h} = \rho \hat{Z}_{i,h-1} + \hat{\xi}_{i,h}, \quad \mathbb{E}(\hat{\xi}_i) = 0,\ Var(\hat{\xi}_{i,h}) = \sigma^2_{\hat{\xi}}. \tag{1.12}$$

$$\hat{y}_{i,0} = \alpha_i + \hat{\iota}_{i,0} + \hat{\xi}_{i,0} \quad \mathbb{E}(\hat{\iota}_{i,0}) = 0,\ Var(\hat{\iota}_{i,h}) = \sigma^2_{\hat{\iota}_0} \tag{1.13}$$

In this model, either the autocovariance function of residual earnings growth or the covariance function of log residual earnings over the life-cycle identify the model moments. Heathcote et al. (2010) show that what they refer to as *Micro* estimation (targeting the autocovariance function of earnings growth) leads to substantially larger persistent shocks than a *Macro* estimation (targeting covariances of cross-sectional inequality). As a consequence, simulations of the *Micro* estimates lead to a too large increase in cross-sectional inequality over the life-cycle and simulations of the *Macro* estimates imply a too negative first autocovariance of earnings growth, i.e., too much of the average shock is off-set the following year.[17] Columns (4) and (5) present the parameter estimates resulting from GMM estimators of the two identification strategies.[18] As expected, the standard deviation of persistent shocks is about twice as large in the *Micro* approach.

In the estimation of our full model, we target the autocovariance function of residual earnings growth and the cross-sectional variance of residual log earnings over the life-cycle. Figure 1.A5 in the Appendix shows that our full model jointly matches these moments. The reason for the relatively modest increase in earnings inequality over the life-cycle (compared to the *Micro* model) is not that persistent shocks have little dispersion. Conditional on receiving such a shock, the age-averaged variance is similar to the *Micro* estimation (0.0280). Instead, the fact that in a given year a substantial fraction of workers receives no persistent shock is key.

[17] Daly et al. (2016) show that eliminating beginning and end of earnings spell observations helps to reconcile the two approaches within this framework.

[18] For the *Macro* estimation, we use the variance and first ten covariances of log residual earnings. We have also estimated a just identified model with only the first two covariances, and the results remain quantitatively very similar.

1.5 Life-Cycle Consumption and Savings Model

We now turn to the implications of our earnings process for consumption and wealth inequality and the degree to which workers can insure against idiosyncratic earnings shocks. To this end, we introduce the estimated earnings uncertainty into a structural model with incomplete insurance markets.

For simplicity, we consider a partial equilibrium model with exogenous earnings and interest rates. Individuals work for H_W years in the labor market and die with certainty at age $H > H_W$. They have CRRA preferences over consumption with a risk aversion parameter γ, and they discount the future with factor β. There exists a one-period risk free asset a that pays certain returns $1+r$. Individuals face a zero borrowing constrained $a_{h+1} \geq 0$ and make consumption decisions to maximize expected lifetime utility:

$$\max_{c^i_{h=1\ldots H}, a^i_{h=1\ldots H}} \left\{ \mathbb{E}_0 \sum_{h=1}^{H} \beta^{h-1} \frac{c_{i,h}^{1-\gamma}}{1-\gamma} \right\}$$

$$a^i_{h+1} = (1+r)a^i_h + Y^i_h - c^i_h$$

$$a^i_{h+1} \geq 0, \;\; c^i_h \geq 0$$

where Y^i_h are post tax earnings of individual i at age h. During working life, log gross earnings follow the sum of a common deterministic and an individual specific stochastic component:

$$E^i_h = exp(d_h + v_{i,h}) \;\; \text{if } h \leq H_W. \tag{1.14}$$

The government reduces earnings inequality by applying a progressive income tax schedule. We apply the statutory income and social security tax schedule from Germany to map gross earnings into after tax income:

$$Y^i_h = G(E^i_h) \;\; \text{if } h \leq H_W. \tag{1.15}$$

During retirement, workers face no further uncertainty and receive social security benefits. To avoid keeping track of individuals' average earnings, we assume social security benefits depend only on the fixed type α_i:[19]

$$Y^i_h = F(\alpha_i) \text{ if } h > H_W. \tag{1.16}$$

1.5.1 Calibration

We calibrate the coefficient of relative risk aversion and the interest rate outside of our data. The former, γ, is set to 1.5, consistent with Attanasio and Weber (1995). Following Siegel (2002), we fix the value of r to imply a yearly interest rate of 4%. To ensure that households have on average an adequate level of self-insurance, we match median wealth to earnings ratios using data for

[19] Bundesministerium (2015) shows that the retirement replacement rate has decreased over the last decades and is projected to continue to do so. We assume households expect the replacement rate from 2010.

Germany from the *Eurosystem Household Finance and Consumption Survey* (see Eurosystem Household Finance and Consumption Network (2013)). To make the data comparable to the *SIAB*, we restrict the sample to males aged 24-55, who are employees and have positive earnings.[20] We calibrate β to match a median wealth-to-earnings ratio of 4.3 at age 55 leading to a value of 0.9725. As in the data, we assign individuals' initial assets equal to 71% of initial earnings.

Workers work until the age of 55 and, after that, spend twenty years in retirement. We match average earnings during working life, d_h, by estimating cohort averaged age profiles as in Deaton and Paxson (1994). In what we call the age-varying risk model (*AVRM*), the stochastic log earnings component, $v_{i,h}$, follows the process estimated in Column (1) of Table 1.1. For simplicity, we impose $\theta^+ = \theta^- = 0$. We compare the implications of this model to those from the *Macro* approach. To ensure that income inequality is the same in the two models, we estimate the latter model on the variance and the first ten covariances of log earnings implied by the *AVRM*, instead of the data. Figure 1.6b shows that the resulting variance of log earnings over the life-cycle closely tracks the *AVRM*. We refer to this model as the age-invariant risk model (*AIRM*). As it is common in the literature, we assume shocks follow normal distributions in the *AIRM*. We assure that mean earnings are the same in both models at each age by adjusting the path of deterministic earnings over the life-cycle accordingly in the *AIRM*. Finally, we recalibrate β to match the median wealth-to-earnings ratio of 4.3 at age 55 which leads to a somewhat larger value (0.9785) than in the *AVRM*.

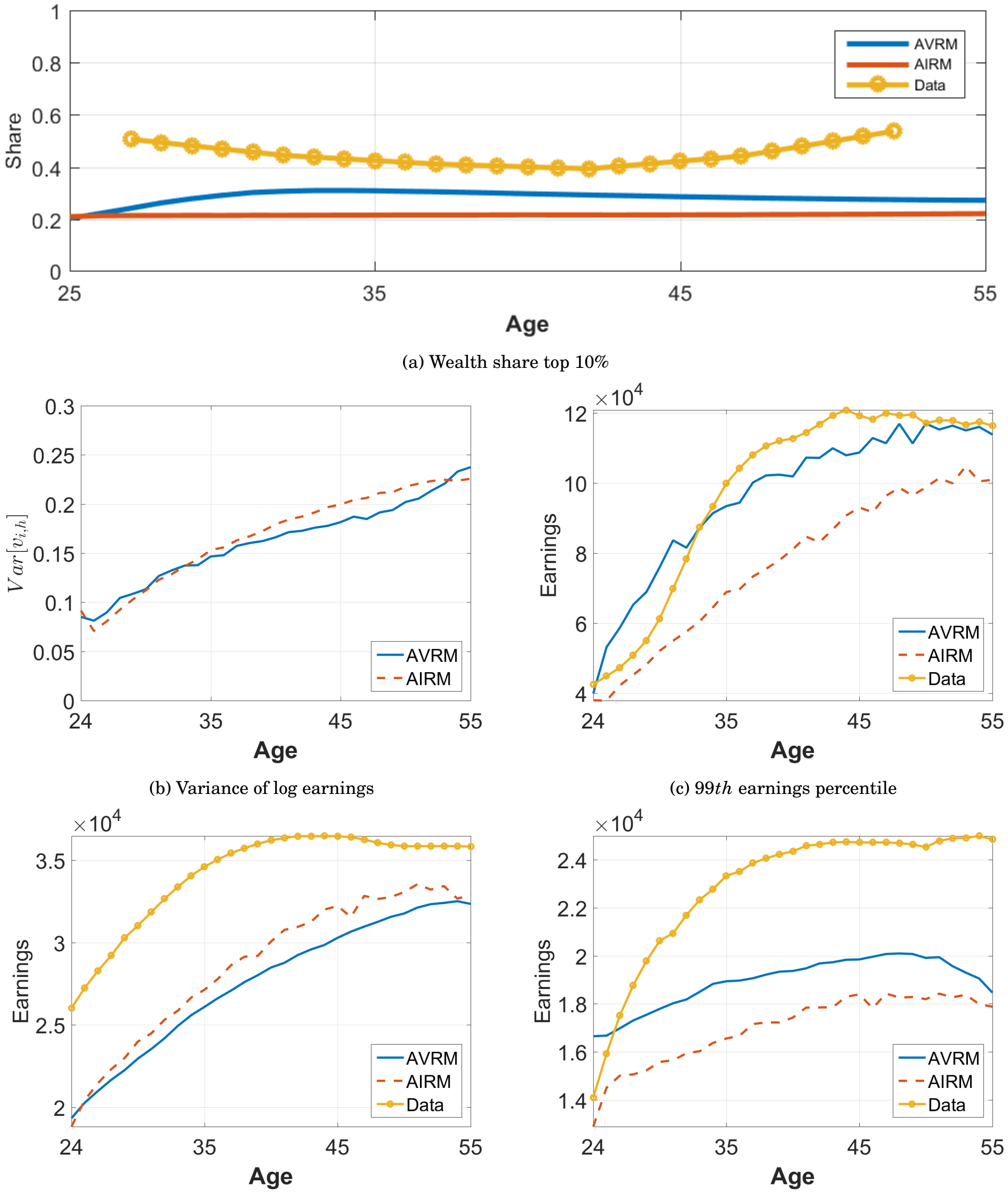
1.5.2 Wealth Inequality

De Nardi et al. (2019) show that existing life-cycle models fail to rationalize sufficient cross-sectional wealth inequality given the observed earnings inequality in the US data. Particularly, the models imply too little wealth holdings by the very top of the wealth distribution. Wealth is also top-concentrated in our German sample of workers: the top 1% own 18.5% of net wealth, and the bottom 50% only own 6.8% of net wealth. The *AVRM* implies wealth shares of 8.8% and 13.6%, respectively. Therefore, wealth inequality is still much lower than in the data, but it is higher than in the *AIRM* which implies wealth shares of 5.5% and 15.6%, respectively.[21] Figure 1.6a shows the share of wealth held by the top 10% over the entire life-cycle. After age 35, the share is around 0.37 in the *AVRM*, compared to 0.22 in the *AIRM*. The figure also highlights that the model, in part, falls short of the data because the calibration restricts wealth inequality to equal earnings inequality at age 24.

The models feature wealth heterogeneity for two reasons. The first is heterogeneity in life-cycle savings. Retirement benefits are lower than average earnings; hence, workers accumulate wealth to smooth consumption. Put differently, heterogeneity in lifetime earnings translate into

[20] The survey imposes that earnings are larger than 1100 Euro per year to be considered employed, which is somewhat more than our restriction in *SIAB*.

[21] Cagetti and Nardi (2006) show that a model with entrepreneurial choice is one possibility to match the right tail of the wealth distribution of workers because former entrepreneurs have high wealth holdings.



(a) Wealth share top 10%

(b) Variance of log earnings

(c) 99th earnings percentile

(d) 50th earnings percentile

(e) 10th earnings percentile

Notes: Panel (a) displays the share of wealth held by the top 10%. *AVRM*: Age-varying risk model. *AIRM*: Age-invariant risk model. *Data*: Eurosystem Household Finance and Consumption Survey. Panel (b) displays the variance of log earnings in the models. Panels (c) to (e) compare selected percentiles of earnings in the models and the data from the *SIAB*.

Figure 1.6: Wealth and Earnings Inequality over the Life-Cycle

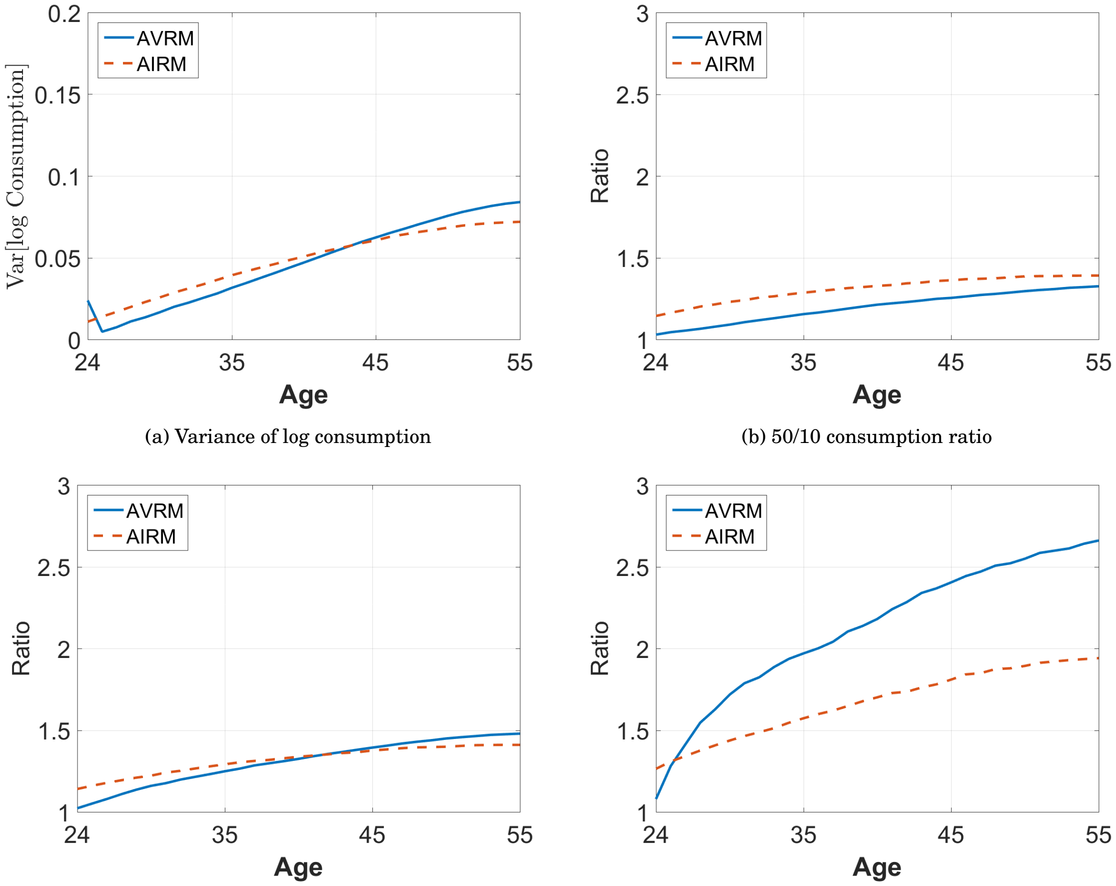
heterogeneity in retirement savings. This channel is particularly potent to explain large top wealth inequality when large earnings differences at the top of the distribution arise early in the working life and are persistent; hence, they translate into large differences in lifetime earnings. Figure 1.6c shows that top earnings (99th percentile) are much higher in the *AVRM* than in the *AIRM*, and they match almost perfectly the data. At the beginning of the life-cycle, top earnings are almost identical in the two models and the data, but they grow much more rapidly in the *AVRM* than in the *AIRM* afterward. This rapid growth in top earnings results from the rare but persistent and fat-tailed positive shocks early in life. Figure 1.6d shows that median earnings are almost identical in the two models; thus, the top 1% have much higher earnings relative to the median worker in the *AVRM*. Finally, Figure 1.6e shows that bottom earnings (the 10th percentile of earnings) are higher and, thus, closer to median earnings in the *AVRM* (and in the data) than in the *AIRM*, therefore, rationalizing that the two models feature the same variance in log earnings over the life-cycle.

The second mechanism that generates wealth inequality is precautionary savings. Castañeda et al. (2003) show that this mechanism contributes strongly to top wealth inequality when there exists a “superstar" earnings state that occurs infrequently and is mildly persistent. When the state is only mildly persistent, workers have incentives to save most of the temporary earnings increase because their earnings are expected to soon be lower. Though rare and large positive shocks early in life have some flavor of this type of shock, these shocks are highly persistent in the *AVRM*. Given their persistent nature, households increase consumption and the effect on precautionary savings is small. Large and persistent negative shocks late in life do increase the need for precautionary savings. Yet, as Civale et al. (2017) show, negative skewness in the shock distribution increase precautionary savings most at the left tail of the wealth distribution; thus, they decrease wealth inequality. Consistent with this, the 10th percentile of the wealth distribution is higher in the *AVRM* than in the *AIRM*, particularly after the age of 45.[22] Measuring overall wealth inequality by the Gini-coefficient of wealth, we find that the increase in top wealth inequality outweighs the decrease in bottom inequality. That is, the Gini-coefficient of wealth is 0.54 in the *AVRM* and 0.49 in the *AIRM* (0.64 in the data).

1.5.3 Consumption Inequality and Insurance

Figures 1.7b to 1.7d compare the consumption distributions in the *AVRM* and the *AIRM*. Bottom inequality (50/10 consumption ratio) grows by a similar amount in the two models over the life-cycle. However, it is somewhat higher in the *AIRM* throughout the life-cycle. To understand higher bottom inequality, the timing and composition of shocks play a key role. Regarding the timing, note that at the beginning of life, when self-insurance is at its lowest, the *AVRM* features relatively few large negative shocks and, thus, relatively few catastrophic events that lead to a large downward consumption adjustment. Regarding the composition, remember that relatively many large negative shocks are transitory and, thus, relatively easy to insure in the *AVRM*

[22]Consistent with precautionary savings being relatively unimportant for top wealth inequality, we find that fixing earnings uncertainty beyond age 40 to the process workers face at age 40 leaves top wealth inequality almost unchanged.



(a) Variance of log consumption

(b) 50/10 consumption ratio

(c) 90/50 consumption ratio

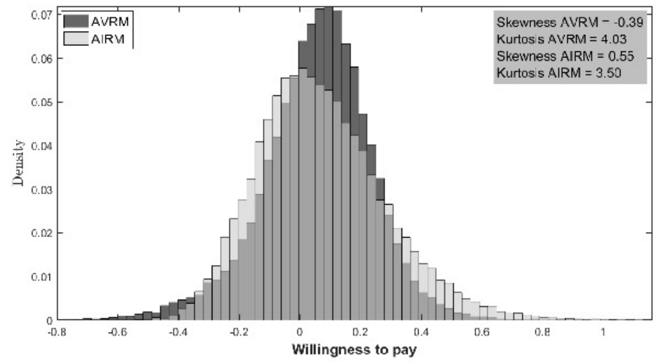
(d) 99/50 consumption ratio

Notes: Panel (a) displays the variance of log consumption by age. Panels (c)-(d) display selected percentile ratios of consumption by age. *AVRM*: Age-varying risk model. *AIRM*: Age-invariant risk model.

Figure 1.7: Consumption Inequality over the Life-Cycle

and this is particularly the case at the beginning of the life-cycle. In contrast, in the *AIRM*, the fraction of large shocks that are negative is age-invariant, and the fraction of large shocks that are transitory is the same for positive and negative shocks.[23] Upper consumption inequality (90/50 ratio) grows somewhat faster in the *AVRM*, but the overall level is similar in the two models. The main difference between the two models is, again, top inequality (99/50 ratio). Mirroring top earnings and wealth inequality, top consumption inequality grows much more rapidly with age in the *AVRM*, and it is substantially higher on average than in the *AIRM*. That is, large and persistent positive shocks early in life allow a few lucky workers to enjoy particularly high consumption levels. Those in the top 1% consume 2.6 times more than the median at age 55 in

[23]We opt for a model with age-invariant shocks as the comparison to the *AVRM* because it is the most widely used framework. Alternatively, one could estimate age-varying variances for transitory and persistent shocks and assume that these shocks are normally distributed. This extension would allow the fraction of large shocks and the fraction of large shocks that are transitory to vary, but, by assumption, these fractions would be the same for positive and negative shocks.



Notes: The figure displays the densities of lifetime consumption equivalences that a worker is willing to pay to receive the consumption stream from the social planner solution instead of his realized stream. Values greater than zero imply the worker is worse off than in the social planner solution. *AVRM*: Age-varying risk model. *AIRM*: Age-invariant risk model.

Figure 1.8: Consumption Equivalences

the *AVRM*, but only 1.9 times more in the *AIRM*.

These consumption dynamics have qualitative ambiguous effects on the welfare costs of incomplete insurance markets. On the one hand, fewer catastrophic consumption events, i.e., less consumption inequality at the bottom of the distribution, imply lower welfare costs from incomplete markets in the *AVRM*. On the other hand, more resources allocated to the top 1% imply that the typical household has lower average consumption and, thus, implies higher welfare costs arising from incomplete markets. Figure 1.8 displays these two effects graphically. It displays the densities of lifetime consumption equivalences an individual worker is willing to pay to receive the consumption stream from the social planner solution instead of his realized stream.[24] The figure shows that poor individual outcomes, values greater than zero, are more likely in the *AIRM*. Put differently, even on a lifetime utility basis, the left tail of the consumption distribution is more dispersed in the *AIRM* leading to lower welfare. At the same time, the *AVRM* has a higher probability of lifetime utility outcomes that are much better than the social planner solution (large negative values in the graph). An outcome that is better than 130% of the social planner solution occurs with 2.5% in the *AIRM* and 3.6% in the *AVRM*. Again, this is a different way of saying that the fatter right tail in the consumption distribution translates to a fatter left tail in the distribution of lifetime consumption equivalences in the *AVRM*. Given that the two models have the same amount of total labor income, the resources used to finance these tail events must come from workers in the center of the distribution. Indeed, the center of the distribution of willingnesses to pay is shifted to the right and is thinner in the *AVRM* relative to the *AIRM*. This also manifests in a kurtosis of the distribution that is larger in the former. Shifting resources from median lifetime consumption outcomes towards high lifetime

[24]We define the social planner solution as the discounted utility resulting from optimal choices when earnings are pooled across all agents at each age, but they are not pooled across ages. That is, the social planner cannot use future labor income to finance today's consumption.

consumption reduces welfare. We find that this effect dominates the effect of less catastrophic outcomes, i.e., welfare is lower in the *AVRM*. An unborn worker is willing to pay 4.4% of lifetime consumption to avoid the idiosyncratic earnings risk in the *AVRM* and 3.7% in the *AIRM*.

Finally, we inspect in more detail the differences between the two models concerning the dynamics of cross-sectional consumption inequality over the life-cycle. Guvenen (2007) shows that the shape of this moment is informative about the age-varying insurance that households have against earnings risk. More specifically, he shows that standard earnings risk models generate a concave profile of consumption inequality over the life-cycle because earnings shocks become effectively more transitory as workers approach retirement. He shows that learning about deterministic differences in individual earnings growth profiles can reconcile the model with the more linear increase in US data.[25] Fuchs-Schúndeln et al. (2010) find that the German data also displays close to a linear increase in the variance of log consumption over the life-cycle.

Figure 1.7a shows that consumption inequality over the life-cycle also shows a concave shape in the *AIRM* calibrated to German data. The model implies a total increase in the variance of log consumption of 0.05 from age 25 to age 55 which is consistent with the consumption data analyzed by Fuchs-Schúndeln et al. (2010).[26] The total increase in consumption inequality over the life-cycle is similar in the *AVRM*, however, the shape of the increase is somewhat different. In particular, after an initial fall, the increase is steeper than in the *AIRM* and shows less flattering at old age.

To understand how age-varying risk affects cross-sectional consumption inequality over the life-cycle, we compute at each age the average consumption responses to different shocks using a linear regression:[27]

$$\Delta log(c_{i,h}) = \varphi_{0,h} + \varphi_{\xi^+,h}(\xi_{i,h}|\xi_{i,h} > 0) + \varphi_{\xi^-,h}(\xi_{i,h}|\xi_{i,h} < 0) + \varphi_{\iota^+,h}(\iota_{i,h}|\iota_{i,h} > 0) + \varphi_{\iota^-,h}(\iota_{i,h}|\iota_{i,h} < 0) + \varsigma_{i,h}.$$

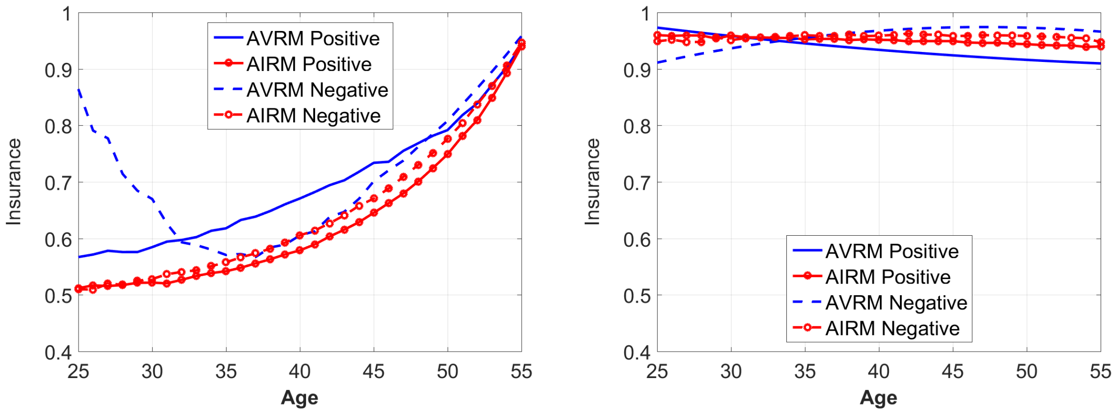
Thus, $1 - \varphi_{\xi^+,h}$ measures how much of a persistent positive shock does not translate into consumption, i.e., how much of a shock is insured. We calculate these insurance coefficients for the four types of shocks. In case of uncorrelated shocks, as in the *AIRM*, and without conditioning on the sign of the shocks, these insurance coefficients are equal to those calculated by Kaplan and Violante (2010).

Figure 1.9 shows that consumption responds more strongly to positive than to negative shocks, particularly late in life. The asymmetry is larger in the *AVRM* because precautionary savings are higher late in life. Hence, when a positive shock occurs, a worker requires fewer precautionary savings for the rest of the working life; hence, he can consume these. The figure also shows that

[25] De Nardi et al. (2019) come to a different conclusion regarding the shape of this moment in US data. Their results imply a concave shape.

[26] Similar to wealth data, consumption data is only available at the household level in Germany.

[27] In the *AVRM*, transitory shocks include those from the mean zero component of earnings, that tend to be small and those draws from the positive and negative components that tend to be large. Because the former have almost no dispersion and, thus, almost no quantitative effect on consumption, we only focus on the latter.



(a) Persistent Shocks

(b) Transitory Shocks

Notes: Panel (a) displays the fraction of persistent shocks that do not translate into consumption changes. Panel (b) displays the the same for transitory shocks.*AVRM*: Age-varying risk model. *AIRM*: Age-invariant risk model.

Figure 1.9: Insurance against Shocks

consumption responds more to persistent than to transitory shocks in both models.[28] Remember that the probability to receive a positive shock and the probability to receive a persistent shock are increasing late in life in the *AVRM*. As a consequence, consumption responses become relatively large in this model leading to a relatively rapidly growing consumption inequality.[29]

1.6 Conclusion

This paper estimates explicit age-varying distributions of positive and negative (transitory and persistent) earnings shocks in Germany. Early in the working life, workers experience rare but large positive shocks, both transitory and persistent. As workers move into prime-age, earnings risk decreases, both because earnings fluctuate less and fluctuations are more transitory on average. For elderly workers, rare but large (persistent and transitory) negative earnings shocks become a major source of risk. Our parametric earnings process is simple enough to introduce it into a model of consumption decisions with incomplete financial markets. The age-varying risk structure helps us to reconcile two stylized facts from the data. First, relative to a model with an age-invariant $AR(1)$ process and Gaussian shocks, wealth is more concentrated at the top of the

[28] In either model, average consumption responses are weaker than those found by Kaplan and Violante (2010) for a US calibration. For one, the differences arise from their model featuring permanent shocks (shocks are highly persistent in our case). Moreover, taxes are more progressive in Germany leading to smaller net earnings changes and, thus, consumption changes, given a gross earnings change. Relative to their findings, consumption responds particularly weak at the beginning of life. Different from them, workers start with positive assets in our model which weakens consumption responses, particularly of persistent negative earnings shocks. Moreover, net income growth is smaller in Germany over the life-cycle which weakens initial consumption responses to positive shocks.

[29] We find that when decreasing the variance of shocks after age 45 by 30% and recalibrating the location parameters of the distributions to ensure that the conditional means of the shocks are unchanged results in a flattering in the growth rate of consumption inequality late in life.

wealth distribution. Large persistent positive shocks early in life imply high lifetime incomes for a small group of workers. These workers have incentives to accumulate large savings for life-cycle purposes. As a result, the share of wealth held by the top one percent increases by a factor of 1.6. Second, cross-section consumption inequality grows relatively more rapidly close to retirement in our model. This results from positive and persistent shocks becoming relatively more likely at the end of working life and consumption responding relatively strongly to these types of shocks. As individual consumption responses become stronger, the variance of consumption inequality increases.

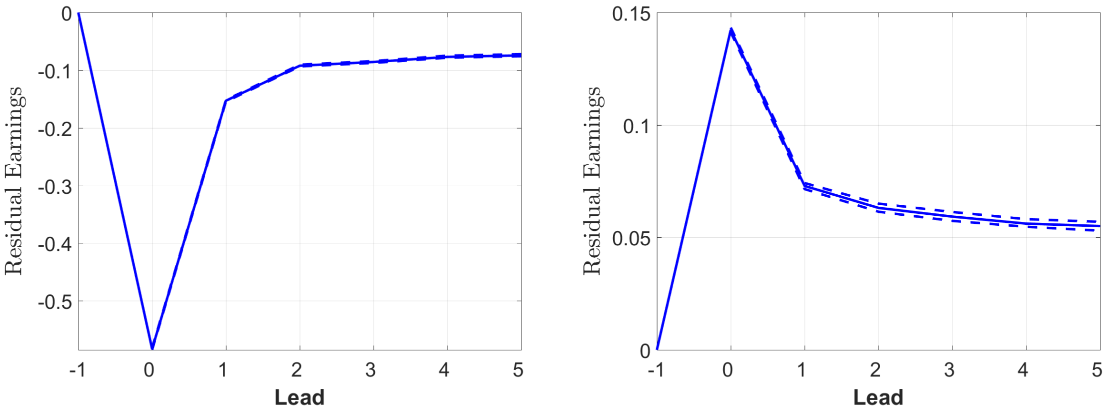
Our analysis restricts itself to male workers with a high attachment to the labor force, mainly, because our data do not allow us to identify workers' participation decisions upon shocks as in Low et al. (2010). Studying age-varying, non-normally distributed earnings risk while allowing at the same time for employment decisions resulting from shocks promises further insights into the welfare costs of incomplete insurance markets. Similarly, little is known about the effects that this richer risk structure has on joint household decisions of labor supply, consumption, and fertility.

Age-varying risk also raises several questions regarding social insurance. On average, earnings risk is negatively skewed, implying that insurance against catastrophic events is highly valuable to society. Yet, early in life, when self-insurance is lowest, earnings risk is positively skewed, thus, decreasing the need for insurance. What is more, most large shocks early in life are transitory. The optimal size and design of the welfare state is, therefore, an even more complex question than that of age-independent Gaussian shocks.[30] Finally, the risk structure also has implications on the level of attainable private (and public) insurance. Krueger and Perri (2006) analyze privately efficient risk-sharing contracts. We show that prime-aged workers face little risk; thus, they have little incentives to enter into any private risk-sharing contract or support large public risk-sharing contracts.

[30] Golosov et al. (2016) is a recent example that studies optimal redistribution with non-normally distributed earnings shocks.

1.7 Appendix

1.7.1 Residual Earnings Dynamics After Observable Events



(a) Unemployment

(b) Job-to-Job

The figure displays mean residual log earnings around observable labor market events. We normalize mean residual log earnings to zero in the year before the event. Panel (a) shows the case of workers becoming unemployed, and panel (b) shows the case of a job-to-job transition that resulted in an earnings gain in year one. The dashed lines display bootstrapped 95% confidence intervals.

Figure 1.A1: Residual Log Earnings after Unemployment and Job-to-Job Transition

Figures 1.A1a and 1.A1b display residual log earnings around observable labor market events. In constructing these figures, we first obtain residual log earnings by regressing for each age the log earnings on workers' observable characteristics. Next, we define an unemployment event as a worker working less than 300 natural days in a given year while in the previous year he has worked more than 300 natural days. Moreover, we define a job-to-job transition as a worker working more than 300 natural days in two consecutive years while he changes his establishment. Tjaden and Wellschmied (2014) show that about one third of job-to-job transitions result in a downward move in the job ladder. To avoid this complication, we condition on job-to-job transitions that lead to an earnings gain in the initial year. We normalize a worker's residual log earnings to zero in the year before the labor market event occurs and trace average residual log earnings for the consecutive five years.

Figure 1.A1a shows that residual log earnings fall by about 0.57 log points in the year of an unemployment spell. However, they partially recover during the consecutive years leaving a worker with 8% lower earnings on average after 5 years. This pattern is qualitatively consistent with the US data analyzed by Jacobson et al. (1993). The reduction in workdays during the first year of unemployment contributes to the initially large decline in earnings. As workers find work and reclimb the job-ladder, their earnings return towards their pre-unemployment level. Figure 1.A1b shows that job-to-job transitions show a similar pattern. On average, residual log earnings rise by 0.14 log points in the year of the transition but fall during subsequent years resulting in

an average increase in log earnings of 0.06 after 5 years. A possible explanation for the initial overshooting of residual earnings are signing bonuses paid upon hiring.

1.7.2 Constructing the Moments

We model log earnings as the sum of deterministic and stochastic components that may depend on cohort and time effects. Let $Y^c_{i,h,t}$ be the log earnings of individual i, at age h, belonging to the birth cohort c, in year t:

$$Y^c_{i,h,t} = f(X_{i,h,t}) + y^c_{i,h,t}, \tag{A.17}$$

where $f(X_{i,t,h})$ is a function representing observable differences among workers ($X_{i,h,t}$) such as education, region, age and industrial sector, and year effects. $y^c_{i,h,t}$ represents the unobserved component of earnings. Rewriting the above process in first differences yields

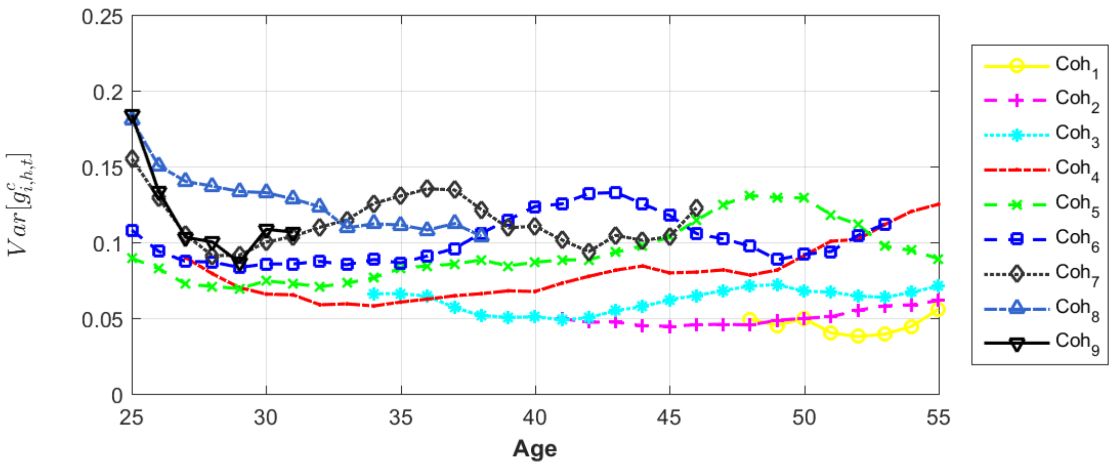
$$\Delta Y^c_{i,h,t} = \Delta f(X_{i,t,h}) + \Delta y^c_{i,h,t}. \tag{A.18}$$

First, we remove predictable changes in log earnings, such as education, by running for each age cross-sectional regressions. The regressions control for a dummy of workers' education, year, region of residence, and 14 major industries. Denote the corresponding residual by $g^c_{i,h,t}$:

$$g^c_{i,h,t} \equiv \Delta y^c_{i,h,t} \tag{A.19}$$

So far, our specification allows the moments of residual earnings growth to be calendar year and birth cohort specific. As an illustration of such effects, Figure 1.A2 shows the variance of residual earnings growth for each of our 9 cohorts. There are two salient features. First, there is a calendar year effect with large variances for all cohorts about 5 years after the German reunification. For example, for the 5th birth cohort, born between 1951-1957 (green line) the German reunification occurs at ages 34-40, and the time effect increases the variances after age 45. Second, there is also a visible cohort effect, with later cohorts facing substantial higher variances than earlier cohorts. We follow Blundell et al. (2015) and eliminate these effects by averaging all moments (variance, skewness, kurtosis, etc.) across cohorts, assigning equal weight to each. Therefore, our results can be interpreted as the risk a typical cohort faces.

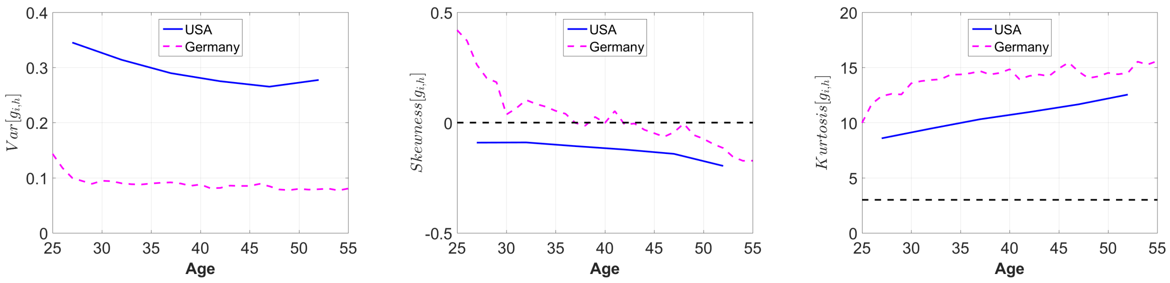
To compute the cross-sectional earnings inequality over the life-cycle, $Var(y_{i,h})$, we follow Deaton and Paxson (1994) and regress the cross-sectional variance of log earnings on a full set of age and cohort dummies. We compute the intercept (age 24) as the mean effect across cohorts.



Notes: Figure 1.A2 displays the variance of residual earnings growth by age and birth cohorts. Birth cohorts 1-9 belong to years of birth 1923-1929,1930-1936,...,1980-1986, respectively.

Figure 1.A2: By Cohort Variance of Residual Earnings Growth

1.7.3 US Comparison



(a) Variance (b) Skewness (c) Kurtosis

Panels (a), (b), and (c) display, respectively, the variance, skewness and kurtosis of residual earnings growth by age for the US and Germany. Section 1.2 describes the German data. For the US, we compute for each age groups (25-29,...,50-54) the average over the percentiles reported in Guvenen et al. (2016).

Figure 1.A3: US and German Higher Order Moments

Figure 1.A3 compares the variance, skewness and kurtosis of residual earnings growth. The German data refers to labor earnings from the *SIAB* sample described in Section 1.2. For the US, we compute for each age groups (25-29,...,50-54) the average over the percentiles reported in Guvenen et al. (2016). Different from the *SIAB* data, the latter includes self-employment income.

1.7.4 Moments Selection and Estimation

We simulate life-cycle employment histories for 20,000 workers who enter the labor market at the age of 24 and work until the age of 55. The resulting simulated minimum distance estimator is given by:

$$\hat{\theta} = \underset{\theta}{argmin} \boldsymbol{F}(\theta)' \boldsymbol{I} \boldsymbol{F}(\theta) \tag{A.20}$$

$$F(\theta)_n = \frac{f_n(\theta) - m_n}{\omega_n}, \tag{A.21}$$

where $f_n(\theta)$ is the n^{th} model moment, and m_n is the corresponding n^{th} data moment. Similar to Guvenen et al. (2016), we employ moment specific adjustment factors, ω_n. We use these adjustment factors to jointly deal with two issues presented by the data. First, the moments are measured on different scales. For example, kurtosis is in absolute value about 500 times larger than the first autocovariance. If we had minimized the sum of absolute squared deviations ($\omega_n = 1$), the optimization would not have had put any emphasis on moments with low absolute sizes. At the same time, we have several moments which are close to zero, such as the autocovariance function, but fluctuate substantially in relative terms from one age to the next. Hence, if we had minimized the sum of relative squared deviations ($\omega_n = abs(m_n)$), the optimization would have concentrated almost exclusively on these large relative deviations close to zero that are likely the result of a small sample.

Using moment specific adjustment factors allows us to use absolute deviations but reduce the emphasis on moments with large absolute numbers. Unfortunately, it gives us a degree of discretion. We choose the adjustment factors in an iterative fashion such that the implied loss function displayed in Table 1.A2 is consistent with the model fit we observe in Figure 1.A5. We opt to give the variance of log earnings over the life-cycle and the mean earnings growth by age (which is zero by construction in the data) somewhat larger weights as we want to ensure a good fit with these moments. We keep the adjustment factors fixed when estimating restricted versions of the model.

Most sets of moments contain 31 year moments. This is the case for the skewness, kurtosis, fraction of positive shocks, fraction of shocks above 5%, unconditional mean, unconditional autocovariance, conditional mean and conditional variance. This amounts to $31 \times 10 = 310$ moments. The conditional first autocovariance are observed for 30 years. These amount to $30 \times 2 = 60$ moments. The conditional second autocovariance are observed for 29 years, amounting to $29 \times 2 = 58$ moments. The variance of log earnings amount to 32 moments. Lastly, the initial mean of log residual earnings at age 24 amounts to 1 moment. The total number of moments that we target is then $N = 310 + 60 + 58 + 32 + 1 = 461$.

Given our large parameter set, the issue of finding a global minimum arises. We first obtain reasonable starting values by experimenting with different combinations of parameters. We tested different global minimum algorithms and a pattern search algorithm performed best in finding a minimum. Provided the optimal parameters, we compare the minimum to (possibly) other minima where we start the algorithm from different starting points. We find that the pattern search algorithm, in general, is able to converge to the same minimum from different starting points.

We obtain standard errors by 100 block bootstraps. Using a global search algorithm in each iteration is infeasible numerically. Therefore, we use a local optimizer, a sequential quadratic programming algorithm. Implicitly, we assume that a change in the data sample does not lead to a too large change in our estimates, therefore, possibly downward biasing the standard errors.

1.7.5 Growth Rate Heterogeneity

Our baseline specification omits heterogeneity in individual earnings growth rates. Guvenen (2009) (and the citations within) show that this type of heterogeneity is potentially an important source of individual earnings dynamics. In particular, this line of literature finds that the increase in the cross-sectional inequality of earnings over the life-cycle is driven partly by this type of heterogeneity. Moreover, this literature finds that shocks to earnings, instead of featuring a close to permanent component as in our baseline results, are only mildly persistent. To show the robustness of our results, we estimate the following augmented version of the model:

Table 1.A1: Growth Rate Heterogeneity

ρ^-	ρ^+	θ^-	θ^+	σ_α	σ_κ
0.9795	0.9767	0.0497	0.1516	0.0224	0.0025
Obj. Function 81.95					

Notes: The table displays selected parameter estimates of the earnings process described by Equation (A.22). The process is estimated by the method of simulated moments. We use the sample from *SIAB* described in Section 1.2.

$$y_{i,h} = \underbrace{\alpha_i + \kappa_i h}_{\text{initial heterogeneity}} + \underbrace{u_{i,h}}_{\text{stochastic component}}, \tag{A.22}$$

where $\alpha_i \sim N(0,\sigma^2_\alpha)$, $\kappa_i \sim N(0,\sigma^2_\kappa)$, and $COV(\alpha_i,\kappa_i) = 0$. Our moments identify σ^2_κ in two distinct ways. First, Guvenen (2009) shows that heterogeneous growth rates imply that the cross-sectional variance of residual earnings growth increases in a convex fashion over the life-cycle. Second, Hryshko (2012) shows that the the autocovariance function of residual earnings growth converges at distant lags towards σ^2_κ.

Table 1.A1 shows the results from estimating this model. The resulting change in the objective function is small, and we find little unobserved heterogeneity in individual earnings growth rates. Within two standard errors, the variance is smaller than 0.000013 which is by an order of magnitude smaller than the values found by the literature that estimates this parameter jointly with modestly persistent earnings shocks. These results are consistent with those in Blundell et al. (2015) who, similar to us, identify the parameter from the autocovariance function of earnings growth with sufficient long lags. The tight estimate of the parameter may be surprising at first, given the large noise in this moment even in administrative data (see Figure 1.4c). Hryshko (2012) uses simulation exercises to show that a minimum distant estimator closely identifies σ^2_κ when it takes all, though noisy, autocovariances into account.

In a simpler model, Hryshko (2012) also shows that omitting transitory shocks downward biases the estimate for persistent shocks and upward biases σ^2_κ. Following this idea, we reestimate the model without transitory shocks.[31] We find much lower $AR(1)$ estimates and a larger estimate of profile heterogeneity, $\sigma_\kappa = 0.0112$. The intuition is simple. When neglecting transitory shocks, the moments estimator implies $\rho << 1$ to match the negative autocovariance function at lag one. Yet, $\rho << 1$ alone implies that the autocovariance function is negative at intermediate lags. To obtain an autocovariance function which is closer to zero at those lags, $\sigma_\kappa >> 0$ is required.

[31]The results are available upon request from the authors.

1.7.6 Estimation Results

Table 1.A2: Objective Function Decomposition

Model:	(1)	(2)	(3)	(4)
	Full	No	No	Heterog.
Moments	Model	h	ι	Growth
$\mathbb{E}[g^+]$	11.42	23.61	21.03	11.06
$\mathbb{E}[g^-]$	4.78	7.65	6.36	4.87
$Var[g^+]$	4.19	15.02	6.65	4.29
$Var[g^-]$	6.51	6.10	5.95	6.31
$Kelly's\ Skewness[g]]$	3.48	6.48	7.73	3.43
$Kurtosis[g]$	5.87	9.14	10.89	5.91
% of Positive Innovations	10.28	22.33	16.54	10.12
$E[g_h^- g_{h+1}]$	5.49	12.07	-	5.41
$E[g_h^+ g_{h+1}]$	6.54	7.26	-	6.62
$E[g_h^- g_{h+2}]$	1.30	1.37	-	1.31
$E[g_h^+ g_{h+2}]$	2.04	9.11	-	2.10
$E[g]$	7.15	6.98	5.45	6.97
% of Innovations > 5%	5.32	4.38	6.70	5.26
Uncond. Autocovariance	2.63	11.33	8.58	2.61
Initial $\mathbb{E}$[log earnings]	1.29	5.53	1.92	1.28
Var[log earnings]	4.42	47.70	41.00	4.40
Total	**82.70**	**196.05**	**138.79**	**81.95**

Notes: The table displays a decomposition of the loss function. The process is estimated by the method of simulated moments. We use the *SIAB* sample selection described in Section 1.2. Column (1) estimates our *Baseline* specification outlined in 3.3. Columns (2)-(3) shut down age-dependence and transitory shocks, respectively. Column (4) includes heterogeneity in deterministic individual earnings growth.

1.8 Online Appendix

1.8.1 Identification

In the following, we provide additional intuition for the identification of the parameters discussed in Section 4.4 of the paper. To this end, we perform two related simulation exercises. First, we

Table 1.A3: Additional Parameter Estimates from Table 1.1

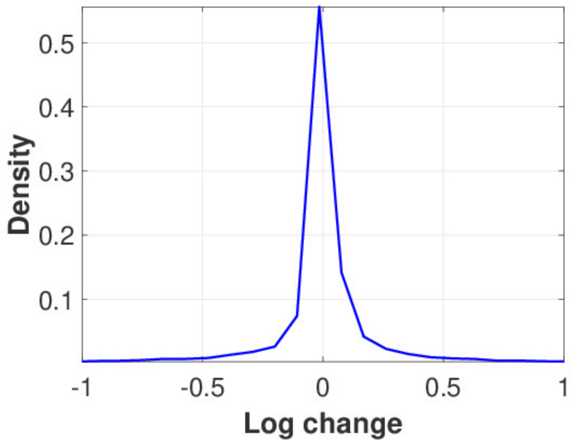
Model:	(1)	(2)	(3)	(4)
	Full	No	No	Heterog.
Parameters	Model	h	ι	Growth
δ_I^-	0.4844	0.5442	0.5621	0.4844
δ_{II}^-	-0.0294	-0.0280	-0.0294	-0.0294
δ_{III}^-	0.0006	0.0006	0.0006	0.0006
δ_I^+	0.1132	0.2521	0.2727	0.1132
δ_{II}^+	-0.0030	-0.0053	-0.0031	0.0030
δ_{III}^+	0.0004	0.0004	0.0004	0.0004
γ_{a,ι^+}	0.7989	1.7753	-	0.8033
γ_{b,ι^+}	0.0267	-	-	0.0266
γ_{a,ι^-}	1.4259	1.3716	-	1.4270
γ_{b,ι^-}	-0.0133	-	-	-0.0131
γ_{a,ξ^+}	0.8798	1.9205	1.1186	0.8729
γ_{b,ξ^+}	0.0171	-	0.0203	0.0170
γ_{a,ξ^-}	0.6636	1.5199	1.3826	0.6702
γ_{b,ξ^-}	0.0007	-	-0.0012	-0.0001
λ_a^+	-1.6661	-4.5968	-1.8372	-1.6632
λ_b^+	-0.0747	-	-0.0694	-0.0748
λ_a^-	-3.0584	-3.4462	-3.4936	-3.0584
λ_b^-	0.0451	-	0.0380	0.0448
σ_{ι^n}	0.0169	0.0068	0.0083	0.0168
σ_{0,ι^+}	0.5807	0.0000	0.0000	0.5794
σ_{0,ι^-}	1.5764	1.4263	1.7277	1.5770
$\mu_{0,\iota}^+$	-1.1461	-1.8582	-1.2565	-1.1535
$\mu_{0,\iota}^-$	-3.0737	-2.6942	-3.8262	-3.0820
Objective Function	**82.70**	**196.05**	**138.79**	**81.95**

Notes: The table displays additional estimates to Table 1.1. The last column refers to Table 1.A1.

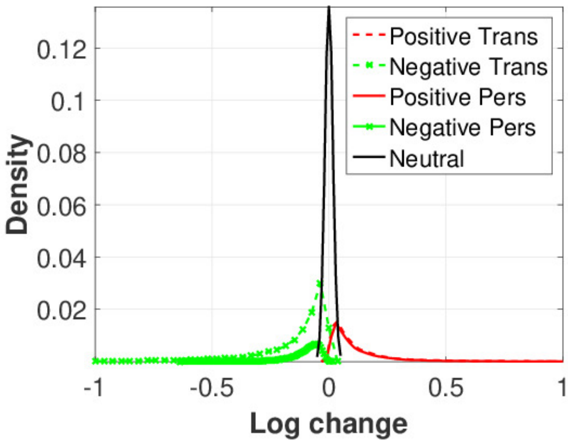
Table 1.A4: Standard Errors, Table 1.A1.

Parameters	SE	**Parameters**	SE
ρ^-	0.0000***	δ_I^-	0.0000***
ρ^+	0.0000***	δ_{II}^-	0.0000***
σ_α	0.0000***	δ_{III}^-	0.0000***
σ_{ι^n}	0.0002***	δ_I^+	0.0000***
$\gamma_{a,\iota}^+$	0.0001***	δ_{II}^+	0.0000***
$\gamma_{b,\iota}^+$	0.0009***	δ_{III}^+	0.0000***
$\gamma_{a,\iota}^-$	0.0002***	$\gamma_{a,\xi}^-$	0.0007***
$\gamma_{b,\iota}^-$	0.0013***	$\gamma_{b,\xi}^-$	0.0016
λ_a^+	0.0063***	$\gamma_{a,\xi}^+$	0.0002***
λ_b^+	0.0010***	$\gamma_{b,\xi}^+$	0.0006***
$\gamma_{0,\iota}^+$	0.0000***	λ_a^-	0.0058***
$\gamma_{0,\iota}^-$	0.0000***	λ_b^-	0.0020***
$\lambda_{0,\iota}^-$	0.0004***	$\lambda_{0,\iota}^+$	0.0001***
θ^+	0.0000***	θ^-	0.0000***
σ_κ	0.0005***		

Notes: The table displays standard errors for the estimates of our full model, displayed in Column (1) of Tables 1.1. Standard errors are obtained by 100 block bootstraps. Estimates with superscripts {*,**,***} imply the parameter is different from zero at the 10, 5, and 1 percent significance level, respectively.



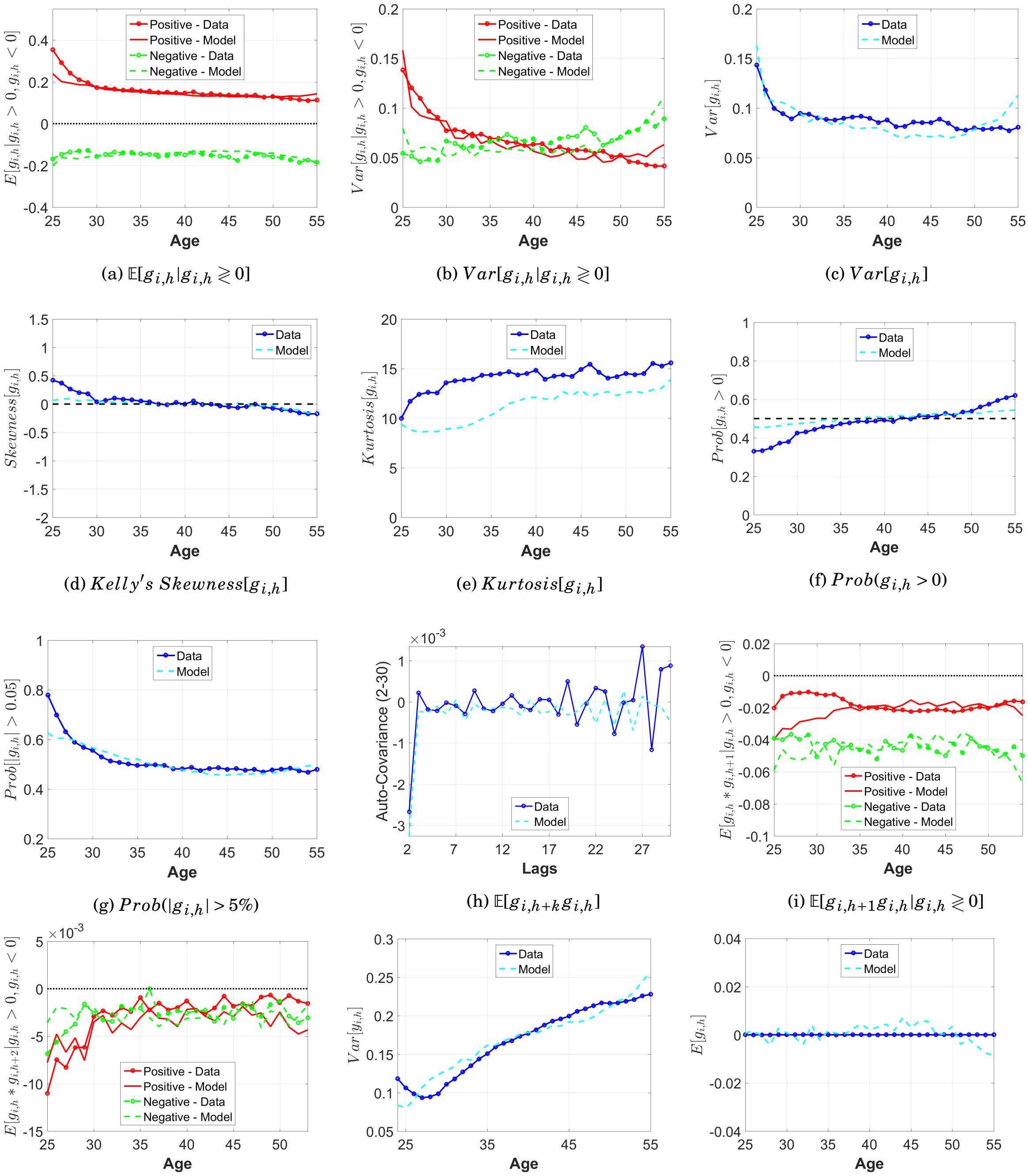
(a) Data



(b) Model

Figure 1.A4a displays the kernel distribution of residual earnings growth at the age of 36 in our data described in Section 1.2. Figure 1.A4b displays the densities of shocks from the model described in Section 3.3 at age 36. We weigh the individual densities with the probability that each shock occurs.

Figure 1.A4: Density of Residual Earnings Growth



(a) $\mathbb{E}[g_{i,h}|g_{i,h} \gtrless 0]$ (b) $Var[g_{i,h}|g_{i,h} \gtrless 0]$ (c) $Var[g_{i,h}]$

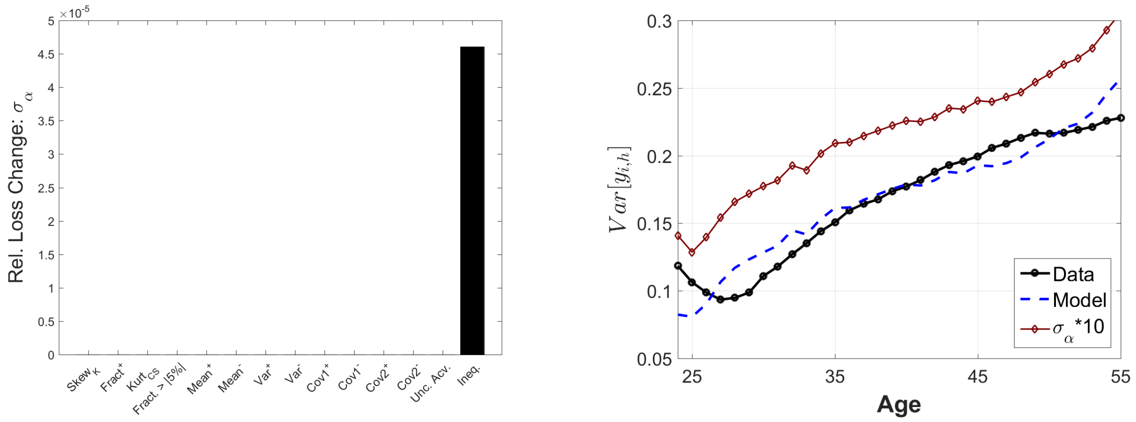
(d) $Kelly's\ Skewness[g_{i,h}]$ (e) $Kurtosis[g_{i,h}]$ (f) $Prob(g_{i,h} > 0)$

(g) $Prob(|g_{i,h}| > 5\%)$ (h) $\mathbb{E}[g_{i,h+k}g_{i,h}]$ (i) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$

(j) $\mathbb{E}[g_{i,h+2}g_{i,h}|g_{i,h} \gtrless 0]$ (k) $Var[log\ Earnings]$ (l) $\mathbb{E}[g_{i,h}]$

Figure 1.A5: Model Fit - Column (1), Table 1.1

highlight the relationship between a particular model parameter and the different data moments. To this end, we simulate a 1% change in a model parameter from its optimum holding all other parameters fixed and plot the resulting change in the age averaged model moments relative to their minimum.[32] Second, to highlight those moments providing most of the identification of a particular parameter, we plot the non-aged average change in those model moments as a response to a change in the model parameter from its optimum. In this exercise, we select changes in parameter values at discretion to make the effects best visible.



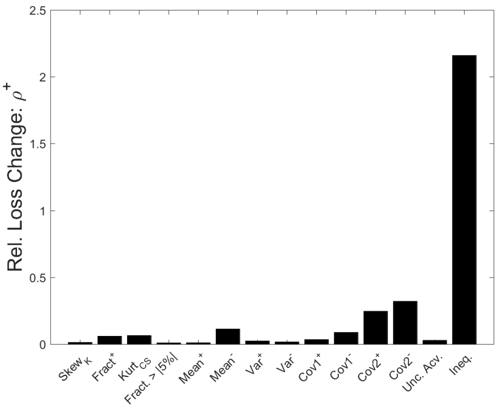
(a) Sensitivity

(b) Simulation

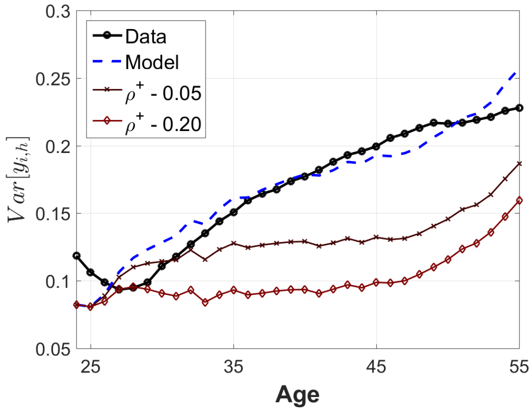
Panel A displays the moment responses to a 1% increase in the standard deviation of permanent heterogeneity, $\hat{\sigma}_\alpha$. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panel B displays the simulated cross-sectional inequality resulting from a ten-fold increase relative to the optimum.

Figure 1.A6: Permanent initial heterogeneity

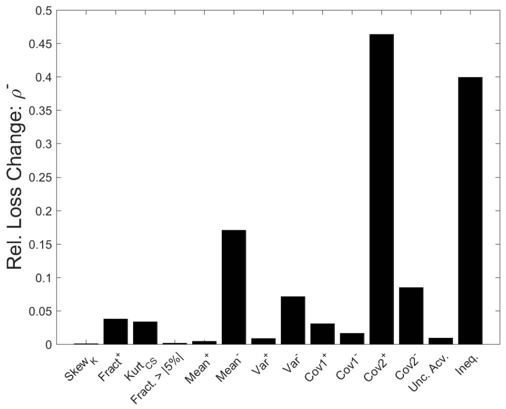
[32] All parameter changes affect the mean of log earnings and log earnings growth, and we choose to omit these responses in our graph for illustration purposes.



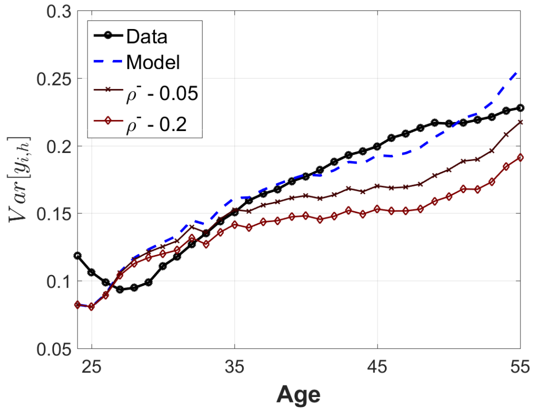
(a) Sensitivity



(b) Simulation



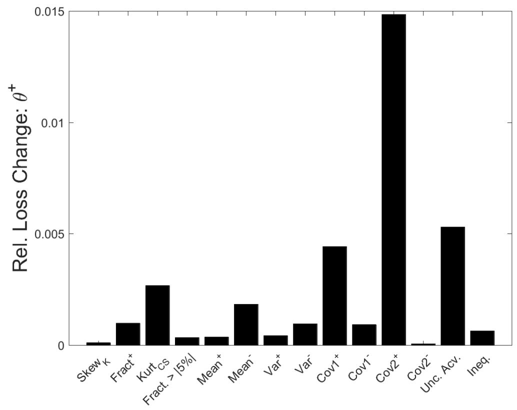
(c) Sensitivity



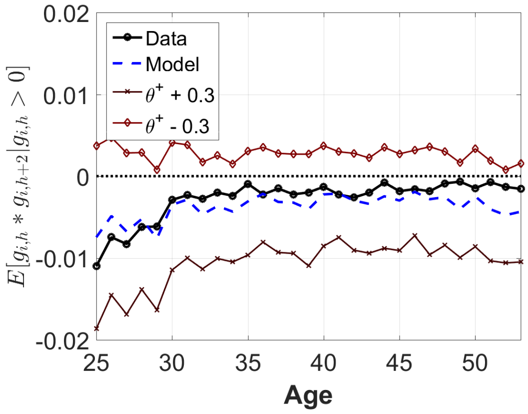
(d) Simulation

Panels A and C display the moments response to a 1% increase of the autocorrelation parameters, $\hat{\rho}^+$ and $\hat{\rho}^-$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated cross-sectional inequality of selected parameter values that are of moderate persistence ($\rho \approx 0.8$).

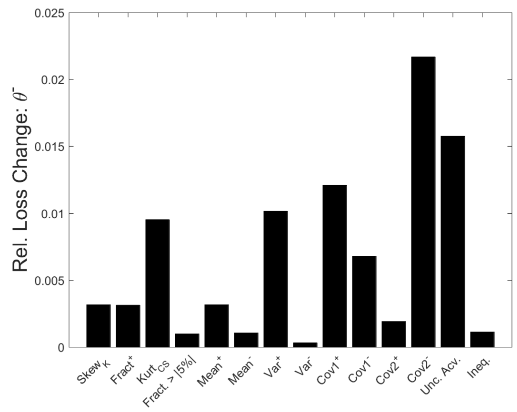
Figure 1.A7: Autocorrelation of persistent shocks



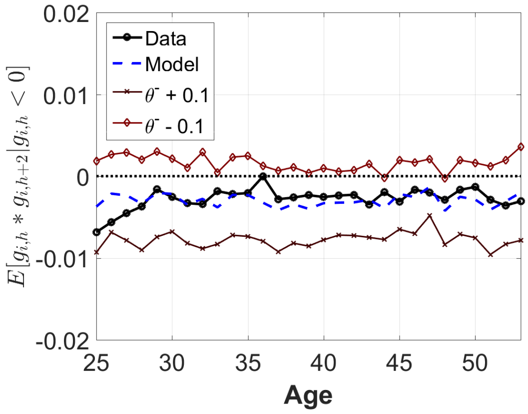
(a) Sensitivity



(b) Simulation



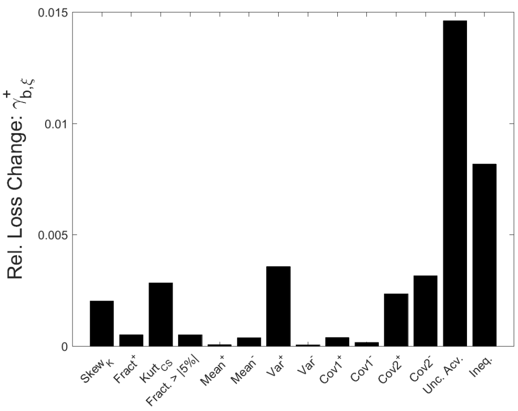
(c) Sensitivity



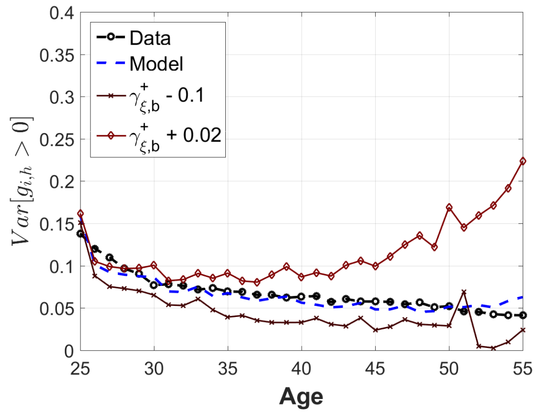
(d) Simulation

Panels A and C display the moments response to a 1% increase in the parameters guiding the persistence of transitory shocks, θ^+ and θ^-, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated positive and negative first autocovariances, respectively, resulting from increasing and decreasing these parameters.

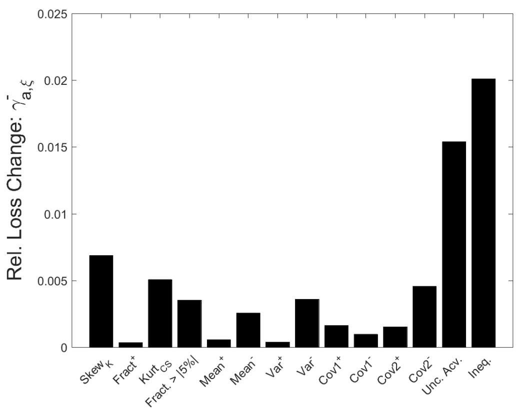
Figure 1.A8: Persistence of transitory shocks



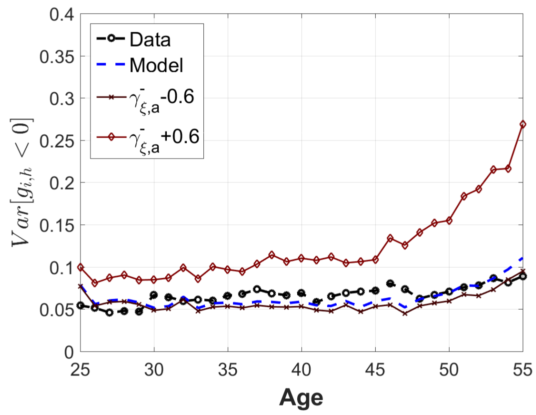
(a) Sensitivity



(b) Simulation



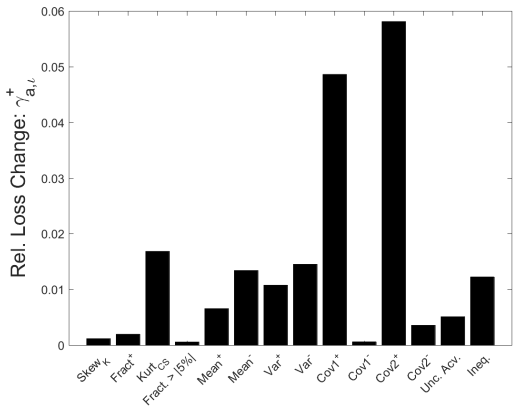
(c) Sensitivity



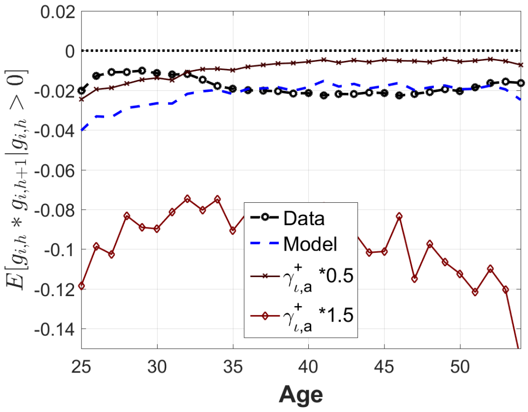
(d) Simulation

Panels A and C display the moments response to a 1% decrease in the parameters guiding the variances of persistent shocks, $\hat{\gamma}^+_{\xi,b}$, and a 1% increase of $\hat{\gamma}^-_{\xi,a}$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated positive and negative variance, respectively, resulting from increasing and decreasing these parameters.

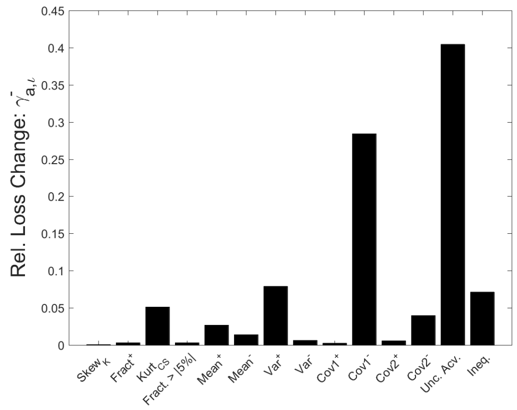
Figure 1.A9: Variance of persistent shocks



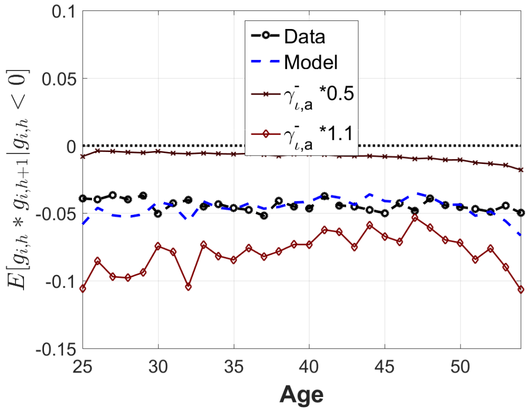
(a) Sensitivity



(b) Simulation



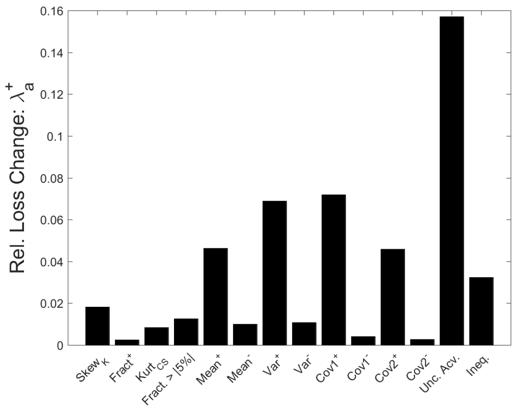
(c) Sensitivity



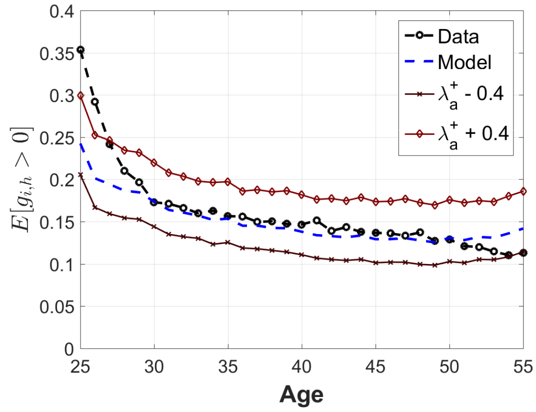
(d) Simulation

Panels A and C display the moments response to a 1% increase in the parameters guiding the variances of transitory shocks, $\hat{\gamma}_{a,\iota^+}$ and $\hat{\gamma}_{a,\iota^-}$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated positive and negative first autocovariance, respectively, resulting from increasing and decreasing these parameters.

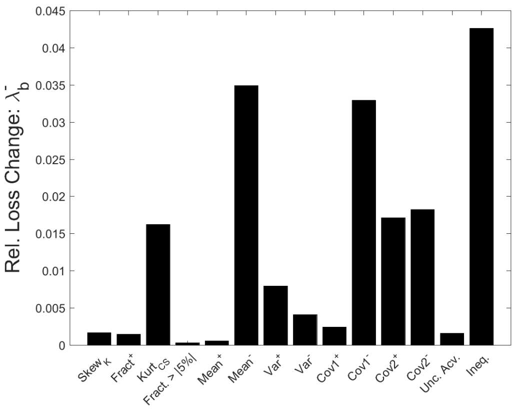
Figure 1.A10: Variances of transitory shocks



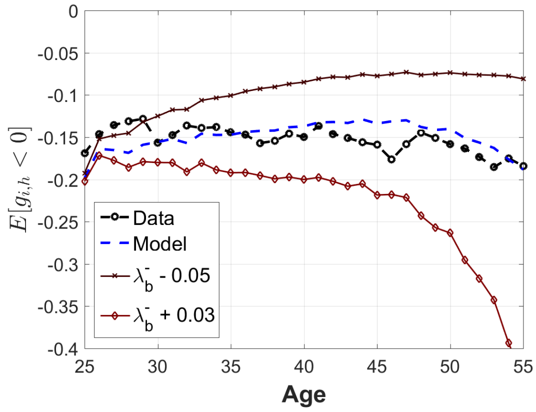
(a) Sensitivity



(b) Simulation



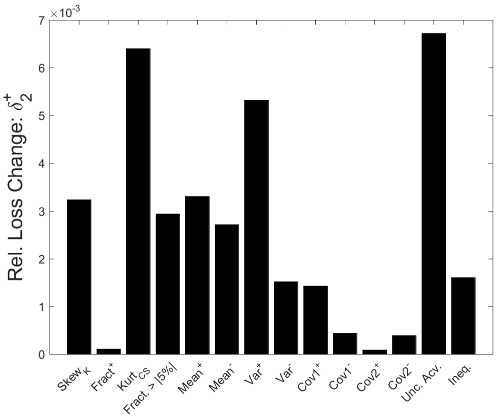
(c) Sensitivity



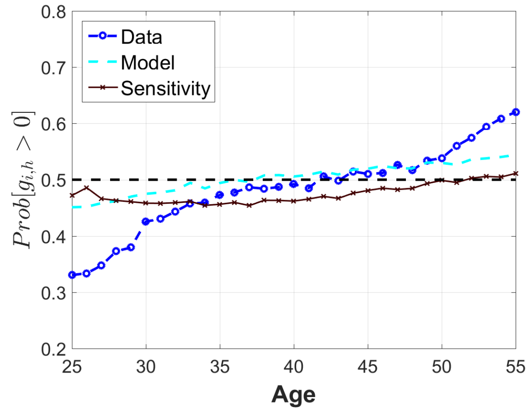
(d) Simulation

Panels A and C display the moments response to a 1% increase in the parameters guiding the means of shocks, $\hat{\lambda}_a^+$ and $\hat{\lambda}_b^-$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated positive and negative mean, respectively, resulting from increasing and decreasing these parameters.

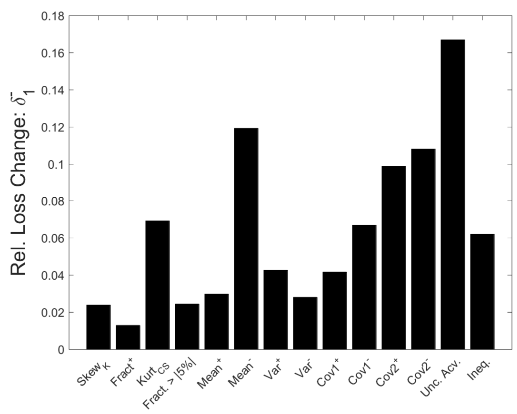
Figure 1.A11: Means of shocks



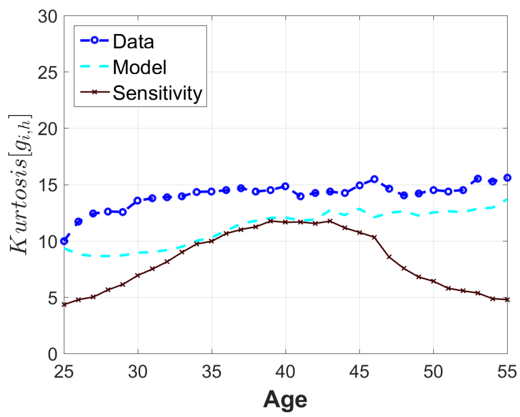
(a) Sensitivity



(b) Simulation



(c) Sensitivity

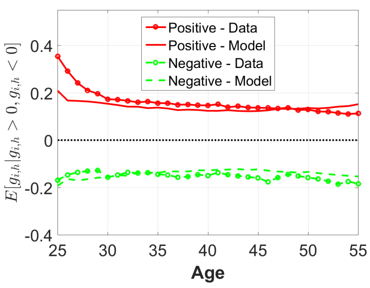


(d) Simulation

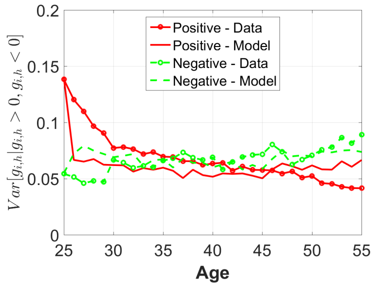
Panels A and C display the moments response to a 1% increase in the parameters guiding the sampling probabilities of shocks, $\hat{\delta}_2^+$ and $\hat{\delta}_1^-$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display selected parameters guiding the probability of positive and negative shocks and the corresponding simulated fraction of positive innovations and kurtosis.

Figure 1.A12: Sampling probabilities

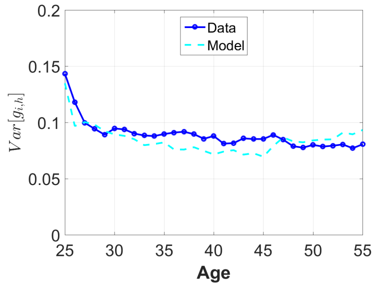
1.8.2 Model Moments for Alternative Models



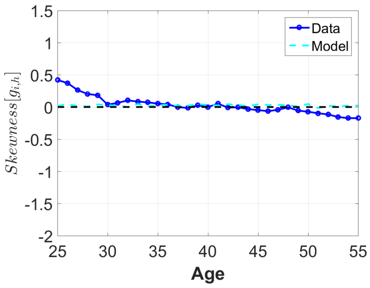
(a) $\mathbb{E}[g_{i,h}|g_{i,h} \gtrless 0]$



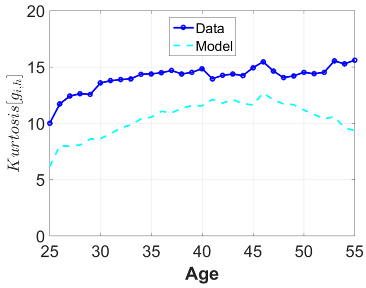
(b) $Var[g_{i,h}|g_{i,h} \gtrless 0]$



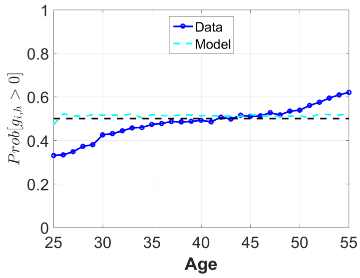
(c) $Var[g_{i,h}]$



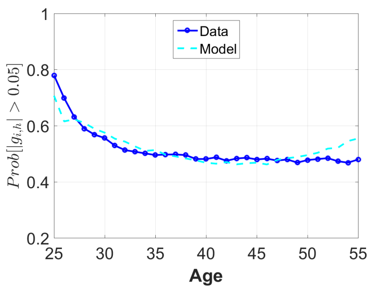
(d) $Kelly's\ Skewness[g_{i,h}]$



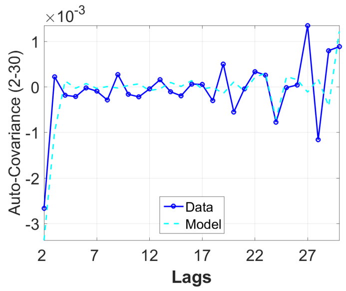
(e) $Kurtosis[g_{i,h}]$



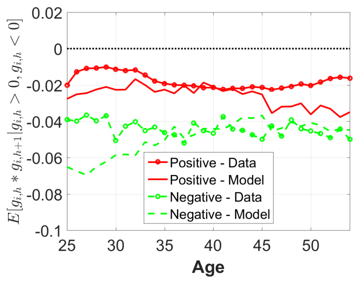
(f) $Prob(g_{i,h} > 0)$



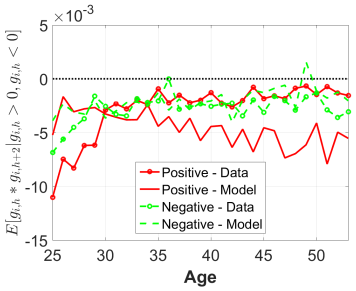
(g) $Prob(|g_{i,h}| > 5\%)$



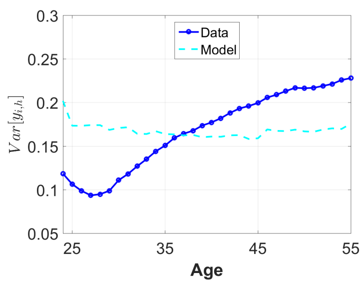
(h) $\mathbb{E}[g_{i,h+k}g_{i,h}]$



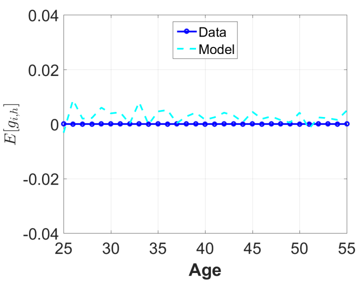
(i) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$



(j) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$

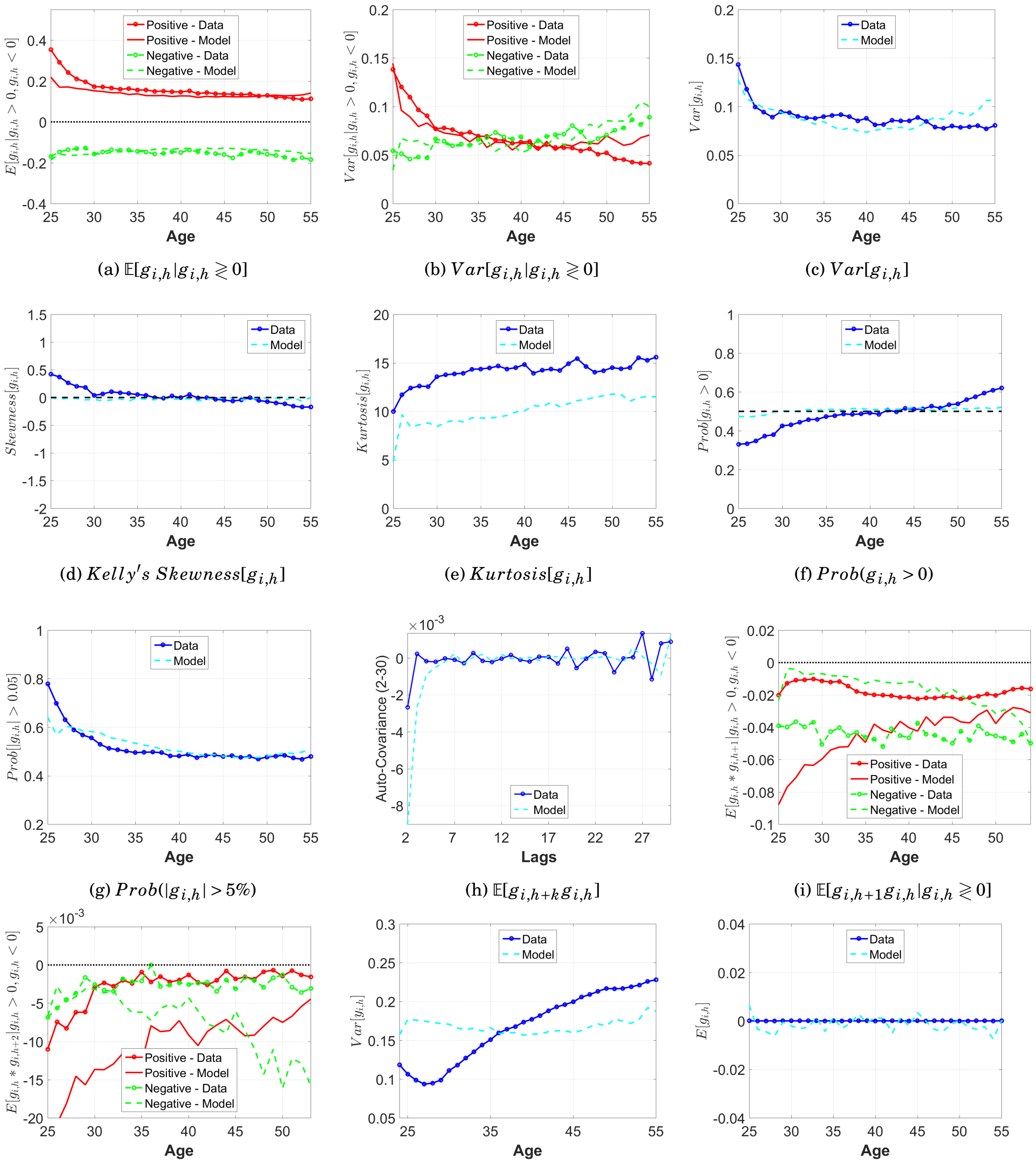


(k) $Var[log\ Earnings]$



(l) $\mathbb{E}[g_{i,h}]$

Figure 1.A13: Model Fit - Column (2) of Table 1 in the Paper



(a) $\mathbb{E}[g_{i,h}|g_{i,h} \gtrless 0]$ (b) $Var[g_{i,h}|g_{i,h} \gtrless 0]$ (c) $Var[g_{i,h}]$

(d) $Kelly's\ Skewness[g_{i,h}]$ (e) $Kurtosis[g_{i,h}]$ (f) $Prob(g_{i,h} > 0)$

(g) $Prob(|g_{i,h}| > 5\%)$ (h) $\mathbb{E}[g_{i,h+k}g_{i,h}]$ (i) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$

(j) $\mathbb{E}[g_{i,h+2}g_{i,h}|g_{i,h} \gtrless 0]$ (k) $Var[log\ Earnings]$ (l) $\mathbb{E}[g_{i,h}]$

Figure 1.A14: Model Fit - Column (3) of Table 1 in the Paper

CHAPTER 2

Lifetime Earnings Inequality and Marginal Employment in Germany

Additional acknowledgments: I thank Annette Bergemann, Etienne Lalé, Hélène Turon, Sekyu Choi, Diego Lara de Andrés and participants of the Economics Department at the University of Bristol for helpful comments and suggestions.

2.1 Introduction

During the second half of the XXth century, individuals entering the labour market in Germany experienced larger lifetime earnings inequality than their predecessors. Bönke et al. (2015) document a striking secular rise of intergenerational inequality in lifetime earnings[1]: West German men born in the early 1960s were likely to experience about 85% more lifetime inequality than their fathers.[2] This increase in lifetime earnings inequality has happened both at the upper tail and at the bottom tail of the earnings distribution, although the rise has been larger at the bottom.

In line with the dramatic rise of intragenerational inequality in lifetime earnings in Germany, there has been a rapid escalation of atypical work arrangements for newer cohorts entering the labour market: (i) the fraction of part-time employees has increased in a linear fashion from 11 percent in 1985 to 22 percent in 2010, (ii) the share of workers with working time accounts[3]

[1] As opposed to "intragenerational", which refers to *within* a generation, "intergenerational" refers between generations. One generation after another.

[2] Measured in Gini coefficients on lifetime earnings.

[3] The concept of flexible working time accounts (WTAs) is to establish labor-self accounts, and labors can save their working hours, just like saving money, into their own accounts. The working hours in their accounts are their assets, so that employers and workers both sides can increase or decrease the work required by each other without affecting the salaries and welfare. Burda and Hunt (2011) argue that up to 40% of the ‚Äúmissing‚Äù decline in employment in

rose between 1998 to 2005 from 33 to 48 percent, and (iii) the number of marginal part-time work ('mini-jobs') has increased in recent years, reaching the second most important form of employment with around half of the establishments using some type of marginal employment.[4] These changes in the labour market occurred after the implementation of the *Hartz reforms*, named after Peter Hartz, the head of the committee which recommended changes to the German labour system. The reforms increased the efficiency of the job search process, cut unemployment benefits for long term unemployed (reducing reservation wages)[5] and allowed more flexible forms of employment.

In this paper, I study how more flexible forms of labour contracts affect the distribution of lifetime earnings. As opposed to cross-sectional studies, I opt for a cohort-based analysis of the mobility experienced over long panels as it helps to understand the consequences of introducing more flexible forms of employment. Thus, my goal is to develop a methodology targeting lifetime earnings using employment transitions and wage mobility in a model that captures positional earnings mobility, employment risk, and that allows for an assessment of how different forms of employment affect the degree of inequality in lifetime earnings.

The expansion in alternative work arrangements has mixed implications for workers. On one hand, flexible working time contributes to the decentralization of employment relations on multiple levels, decreasing the bargaining power of workers and individualizing working conditions.[6] This is often seen as a more precarious and lower paid work than regular open-ended employment. On the other hand, flexible working time serves as an alternative to forced redundancies and can be used by employees as a mean to insure against unemployment and labour income risk. Workers under marginal employment can use their flexible working time to embark on training programs, having more time and being better prepared to search for stable jobs. Flexible work arrangements may thus act as as a stepping stone to permanent work.[7]

Not much is known about the impact that new, flexible forms of employment have on lifetime earnings inequality. Bönke et al. (2015) show that the increase in lifetime earnings inequality in Germany has been greater at the bottom of the distribution. However, their data set does not contain information about the attachment of workers to the labour market, so they cannot distinguish between inequality in wages and inequality in hours worked. The way in which flexible employment status can explain the rise of intragenerational lifetime earnings inequality is *a priori* unclear and merits an in-depth investigation.

Studies on lifetime earnings are scarce since only a few countries provide access to sufficiently long panel data sets. In this paper, I use the German R-SIAB 7514 data set, which is one of those

Germany for the recession period (2008-2009) can be largely explained by the WTAs. Balleer et al. (2017) call into question that WTAs were the key driver of the unusually small increase in German unemployment in the Great Recession.

[4]OECD Data, Gross and Schwarz (2006), Bechmann et al. (2010), Carrillo-Tudela et al. (2015), Galassi (2017).

[5]The reduction of reservation wages after a cut in unemployment benefits is the hypothesis of Burda and Hunt (2011) and Lietzmann et al. (2016), among others. It is also consistent with the model estimation of Price (2016).

[6]For empirical evidence that suggests those conclusions, see Addison et al. (2015) and Dustmann et al. (2014).

[7]Berg (2008), Seifert (2006), Caliendo et al. (2016), Booth et al. (2002).

exceptions. The sample size is large and earnings are precisely measured (administrative records). The data set provides information on earnings for as long as an individual is subject to social security records. Individuals may appear from several days up to entire lifetime earnings. The data set also provides unique information regarding the intensity of attachment of workers to the labour market, as in full-time work, part-time work, marginal employment or unemployment spells, something that is key to quantify the contribution of employment status on lifetime earnings inequality. Finally, panels are sufficiently long, which is a necessary condition to avoid the bias that results from omitting unobserved heterogeneity in employment continuity.[8]

In the data, I show that full-time employment, as a fraction of total gender-specific employment, decreases during the sample period. Male full-time employment decreased from 97% in 1999 to 86% in 2014. Female full-time employment decreased from around 65% in 1999 to 47% in 2014. Part-time, marginal employment and concurrent employment increase throughout the sample period. Concurrent employment appears as a new form of employment after the Hartz reforms in 2003. It is close to 0% in 1999 and around 4% (6%) for male (female) workers in 2014. Also, there is significant heterogeneity between males and females: while males concentrate around full-time employment , females take more part-time employment.[9]

The empirical model that I estimate captures these employment dynamics, but also the different wage dynamics after the Hartz reforms. Wages in full-time decrease, earnings in marginal employment increase, and persistence in the earnings rank decreases, generating more transitions along the earnings distribution. Making use of the estimated model, I simulate employment and earnings trajectories and obtain lifetime values of earnings.[10] This estimation shows that there is a shift to the left (lower earnings) in the distribution of lifetime values after the Hartz reforms, for both males and females. For females, both lower wages and a larger hazard of falling and remaining in flexible forms of employment explain the lower lifetime earnings. For males, the lower lifetime earnings comes as a result of significantly lower wages in full time employment. Because in the empirical model I have the contribution of the employment state and the wage separately, I am able to decompose the contribution that each factor has on lifetime earnings. In a counterfactual exercise I show that the new wage dynamics after the Hartz reforms have twice the impact in lower lifetime earnings than the expansion of marginal employment after the Hartz reforms.

The findings of this paper contribute to the vast literature on marginal employment in Germany, but also to the less prolific literature in lifetime earnings inequality. Bönke et al. (2015) show that lifetime earnings inequality has increased for male cohorts born in 1935-1969 in West

[8]See Addison and Surfield (2008) and Bowlus and Robin (2010).

[9]Taxation, unemployment benefits and social assistance in Germany is means tested at the household level. In my analysis I can only look at individual earnings. In addition, I am unable to evaluate the interactions of these policy reforms at the household level.

[10]The sample before the Hartz reforms uses 4 years of data, while the sample after the Hartz reforms uses 5 years of data. When I construct lifetime values, I assume that the young person expects the old person's labour market from their corresponding sample.

Germany (using VSKT records). They use monthly wages observations for full-time employment. The rise in lifetime earnings inequality comes from both the bottom and the top of the distribution, but the rise has been stronger at the bottom. Their proximate causes are: 1. Longer unemployment spells of workers at the bottom of the distribution of younger cohorts contribute to explaining 20%-40% of the overall increase in lifetime earnings inequality, 2. 60%-80% of the overall increase is due to an increase of intragenerational wage inequality. The question they do not address is how part-time and marginal employment becoming more popular have affected long term earnings. I show that higher persistence of flexible forms of employment plus lower wages in full-time both contribute to the increase in bottom earnings inequality.

Carrillo-Tudela et al. (2018) do the most comprehensive and extensive analysis on cross-sectional employment transitions after the Hartz reforms. They show that the increase in German total employment during the great financial recession comes as a result in the increase of part-time employment and concurrent employment. Mini jobs and full-time employment account for the majority of concurrent employment observations. A large part of the increase in marginal employment comes from full-timers taking up a marginal job as a secondary form of employment. The majority of workers in exclusive marginal employment have a non-participation to mini-job cycle (dead end). I make use of some their flow study and extend their work by putting a higher emphasis to wage dynamics, combining the earnings and mobility process into an empirical model and looking into the permanent lifetime earnings consequences of these different employment dynamics.

Finally, I also contribute to the work of Dustmann et al. (2009). They show that cross-sectional wage inequality increased between 1975-2004 in Germany. In particular, the decade of 1980 displays an increase in top inequality, while the decade of 1990 has both an increase in top and bottom inequality. They suggest technological change (polarization) is behind the increase in top inequality. On the contrary, supply shocks and labour market institutions (decrease in unionization) can explain the increase in bottom inequality. I show that the period 1999-2014 displays an increase in bottom inequality. Most of this increase in inequality happens at the bottom of the distribution for males in full-time.[11]

There is, in fact, a vast literature studying the consequences of the Hartz reforms in Germany. Krause and Uhlig (2012), Krebs and Scheffel (2013), Launov and Wälde (2013) and Bradley and Kügler (2019) are among those who calibrate macroeconomic search models simulating the effect of the reforms with a focus on the effect on unemployment. Fahr and Sunde (2009), Klinger and Rothe (2012) and Hertweck and Sigrist (2012) show the improvement of the matching process after the Hartz I-III reforms by estimating matching functions. Gehrke et al. (2019) analyze the role of different shocks and institutions during the Great Recession in Germany through the lens of an estimated dynamic stochastic general equilibrium model. Engbom et al. (2015) use a regression framework to identify the impact of the reforms on earnings, and Rothe et al. (2013)

[11]As a matter of fact, Dustmann et al. (2014) document that bottom wage inequality has been increasing in Germany, although they do not disentangle the contribution by employment form.

use a similar interval regression model during 1998-2010 in Germany. The interaction between marginal employment, unemployment and full time employment transitions for different aspects of the Hartz reforms are studied through diverse microeconometric models in Caliendo and Wrohlich (2010), Caliendo et al. (2016) and Price (2016). The "churning" effect of the expansion of earnings subject to marginal employment status is studied in Galassi (2017), Tazhitdinova (2017) and Gudgeon and Trenkle (2019). Additionally, Möller (2010), Burda and Hunt (2011), Dustmann et al. (2014) and Bellmann et al. (2016) address the 'German labour market miracle' during the great recession.

My paper contributes to the literature on marginal employment in Germany by looking at the long term effect in earnings of the Hartz reforms. The existing work on marginal employment earnings (Dustmann et al. (2009) and Carrillo-Tudela et al. (2018)) has looked at the cross-sectional changes in earnings but it does not construct a measure of the long term impact. Hence, it is not clear whether the proliferation of marginal employment will lower long term earnings (as it is a form of low-pay employment) or if the new wage dynamics after the Hartz reforms are more important. With my empirical model, I am able to construct lifetime values and see the change in the long term distribution of earnings. In addition, I can account for which of the effects, either the new employment mobility or the new wage dynamics, is stronger in explaining the new lifetime earnings distribution. The results of my approach and the counterfactual exercises suggest that the new wage dynamics in full time employment (namely lower full time wages after the Hartz reforms) are behind the lower long term distribution of earnings.

The rest of the paper is organized as follows. Section 2 describes the German data set. Section 3 presents the empirical model. Section 4 discusses the parameter estimates and model fit. Finally, Section 5 uses the model to construct lifetime values and performs a counterfactual exercise by changing the employment (wage) parameters of the model estimated before the Hartz reforms.

2.2 Data

2.2.1 Institutional framework

During the sample period, a series of reforms in the labour market were implemented in Germany. The intention is to study the transitions and long term earnings before and after the implementations of these reforms. These reforms receive the name of *Hartz reforms*, named after Peter Hartz, the head of the committee which recommended changes to the German labour system. The reforms increased the efficiency of the job search process, cut unemployment benefits for long term unemployed (reducing reservation wages) and allowed more flexible forms of employment. Worldwide, the Hartz reforms are known for the introduction of marginal employment, which partially explain the German labour market miracle during the Great Recession.[12] I briefly summarize descriptions of the reforms that otherwise are plentiful in the literature. The work of

[12] See Burda and Hunt (2011) and Dustmann et al. (2014).

Carrillo-Tudela et al. (2018) provides one of the most recent and complete description of these reforms.[13]

The creation of marginal employment in Germany goes back to the 1960s. Back then, marginal employment was envisaged to facilitate non-participants engage in some form of paid work. The labour contracts under marginal employment are designed for low-pay employment and have a cap on hours and pay. However, labour earnings under marginal employment are exempted from income tax and social security contributions. Likewise, these earnings do not contribute to the unemployment benefits and pension payments at retirement are reduced.[14] Unemployment insurance in Germany consisted of three layers: unemployment benefits, unemployment assistance and social assistance (a means-tested lump-sum transfer designed for those workers that did not qualify for unemployment benefits or unemployment assistance). In April 1999, the German Federal Employment Agency set the wage cap to 325 euros per month with a working time restriction of 15 hours per week, and temporary employment contracts were restricted to a two-month maximum, or 50 working days by year. Employees were exempted from social security contributions and employers paid a fixed 22% rate. In the following paragraphs, I briefly detail each implementation of the Hartz reforms.

The Hartz reforms were implemented gradually and in different stages. Hartz I, which was introduced in January 1st, 2003, changed labour regulation in order to enhance temporary employment, implemented occupational training programmes, introduced subsistence payments on behalf of employment agencies, and created new forms of employment for elderly workers. It also introduced the setup of the Personel Service Agencies (PSAs). These were placement-oriented temporary employment agencies that increased the efficiency of the matching process.

Hartz II was introduced in different dates, on January 1st, 2003, and April 1st, 2003. This staged defined formally two types of labour contracts under marginal employment: mini-jobs and midi-jobs. Mini-jobs paid up to 400 euros per month (450 euros in 2013), while midi-jobs paid between 400 and 800 euros per month. Hartz II introduced three legal modifications: (i) it lifted the threshold for the minijobs' maximal wage to 400 euros, (ii) it eliminated the cap on 15 working hours per week, and (iii) it facilitated the adoption of marginal employment as a secondary form of employment by extending the income tax and social security exemptions for mini-jobs held as a secondary job. Hartz II also made it easier for firms to use marginal employment from an administrative point of view. It facilitated the adoption of marginal employment and the payments of the corresponding taxes and social security contributions.[15] The social security

[13]The literature has discussed in detail these series of reforms. See Burda and Hunt (2011) and Engbom et al. (2015) for a general discussion of all the reforms. Fichtl (2015) for more in-depth discussion of the technical details of the reforms. Caliendo et al. (2016) for a discussion and an application on Hartz I-III reforms. Lietzmann et al. (2016) and Price (2016) for a discussion and an application on the Hartz IV reform.

[14]Before the implementations of the Hartz reforms, unemployed workers who received benefits were allowed to work in marginal employment to top up their benefits, as long as their jobs did not pay more than 165 euros per month.

[15]The *Minijobzentrale* was created to serve as a unique legal entity responsible for registering marginally employed workers. It also served to deal with all the tax and social security matters related to marginal employment.

contribution of employers increased slightly to 23%. Employees only paid 2% income tax. For midi-jobs, employees paid higher income tax and social security contributions. The aim of the reform was to create incentives to take up marginal employment that led to regular employment and to reduce unregistered work.

Hartz III, introduced in January 1st, 2004, restructured the Federal Employment Agency as an entity improving the efficiency in job offer mediation to unemployed workers. The elements of Hartz III increased the efficiency in the matching process and also restructured hiring subsidies to incentivise employers to hire hard-to-place workers, such as older and disabled people.

The Hartz IV reform, introduced in January 1st, 2005, tightened conditions on unemployment benefit recipients. In particular, it merged the long-term unemployment assistance benefits with social assistance benefits into one new transfer (ALG II). ALG II benefits became means-tested at the household level, affecting the eligibility of the long-term unemployed. On the contrary, unemployment benefits remained largely unmodified. In other words, most workers who qualified for unemployment assistance in the old system experienced a dramatic cut in benefits if they remained in unemployment. In addition, Hartz IV could potentially cut benefits by 30% for 12 weeks if a person who was able to work refused to enter the activation program. It also cut benefits if a suitable offer of work proposed by the case worker was rejected by the worker. In practice, Hartz laws explicitly stated that about any work was then considered suitable. Hence, refusal to accept employment led to benefit cuts. Finally, the Hartz IV reform also introduced the so called One-Euro-jobs , which was a form of low pay employment intended to activate benefit recipients by taking up at least some employment in exchange for the ALG II benefits. These jobs paid one Euro per hour worked on top of the unemployment asistance, hoping to attract long term unemployed workers into regular employment.

2.2.2 Data source

My data source is the *Sample of Integrated Labour Market Biographies (SIAB)* provided by the Institute for Employment Research (IAB) for the years 1975-2014. The SIAB is a 2% random sample drawn from the Integrated Employment Biographies (IEB). The variables captured in this dataset include gender, year of birth, education, working spells recorded at the day level, unemployment benefits and gross daily wages pertaining to jobs covered by social security. Civil servants, self-employed and regular students are excluded from this dataset. East Germany observations appear after 1991. In total, the raw dataset has information on 1,707,228 individuals and 51,987,959 unique person-year records.

The dataset reports employment observations from either full-time or part-time spells, but marginal employment is classified as a separate category since April 1999. This is why in my analysis I restrict the data to the period 1999-2014. The datasets provides employment spells with the starting and the ending date of the spell. Therefore, I am able to observe different employment transitions for the individual within a year. I then construct monthly job spells with the aggregate individual's earnings of the employment observations and deflate earnings using the German consumer price index of 2010.[16] The primary source of analysis is daily earnings, which are top-coded by the limit liable to social security.[17] I follow Daly et al. (2016) and impute daily earnings from an extrapolated Pareto density fitted to the non-top-coded upper-end of the observed distribution for each year, separately for year periods, gender, full-time and part-time observations.

I keep workers with different employment status to allow for different attachment to the labour market among workers. I only drop from the sample workers in apprenticeships and partial retirement. Moreover, I consider different samples by gender. Furthermore, the age consideration is of some importance. While young workers may be using marginal employment to support their studies, elder workers may retire early due to generous unemployment benefits to high tenured workers.[18] Therefore, I keep workers between 22 and 56 years old in the sample. Section 2.7.1 in the Appendix provides further details on the sample construction.

Following Carrillo-Tudela et al. (2018), I categorize workers by their employment type. I use the following five categories: exclusive full-time, *FT*, exclusive part-time, *PT*, exclusive marginal employment, *ME*, non-employment, *NE*, and concurrent employment, *CE*. Full-time and part-time spells are specified in the *SIAB* data. The variable *occupational status* distinguishes between full-time and part-time. The decisive factor is the ratio between the contracted hours and the usual working hours in the establishment.[19] Also, marginal employment is specified for mini-jobs and midi-jobs (to be defined in the next subsection)[20]. For non-employment, I assign to this state workers receiving some form of unemployment benefit/assistance and those workers that are not observed in unemployment or registered employment (non-participants). With this sample selection, I can observe the transitions from non-employment to employment and whether the policy change of marginal employment across samples have had a significant impact on non-participants. Finally, concurrent employment are those employment observations where the worker holds a marginal employment as a secondary form of employment, together with a main full-time or part-time spell.

2.2.3 Descriptive statistics

One of the objectives of this paper is to estimate the separate contribution that wages, employment status and transitions have had on cross-sectional and lifetime earnings inequality in Germany before and after the Hartz I-IV reforms. The sample period comprises 16 years (1999-2014) and

[16] `https://data.oecd.org/price/inflation-cpi.htm`

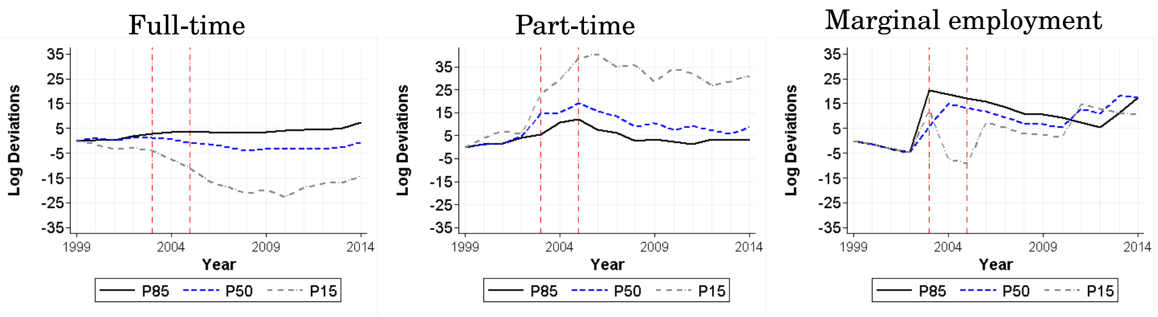
[17] In the 1999-2002 (2005-2009) sample, 3% (2.3%) of the observations are top-coded.

[18] See Arnds and Bonin (2002).

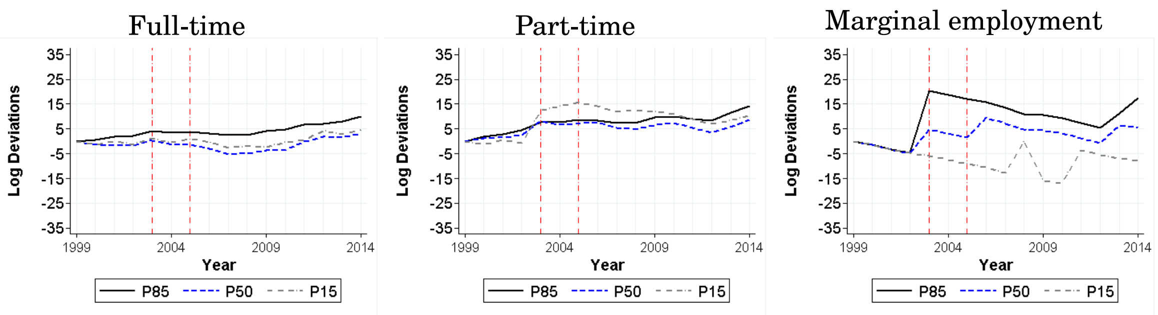
[19] To the best of my knowledge, there is no misclassification between full-time, part-time and marginal employment in the *SIAB* data.

[20] Technically, marginal employment relates to mini-jobs only. Mini-jobs are regulated in the Article 8, paragraph 1 in the German Social Code, Book IV midi-jobs are regulated in the Article 20 German Social Code, Book IV.

Males:



Females:



Notes: the figure displays percentiles of log real daily wage for full-time, part-time and marginal employment for male and female workers, deviated from the value of the same percentile in 1999 and multiplied by 100. The vertical red dashed lines denote years 2003-2005 where the Hartz reforms took place.

Figure 2.1: Real Log Daily Wage Inequality

contains the Hartz reforms in the middle of the period (years 2003-2005), thus providing several year-observations *before* and *after* the change in policy.

I start the analysis by having a look at the trends in wage inequality during the sample period. I analyze changes in inequality at the bottom, the median and the top of the distribution. Figure 2.1 plots the wage growth (log deviations) of the 15th, 50th and 85th percentiles of the wage distribution relative to year 1999. I make a distinction by employment type and gender. For men, the 15th percentile of wages for full-time employment decreased reaching a low of -25%. The ratio between the 85th percentile and the 15th percentile (P85/P15 gap) in wages widened to reach a 20% increase at the end of the period. Meanwhile, median wages (50th percentile) for males in full-time remained stagnant. Part-time wages for males saw an increase at the bottom of the distribution. The bottom 15th percentile in part-time reached about a 30% increase at the end of the period. In contrast, marginal employment saw an increase at the top of the distribution. Wages increased between 15-20% for the 85th and the 50th percentile at 2003-2004 years when the Hartz reforms increased the maximum threshold for marginal employment. Thereafter, real wages in marginal employment started declining due to inflation.

The picture is somewhat different for females. First, volatility is generally lower. Females in full-time show a P85/P15 gap that never exceeds 10% during the period. Unlike men, women in full-time do not see an increase in bottom inequality: both the 15th and 50th percentiles in full-time remain around 0% during the period. Only top inequality in full-time eventually increases to reach a 10% increase at the end of the period, 5 points larger than the increase for males. The feature found for males, where there is an increase of bottom wages in part-time and an increase of top wages in marginal employment is also present for females. The 15th percentile for part-time wages increase around 15% at the 2003-2004 years, together with an increase of 20% for the 85th percentile of marginal employment wages. Only females present a widening of marginal employment wages. The bottom 15th percentile of wages in marginal employment see a total decrease around 10%.

The findings of widening bottom inequality in full-time for males are documented in Card et al. (2013) for the 1996-2009 period.[21] I have extended the analysis for part-time and marginal employment and show that part-time wages concentrate as bottom wages increased during the period, while marginal employment wage inequality increases (for females).

Next, I turn to the stock of employment. The aim is to observe the changes in employment stock before and after the Hartz reforms. Figure 2.2 plots the trend of the stocks of the different forms of employment: full-time, part-time, marginal employment and concurrent employment. The figure shows the different evolution of employment by gender. The chart has several takeaways:

1. Full-time employment, as a fraction of total employment, decreases during the sample period. Male full-time employment decreased from 97% in 1999 to 86% in 2014. Female full-time employment decreased from around 65% in 1999 to 47% in 2014.

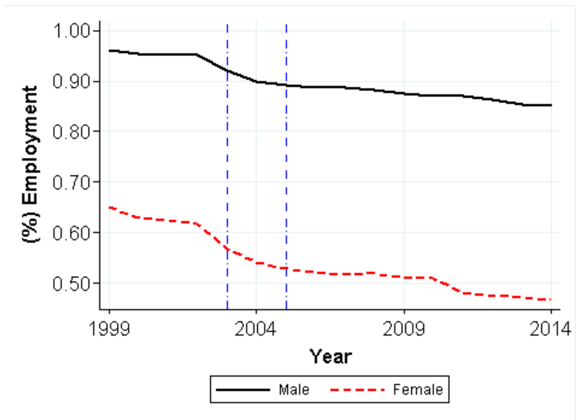
2. Part-time, marginal employment and concurrent employment increase throughout the sample period.

3. Concurrent employment appears as a new form of employment after the Hartz reforms in 2003. It is close to 0% in 1999 and around 4% (6%) for male (female) workers in 2014.

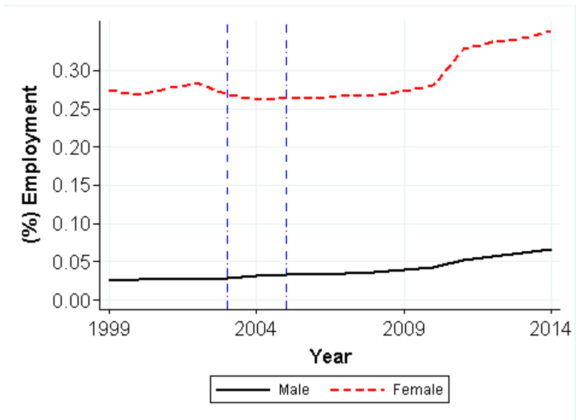
4. There is significant heterogeneity between males and females: while males concentrate around full-time employment (around 90% of employment stock), females take more flexible forms of employment (only between 50-60% of full-time employment stock).

As shown by Galassi (2017) and Carrillo-Tudela et al. (2018), part-time and marginal employment contracts are predominantly both a female and a low education phenomenon for prime age workers. Also, the closest group in worker characteristics to marginal employment workers

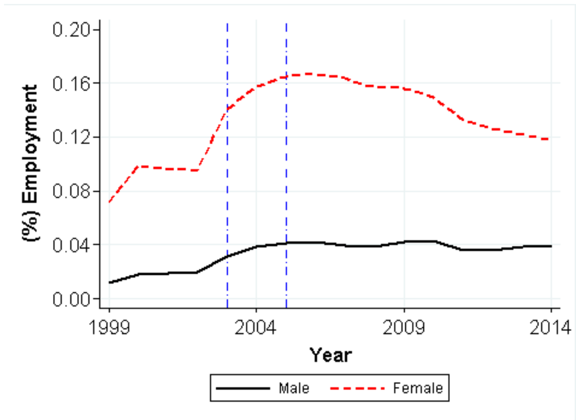
[21] Wage inequality is a larger, ongoing phenomenon in Germany. Also, wage flexibilization at the bottom of the distribution for full-time employment is discussed as one possible explanation for the German labour market miracle in the great recession (Burda and Hunt (2011)). See Dustmann et al. (2009) for a wage inequality study for the 1975-2004 period.



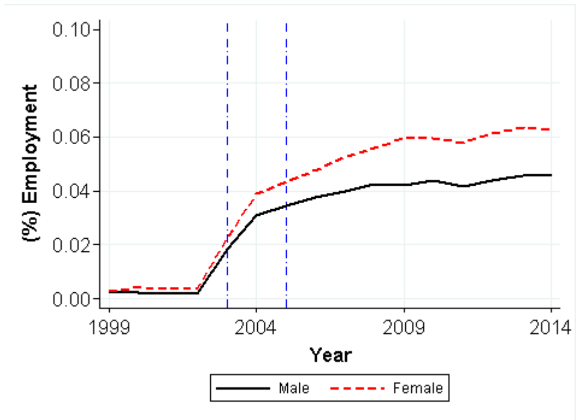
(a) full-time



(b) part-time



(c) marginal employment



(d) concurrent employment

Notes: the figure displays aggregate stocks of employed workers, separated by gender. The vertical blue lines denote years 2003-2005 where the Hartz reforms took place.

Figure 2.2: Employment Stock

are non participants. Taken together, the employment stock figure depicts a labour market that transits from a (close to) full-time labour market to a more flexible labour market after the implementation of the Hartz reforms.

Another important aspect of the different labour market after the Hartz reforms is the transitions between employment types. Figure 2.3 depicts monthly worker flows in and out of each employment type, by gender and for different sample periods. I select years 1999-2002 to account for the labour market before the change in policies, and years 2005-2009 for the labour market after. Rows refer to the employment type of the worker during the previous month and columns refer to the destination state (current month). Each cell is an average for the corresponding sample period. For example, 98% of the males in the 1999-2002 sample that were in full-time (FT_{t-1}) the previous month remained in FT_t the next month. Only 1% of male workers in FT_{t-1} in the previous month transited to non employment (NE_t).

The tables illustrate the changes in labour market mobility before and after the policy change:

Males:

1999-2002

	FT	PT	ME	NE
FT (t-1)	98	0	0	1
PT (t-1)	2	95	0	4
ME (t-1)	4	1	88	8
NE (t-1)	8	1	1	90

2005-2009

	FT	PT	ME	NE	CE
FT (t-1)	98	0	0	1	0
PT (t-1)	1	96	0	2	1
ME (t-1)	3	1	89	6	1
NE (t-1)	8	1	2	89	0
CE (t-1)	5	0	1	0	93

Females:

1999-2002

	FT	PT	ME	NE
FT (t-1)	98	0	0	2
PT (t-1)	1	98	0	1
ME (t-1)	1	1	94	4
NE (t-1)	4	2	3	90

2005-2009

	FT	PT	ME	NE	CE
FT (t-1)	98	0	0	1	0
PT (t-1)	0	98	0	1	0
ME (t-1)	1	1	95	3	1
NE (t-1)	4	2	4	90	0
CE (t-1)	3	2	1	0	94

Notes: the figure displays monthly average transitions in and out of each of the employment states for selected sample years and by gender. For each table, rows correspond to the previous monthly employment state and columns correspond to the current monthly employment state.

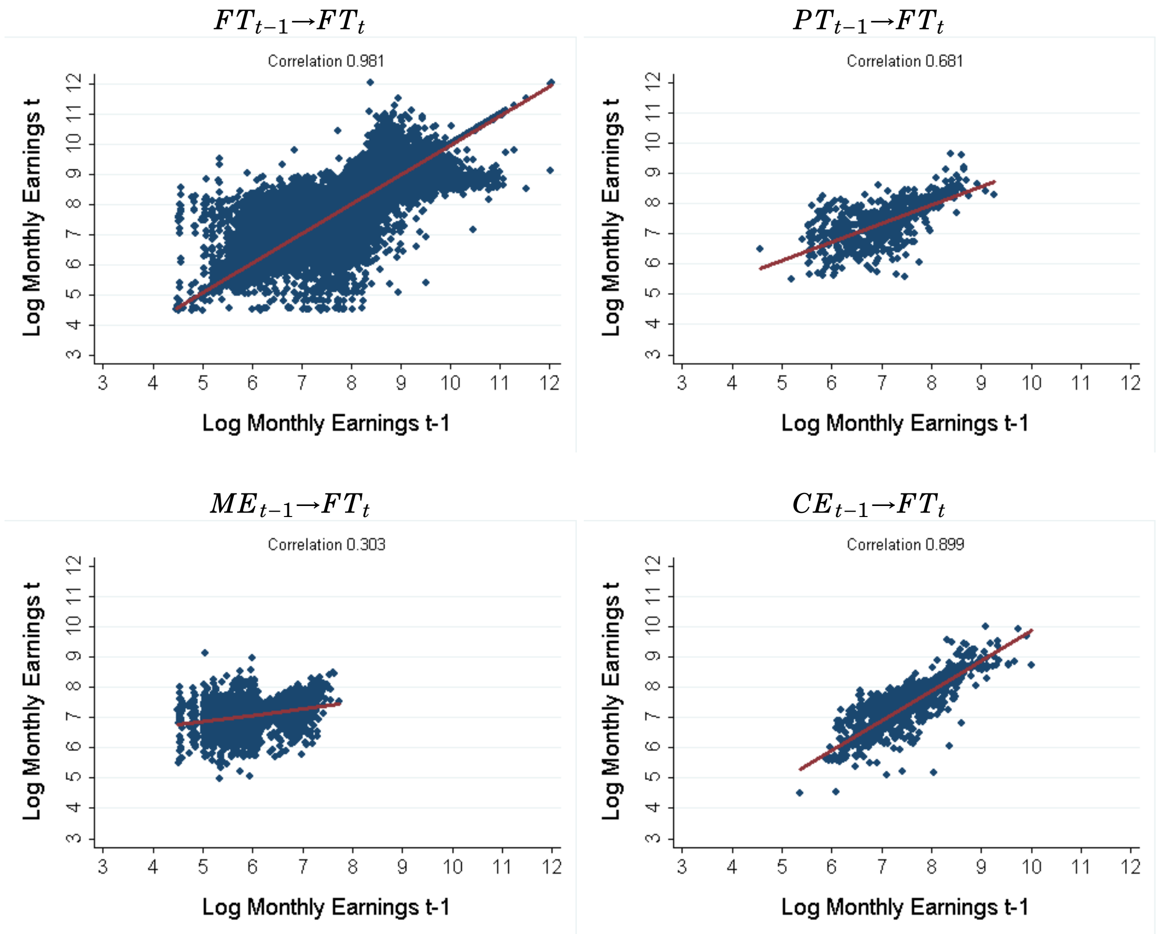
Figure 2.3: Worker Flows

i) Marginal employment becomes more persistent after the Hartz reforms. During the 1999-2002 year sample, 88% (94%) of male (female) workers in marginal employment continued in the same employment type next month. These rates increase to 89% (95%) for the 2005-2009 year sample.

ii) Marginal employment to non-employment transitions decreased after the Hartz reforms. They go from 8% (4%) in years 1999-2002 to 6% (3%). More generally, there are lower flows into non-employment after the Hartz reforms.

iii) There is more non-employent to marginal employment transitions after the Hartz reforms, while transitions out of non-employment to other types are similar. The transitions out of non-employment to marginal employment go from 1% (3%) in 1999-2002 years to 2% (4%) in 2005-2009 years for male (female) workers. Together with the previous results, this supports the *stepping stone* hypothesis.

Finally, I study the persistence of earnings across all flows of employment states. This informs of the correlation of workers' earnings across different employment status. For instance, it is not clear whether salaries in any of the flexible forms of employment will correlate with workers' earnings in full-time employment. This persistence of earnings between a worker's full-time job and other forms of employment should play a big role in shaping the worker's long term earnings, especially when flexible forms of employment become more popular and their likelihood increases.



Notes: The figure displays a scatter plot of log monthly earnings in the previous month (x-axis) against log monthly earnings in the current month (y-axis) for male workers during the 2005-2009 period. Employment states are defined in Section 2.2.2. For illustration purposes, only transitions into full-time are shown.

Figure 2.4: Autocorrelation, Male workers 2005-2009

Figure 2.4 plots selected scatter plots of log monthly earnings for different employment flows. The graph shows flows into full-time only for simplicity. The x-axis represents earnings in the previous month, and the y-axis earnings in the current month. The top-left graph shows log monthly earnings for male workers that remain in full-time between any two consecutive months during the 2005-2009 years. Figure 2.A3 in the Appendix shows the corresponding graph for females. As expected, the correlation of monthly earnings for full-time transitions is high: the coefficient is equal to 0.981.[22] Figure 2.A4 in the Appendix shows that the autocorrelation of earnings for workers remaining in their employment type is generally high. It amounts to 0.980 for part-time flows, 0.938 for marginal employment, and 0.980 for concurrent employment.

What is more interesting to observe is the correlation of earnings between other forms of em-

[22]This high correlation is in part by construction since in the SIAB data I observe working spells that last longer than a month. In these cases, since daily wages are averaged across the entire employment period, correlation should be 1.

ployment and full-time employment. Figure 2.4 illustrates that part-time to full-time transitions preserve a moderate persistence in earnings (0.681), marginal to full-time employment has the lowest persistence (0.303) and concurrent to full-time employment has highest persistence (0.899). The low correlation in earnings from marginal employment to full-time is expected given the tight nominal thresholds that marginal employment has. However, the high correlation between concurrent and full-time employment shows that earnings between these two employment states are not very different. Carrillo-Tudela et al. (2018) show that workers at the medium/bottom of the earnings distribution use secondary earnings as a form of topping-up their main earnings stemming from full-time. They study a causal relationship and find evidence in favor of this "moonlighting" effect. Therefore, a similar distribution of earnings between full-time and concurrent employment is expected.

2.3 Model

In this section, I present the empirical model that will be estimated. The model has a mobility process from which workers draw their employment state. Then there is a salary drawn from a bivariate normal distribution. Worker characteristics and unobserved heterogeneity affect both workers mobility and earnings. I follow Postel-Vinay and Turon (2005) and Dickson et al. (2014) in building a likelihood function with contributions from the wage process, employment state and unobserved heterogeneity, but I consider a larger number of employment types.

2.3.1 Main framework

I follow N workers, indexed $i = 1,...,N$ up to T consecutive months[23]. My data has information on employment spells and unemployment benefits/assistance. Unfortunately, I lack information about the participation (labour force) decision at the worker level. That means I track workers from their first to their last employment spell that is observed in the sample. For each individual in the sample I observe the length of employment spells, earnings in that spell and worker characteristics. A typical observation for any individual i will be represented as a vector $\mathbf{x}_i = (\mathbf{w}_i, \mathbf{e}_i, \mathbf{c}_i^v, c_i^u)$, where:

- $\mathbf{w}_i = (w_{i1},...,w_{iT})$ is the observed sequence of individual i's log monthly real wage flows.

- $\mathbf{e}_i = (e_{i1},...,e_{iT})$ is individual i's observed sequence of employment states in the finite state space $E = \{FT, PT, ME, CE, NE\}$, where FT stands for full-time, PT stands for part-time, ME stands for marginal employment, CE stands for concurrent employment (multiple job holding), and NE stands for non-employment. These employment states have been defined in Section 2.2.2.

[23] up to 48 months for the 1999-2002 sample, and up to 60 months for the 2005-2009 and 2010-2014 samples.

- $\mathbf{c}_i^v = (c_{i1}^v, ..., c_{iT}^v)$ is the observed varying sequence of individual i's characteristics. In the model, I only consider labour market experience, $\mathbf{c}_i^v = (\exp_{i1}, ..., \exp_{iT})$.

- c_i^u is the observed set of individual i's characteristics that are constant. I include education and labour market cohort[24].

In addition to the observed individual heterogeneity captured by $\mathbf{c}_i^v$ and c_i^u, I allow for unobserved heterogeneity which may influence wages and/or the selection of employment states. I append a set k_i of (time-invariant) unobserved characteristics. The goal is to estimate simultaneously transitions between different types of employment and earnings trajectories. I define the likelihood of $(\mathbf{x}_i, k_i)$ as follows:

$$\mathscr{L}_i(\mathbf{x}_i, k_i) = \ell_i(\mathbf{w}_i | \mathbf{e}_i, \mathbf{c}_i^v, c_i^u, k_i^w) \cdot \ell_i(\mathbf{e}_i | \mathbf{c}_i^v, c_i^u, k_i^e) \cdot \ell_i(k_i | c_i^u) \cdot \ell_i(c_i^u). \tag{A.1}$$

The likelihood for the typical individual is decomposed into four terms. Starting from the right, $\ell(c_i^u)$ is the sample distribution of observed individual fixed (unvarying) characteristics. This distribution is observed in the data. Next, $\ell_i(k_i | c_i^u)$ is the distribution of unobserved heterogeneity given observed characteristics c_i^u. Next, $\ell_i(\mathbf{e}_i | \mathbf{c}_i^v, c_i^u, k_i^e)$ is the contribution to the likelihood function from job spells. Finally, $\ell_i(\mathbf{w}_i | \mathbf{e}_i, \mathbf{c}_i^v, c_i^u, k_i^w)$ is the likelihood contribution from a sequence of wage observations over job spells. Hence, individual i's contribution to the complete likelihood comes from three components, pertaining to wage history, labour market status history and unobserved heterogeneity. The underlying assumption of the model is that dynamics of job spells are independent of the wage sequence, given (c_i^v, c_i^u, k_i). In the following subsections, I provide further details of each component of Equation (A.1).

2.3.2 Unobserved Heterogeneity

I consider two types of unobserved heterogeneity, $k_i = (k_i^e, k_i^w)$. The first type, k_i^e, relates to heterogeneity in terms of the propensity to be in each employment state (called mobility classes hereafter). In particular, k_i^e is the unobserved factor that conditions the parameters relating to employment state history. The second type, k_i^w, relates to heterogeneity in terms of wage (called wage classes hereafter) through its impact on wage levels and wage dynamics. k_i^w conditions the parameters relating to wage distribution.[25] Both types of heterogeneity are time-invariant individual random effects, which are independent one from each other.

[24] Labor market cohort is understood as the year in which the individual first entered the labour market. Therefore, c_i^v is deterministic conditional on c_i^u.

[25] This type of heterogeneity increases the persistence of income ranks, which is found to be underestimated otherwise. See Shorrocks (1976).

I refer to wage and mobility classes as I use a finite mixture approach to model unobserved heterogeneity where an individual can belong to one of K^e employment classes and K^w wage classes. The probability of belonging to any latent class depends on observed individual heterogeneity $(\mathbf{c}_i^v, c_i^u)$, as follows:

$$\ell_i(k_i|c_i^u) = \Pr\{k_i^w|c_i^u\} \cdot \Pr\{k_i^e|c_i^u\}. \tag{A.2}$$

The previous two components are modeled as multinomial logits:

$$\Pr\{k_i^e = k_p^e|c_i^u\} = \frac{\exp[c_{p,i}^u \omega_p^e]}{\sum_{p=1}^{k^e} \exp[c_{p,i}^u \omega_p^e]} \text{ and } \Pr\{k_i^w = k_p^w|c_i^u\} = \frac{\exp[c_{p,i}^u \omega_p^w]}{\sum_{p=1}^{K^w} \exp[c_{p,i}^u \omega_p^w]} \tag{A.3}$$

with (K^e, K^w) outcomes. (k_1^e, k_1^w) are taken as the reference category; i.e. they are normalized to zero.

2.3.3 Labor Market States

Transition probabilities between the five distinct labour market states are assumed to depend only on the individual's state at the previous month and on observed and unobserved heterogeneity - i.e. labour market states are assumed to follow a (conditional) first order Markov chain:

$$\ell_i(\mathbf{e}_i|\mathbf{c}_i^v, c_i^u, k_i^e) = \ell_i(e_{i1}|c_{i1}^v, c_i^u, k_i^e) \prod_{t=2}^{T} \ell_i(e_{it}|e_{it-1}, c_{it}^v, c_i^u, k_i^e). \tag{A.4}$$

Specifically, the hazard rate for a worker to be in a employment state j can be expressed by the following multinomial logit:

$$\begin{aligned} &\ell_i(e_{it}|e_{it-1}, c_{it}^v, c_i^u, k_i^e) = \frac{\pi_j}{\sum_{j'} \pi_j}; \\ &\pi_j = \pi(e_{it} = l|e_{it-1} = j, c_{it}^v, c_i^u, k_i^e) = \exp[\beta_j + \alpha_j k_i^e + h(c_{it}^v, c_i^u)\gamma_j], \end{aligned} \tag{A.5}$$

where $h(\cdot)$ is a function of the observed covariates and γ_j is a conformable coefficient vector, β_j is the hazard intercept and $\alpha_j k^e$ is the unobservable component specified as the product of the factor loading α_j and the fundamental unobserved factor k^e. $\ell_i(e_{i1}|c_{it}^v, c_i^u, k_i^e)$ is the initial condition of individual i's labour market history, which is specified as a multinomial logit:

$$\ell_i(e_{i1}|c_{i1}^v, c_i^u, k_i^e) = \frac{\exp[h(c_{i1}^v, c_i^u, k_i^e)\gamma_e]}{\sum_{e=FT}^{NE} \exp[h(c_{i1}^v, c_i^u, k_i^e)\gamma_e]}, \tag{A.6}$$

where $e = \{FT, PT, ME, CE, NE\}$[26].

2.3.4 Wage Process

I consider log (real) wage w_{it} both in employment and in non-employment[27] and assume wage trajectories w_i to be the realization of a first-order Markov process of continuous random variables W_t. I use the terms *wage* and *monthly earnings* interchangeably, but it should be understood as the same measure of interest. The likelihood of a given wage trajectory over T periods will be written as:

$$\begin{aligned}\ell_i(\mathbf{w}_i|\mathbf{e}_i, c^v_{it}, c^u_i, k^w_i) &= \ell_i(w_{i1}|e_{i1}, c^v_{i1}, c^u_i, k_i)\prod_{t=2}^{T}\ell_i(w_{it}|w_{it-1}, e_{it}, e_{it-1}, c^v_{it}, c^u_i, k^w_i) \\ &= \ell_i(w_{i1}|e_{i1}, c^v_{i1}, c^u_i, k_i)\prod_{t=2}^{T}\frac{\ell_i(w_{it}, w_{it-1}|e_{it}, e_{it-1}, c^v_{it}, c^u_i, k^w_i)}{\ell_i(w_{it-1}|e_{it-1}, c^v_{it}, c^u_i, k^w_i)},\end{aligned} \tag{A.7}$$

where $\ell_i(w_{it-1}|\cdot)$ follows a univariate standard normal and $\ell_i(w_{it}, w_{it-1}|\cdot)$ follows a bivariate standard normal with correlation $\tau_{i,t,t-1}$ between employment states at dates t and $t-1$.

I assume marginal wage distributions to be normal, conditional on observed and unobserved individual heterogeneity. That is, both mean and variance are allowed to depend on observed and unobserved heterogeneity as well as on current labour market status:

$$\begin{aligned}&w_{it}|c^v_{it}, c^u_i, e_{it}, k^w \sim N(\mu_{it}, \sigma^2_{it}), \\ &\mu_{it} = \mu(c^v_{it}, c^u_i, e_{it}, k^w_i) \text{ and } \sigma_{it} = \sigma(c^v_{it}, c^u_i, e_{it}, k^w_i), \\ &\mu(c^v_{it}, c^u_i, e_{it}, k^w_i) = c^u_i\mu_0 + [c^v_{it} * e_{it} * k^w_i]\mu_1, \\ &\sigma(c^v_{it}, c^u_i, e_{it}, k^w_i) = \sqrt{\exp\{c^u_i\sigma_0 + [c^v_{it} * e_{it} * k^w_i]\sigma_1\}},\end{aligned} \tag{A.8}$$

where the notation $x * y$ stands for all main effects and interactions of variables x and y and both μ_1 and σ_1 are conformable coefficient vectors. I force the variance of log wage to be positive by specifying it as an exponential.

Next, I normalized log wages; $\tilde{w}_{it} = \frac{w_{it} - \hat{w}_{it}}{\hat{\sigma}_{it}}$. By doing so, the pair $(\tilde{w}_{it}, \tilde{w}_{it-1})$ is a Gaussian vector with correlation matrix

[26] For the estimation of the 1999-2002 sample I do not include the *CE* state since it was not possible to observe in the data given labour market institutions.

[27] For non-employment, I consider as wage the unemployment benefits/assistance of the worker. Otherwise, if the worker is non-employed and does not receive unemployment benefits/assistance, the wage is set to missing.

$$\Gamma = \begin{bmatrix} 1 & \tau_{i,t,t-1} \\ \tau_{i,t,t-1} & 1 \end{bmatrix}. \tag{A.9}$$

$\tau_{i,t,t-1}$ is allowed to vary with observed and unobserved heterogeneity and with employment status at dates t and $t-1$:

$$\begin{aligned} &\tau_{i,t,t-1} = \tau(c^v_{it}, c^u_i, e_{it}, e_{it-1}, k^w_i), \\ &\tau(c^v_{it}, c^u_i, e_{it}, e_{it-1}, k^w_i) = \\ &\qquad = -1 + 2\Lambda\Big\{c^v_{it} * k^w_i \cdot \xi_0 + c^u_{it} * k^w_i \cdot \xi_1 + e_{it} * k^w_i \cdot \xi_2 + e_{it-1} * k^w_i \cdot \xi_3\Big\}. \end{aligned} \tag{A.10}$$

Letting $\Lambda[x] = (1+e^x)^{-1}$ designate the logistic cdf, I apply the transformation $-1+2\Lambda$ to constrain the correlation coefficient $\tau(\cdot)$ between $[-1,1]$. Temporarily omitting any conditioning variable, the likelihood of the typical individual's wage trajectory w_i defined in Equation (A.7) becomes:

$$\ell_i(\mathbf{w}_i|\cdot) = \Big(\prod_{t=1}^{T} \frac{1}{\sigma_{it}}\Big) \times \frac{\prod_{t=2}^{T} \phi(\tilde{w}_{it}, \tilde{w}_{it-1}; \Gamma|\cdot)}{\prod_{t=2}^{T} \phi(\tilde{w}_{it-1}|\cdot)}, \tag{A.11}$$

where $\phi(\cdot;\Gamma|\cdot)$ is the bivariate normal pdf with mean 0 and covariance matrix Γ.

2.3.5 Likelihood Maximization

Since I do not know the unobserved characteristics for an individual i, the unconditional log-likelihood contribution corresponds to the weighted sum of contributions corresponding to the (K^e, K^w) points of support. The sample log-likelihood is given by:

$$\ln \mathscr{L} = \sum_{i=1}^{N} \ln \sum_{k^e_i=1}^{k^e} \sum_{k^w_i=1}^{K^w} \mathscr{L}_i[\mathbf{x}_i, (k^e_i, k^w_i)], \tag{A.12}$$

where individual random effects $k_i = (k^e_i, k^w_i)$ are integrated out of the complete likelihood of Equation (A.12). I obtain parameter estimates by maximizing the above log-likelihood function.

2.4 Results

In this section I present the parameter estimates and discuss the model fit with respect to the data.

2.4.1 Estimates

I start the discussion of parameter estimates with the parameters from the labour market status component, $\ell_i(\mathbf{e}_i|\mathbf{c}_i^v, c_i^u, k_i^e)$. Table 2.1 displays parameter estimates for the sample before the Hartz reforms (years 1999-2002) and the sample after (years 2005-2009). The table displays parameter estimates separately by gender. Remember that in Section 2.3.3 I stated that the probability to belong to each employment state was modeled as a multinomial logit. The reference probability is the probability to be in full-time employment, hence I estimate parameters for the contribution of the remaining employment states, {*PT,ME,NE,CE*}, whenever possible.[28] Each column of the table correspond to the multinomial probability of current month's employment state. Rows correspond to the explanatory variables, where I control for labour market experience, education and previous month employment state.

Looking at the constant parameters, the estimates show that part-time, marginal employment and non-employment are less likely than full-time employment since their sign is negative, both for males and females, for the 1999-2002 sample. Non-employment is the most likely labour market state after full-time (the constant parameter is highest among $\{PT, ME, NE\}$. This feature is sustained after the Hartz reforms in the 2005-2009 sample. After the reforms, concurrent employment appears as a new form of employment, but it is nevertheless the least likely state; its constant parameter estimate is the smallest among $\{PT, ME, NE, CE\}$.

Experience in the labour market is allowed to be quadratic to adopt non linearities over workers' life-cycle employment status. The estimates show, for example, that the likelihood of part-time for females is concave through the life-cycle, while marginal employment for males is convex, for both yearly samples. This denotes that female part-time likelihood reaches its peak in the middle of the life-cycle, while male marginal employment reaches its bottom.[29]

With respect to education variables, it is worth highlighting two salient effects. First, For the 1999-2002 sample, higher educated males have a lower propensity to fall into any of the {*PT,ME,NE*} states relative to females. This is shown comparing education parameter estimates across gender. Second, after the Hartz reforms (sample 2005-2009), and for both males and females, the effect that higher education has on any of these employment status diminishes. In other words, higher educated male and female workers have a relatively higher likelihood to adopt any of these employment states after the change in policy. This means that other employment status rather than full time become more likely for higher educated individuals.

State dependence, which is denoted by controlling for each employment state {*PT,ME,NE,CE*} at time *t-1* with respect to the destination state at time *t*, can be studied in the model. In the table, state dependence shows that remaining in the same state has the highest likelihood for each employment status. For example, $NE_{t-1} \rightarrow NE_t$ transitions for males in 2005-2009 have a parameter estimate of 5.79. This effect is larger than any other outflows from NE_{t-1}. To finalize

[28] As discussed previously, the sample 1999-2002 does not consider concurrent employment.
[29] Again, there could be household decisions that I am not able to observe in my analysis.

Table 2.1: Labour Market State Estimates

	Males				Females			
1999-2002	$\mathbf{PT}_t$	$\mathbf{ME}_t$	$\mathbf{NE}_t$	$\mathbf{CE}_t$	$\mathbf{PT}_t$	$\mathbf{ME}_t$	$\mathbf{NE}_t$	$\mathbf{CE}_t$
Constant	-6.17	-5.74	-2.56	-	-6.18	-5.90	-3.05	-
Exper. ($\times$100)	0.3	-5.8	-2.3	-	9.1	4.0	1.6	-
Exper.^{2}($\times$1000)	0.2	1.5	0.7	-	-1.3	-0.5	-0.2	-
High-School	-4.74	-4.87	-3.68	-	-2.09	-3.45	-1.83	-
College	-4.26	-4.41	-3.54	-	-2.27	-3.88	-1.82	-
PT_{t-1}	9.00	3.44	3.17	-	9.00	3.06	3.96	-
ME_{t-1}	3.69	9.00	3.60	-	3.55	9.00	4.21	-
NE_{t-1}	3.34	4.34	5.55	-	4.18	5.28	6.64	-
$k^e = 2$	4.85	4.60	3.00	-	3.11	4.02	1.45	-
2005-2009								
Constant	-5.96	-5.24	-2.66	-6.04	-5.49	-5.08	-2.76	-5.63
Exper.($\times$100)	-0.6	-6.8	-3.5	3.0	4.3	-0.4	-0.7	1.0
Exper.^{2}($\times$1000)	0.4	1.6	0.9	-0.0	-1.0	-0.3	-0.3	-0.0
High-School	-4.00	-3.94	-3.07	-1.56	-0.47	-1.11	-1.32	-0.19
College	-3.60	-3.57	-2.93	-1.90	-0.50	-1.24	-1.23	-0.42
PT_{t-1}	9.00	3.35	3.15	3.33	9.00	2.99	3.89	4.20
ME_{t-1}	3.74	9.00	3.94	4.20	3.56	9.00	4.26	4.35
NE_{t-1}	3.36	4.60	5.79	0.42	4.22	5.13	6.77	1.19
CE_{t-1}	3.27	4.48	0.74	8.07	3.96	4.11	1.27	8.62
$k^e = 2$	4.19	3.69	2.49	2.30	2.93	3.28	2.31	2.39

Notes: the table displays parameter estimates from the employment component of the model presented in Section 2.3.3. The model is estimated separately by sample year and across gender. The omitted categories are workers high-school dropouts in full time. Additional parameter estimates of the initial state are displayed in Table 2.A1 in the Appendix.

with the parameters estimates for the mobility component, note that across all samples and gender, the factor for unobserved heterogeneity for the mobility class $k^e = 2$ is estimated with a positive sign, indicating that mobility class $k^e = 2$ has a higher likelihood to fall into any {*PT,ME,NE,CE*} states and hence lower probability to experience FT employment.

Lastly, I discuss about the likelihood of finding full time employment coming from non-employment and marginal employment. This is important to discriminate between a *stepping stone* argument and a *dead-end* argument. Estimates comparisons before and after the Hartz reforms should be taken considering that after the Hartz reforms there is a new form of employment: concurrent employment. Looking at rows {ME_{t-1},NE_{t-1}} estimates for the 1999-2002 and the 2005-2009 samples, it can be seen that there is barely no change within the estimates across states {$\mathbf{PT}_t$,$\mathbf{MJ}_t$,$\mathbf{NE}_t$}. Namely, the model generates similar outflows from {ME_{t-1},NE_{t-1}} states

Table 2.2: Wage Distribution Estimates

	Males			Females		
1999-2002	μ	σ	τ	μ	σ	τ
Constant	7.13	-0.50	-8.00	7.03	-0.50	-7.97
High-School	0.22	-	-	0.11	-	-
College	0.56	-	-	0.40	-	-
Exper.($\times$100)	3.4	-1.7	0.7	2.7	-1.8	0.4
Exper.^{2}($\times$1000)	-1.0	-	-	-0.0	-	-
PT$_t$	-0.43	-0.07	-	-0.27	-0.22	-
ME$_t$	-1.17	0.72	-	-1.40	0.31	-
NE$_t$	-0.92	0.20	-	-0.94	0.52	-
$k^w = 2$	0.04	0.22	-4.23	0.09	0.18	-4.00
2005-2009						
Constant	6.74	-1.00	-3.83	6.69	-1.15	-3.81
High-School	0.30	-	-	0.24	-	-
College	0.60	-	-’	0.43	-	-
Exper.($\times$100)	3.6	0.2	-1.3	2.2	0.5	-2.3
Exper.^{2}($\times$1000)	-1.0	-	-	-0.0	-	-
PT$_t$	-0.45	0.23	-	-0.17	0.27	-
ME$_t$	-1.48	0.57	-	-1.34	0.06	-
NE$_t$	-0.85	1.11	-	-0.88	1.23	-
CE$_t$	0.11	-0.20	-	0.12	-0.17	-
$k^w = 2$	0.28	-0.75	-3.37	0.23	-0.38	-3.60

Notes: the table displays parameter estimates from the wage component of the model presented in Section 2.3.4. The model is estimated separately by sample year and across gender. Additional parameter estimates from the autocorrelation function are displayed in Table 2.A2 in the Appendix.

into {$\mathbf{PT}_t$,$\mathbf{MJ}_t$,$\mathbf{NE}_t$} before and after the Hartz reforms for both males and females. What is new is that there are non-negligible outflows from ME$_{t-1}$ to $\mathbf{CE}_t$ in the 2005-2009 sample. This means that the stepping stone argument from marginal employment is shared into outflows to $\mathbf{FT}_t$ and $\mathbf{CE}_t$. In other words, marginal employment after the Hartz reforms serves as a stepping stone to earn secondary earnings with another form of employment. Transitions from ME$_{t-1}$ to $\mathbf{CE}_t$ are larger than to $\mathbf{NE}_t$, which suggests that marginal employment has a stronger effect in engaging workers into the labour market by holding multiple forms of employment than by using marginal employment as a dead-end.

Table 2.2 provides parameter estimates for the wage component of the model presented in section 2.3.4. Again, it is estimated separately by gender and before and after the Hartz reforms (years 1999-2002 and 2005-2009). Only looking at the constant parameter of mean earnings, the table shows the gender gap, 7.13 vs 7.03 and 6.74 vs 6.69 in average log monthly earnings estimated before and after the Hartz Reforms. Education parameters show that returns to higher education amplify the gender gap; the effect that higher education has on males has a greater impact on average monthly earnings for males relative to females.

Earnings profiles are estimated to be concave in labour market experience as expected, and as can be observed by looking at the experience coefficients. Also, the effect that {*PT,ME,NE*} have on wages is negative, with ME having the lowest loss in earnings.[30] Noticeably, and in line with the findings of Carrillo-Tudela et al. (2018), CE has a positive effect on earnings. They argue that concurrent employment is used as a form to top up a worker's earnings that is at the bottom of the distribution. My model reflects that CE has a positive effect on earnings with the estimate on earnings being positive.

In terms of volatility, before the Hartz reforms volatility decreases with age while after the reforms volatility increases with age. This effect is net of the contribution of the expansion of flexbile forms of employment, since I controlled for the contribution that each employment state has on the dispersion of earnings. For example, $\{ME, NE\}$ have larger dispersion on earnings both before and after the Hartz reforms and on both males and females. Part-time earnings become more volatile after the reforms, and the volatility of concurrent employment is relatively small.

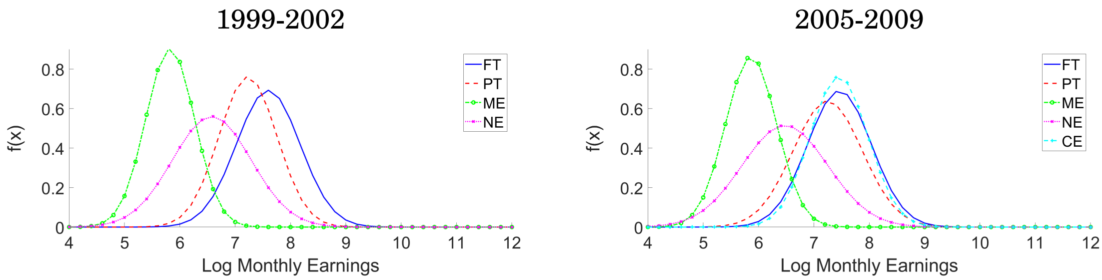
With regard to the autocorrelation of earnings, the life-cycle profile in the autocorrelation of earnings changes significantly before and after the reform. Before the Hartz reforms, earnings had a constant with high persistence (around 0.99 for both males and females) and it declined with labour market experience. Instead, after the Hartz reforms the constant is of lower persistence (around 0.90) and increases with experience. The contribution to the persistence from transitions among different employment states can be seen in Table 2.A2 in the Appendix.

Finally, with respect to unobserved heterogeneity factors, it should be noted that the wage class $k^w = 2$ has a lower volatility in earnings across gender after the Hartz reforms. It also generates slightly higher mean wages for both males and females across samples. Also, this wage class $k^w = 2$ also acts increasing persistence for both males and females across samples.

In the Appendix, I present parameter estimates from unobserved heterogeneity in Table 2.A3 and from the composition of unobserved heterogeneity in Table 2.A4. Table 2.3.2 shows that high-school and college individuals generally have a higher probability to belong to the mobility class $k^e = 2$, across samples. Males with higher experience also have a higher probability to belong to this class. This is relevant since it has been shown that the mobility class $k^e = 2$ has a higher probability to experience flexible forms of employment {*PT,ME,NE,CE*}, as shown in Table 2.1.

Figure 2.A2 in the Appendix shows the gender wage gap distribution for different samples. The figure shows the shift to the left (lower mean earnings) of the gender specific distributions

[30] ME actually pays less than NE, since earnings observed for non-employment come from unemployment benefits/assistance.



Notes: the figure displays kernel density estimates of log monthly earnings simulated from the model. It displays kernel density estimates of each employment state for all workers, before and after the Hartz reforms.

Figure 2.5: Cross-sectional Log Monthly Earnings

as the mean parameter of the respective distributions have fallen after the Hartz reforms. This is reflected in the parameters estimates of the wage process from Table 2.2. Volatility has also increased in the cross-sectional distribution after the Hartz reforms. This is in part because experience profiles in volatility are increasing after the change in policy. Lastly, Figure 2.5 shows the model cross-sectional kernel density estimates of the distributions of log monthly earnings by employment state. The figure shows that there is larger dispersion within employment state after the Hartz reforms and as can be observed from parameter estimates of the wage process from Table 2.2.

2.4.2 Model Fit

In this subsection I simulate the model and compare against the data, in order to evaluate the fit of the model. Figure 2.3 in Section 2.2.3 presented employment transitions in the data, while Table 2.3 presents the employment transitions from the model. I compare the three main takeaways from the data and the model that I summarize here:

i) Marginal employment becomes more persistent after the Hartz reforms. In the data, $ME_{t-1} \rightarrow ME_t$ goes from 87 (93) to 88 (94) percent monthly transitions for males (females) before and after the Hartz reforms. In the model, these numbers are 85 (93) to 86 (93), close in levels and generating higher persistence in ME after the reform (for males).

ii) Marginal employment to non-employment transitions decreased after the Hartz reforms. More generally, there are lower flows into non-employment after the Hartz reforms. In the data, $ME_{t-1} \rightarrow NE_t$ goes from 10 (5) to 8 (4) percent monthly transitions for males (females) before and after the Hartz reforms. In the model, these numbers are 8 (4) to 6 (3), close in levels and generating lower $ME_{t-1} \rightarrow NE_t$ transitions after the reform.

iii) There is more non employent to marginal employment transitions after the Hartz reforms. In the data, $NE_{t-1} \rightarrow ME_t$ goes from 1 (3) to 2 (3) percent monthly transitions for males (females) before and after the Hartz reforms. In the model, these numbers are 1 (3) to 2 (4), close in levels and generating higher $NE_{t-1} \rightarrow ME_t$ transitions after the reform.

Table 2.3: Model Job Mobility

	Males					Females				
1999-2002	**FT**$_t$	**PT**$_t$	**ME**$_t$	**NE**$_t$	**CE**$_t$	**FT**$_t$	**PT**$_t$	**ME**$_t$	**NE**$_t$	**CE**$_t$
FT_{t-1}	98	0	0	1	-	97	0	0	2	-
PT_{t-1}	4	92	0	4	-	1	97	0	1	-
ME_{t-1}	5	1	85	8	-	2	1	93	4	-
NE_{t-1}	8	1	1	89	-	5	2	3	89	-
2005-2009										
FT_{t-1}	96	0	0	1	0	96	0	0	1	0
PT_{t-1}	3	92	0	2	1	1	96	0	1	0
ME_{t-1}	4	1	86	6	1	1	1	93	3	1
NE_{t-1}	8	1	2	87	0	4	2	4	88	0
CE_{t-1}	5	0	1	0	91	3	2	1	0	92

Notes: the table displays employment transitions (in percentage ×100) simulated from the model, across different year samples and by gender. The data counterpart is shown in Figure 2.3.

In general, the likelihood component of employment status is very flexible in matching the data. The model is very close to the data. It matches the high persistence of any given employment state, but also matches transitions to other employment status. A key element to obtain this good fit is state dependence; namely, having as a factor in the likelihood the worker's last month employment status. The model generates high persistence of a worker's current employment status by giving more weight to the worker last month's employment status. And when there is a transition, the different weights to fall in each employment status generate the transition outflows similar to the data. This goodness of fit is generated for all samples and gender. How state dependence has changed before and after the Hartz reforms -how parameters have changed from one labour market to another- is key in making the model replicate the data.

Next, I turn to earnings dynamics. Table 2.4 displays data quantiles transitions from the monthly earnings distribution and the corresponding simulation from the model, across different year samples and by gender. I compare two main takeaways from the data and the model that I summarize here:

I) Persistence in the position of the ranking of workers' earnings is increasing with the level of earnings (especially for the 1999-2002 sample). The model is close the levels in the data and generates the increasing pattern in the persistence of earnings.

II) After the Hartz reforms, persistence in the earnings rank decreases, generating more transitions across the earnings distribution. The model generates lower persistence for

Table 2.4: Fit to Income Mobility

	Males					Females				
1999-2002										
Model	$Q_{1,t}$	$Q_{2,t}$	$Q_{3,t}$	$Q_{4,t}$	$Q_{5,t}$	$Q_{1,t}$	$Q_{2,t}$	$Q_{3,t}$	$Q_{4,t}$	$Q_{5,t}$
$Q_{1,t-1}$	91	2	1	1	1	92	2	1	1	0
$Q_{2,t-1}$	2	93	2	1	1	2	93	2	1	1
$Q_{3,t-1}$	1	2	93	2	1	1	2	92	2	1
$Q_{4,t-1}$	1	1	2	93	2	1	1	2	93	2
$Q_{5,t-1}$	0	0	1	2	95	0	0	1	2	95
Data										
$Q_{1,t-1}$	90	6	1	0	0	90	5	1	1	0
$Q_{2,t-1}$	6	90	1	0	0	6	89	2	1	00
$Q_{3,t-1}$	1	2	96	1	0	1	3	94	1	00
$Q_{4,t-1}$	0	1	1	97	1	0	1	1	96	01
$Q_{5,t-1}$	0	0	0	1	99	0	0	0	1	98
2005-2009										
Model										
$Q_{1,t-1}$	91	3	1	1	0	93	2	1	1	0
$Q_{2,t-1}$	2	92	2	1	1	1	93	2	1	0
$Q_{3,t-1}$	1	2	92	2	1	1	2	92	2	1
$Q_{4,t-1}$	1	1	2	93	2	1	1	2	92	3
$Q_{5,t-1}$	0	0	1	2	95	0	0	1	2	95
Data										
$Q_{1,t-1}$	90	6	1	1	0	91	5	2	1	00
$Q_{2,t-1}$	07	89	02	1	0	05	89	3	1	00
$Q_{3,t-1}$	01	2	94	1	0	2	3	93	2	00
$Q_{4,t-1}$	0	1	1	96	1	1	1	1	95	01
$Q_{5,t-1}$	0	0	0	1	98	0	0	0	1	98

Notes: the table displays quantile transitions within the earnings distribution simulated from the model and in the data, across different year samples and by gender.

males across the entire distribution. For females, it generates lower persistence of earnings at the top of the distribution.

In general, the model fits well the mean log monthly earnings at the cross-section and separated by employment state, for all samples and gender (see Figure 2.A1 in the Appendix). The variance of log monthly earnings is, however, the most difficult moment to match with the data. While the model is not far from the data, it is difficult to approximate. This is possibly due to the distributional normality assumption in the model and the existence of outliers/noise in the data. Finally, the persistence of earnings across states is generally satisfactory (see Figure 2.A4 in the Appendix). It matches best persistence for earnings across the same employment status and it is somewhat less close to the data for transitions across employment states, also possibly due to the existence of outliers/noise and a relatively smaller number of observations.

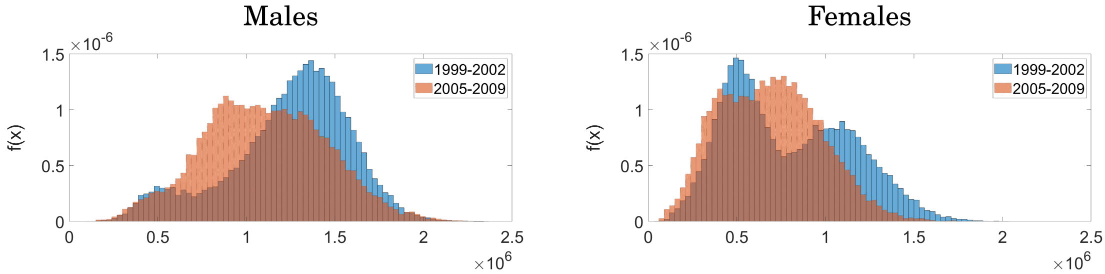
Two elements in the model help to generate the persistence of earnings: the autocorrelation component and the latent factors of the unobserved heterogeneity component. While the model does not have shocks depending on the position in the ranking of earnings, it is capable of approximating the levels and the dynamics of earnings persistence along the ranking distribution. For example, the observed high persistence of earnings at the top of the distribution in the data are generated in the model from the realization of high earnings drawn from the bivariate normal distribution. These high earnings realizations, together with a close to permanent autocorrelation parameter, accumulate over time and are carried over to the successive periods. The high persistence of earnings at the top of the distribution comes from workers who are in full-time and remain in that state. This is because full-time employment has the highest mean earnings in the model. Also, the less persistent earnings in the middle and at the bottom of the distribution come as a result of having more transitions within employment states, with less persistence in earnings carried over to the next period.

With the already satisfactory model fit, I did not further explore the possibility of using a Markov process of higher order. Alternatively, I have neither explored adding more unobserved heterogeneity, given that it would add model complexity.

2.5 Lifetime Values

In this section, I adopt a life-cycle approach to the labour market changes before and after the Hartz reforms. To this end, I construct lifetime values before and after the Hartz reforms. The goal is to analyze the differences in the distribution of lifetime values decomposing the employment and wage effects. In order to do so, I will perform a series of counterfactual exercises.

The measure of lifetime value that I will use is simply the present discounted sum of future earnings flows, taking into account labour market trajectories. This would be be the relevant measure if one assumes that individuals are risk neutral and can perfectly insure. I further assume that the environment is the same, before and after Hartz reforms. Since samples are unbalanced,



Notes: the figure displays kernel density estimates of lifetime earnings, across different year samples and by gender.

Figure 2.6: Lifetime Values Distributions

I assume that workers that are unobserved for a certain number of years in the sample will follow the same process as those who are observed. Making use of the estimated parameters for the wage distribution, employment status and unobserved heterogeneity, I simulate employment and earnings trajectories for the individuals in the different samples along the working age. The lifetime value at experience level t of an individual's simulated future earnings trajectory $\mathbf{w}_{s\geq t}$ is denoted as:

$$V_t(\mathbf{w}_{s\geq t}) = \sum_{s=t}^{T} \beta^{s-t} \exp(w_s), \tag{A.13}$$

where $\beta \in (0,1)$ is a discount factor, $\exp(w_s)$ is the earnings flow that the individual receives at experience level s and the individual retires at period T. Remember that w_s is conditional on the individual's characteristics and labour market state as specified in Section 3.3. In this context, the only parameter to be calibrated is β, which I set to 1.00 to give equal weight to all years through the life-cycle[31].

I assume the environment to be stationary. That is, individuals anticipate their life cycle path and receive their earnings and changes to their labour market status given their current state, but do not expect any of the parameters to be changed over the rest of their working life. This assumption is most credible if the sample period from which I obtained the estimates of the model is representative of an average state of the business cycle and the change in policy could not be anticipated. As explained in Section 3.2, I took a sufficient number of years before and after the change in policy to approximate such average state of the business cycle.

Figure 2.6 displays the histogram of the lifetime values before and after the Hartz reforms by gender. In general, it can be seen that there is a shift to the left in the distribution of lifetime

[31] Unlike Postel-Vinay and Turon (2005) and Dickson et al. (2014), I do not discount future income flows to construct present discounted lifetime values in this exercise. By setting β=1.00, I give equal weight to all income streams to make a steady-state comparison of two different stationary worlds: the labour market before and after the Hartz reforms. For a comparison of present discounted lifetime values, see Figure 2.A7 in the Appendix which uses counterfactuals introduced in the next subsection. Results are qualitatively similar.

values after the Hartz reforms. The mean of lifetime values goes from 1.24 (0.79) to 1.10 (0.69) million € for males (females) before and after the Hartz reforms.[32]

Another noticeable aspect of the distributions is that females before the Hartz reforms have a bimodal distribution, with the highest density concentrating around 0.5 million €. This is expected given that the employment stocks in Figure 2.2 showed that the female sample is more fragmented between those working in full-time and those in part-time, relative to males. For males, the concentration of earnings around 0.5 million € is significantly lower relative to females. After the Hartz reforms, the distribution of lifetime values for females somewhat concentrates around its mean conforming a less bimodal distribution and closer to a unimodal distribution. Therefore, the dispersion of lifetime values decreases for females after the Hartz reforms. The standard deviation of female log lifetime values goes from 0.4848 to 0.4698. For males, the dispersion of lifetime values increases after the Hartz reforms from a measure of 0.3508 to 0.3564.[33]

While for females the distribution of lifetime values is positively skewed both before and after the Hartz reforms (with a skewness of 0.3564 vs 0.2749), male skewness changes its sign. Before the Hartz reforms the distribution of lifetime earnings concentrates around 1.5 million €, with a small left tail making the distribution negatively skewed (-0.6402). Instead, after the Hartz reforms lifetime values shift to the left concentrating around 1 million €, and the distribution becomes slightly positively skewed (0.0876).

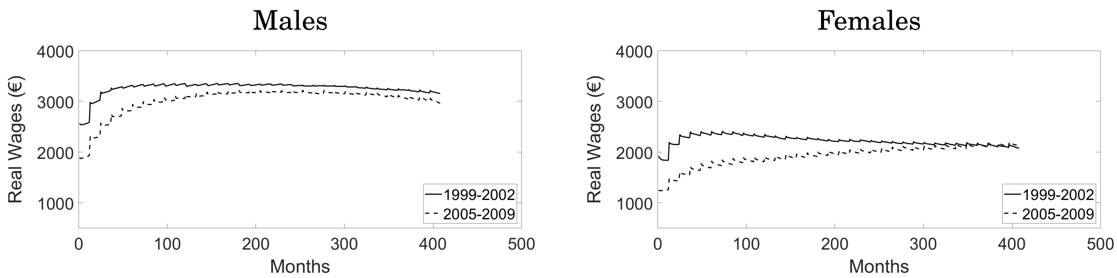
In the model, I have two components that can rationalize the shift to the left in the distribution of lifetime values: wage dynamics and employment transitions. The estimation of the model in Section 2.4.1 after the Hartz reforms shows that

- lower wages
- lower full-time incidence
- higher persistence in marginal employment
- less inflows into non employment
- higher outflows from non employment

can explain the shift to the left in lifetime values. Therefore, it remains to assess the contribution that each factor has had to generate the lower lifetime earnings. In other words, I want to know if the expansion in marginal employment is what is driving long term earnings down, or if, by contrary, lower wages play a bigger role. I try to address these questions in the following subsection.

[32] Figure 2.A6 in the Appendix compares the 2005-2009 lifetime values against the same model estimated and simulated for the 2010-2014 sample. The figure shows that the lifetime values from the model in 2005-2009 and 2010-2014 differ little. If any, the right tail for female values expands in 2010-2014.

[33] Figure 2.A6 in the appendix shows that for years 2010-2014, lifetime values desities are unimodal for Females.



Notes: the figure displays life-cycle monthly earnings simulated from the model before and after the Hartz reforms, separated by gender. "Counterfactual - Employment" uses the estimation from the 1999-2002 sample with the employment parameters from the model estimated for the 2005-2009 sample. "Counterfactual - Wage" uses the estimation from the 1999-2002 sample with the wage parameters from the model estimated for the 2005-2009 sample.

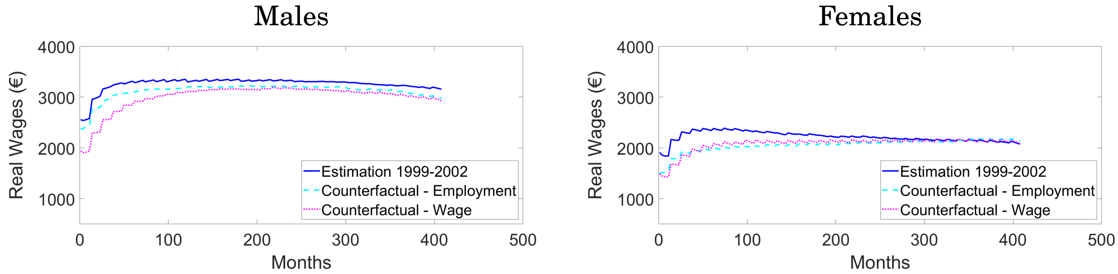
Figure 2.7: Life-cycle Earnings Profiles

2.5.1 Counterfactual Exercise: Wage vs Employment

Firstly, I start having a look at the estimated earnings profiles over the life cycle by sample and gender. On Figure 2.7 I show the model's estimated monthly average life cycle earnings profiles before and after the Hartz reforms, separated by gender. The figure displays the usual concave pattern in earnings, with the feature that for the 1999-2002 sample this concave pattern is somewhat flatter. The comparison between the 1999-2002 and 2005-2009 samples shows that after the Hartz reforms the average profiles are shifted down for both males and females. This means that earnings are generally lower after the Hartz reforms, as previously shown for lifetime earnings in Figure 2.6. Most of the lower earnings occur between the beginning and the middle of the life-cycle. Elder workers before and after the Hartz reforms somewhat keep similar earnings trajectories. Together with the lifetime values displayed in Figure 2.6, the pronounced decline in earnings profiles was to be expected.

The question that I can address with the estimation of the model is how much of the relative contribution to the decrease in lifetime earnings comes from changes in employment trajectories or changes in wage dynamics given the old employment trajectories. In order to address this, I will perform a counterfactual exercise. I use the 1999-2002 estimated model as the baseline, with all wage, employment and unobserved heterogeneity components estimated from the 1999-2002 sample. Then, I replace the wage (employment) component using the 2005-2009 estimated model. Put differently, I swap parameter estimates between models to see the partial response that each component has on life cycle earnings. This counterfactual exercise can be informative of the wage (employment) relative contribution to the estimated decline in lifetime earnings. I assume, again, the environment to be stationary, and that the only change in the labour market comes from the different parameter estimates of the respective component.

Figure 2.8 displays the counterfactual exercise. The solid blue line presents the estimated life cycle earnings profile for the sample 1999-2002. The dashed cyan line presents the 1999-2002 model with the employment component parameters of the model estimated for the sample 2005-2009. The dotted magenta line presents the 1999-2002 model with the wage component



Notes: the figure displays life-cycle monthly earnings counterfactuals separated by gender.

Figure 2.8: Wage and Employment State counterfactuals

parameters of the model estimated for the sample 2005-2009. Both counterfactuals lie below the 1999-2002 estimated model, which means that the changes in employment transitions and wage dynamics have both negatively affected life cycle earnings profiles for males and females. The difference across gender lies in the relative importance of each factor. While for females the employment and the wage component have a similar impact in the decrease of lifetime values (both counterfactuals shift down very closely), for males, the contribution of the wage component has a larger effect. If I compute the average monthly loss in earnings between each counterfactual and the 1999-2002 estimation, I obtain that for females the wage (employment) loss is of 140.87 (163.63) €/monthly. For males, these numbers are 256.13 (138.41) €/monthly. This indicates that the introduction of the Hartz reforms had a similar effect in decreasing long term earnings for females stemming from both the wage and the mobility component. However, for males, the different wage distribution (in particular, lower mean wages) after the Hartz reforms had around two times the impact in lowering long term earnings than the changes in the new employment transitions stemming from the Hartz reforms.

2.6 Conclusion

In this paper, I have studied how more flexible forms of labour contracts affect the distribution of lifetime earnings. I used a change in policy (the Hartz reforms) that transformed the labour market in Germany and expanded the adoption of marginal employment. At the estimation of the empirical processes, I take a sufficient number of years in order to capture the stationary labour market before and after the Hartz reforms. The empirical models reflect lower incidence of full time employment, higher propensity to use marginal employment (exclusive or as a secondary job), lower full time earnings and less persistence in the position in the ranking of earnings at the bottom of the distribution, after the Hartz reforms. Assuming at each sample that the young person anticipates the old person's labour market, I am able to construct lifetime employment and earnings trajectories. I construct lifetime values and show that the distribution has shifted to the left, hence generating lower long term earnings. In a counterfactual exercise I use the estimation of the model for the 1999-2002 sample as a baseline and input the employment (wage) parameters from the estimation of the model in the 2005-2009 sample for both males and females. This exercise shows that both the new employment and the wage dynamics contribute to the lower long term earnings after the Hartz reforms. However, for males, the effect is twice larger for the wage component relative to the employment one. In other words, the decrease in wages has a higher impact in long term earnings than the new employment dynamics.

The findings of this paper contribute to the ongoing debate on the German labour market miracle during the great recession. In particular, the importance of the wage component in lowering long term earnings adds to the arguments of Burda and Hunt (2011) who argue that wage flexibilization at the bottom of the distribution for full time employment is discussed as one possible explanation for the German labour market miracle in the great recession. In addition, the findings of this paper also contribute to the argument developed in Dustmann et al. (2014). Specifically, the Hartz reforms have contributed to the continued increase in wage inequality at the lower end of the distribution, but the specific governance structure of the German system of industrial relations is what paved the way for the exceptional decentralization of the wage formation from the industry level to the level of the single firm or the single worker. The unprecedented decentralization of the wage-setting process from the industry level to the firm level in Germany could have been the mechanism that allowed for wage restraints and the dramatic decrease in real wages at the lower end of the distribution after the Hartz reforms.

2.7 Appendix

2.7.1 Sample Selection

Unfortunately, some discretionary decisions are necessary to be implemented for the classification of employment spells into the five employment categories that I construct. I perform the following sample restrictions:

- I assign an employment spell to the sample if they do not fully overlap with a spell in unemployment. Otherwise it will be categorized as a non-employment spell.
- A employment spell(s) below 15 natural days in total within the month is classified as a non employment spell.
- In the, unlikely, but possible event of two or more working events within a month, I assign the category which employment days are larger than two times the sum of the remaining working spells. Therefore, if a person has different working spells, but one category does not exceed in days two times the other, is classified as concurrent employment.
- Non employment are coded either if the worker is registered unemployed, earn wages below the minimum subsistence level or are out of the labour force. This implies that some of the non employed workers receive earnings within their months while receiving zero earnings. For the estimation section, I keep track of the person's employment record and I predict their likelihood to receive non employment earnings based on an OLS regression on education, gender, lag employment, lag unemployment benefits and year of birth for the different yearly samples.[34]
- A worker enters the sample if the working spell in any of the yearly samples is at least of 12 months.
- To delete miscoded employment records, I delete working spells that are repeated, employment spells below 3 working days and daily real earnings below 5 €.
- With special relevance for this paper, I drop marginal employment earnings above their maximum legislative threshold.

[34]A probit specification was also considered with, quantitatively, similar results.

2.7.2 Additional Tables

Table 2.A1: Initial Labour Market State Estimates

	Males				Females			
1999-2002	**PT**$_t$	**ME**$_t$	**NE**$_t$	**CE**	**PT**$_t$	**ME**$_t$	**NE**$_t$	**CE**$_t$
Constant	-3.14	-4.80	0.60	-	-2.73	-3.90	0.68	-
Age	0.015	-0.080	-0.014	-	0.079	0.011	0.014	-
High-School	-4.92	-3.51	-5.02	-	-3.06	-3.08	-3.28	-
College	-3.56	-2.37	-4.32	-	-3.20	-3.09	-3.09	-
$k^e = 2$	5.00	5.00	4.17	-	5.00	5.00	3.07	-
2005-2009								
Constant	-3.19	-1.78	0.50	-3.92	-2.15	-0.67	1.14	-2.76
Age	0.024	-0.019	-0.015	0.02	0.013	-0.028	-0.045	-0.01
High-School	-4.72	-5.00	-4.32	-2.61	-0.70	-1.87	-2.13	-0.54
College	-3.57	-3.92	-3.65	-3.01	-0.43	-1.60	-1.50	-0.84
$k^e = 2$	5.00	4.90	3.62	4.02	5.00	5.00	3.79	3.92

Notes: the table displays parameter estimates of the initial state from the employment component of the model presented in Section 2.3.3. The model is estimated separately by sample year and across gender.

Table 2.A2: Additional Wage Distribution Estimates, Autocorrelation τ

1999-2002	Males	Females		Males	Females
$PT_{t-1}\rightarrow FT_t$	4.1758	4.6768	$FT_{t-1}\rightarrow PT_t$	4.1525	4.8218
$ME_{t-1}\rightarrow FT_t$	4.7544	4.9320	$ME_{t-1}\rightarrow PT_t$	4.6063	5.0000
$NE_{t-1}\rightarrow FT_t$	4.4025	4.7585	$NE_{t-1}\rightarrow PT_t$	4.4218	4.8682
$CE_{t-1}\rightarrow FT_t$	-	-	$CE_{t-1}\rightarrow PT_t$	-	-
$FT_{t-1}\rightarrow NE_t$	4.3173	-	$FT_{t-1}\rightarrow ME_t$	4.5870	4.5982
$PT_{t-1}\rightarrow NE_t$	4.4741	-	$PT_{t-1}\rightarrow ME_t$	4.3782	5.0000
$ME_{t-1}\rightarrow NE_t$	4.5193	-	$NE_{t-1}\rightarrow ME_t$	4.5990	4.5796
$CE_{t-1}\rightarrow NE_t$	-	-	$CE_{t-1}\rightarrow ME_t$	-	-
2005-2009					
$PT_{t-1}\rightarrow FT_t$	4.8489	4.7008	$FT_{t-1}\rightarrow PT_t$	5.0000	4.9579
$ME_{t-1}\rightarrow FT_t$	5.0000	5.0000	$ME_{t-1}\rightarrow PT_t$	5.0000	5.0000
$NE_{t-1}\rightarrow FT_t$	4.8197	5.0000	$NE_{t-1}\rightarrow PT_t$	5.0000	5.0000
$CE_{t-1}\rightarrow FT_t$	2.9492	3.4885	$CE_{t-1}\rightarrow PT_t$	4.7261	4.4711
$FT_{t-1}\rightarrow NE_t$	4.5692	4.9124	$FT_{t-1}\rightarrow ME_t$	5.0000	5.0000
$PT_{t-1}\rightarrow NE_t$	4.9602	5.0000	$PT_{t-1}\rightarrow ME_t$	4.9348	5.0000
$ME_{t-1}\rightarrow NE_t$	4.8876	5.0000	$NE_{t-1}\rightarrow ME_t$	4.9226	5.0000
$NE_{t-1}\rightarrow NE_t$	2.8586	2.2715	$CE_{t-1}\rightarrow ME_t$	2.2867	3.1397
$FT_{t-1}\rightarrow CE_t$	2.3727	3.0213			
$PT_{t-1}\rightarrow CE_t$	4.6905	3.9962			
$ME_{t-1}\rightarrow CE_t$	4.9839	5.0000			
$NE_{t-1}\rightarrow CE_t$	5.0000	5.0000			

Notes: the table displays parameter estimates of the autocorrelation function from the wage component of the model presented in Section 2.3.4. In particular, it displays the estimates of the persistence component of wages when there are transitions across employment states. The model is estimated separately by sample year and across gender.

Table 2.A3: Unobserved Heterogeneity Estimates

	Males		Females	
1999-2002	$k^e = 2$	$k^w = 2$	$k^e = 2$	$k^w = 2$
Constant	-5.0000	1.2470	-3.3026	1.1119
Year of Birth	0.0650	0.0200	0.0053	0.0272
High-School	3.2815	-2.1167	3.2367	-2.1855
College	3.4400	-1.3575	3.1015	-1.7222
2005-2009				
Constant	-4.2006	-1.0887	0.1904	-0.6683
Year of Birth	0.0627	-0.0289	-0.0421	-0.0407
High-School	2.7927	1.7900	0.1962	1.4938
College	2.8163	1.1281	-0.1597	1.1715

Notes: the table displays parameter estimates from the unobserved heterogeneity component of the model presented in Section 2.3.2. The model is estimated separately by sample year and across gender.

Table 2.A4: Composition of Unobserved Heterogeneity

1999-2002	Males						Females					
Mobility	CO	HS	DO	Exp	$k^w = 1$	$k^w = 2$	CO	HS	DO	Exp	$k^e = 1$	$k^e = 2$
$k^e = 1$	16	68	15	24	53	46	13	75	11	24	54	45
$k^e = 2$	21	77	00	17	56	43	19	80	00	17	56	43
Wage	CO	HS	DO	Exp	$k^e = 1$	$k^e = 2$	CO	HS	DO	Exp	$k^e = 1$	$k^e = 2$
$k^w = 1$	14	82	03	22	63	36	11	86	02	22	62	37
$k^w = 2$	22	59	18	20	65	34	19	66	14	20	63	36
2005-2009												
Mobility	CO	HS	DO	Age	$k^w = 1$	$k^w = 2$	CO	HS	DO	Age	$k^e = 1$	$k^e = 2$
$k^e = 1$	19	67	12	25	51	48	22	70	06	20	53	46
$k^e = 2$	23	75	01	18	52	47	15	78	06	26	48	51
Wage	CO	HS	DO	Age	$k^e = 1$	$k^e = 2$	CO	HS	DO	Age	$k^e = 1$	$k^e = 2$
$k^w = 1$	25	61	12	21	58	41	22	67	09	20	63	36
$k^w = 2$	17	80	02	24	59	40	17	80	02	25	57	42

Notes: the table displays the composition of unobserved heterogeneity simulated from the model, across different year samples and by gender.

2.7.3 Additional Figures

Males

	1999-2002				
Males	Mean	FT	PT	MJ	NE
Data	7.843623	7.975314	7.355525	6.118457	6.701506
Model	7.75	7.85	7.36	6.42	6.78

2005-2009					
Mean	FT	PT	MJ	NE	CJ
7.80341	7.946193	7.285395	5.817175	6.641498	7.945496
7.63	7.72	7.27	6.15	6.73	7.79

2010-2014					
Mean	FT	PT	MJ	NE	CJ
7.774604	7.923139	7.178835	5.804833	6.624875	7.929647
7.6	7.68	7.32	6.12	6.71	7.77

	1999-2002				
Males	Std. Dev.	FT	PT	MJ	NE
Data	0.643815	0.509589	0.610577	0.695242	0.393625
Model	0.62	0.54	0.57	0.79	0.58

2005-2009					
Std. Dev.	FT	PT	MJ	NE	CJ
0.700567	0.528072	0.646739	0.504319	0.50648	0.434411
0.71	0.61	0.69	0.8	0.88	0.62

2010-2014					
Std. Dev.	FT	PT	MJ	NE	CJ
0.68254	0.502238	0.677623	0.386991	0.502344	0.433585
0.71	0.62	0.77	0.7	0.91	0.61

Females

	1999-2002				
Females	Mean	FT	PT	MJ	NE
Data	7.361891	7.694648	7.29496	5.802595	6.434809
Moment	7.27	7.5	7.23	5.98	6.49

2005-2009					
Mean	FT	PT	MJ	NE	CJ
7.252844	7.654187	7.275957	5.795255	6.354369	7.516019
7.11	7.33	7.19	6	6.45	7.46

2010-2014					
Mean	FT	PT	MJ	NE	CJ
7.278516	7.693525	7.236265	5.792635	6.406642	7.499186
7.12	7.32	7.2	5.98	6.47	7.44

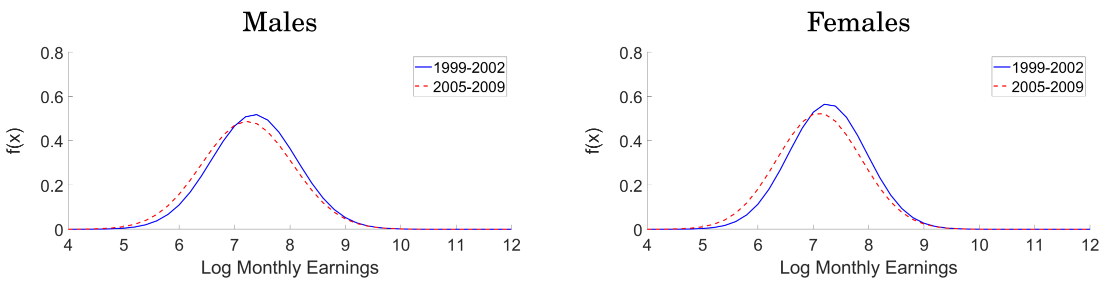
	1999-2002				
Females	Std. Dev.	FT	PT	MJ	NE
Data	0.734363	0.501531	0.463503	0.432503	0.42959
Moment	0.7	0.58	0.55	0.64	0.66

2005-2009					
Std. Dev.	FT	PT	MJ	NE	CJ
0.81487	0.557559	0.52134	0.34264	0.480191	0.478861
0.77	0.59	0.66	0.6	0.88	0.58

2010-2014					
Std. Dev.	FT	PT	MJ	NE	CJ
0.788183	0.534374	0.550644	0.292641	0.472565	0.493381
0.76	0.59	0.68	0.6	0.9	0.58

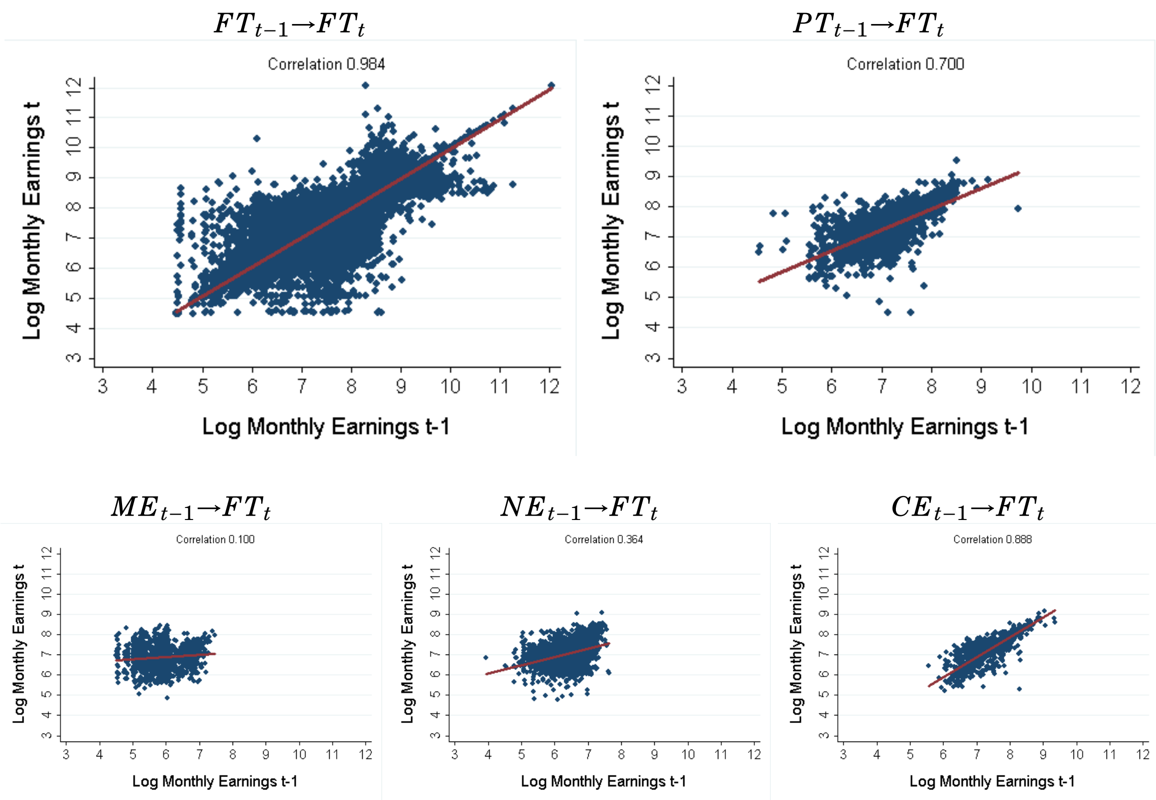
Notes: The figure displays the mean and standard deviation of log monthly earnings in the data and the model, at the cross-section and for all employment states.

Figure 2.A1: Wage Fit



Notes: the figure displays kernel density estimates of log monthly earnings simulated from the model. It displays kernel density estimates of the whole distribution of earnings before and after the Hartz reforms, by gender.

Figure 2.A2: Cross-sectional Wage Gap Distribution



Notes: The figure displays a scatter plot of log monthly earnings in the previous month (x-axis) against log monthly earnings in the current month (y-axis) for female workers during the 2005-2009 period. For illustration purposes, only transitions into full-time are shown.

Figure 2.A3: Autocorrelation, Female workers 2005-2009

1999-2002

t-1	t	Males	Females
FT	FT	0.9798	0.9861
PT	FT	0.5792	0.645
ME	FT	0.1858	0.1648
NE	FT	0.4517	0.4617
FT	PT	0.6353	0.7125
PT	PT	0.9818	0.986
ME	PT	0.3329	0.1615
NE	PT	0.2292	0.3245
FT	ME	0.23	0.3448
PT	ME	0.3654	0.3386
ME	ME	0.9521	0.9425
NE	ME	0.5276	0.5708
FT	NE	0.5988	0.6067
PT	NE	0.3406	0.4628
ME	NE	0.4536	0.4949
NE	NE	0.8741	0.8891

2005-2009

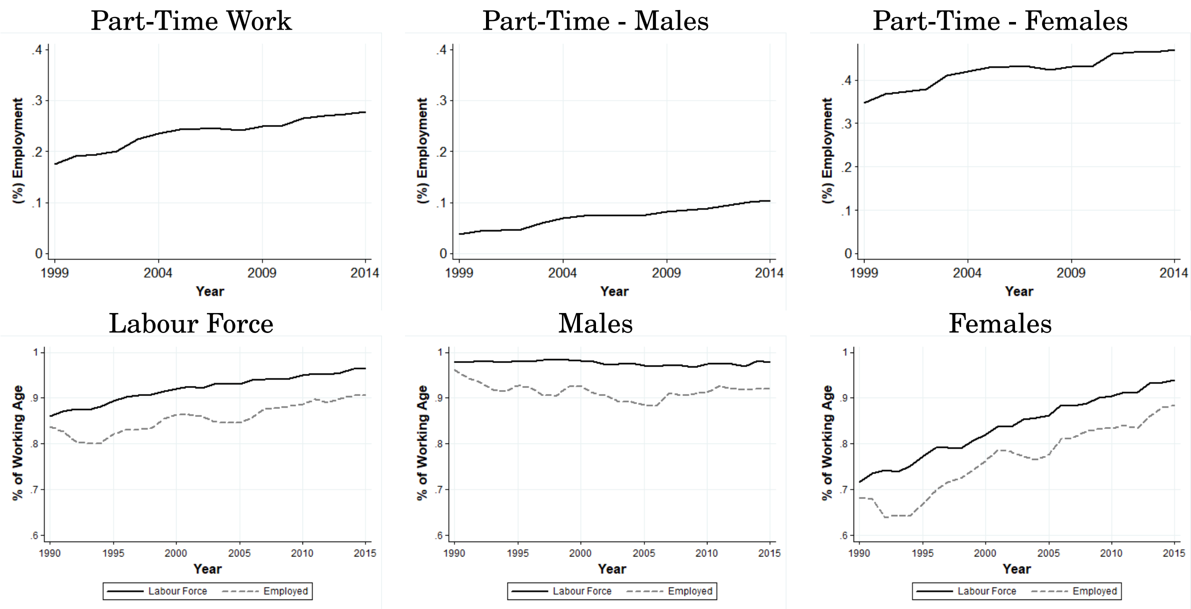
t-1	t	Males	Females
FT	FT	0.9775	0.9844
PT	FT	0.6707	0.7111
ME	FT	0.3096	0.0487
NE	FT	0.3518	0.3474
CE	FT	0.9004	0.885
FT	PT	0.7289	0.7479
PT	PT	0.9799	0.9819
ME	PT	0.155	0.0471
NE	PT	0.3276	0.2729
CE	PT	0.8096	0.8532
FT	ME	0.2827	0.1678
PT	ME	0.3646	0.2756
ME	ME	0.9382	0.9435
NE	ME	0.5917	0.5874
CE	ME	0.2321	0.2405
FT	NE	0.5717	0.6669
PT	NE	0.4953	0.5136
ME	NE	0.5798	0.6101
NE	NE	0.8673	0.8907
CE	NE	0.5923	0.6605
FT	CE	0.9279	0.931
PT	CE	0.7739	0.8745
ME	CE	0.2545	0.1849
NE	CE	0.2099	0.5676
CE	CE	0.9786	0.9883

2010-2014

t-1	t	Males	Females
FT	FT	0.9806	0.983
PT	FT	0.618	0.7161
ME	FT	0.1805	0.144
NE	FT	0.3769	0.435
CE	FT	0.9146	0.88
FT	PT	0.747	0.7827
PT	PT	0.9788	0.9817
ME	PT	0.1384	0.0893
NE	PT	0.3878	0.3147
CE	PT	0.7926	0.8589
FT	ME	0.2451	0.0704
PT	ME	0.28	0.286
ME	ME	0.9194	0.934
NE	ME	0.6341	0.6339
CE	ME	0.2368	0.189
FT	NE	0.5917	0.6695
PT	NE	0.5244	0.5693
ME	NE	0.6119	0.4287
NE	NE	0.8478	0.8768
CE	NE	0.6179	0.7064
FT	CE	0.9335	0.9332
PT	CE	0.7292	0.8791
ME	CE	0.2218	0.2001
NE	CE	0.3339	0.453
CE	CE	0.9864	0.99

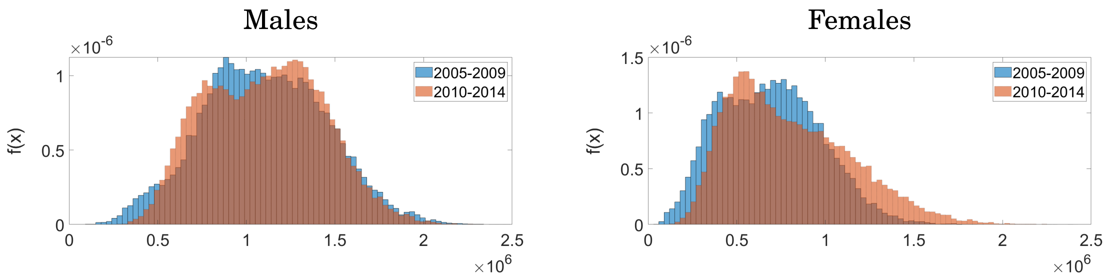
Notes: The figure displays the correlation of log monthly earnings in the previous month (t-1) against log monthly earnings in the current month (t) for all employment types by gender and different yearly samples.

Figure 2.A4: Data Wage autocorrelation



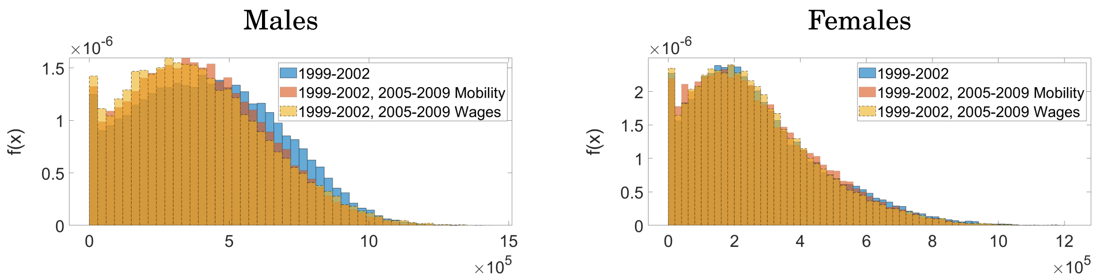
Notes: the figure displays aggregate, and gender specific, stocks of part-time employment (top panel) and labour force participation (bottom panel) in the sample data.

Figure 2.A5: Part-Time and Labour Force trends



Notes: the figure displays kernel density estimates of lifetime earnings, across different year samples and by gender.

Figure 2.A6: Lifetime Values Distributions, 2010-2014



Notes: the figure displays kernel density estimates of present discounted ($\beta = 0.99$) earnings, for the 1999-2002 sample (blue density). The earnings are projected for workers entering the sample for their out-of-sample years until retirement age, assuming their path follows parameters of the 1999-2002 estimated sample. The red and yellow densities display kernel density estimates of present discounted ($\beta = 0.99$) earnings, for workers in the 1999-2002 sample using the estimated parameters of the mobility component of the 2005-2009 sample and the wage component, respectively.

Figure 2.A7: Lifetime Values Distributions, $\beta = 0.99$

CHAPTER 3

On Returns to Effective Experience

with Sekyu Choi and Benjamín Villena-Roldán

Statement on coauthorship: this chapter is joint work with Sekyu Choi and Benjamín Villena-Roldán. I have made significant contributions to all aspects of the work including data cleaning, empirical estimation, placement in the literature and writing up the paper. Structural model estimation and counterfactual analyses were done by myself alone. Benjamín Villena-Roldán contributed to data cleaning and empirical estimation. Sekyu Choi, offered guidance on all aspects of the work as it is his role as PhD supervisor, but also contributed to the construction of the structural model, placement in the literature and interpreting the results.

Additional acknowledgments: we thank Hélène Turon, Diego Lara de Andrés and participants of the Macro online webinar at the University of Bristol for helpful comments and suggestions.

3.1 Introduction

Heterogeneous wage growth over the lifecycle is closely linked to overall earnings inequality.[1] New evidence shows great disparity across countries (Lagakos et al. (2018) and Engbom (2019)).[2] The literature has provided two possible explanations for these facts: (i) labour market frictions, and

[1]Economic research on determinants of wage differentials go back to Mincer (1974). Since then, a large literature emerged on lifecycle wage growth and inequality (Deaton and Paxson (1994) and Storesletten et al. (2004), for example).

[2]For further cross-country cross-sectional comparison studies, see Dabla-Norris et al. (2015) and Tomaskovic-Devey et al. (2020).

(ii) heterogeneity in the human capital accumulation process.[3] In this paper, we study lifecycle wages and the process of general human capital accumulation through on-the-job learning. We analyze a novel dataset from a Chilean internet job board, where we observe a sizeable number of job advertisements, and more importantly, information on expected wages to be paid in each position along with a number of observable characteristics such as offered contracts and required education and experience. The data merge information on applicants, firms, applications, and job ads in a context of heterogeneous workers and positions where required experience is an observable job ad characteristic. To the best of our knowledge, this is a unique feature among databases of this sort that allows us to estimate returns to experience.

As a first contribution, we estimate profiles of log wages over different levels of *required* experience (in years) as measured in job advertisements controling for all possible observables, such as timing of the job posting and fixed effects at the firm and job title levels. We view these profiles as being closer to the true *returns to experience*. This is because the information provided by firms on expected wages to be paid at the advertised position does not depend on any particular worker, who may affect observed (ex-post) wages by way of different mechanisms: wage bargaining, individual match-quality or returns to (unobserved) worker skill, among others.

We further compare our estimated returns to experience (from job advertisement data) with lifecycle wages computed from the worker side. We estimate worker profiles using standard representative surveys of Chilean workers.[4] The comparison shows that returns to experience from job ads grow faster than observed lifecycle wages from workers.

We take the apparent mismatch found in the data as evidence of a failure to accumulate general human capital while working (on-the-job learning). Put it differently, if wage growth through job ads is two times larger than workers wage growth after 8 years, it must be that workers fail to accumulate the demanded level of human capital requested by firms. To rationalize the facts, we put forward a structural labor supply model of the lifecycle, with two types of frictions: standard labor market ones and frictions on the actual process of human capital accumulation. The latter is represented as a simple extension of a Ben-Porath style model, where human capital accumulation is not deterministic but subject to some randomness. Our model also allows for a worker-firm idiosyncratic job match quality so that workers randomly differ in the fit with their corresponding firm.

While simple enough, our model fits Chilean data moments well and serves as a benchmark to start thinking about the failure of human capital accumulation. In the estimation, lifecycle effects

[3] Frictional labour markets build from the work of McCall (1970), Mortensen (1970), Lucas and Prescott (1974), Burdett (1978), Pissarides (1985), Mortensen and Pissarides (1994), Burdett and Mortensen (1998), Hornstein et al. (2007) and Low et al. (2010). For recent applications, see Jung and Kuhn (2016), Engbom (2019). For cross-country differences in human capital see Bils and Klenow (2000), Caselli (2005) and Manuelli and Seshadri (2014). Economists have studied the relationship between human capital accumulation and inequality in Becker (1964), Ben-Porath (1967), Lucas (1988), Keane and Wolpin (1997) and Huggett et al. (2011). Examples of papers combining human capital accumulation and job search are Bowlus and Liu (2013) and Bagger et al. (2014).

[4] We also estimate worker's lifecycle wages using information from job seekers using the job board: both of these approaches give us almost identical results.

are important, especially late in life. The probability of human capital depreciation through unemployment or job-to-job is estimated to be increasing over the lifecycle, while learning on the job (namely, the probability to increase human capital while working) is concave over the lifecycle, closely tracking workers' lifecycle wages.[5]

We use our model to quantify the role that each friction has in the observed wage mismatch. Our simulation exercises show that losing human capital through either unemployment or job-to-job have a lower impact on wage profiles than increasing one's human capital through on-the-job learning. This is in part because a worker spends most of his time at work, hence learning and increasing human capital while working is more relevant. In addition, we eliminate labour market frictions in order to measure their effect to close the gap in wage and job ads growth. Labour market frictions are less important quantitatively than failure-to-learn in explaining the returns gap. Eliminating job separation completely from our model can only close half the gap between worker vs. ads wage growth, while higher on-the-job learning can close the gap fully.

In this vein, we perform a counterfactual exercise where we choose the level of on-the-job learning so that we close the gap in wage and job ads growth. In this counterfactual, the level of on-the-job learning and the level of human capital accumulation are both four times above our baseline estimation. This exercise illustrates that failure-to-learn could be an important factor behind the gap between workers and firms wage growth. A non trivial implication of lower accumulation of human capital throughout the lifecycle is not only that Chilean average wages are lower, but also that inequality of earnings is larger, which should matter for welfare.[6] Although our model does not target Chilean labour earnings inequality measures, the results suggest that failure to accumulate human capital for a large fraction of workers throughout the lifecycle produces larger earnings inequality than in the counterfactual where wage and firms growth equal through higher accumulation of workers' human capital.

During the paper, we interchangeably use the terms "lifecycle wages", "worker side" and "supply side" to refer to workers' lifecycle wage profiles. We use the terms "ads wages", "job ads wages", "firm side" and "demand side" to refer to firms' job ads wage profiles by required experience. The rest of the paper is organized as follows. Section 3.2 describes the data sets, the facts, and the empirical findings. Section 3.3 presents the structural model. Section 3.4 discusses the parameter estimates and model fit. Finally, Section 3.5 uses the model to perform simulation exercises and a counterfactual by closing the worker vs. the ads wage growth mismatch.

3.2 The Facts

Data Source.

[5]It has to be said that although job-to-job creates some depreciation of human capital that increases over the lifecycle, workers still gain from switching jobs as they have incentives to find a good worker-firm job quality match throughout the lifecycle.

[6]Attanasio and Davis (1996).

The main novel facts in this paper are related to the monthly wage profiles estimated using job posting information. This information can be thought of as direct evidence of *demand side* technological requirements in terms of experience profiles.

We use data from `www.trabajando.com`. Our data covers a sample of job postings and job seekers in the Chilean labor market between January 1st 2008 and December 24th, 2016. The raw information in the dataset contains more than 14 million single applications, from around 1.5 million job seekers, to around 270 thousand job ads.[7] In terms of the website's platform, job seekers can use the site for free, while firms are charged for posting ads. Job advertisements are posted for a minimum of 60 days, but firms can pay additional fees to extend this term.

For job seekers, we observe date of birth, gender,[8] nationality, place of residency ("comuna" and "región", akin to county and US state, respectively), marital status, self-reported years of experience, years of education,[9] college major and name of the granting institution of the major.[10] We have codes for occupational area of the current/last job of individuals,[11] information on their salary and both their starting and ending dates.

For each posting, we observe its required level of experience (in years), required college major (if applicable), indicators on required skills (specific, computing knowledge and/or "other") how many positions must be filled, the same occupational code applied to workers, geographic information ("región" only) and some limited information on the firm offering the job: its size (number of employees in brackets) and industry (1 digit code).

Besides this information, recruiters are also asked to record the expected pay for the job posting, and are given the choice whether to make this information visible or not to applicants. Naturally, one could question the reliability of wage information which will be ultimately hidden from the other side of the market. Banfi and Villena-Roldán (2019) address the potential issue of "nonsensical" wage information in job ads by comparing the sample of explicit vs. implicit (job ads without any salary information) postings by firms, and find that observable characteristics predict fairly well implicit wages and vice versa. Moreover, even if employers choose to hide wage offers, they are used in filters of the website for applicant search. Hence, employers are likely to report accurately even if their wage offers are not shown because misreporting may generate adverse consequences. Relatedly, Choi et al. (2020) show that wages in the website (both the explicit and hidden ones) are representative to the rest of the Chilean economy when compared to representative surveys of Chilean workers.

On the other hand, a major caveat of our dataset is the absence of information on activities performed outside the website: individuals seeking for jobs through other means, and more

[7]A complete description of the website and the data is in Banfi and Villena-Roldán (2019).

[8]If we consider a sample of males alone (as in Lagakos et al. (2018), results are similar.

[9]Educational categories are *primary* (one to eight years of schooling), *high school* (completed high school diploma, 12 years), *technical tertiary education* (professional training after high school, usually 2-4 years), *college* (completed university degree, usually 5-6 years) and *post-graduate* (any schooling higher than college degree).

[10]This information is for any individual with some post high school education.

[11]We observe a one-digit classification, created by the website administrators.

Table 3.1: Characteristics of Job Postings

	Average	Std.Dev.
Wage (thousand CLP)	620.70	533.71
Required experience (years)	1.94	1.80
Required education: High School	0.23	0.42
Required education: College	0.31	0.46
Area of job: Business-Management	0.27	0.44
Area of job: Technology	0.16	0.37
Area of job: Not specified	0.42	0.49
Number of obs.	190,280	

importantly, outcomes of job applications. However, for the purpose of our current exercise, this is not a major drawback since we are interested actually in the independent information contained in the website concerning expected salaries to be paid at different job positions and required experience. The main advantage of this wage data is that it is not contaminated by ex-post compensation nor specific worker skills at the individual level.

In the following exercise we impose minimal sample restrictions on the information from the website: we include all information on job advertisements and job seekers, as long as they have non-missing information on the observable characteristics for which we control. We ignore job advertisements that offer less than 100 thousand CLP,[12] or more than 5 million CLP per month (significantly above the 99-th percentile of the worker's salary distribution according to the CASEN survey, a representative survey of Chilean workers.)

Estimates of return to experience (Ads). Using the information on job ads only, we can compute the gradient of wages on *years of required experience*, which is information contained in all job ads on the website. Table 3.1 shows information of the job ads we use in our exercise. Our sample consists of more than 190 thousand individual job ads. In the table we show expected wages to be paid at the position, required experience, required education (only completed high school and college categories) and the main "area of job" categories.

In order to identify wage increases due to experience only, we run linear regressions on log-wages paid at each job ad, controlling for dummy variables for years of required experience and also controlling for a number of observable characteristics: year when the job ad was posted, required education, the area of the job, geographic location and industry of the firm and type of contract (full/part time). We further control for firm's identifier and job title fixed effects. The

[12]This is around 50% less than the minimum wage during 2008, which was set at 151,500 CLP per month.

Table 3.2: Characteristics of Job Seekers

	Average	Std.Dev.
Wage expectations (thousand CLP)	729.00	641.71
Wage at last job (thousand CLP)	697.33	632.98
Potential experience (years)	9.49	8.99
Self-reported experience (years)	6.90	6.71
Age	32.63	8.92
Male	0.55	0.50
Single	0.69	0.46
High School	0.14	0.35
College	0.41	0.49
Area of worker: Business-Management	0.17	0.37
Area of worker: Technology	0.24	0.42
Area of worker: Not specified	0.38	0.49
Employed	0.30	0.46
Unemployed	0.44	0.50
Number of obs.	1,037,493	

latter is constructed as in Banfi and Villena-Roldán (2019).[13]

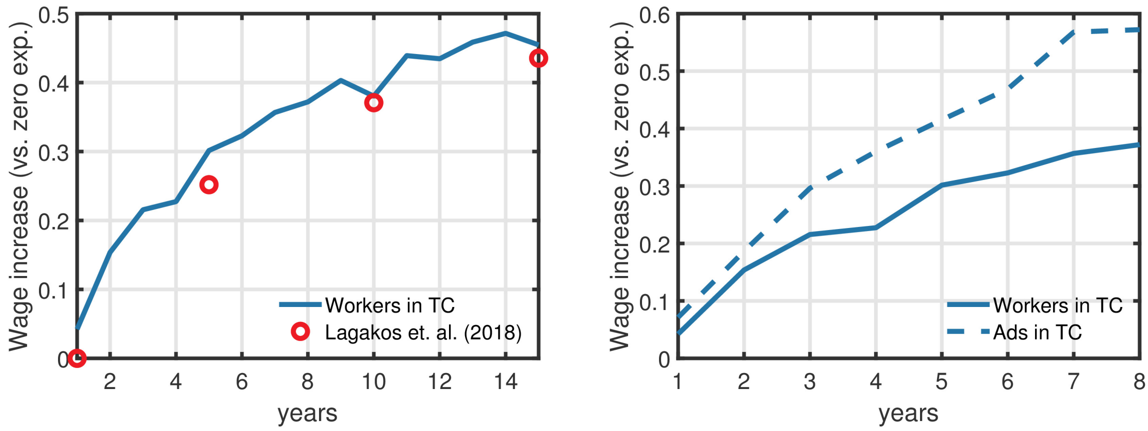
Estimates of life-cycle wages (workers). For life-cycle wages (gradient of salaries with respect to workers' experience), we use the information provided by workers in the website. Table 3.2 shows summary statistics of workers. Compared to the CASEN survey, the sample of workers in the website is younger, more educated and more likely to be male and single than in the entire population of workers.

In Table 3.2 we show monthly salaries at the last job (self-reported by individuals) versus their current salary expectations: the expectations are around 5% higher than their current/last salary, which indicates that the website is used by and large individuals seeking to climb the wage ladder. This is also reflected in the fact that a sizeable number of individuals are currently employed when they use the website for job search (around 30%).

On the hand, we show two indicators of overall experience of workers: a standard calculation of potential experience,[14] and a self-reported experience. This latter number is lower and less disperse than the calculation of potential experience.

[13]We identify the first four words of the job title variable and construct four categorical variables representing a list of words repeated more than 100 times in the whole sample of titles.

[14]This is computed as age minus 18 if total education of the worker is equal or less than 12 years; It is age minus 24 (18+6) otherwise.



Notes: The left panel displays the workers' average wage increase throughout the first 15 years of their lifecycle. The solid line displays the raw data from www.trabajando.com (TC), while the dots display 5-year data intervals from Lagakos et al. (2018). The right panel displays the workers' average wage increase throughout the first 8 years of their lifecycle (solid line) and the average ads' posted wages by required experience from TC data (dashed line), net of firms and job title fixed effects.

Figure 3.1: Worker Side (Supply) vs. Ads Side (Demand)

To compute the experience profile on the worker (supply) side, we use self-reported information on salaries at their last job and the self reported years of experience. More specifically, we run regressions with log-wages as dependent variables, on dummies for years of self-reported experience along a number of controls for observable characteristics: year of birth, year when the individual entered the website (creation of online profile), education level, gender, nationality, civil status, geographic area of residence, job area,[15] and employment status.

Empirical Results. From both the job ad and worker sides, we collect the OLS coefficients on years of required experience and self-reported experience respectively as our main estimates of experience profiles from the two sides of the market. This is displayed on the right panel of Figure 3.1. The estimates in that panel are restricted by sample sizes of job ads with different experience requirements, thus we show estimates for only 8 years of experience. The main takeaway from this figure is the significant gap between the returns to experience at the job ad vs. the worker side.

On the left panel of the figure, we show the coefficients related to the first 15 years of self-reported experience using the equation for workers. Because the variables on required experience and self-reported experience may convey different information, we perform a robustness exercise where we compare our estimates with the estimates for the Chilean economy found in Lagakos et al. (2018). Even though it is estimated using a different data set and a different variable for experience (potential instead of self-reported), the figure shows that the estimates are remarkably close.

[15] Self reported.

In the quantitative exercise below, we extrapolate the experience profile of the demand side (job ads). To obtain an estimate of this profile for years 9 onwards, we forecast using the annual life-cycle growth of worker wages implied in the estimates from Lagakos et al. (2018).

3.3 The Model

We develop a lifecycle, labour supply model based on Ben-Porath (1967), Keane and Wolpin (1997), Eckstein and Wolpin (1999) and Huggett et al. (2011) classic models of investment in human capital. Workers face: (i) frictional labour markets, (ii) uncertainty with respect to match quality with firms, and (iii) uncertainty about human capital accumulation on the job. Time is discrete. The model period is one month and is partial equilibrium. Workers are risk neutral and heterogeneous in terms of: (a) age, $t \in \{1,...,T\}$, and (b) human capital, $x \in \{1,...,X\}$. Workers transit between employment (E) and unemployment (U). While being employed (E), workers may switch from one job to another. Transition probabilities are defined as follows:

- Workers in unemployment find jobs with probability f_t
- Jobs are destroyed with probability s_t
- Employed individuals can find jobs with probability f_t^E

The above probabilities are exogenous and age-dependent. When unemployed, workers receive unemployment benefits (outside option) b_t. While working, workers receive wages $w_i(x,\epsilon_i) = y(x)\exp\{\epsilon_{i,t}\}$, $\epsilon_{i,t} \sim N(0,\sigma^2_{\epsilon,t})$. $y(x)$ is a monotonic function and ϵ is a match quality shock (fixed throughout the duration of a match).

While employed, human capital may appreciate by one unit each period with probability $\rho_{x,t}$. Human capital in our model may also depreciate (also by one unit) in two circumstances:

- When unemployed, with probability $\delta_{x,t}$
- When changing jobs, with probability $\kappa_{x,t}$

Workers seek to maximize the present value of earnings over their working life. The value function of a worker in unemployment is given by:

$$U(t,x) = b_t + \beta(1-f_t)\,\mathbb{E}_{x'}U(t+1,x') + \beta f_t\,\mathbb{E}_{x',\epsilon}\max\{U(t+1,x'),W(t+1,x',\epsilon)\} \tag{A.1}$$

Equation A.1 shows that workers form expectations about ϵ. Agents discount the future at a rate β. While unemployed, human capital x can only decrease so $x' \in \{x, \max\{x-1,1\}\}$. The value function of a worker in employment is given by:

$$
\begin{aligned}
W(t,x,\epsilon) = y(x)\exp(\epsilon) & \\
+\beta\,(1-s_t) & \begin{bmatrix} (1-f_t^E)\,\mathbb{E}_{x'}W(t+1,x',\epsilon) \\ \\ +\,f_t^E\,\mathbb{E}_{x',\eta}\max\{W(t+1,x',\eta),W(t+1,x',\epsilon)\} \end{bmatrix} \\
+\beta\,s_t & \begin{bmatrix} f_t\,\mathbb{E}_{x',\eta}\max\{U(t+1,x'),W(t+1,x',\eta)\} \\ \\ +(1-f_t)\,\mathbb{E}_{x'}U(t+1,x') \end{bmatrix}
\end{aligned}
\tag{A.2}
$$

Equation A.2 states that when a worker is employed, human capital can decrease if changing jobs so $x' \in \{x, \max\{x-1, X\}\}$.[16] If the job is destroyed, workers are allowed to search for a job in that period. In a new job, a new realization of the match quality is created $\eta \neq \epsilon$.

In order to maximize the present value of earnings over their working life, workers choose between remaining in unemployment, staying in their current job or jumping to a new job, provided they receive the corresponding exogenous transition shocks. Workers ponder the potential loss in human capital from changing jobs or remaining in unemployment and, importantly, its impact in earnings, when deciding about their employment state. Also, they have uncertainty about the worker-firm quality match, which has an effect on earnings and affects the worker transition decision.

In our model, employment is an opportunity to increase human capital and hence earnings. Unemployment represents both an opportunity cost of increasing human capital while working and the additional cost of depreciating one's human capital. Last, changing jobs represents a trade-off between potentially finding a better worker-firm quality match -and higher earnings- and potentially losing some of the gained human capital during the transition.

3.4 Calibration

We estimate the model using a combination of datasets from the Chilean economy, including a novel dataset on posted job ads. We use a combination of CASEN and the Encuesta Nacional de Empleo (ENE) to compute a number of moments for the Chilean economy. The moments targeted in the estimation are:

- Profile of lifecycle wages (CASEN)[17]; 32 year-observations

[16] This assumption creates job-to-job transitions that result in wage losses.

[17] We take the estimates directly from Lagakos et al. (2018)

- Returns to experience (www.trabajando.com & CASEN)[18]; 32 year-observations
- lifecycle profiles for E→U, U→E and J→J transitions (ENE); 32 year-observations each
- Average wage loss (%) after an E→U→E episode and average wage gain (%) after an J→J transition (ENE); 1 cross-sectional observation each

The profile of lifecycle wages are the percentage increase of wages relative to the wage at the beginning of the lifecycle.[19] The returns to experience are the percentage increase of wages relative to a job with no required experience. Lifecycle profiles for E→U, U→E and J→J transitions are the average yearly hazard of these transitions. The average wage loss (%) after an E→U→E observation is the workers' average percentage difference in wages after observing an E→U→E transition within a year. Finally, the average wage gain (%) after an J→J observation is the workers' average percentage increase in wages after observing an J→J transition within a year. In addition to these moments, we also compute and compare two more moments not targeted at estimation. These are the share of employment and non-employment over the lifecycle (ENE).

We smooth data observations by using polynomials in age of order 5 for all our lifecycle data moments: the profile of lifecycle wages, returns to experience, and lifecycle profiles for E→U, U→E and J→J transitions. The monotonic component of wages, $y(x)$, is specified as a linear interpolation of the ads lifecycle profiles. For the average wage loss (%) after an E→U→E episode, we compare one cross-sectional observation across the entire lifecycle. Also, for the average wage gain (%) after an J→J episode, we compare one cross-sectional observation across the entire lifecycle. For the rest of the moments, we use year-observations.

All parameters in our model are allowed to vary with age. The components $\rho_x, \kappa_x, \delta_x, f, s, f^E, b$ are each specified as a polynomial of order 2 of the form $z_t = z_1 + z_2\,t + z_3\,t^2$. For example, the probability of job separation has the following form: $s_t = s_1 + s_2\,t + s_3\,t^2$ at any given t over the lifecycle. The volatility component of the match quality shock is specified as $\sigma_{\epsilon,t} = \sqrt{\exp\{\sigma_{\epsilon,1} + \sigma_{\epsilon,2}\,t + \sigma_{\epsilon,3}\,t^2\}}$. We estimate parameters to match Chilean data moments to simulated data moments (Simulated Method of Moments). A model period is one month, hence, the lifecycle of an agent in the model consists of 420 periods. We exogenously set $\beta = 0.99$.

In order to choose the parameters, we minimize the distance between the moments generated by the model and their counterpart in the data. In particular, the calibration algorithm aims to minimize the sum of squared distance between earnings and transition profiles and those produced by the model using the simulated method of moments. Because some moments have different scale, or different number of observations, we impute some weights to each component as in Guvenen et al. (2015). See section A.1 in the Appendix for further details. Lifecycle averages

[18] This is a "hybrid" moment: the first 8 years are from www.trabajando.com, while the rest of the years are extrapolated using the growth rates implied in CASEN. See the complete description in Section 3.2.

[19] As in Lagakos et al. (2018).

Table 3.3: Calibration

Moment	Data	Model	Parameter	Average
J→J Hazard	0.15	0.14	$\bar{\rho}_x$	0.0126
U→E Hazard	0.49	0.47	$\bar{\kappa}_x$	0.0791
E→U Hazard	0.03	0.09	$\bar{\delta}_x$	0.0090
Employment Share	0.86	0.81	$\bar{\sigma}_\epsilon$	0.0583
Non-Employment Share	0.13	0.18	$\bar{f}_t$	0.4947
Returns to Experience (%)	0.75	0.76	$\bar{s}_t$	0.0944
Lifecycle Wages (%)	0.36	0.35	$\bar{f}_t^E$	0.9253
E→U→E Loss (%)	-0.06	-0.06	$\bar{b}$	-0.0236
J→J Gain (%)	0.05	0.05		

Notes: The left side of the table displays targeted average model moments against the data. These moments are described in Section 3.4. The right side of the table displays average parameter estimates of the model presented in Section 3.3. Table 3.A1 in the Appendix displays the full set of parameter estimates. We use the SMM estimation method. Section A.1 in the Appendix describes further details about the estimation procedure.

of estimated parameter values over the lifecycle are presented in Table 3.3, along with the model fit. The complete set of calibrated parameters is presented in Table 3.A1 in the Appendix, and the parameters' lifecycle behaviour is displayed in Figure 3.A2 in the Appendix.

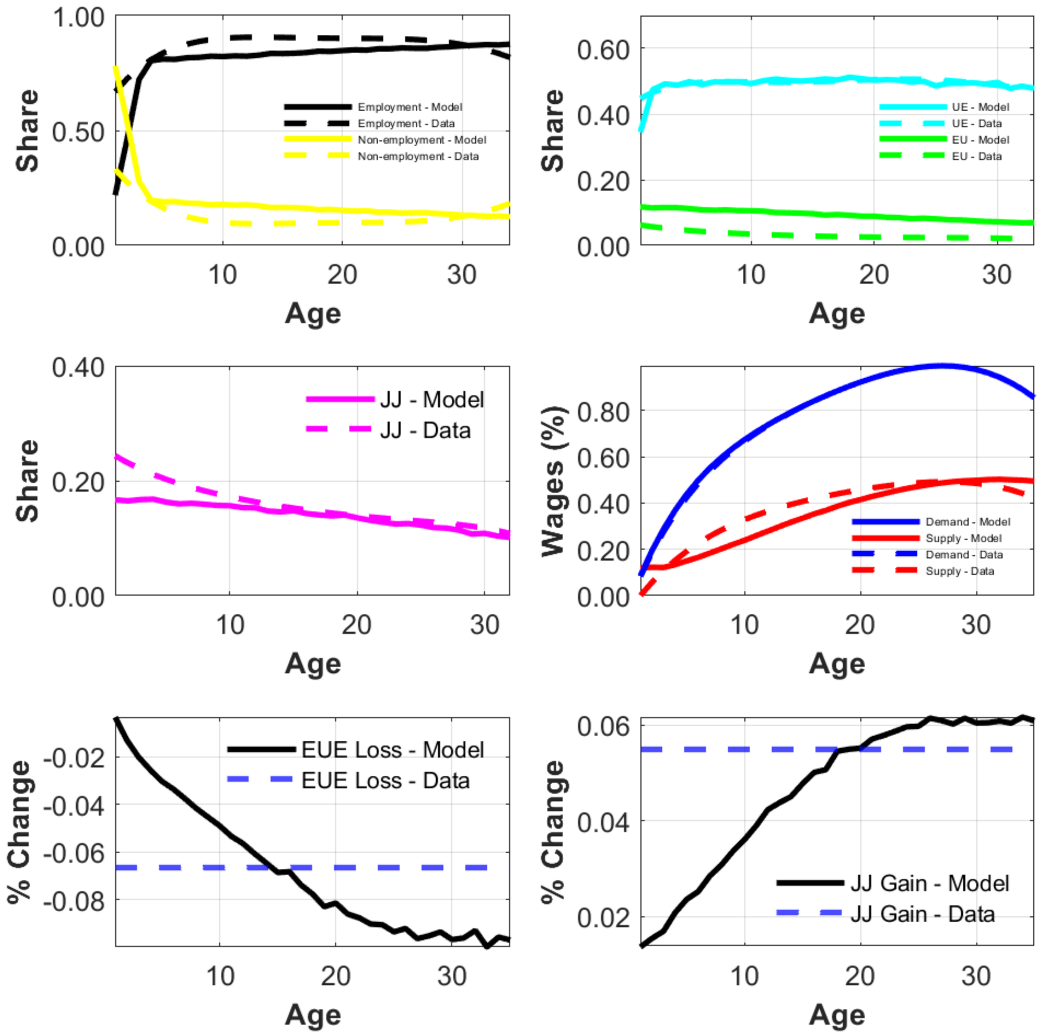
3.4.1 Results

The average parameter of the probability of finding a job from unemployment $\bar{f}_t$, is estimated at 0.4947 and generates a model U→E transition hazard of 47% over the lifecycle. The probability to lose a job, s_t, is estimated to be substantially smaller; the parameters generate an average probability of separation of around 0.0944 over the lifecycle, and it is declining in age. The average probability to upgrade one's human capital over the lifecycle is small, $\bar{\rho}_x = 0.0126$, and has an inverted U-shape. It grows until age 20 in the lifecycle and starts declining thereafter. It never exceeds a 2% monthly probability. By contrast, the probability to lose one unit of human capital from being in unemployment ($\bar{\delta}_x = 0.0090$) is small early in life and increasing throughout the lifecycle, reaching a maximum of around 0.0374 at the end of the working life.

The estimated parameters yield, noticeably, a 100% probability to find a job while being employed (f_t^E) early in life, but the probability declines over age, reaching a minimum of 0.7488 at the end of the working life. In addition, there is an average probability to lose one unit of human capital while experiencing a job-to-job transition of $\bar{\kappa}_x = 0.0791$, which is increasing over age.[20] Finally, the outside option, b_t, is estimated to be decreasing over age. It has a positive

[20]The average fraction of job movers that gain wages when experiencing a J→J transition is 98%. This is because in our model there is a small average probability of $\bar{\kappa}_x = 0.0791$ to suffer a loss of human capital when switching jobs.

value for the first 18 years in the working life, and a negative value thereafter.



Notes: The figure displays model moments fit against the data. The top left panel displays employment and non-employment shares over the lifecycle. The top right panel displays U→E & E→U transition rates over the lifecycle. The bottom middle panel displays J→J transition rates over the lifecycle. The middle right panel displays percentage increase of wages over the lifecycle (supply), and percentage increase of returns to required experience (demand). The bottom left panel displays the model E→U→E wage loss in percentage over the lifecycle (solid line) against the data average (dashed line). The bottom right panel displays the model J→J wage gain in percentage over the lifecycle (solid line) against the data average (dashed line).

Figure 3.2: Model Fit

3.4.2 Model Fit

The model fit can be observed in Table 3.3 and Figure 3.2. The model probabilities, f_t and s_t, generate the E→U and U→E transitions, which are very close to the data.[21] These moments are targeted at estimation. The model also generates the employment and non-employment shares over the lifecycle, close to the data, despite not being a target in the estimation.

Lifecycle earnings profiles, both from the worker side and the ads side, are well matched with our model. The increasing concave profile in these moments is partly obtained by an increasing concave accumulation of human capital (Figure 3.A4 in the Appendix) that is generated both exogenously and endogenously in our model.[22] The bottom left panel of Figure 3.2 shows that we obtain an E→U→E wage loss (%) close to the data on average (around a 6% loss), but our moment is decreasing in age. This is because early in life workers have accumulated little human capital, hence a period of unemployment will not diminish significant human capital from the worker. Late in life it is when workers face the largest risk of losing earnings, as it is the case where they can lose their lifecycle accumulated human capital.

A moment that we also target at estimation is the job-to-job wage gain (%), which in the data is slightly above 5%. Our model is close to the data, particularly late in life. In our model, job-to-job wage gains are only generated from the misallocation between the firm-worker match. Workers endogenously choose to remain in a firm if their earnings, which are affected by the firm-worker match quality shock, are larger than the average draw. Alternatively, they leave a firm if they expect to draw a larger firm-worker match in a new job.

Finally, the model also matches the declining pattern of job-to-job transitions. The middle left panel in Figure 3.2 shows that J→J transitions in the data decline over the lifecycle from 24% at the beginning to 10% at the end.[23] In our model, we generate a somewhat flatter profile from 17% at the beginning to 10% at the end of the lifecycle. Our model generates an average 14% J→J hazard, which is close to the data (13%), despite estimating a high probability to find a job while being employed, $\bar{f}_t^E = 0.9253$. This is because a significant fraction of workers in our model endogenously choose to remain at their current jobs.

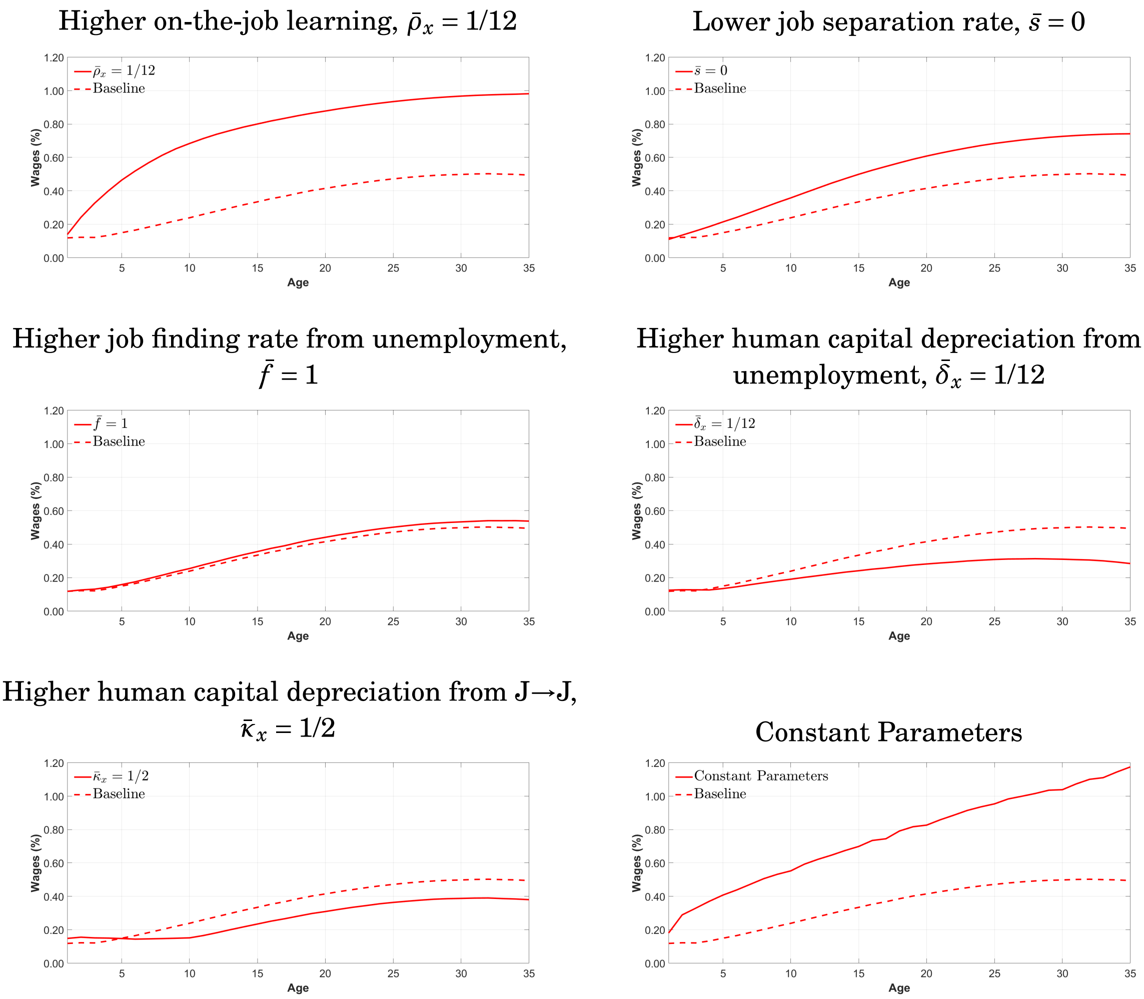
3.5 Counterfactuals

Given that the model performs a reasonable fit across employment transitions and wage growth, in the following we use counterfactual experiments to gain a deeper understanding of the

[21]E→U transitions in Chile are similar to European countries, around 3 percent, and slightly above the US, which is around 1.4 percent. U→E transitions are a bit above the US and European averages (26 and 29 percent, respectively), but close to countries like Denmark and Sweden (42 and 43 percent, respectively). See Ward-Warmedinger and Macchiarelli (2013) and Molloy et al. (2016).

[22]Exogenously as in estimating a concave profile in ρ_x, and endogenously as the worker will choose to avoid unemployment and job-to-job transitions so that human capital does not depreciate.

[23]Chilean J→J transitions are similar to the US, with a 1975-2014 yearly average of 14 percent. See Molloy et al. (2016).



Notes: The figure displays lifecycle wages for different counterfactuals, compared to our baseline estimation. The top left panel displays lifecycle wages for a model with $\bar{\rho}_x = 0.1$ over the lifecycle. The top right panel displays lifecycle wages for a model with $\bar{\delta}_x = 0.1$ over the lifecycle. The middle left panel displays lifecycle wages for a model with $\bar{\kappa}_x = 0.5$ over the lifecycle. The middle right panel displays lifecycle wages for a model with all parameters constant (at their lifecycle average value) over the lifecycle. The bottom left panel displays lifecycle wages for a model with $\bar{s} = 0$ over the lifecycle. The bottom right panel displays lifecycle wages for a model with $\bar{f} = 1$ over the lifecycle.

Figure 3.3: Simulations

contribution of different factors to the mismatch in the worker vs. ads wage growth. More specifically, we perform simulation exercises using our baseline estimated model in order to disentangle the contribution to the workers vs. job ads growth gap that is due to frictions in the "learning" process or standard labour market frictions.

3.5.1 Simulations

We take our calibrated model and input different parameters (one at a time) to the different components of our model in order to see how lifecycle wages respond. We perform a total of six simulation exercises.

First, we increase the average probability of increasing one unit of human capital, ρ_x, to see how higher accumulation of human capital affects wages. In our estimation, ρ_x was estimated at a lifecycle average monthly probability of 0.0126. In annual terms, this would be a probability of 0.15 on average to increase human capital in any given year. In this simulation exercise we increase this probability from 0.0126 to 1/12 so that the annual probability to increase one's human capital is equal to 1. As Figure 3.3 shows, lifecycle wages jump upwards, reaching around twice the wage growth at the end of the lifecycle relative to our estimated model (and the data), specifically 90 percent. Interestingly, the resulting lifecycle profile is similar to the profiles documented for the US and UK in Lagakos et al. (2018).[24] Figure 3.A5 in the Appendix shows that this increase in wages comes through higher human capital accumulation. In our model, wages are directly affected by a monotonic function on human capital, $y(x)$. Hence, the higher the probability to accumulate and increase human capital, the higher lifecycle wage growth.

Second, we decrease the average probability of job separation, s_t, to an extreme to see how labor market frictions affect wages. In our estimation, s_t was estimated at a lifecycle average monthly probability of 0.0944. This is a high separation rate. It implies that jobs are destroyed every 10 months on average. In this simulation exercise we decrease this probability from 0.0944 to 0, so that there is no job destruction whatsoever. As Figure 3.3 shows, lifecycle wages jump upwards, but only from 0.50 in our model to 0.76 at the end of the lifecycle. In this simulation, workers remain at work and, at most, switch from job-to-job, but never experience unemployment (see Figure 3.A6 in the Appendix). Therefore, there is no depreciation in human capital from unemployment and, as workers remain permanently employed, an ongoing probability to increase human capital on the job produces this result. Noticeably, the complete elimination of this type of labour market friction produces a relatively lower effect on wages than the simulation with $\bar{\rho}_x = 1/12$.

Our third simulation also eliminates labour market frictions through the job finding rate from unemployment, f_t. In our estimation, f_t was estimated at a lifecycle average monthly probability of 0.4947. Again, this is a high finding rate. It implies that an unemployed worker finds a job once every two periods in unemployment, on average. In this simulation exercise we increase this probability to 1, so that workers do not remain in unemployment beyond 1 period. Figure 3.3 shows a small lifecycle wage jump upwards. Given that the finding rate was already high at the estimation, the small effect of the simulation is not surprising. Most moments of our model barely change (see Figure 3.A7 in the Appendix). As a consequence, eliminating frictions from job separation, and not job finding, is more important for lifecycle wages in Chile.

Fourth, we increase the average probability of losing one unit of human capital when workers are unemployed, $\delta_{x,t}$, to see how the loss of human capital through unemployment affects wages over the lifecycle. In our estimation, $\delta_{x,t}$ was estimated at a lifecycle average monthly probability

[24] Their panel A of Table 2 reports summary statistics for developed countries. Germany's profile is the steepest, reaching 105 percent by 20-24 years of experience. This is followed by the United States (90 percent), the United Kingdom (85 percent), and Canada (80 percent).

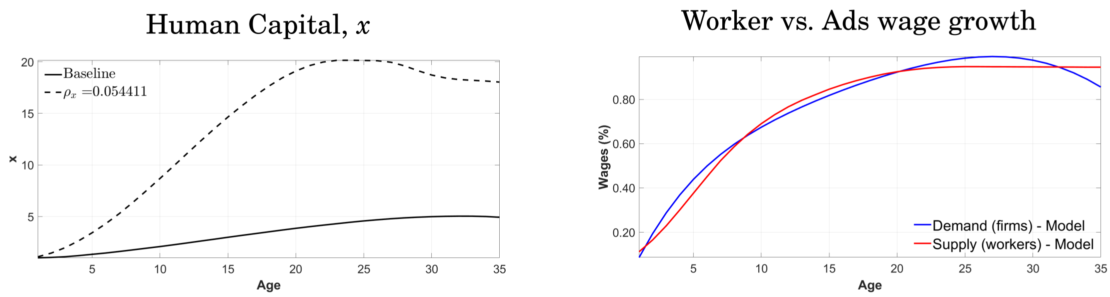
of 0.0090. In this counterfactual exercise we increase this probability to 1/12 so that the annual average probability to decrease one's human capital is equal to 1 if the worker remains the whole year in unemployment. As Figure 3.3 shows, lifecycle wages shift downwards, around half the wage growth at the end of the lifecycle relative to our estimated model (and the data). Figure 3.A8 in the Appendix shows that this decrease in wages comes through lower accumulation of human capital over the lifecycle, since the hazard of unemployment can destroy part of the accumulated human capital. Hence, the higher the chance to lose human capital when unemployed, the lower lifecycle wage growth.

Fifth, we increase the lifecycle average monthly probability of losing one unit of human capital when workers transit from job-to-job, $\kappa_{x,t}$, to see how the loss of human capital through job-to-job affects wages over the lifecycle. In our estimation, $\kappa_{x,t}$ was estimated at a lifecycle average of 0.0791. In this simulation exercise we increase this probability to 1/2 so that one in two J→J transitions decrease a unit of human capital. As Figure 3.3 shows, lifecycle wages shift downwards, above half the wage growth at the end of the lifecycle relative to our estimated model (and the data). Figure 3.A9 in the Appendix shows that this decrease in wages comes from lower human capital accumulation over the lifecycle, but also generates lower Employment shares and J→J transitions early in the lifecycle. Hence, the higher the chance to lose human capital when switching jobs, the lower the lifecycle wage growth. The loss in wage growth from a higher probability to lose human capital through a job-to-job transition, $\kappa_{x,t}$, appears to be substantially lower than the wage loss from a higher probability to lose human capital at the incidence of unemployment, $\delta_{x,t}$. This is because job-to-job transitions are rare, while unemployment is more likely and can persist for a few months.

Finally, in our sixth simulation we use constant, age-unvarying parameters for all the components of our model, to see how the model wages would respond over the lifecycle. In this counterfactual exercise we use constant parameters at their lifecycle average values. As Figure 3.3 shows, lifecycle wages grow linearly, without a concave shape over the lifecycle. Figure 3.A10 in the Appendix shows that this linear behavior is the case for most moments of the model over the lifecycle. Thus, without age varing parameters the model is unable to capture well the nonlinear patterns in the data. We interpret this simulation as an indication that there are lifecycle effects in the accumulation of human capital. In particular, lifecycle effects are important for the shape of wages in the latter parts of the lifecycle.

3.5.2 Counterfactual: closing the worker vs. ads wage growth mismatch

We also want to look at how much of the worker vs. ads wage growth mismatch can be attributed to failure-to-learn. In the data, job ads wage growth is two times the worker wage growth at the end of the lifecycle. We pose the following question: how much can failure-to-learn explain this fact? We address this question by closing the gap between worker vs. ads wage growth over the lifecycle. Failure-to-learn in our model corresponds to the probability to increase human capital,



Notes: The left panel displays human capital accumulation, x, over the lifecycle for our baseline model (solid line) and the counterfactual exercise ($\rho_x = 0.0544$) where worker and ads wage growth are equal (dashed line). The right panel displays worker (red) vs. ads (blue) wage growth for this counterfactual exercise ($\rho_x = 0.0544$).

Figure 3.4: Counterfactual: closing the worker vs. ads wage growth mismatch

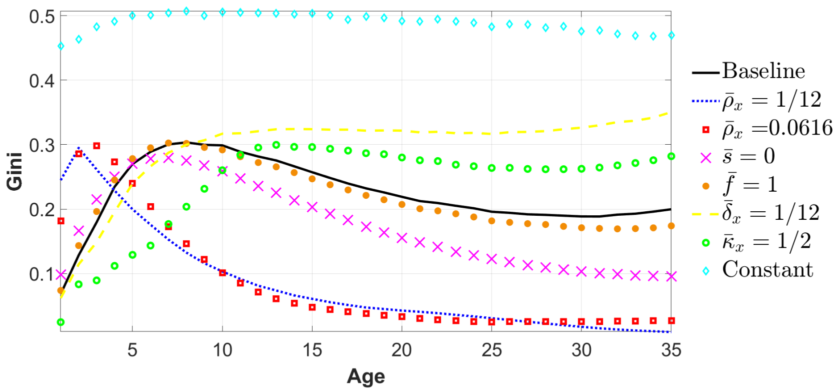
$\rho_{x,t}$. The estimation of our model yielded an average 0.0126 monthly probability to increase human capital over the lifecycle in our model. Thus, we keep all parameters from the baseline estimation fixed and change the parameters pertaining to the accumulation of human capital, $\rho_{x,t}$, such that lifecycle wages from the worker side match the increase of wages from the demand side (job ads).

This exercise yields a lifecycle average monthly probability of $\bar{\rho}_x = 0.0616$, around four times the probability to increase one's human capital in our baseline estimation, but slightly below 1/12. The counterfactual exercise matches supply (workers) and demand (firms) wage growth over the lifecycle as can be seen on the right panel of Figure 3.4. Strikingly, human capital accumulation grows much more faster in this model relative to our baseline. The left panel of Figure 3.4 shows that human capital accumulation in this model reaches a value of slightly above 20 at the end of the lifecycle. Again, four times higher relative to our baseline estimation. The rest of the moments of this counterfactual model remain barely unchanged (see figure 3.A11 in the Appendix). Hence, failure-to-learn, 4 times below the level where worker vs. ads wage growth matches can explain this gap.

Both the simulation (in the previous subsection) and the counterfactual exercises show that on-the-job learning, human capital depreciation and labour market frictions affect wages. What is less known is how inequality of earnings is affected by the process of accumulation of human capital. The amount of inequality of lifetime earnings is important as it has implications for welfare.[25] Our model allows us to observe the distribution of workers' earnings throughout the lifecycle. We are now interested in looking at the Gini coefficients of each simulated (and counterfactual) economy.

Figure 3.5 displays such coefficients. Our baseline estimation (black solid line) generates an increase of inequality during the first 5-7 years starting from a coefficient of 0.07 to a coefficient of 0.30. After the first 5-7 years, inequality starts declining throughout the lifecycle, reaching a final value of 0.20. In our model, inequality increases during the first years since all workers start

[25] The question of welfare costs was first addressed by Attanasio and Davis (1996).



Notes: The figure displays gini coefficients for workers' wages over the lifecycle for each simulated (counterfactual) economy. The black solid line (—) displays the gini coefficients for our baseline estimation. The blue dotted line (····) displays the gini coefficients for the economy where $\rho_x = 1/12$. The yellow dashed line (- - -) displays the gini coefficients for the economy where $\bar{\delta}_x = 1/12$. The green circles (O) display the gini coefficients for the economy where $\bar{\kappa}_x = 1/2$. The cyan diamonds (◇) display the gini coefficients for the economy where parameters are constant (equal to their lifecycle average). The magenta crosses (× display the gini coefficients for the economy where $\bar{s} = 0$. The orange circles (●) display the gini coefficients for the economy where $\bar{f} = 1$. The red squares (□) display the gini coefficients for the economy where $\bar{\rho}_x = 0544$ (no gap between worker vs. ads wage growth).

Figure 3.5: Lifecycle Gini coefficients for each Economy

with the same level of human capital, and their only heterogeneity comes from the firm-match idiosyncratic component, ϵ_i. There are no other sources of observed or unobserved heterogeneity.[26] Early in life, the probability to increase human capital is low, so a few lucky workers increase their human capital and hence increase their earnings, while most workers remain with the initial human capital. After year 7, and when the probability to increase human capital becomes larger and larger, most workers start catching-up increasing their human capital, and hence inequality starts to decrease.

The simulated economy with $\bar{\rho}_x = 1/12$ (blue dotted line) has a larger probability to accumulate human capital. This generates larger accumulation of human capital and larger wage growth for workers over the lifecycle. As Figure 3.5 shows, after the initial increase in earnings inequality after 2 years, the gini coefficients start declining over the lifecycle. In this economy, human capital grows the fastest for all workers and, as a consequence, earnings inequality is the lowest at the end of the lifecycle. Also, the economy with the counterfactual exercise where the gap between workers and ads wage growth is closed, $\bar{\rho}_x = 0.0616$ (red squares), displays a similar pattern. This shows that the required level of human capital to meet demand would produce much lower labour earnings inequality throughout the lifecycle. The cross-sectional average of the gini coefficients in our baseline equals 0.22, while in the counterfactual this number is 0.0838, around 3 times lower.

We can also comment on the inequality generated by the economies without labour market frictions. We showed in Section 3.5.1 that the probability of finding a job from unemployment was

[26]See Huggett et al. (2011) for the importance of preexisting initial conditions for earnings and consumption heterogeneity over the lifecycle. Also, Blundell et al. (2015) show the importance of accounting for unobserved heterogeneity by education over the lifecycle.

high in the estimation of our model, hence eliminating this type of friction ($\bar{f} = 1$, ●) generates little changes. Figure 3.5 shows that there are little differences with respect to our baseline estimation. What is in stark contrast is the frictions generated by job separation. When we eliminate job separation ($\bar{s} = 0$, magenta crosses) inequality is reduced. In our model, there is depreciation of human capital when unemployed, besides the opportunity cost of not learning on the job. Still, when this friction is completely eliminated, the decrease of inequality is not as large as in the economy where the probability to augment human capital is slightly larger ($\bar{\rho}_x = 1/12$ or $\bar{\rho}_x = 0.0616$).

The simulated economies where we increase the probability to lose human capital through either unemployment or J→J ($\delta_x = 1/12$, (yellow dashed line), and $\kappa_x = 1/2$, (green circles), respectively) display higher earnings inequality throughout the lifecycle with respect to our baseline. This shows that the loss of human capital by any of these incidences creates higher earnings inequality.

The simulated economy that has parameters constant at their average lifecycle value (cyan diamonds) produces gini coefficients over the lifecycle that are somewhat constant. This simulated economy has the highest inequality of all the economies that we have displayed. The reason is that in our estimated baseline model the lifecycle effect of $\delta_{x,t}$ and $\kappa_{x,t}$ is present late in life and it is almost nonexistent early in the lifecycle. In this constant economy the parameter values are taken to their lifecycle average, meaning that human capital loss can occur equally at any point during the lifecycle. Hence, the constant pattern and the larger earnings inequality.

To sum up, we estimate human capital in our baseline Chilean economy to be 4 times below the level where supply meets demand, generating around 3 times higher labour earnings inequality. Whereas labour market frictions are not capable to close the wage vs. job ads growth gap alone, the on-the-job learning component is a much more important component.

3.6 Conclusion

In this paper, we use a novel dataset that allows us to estimate effective returns to experience. We observe that returns to experience is significantly larger than wage profiles for workers over the lifecycle. We propose a standard structural labour supply model to start thinking about what type of labour market frictions can explain the observed returns vs. wage profiles gap.

The estimation and the simulation exercises that we perform indicate that standard labour market frictions, such as the finding rate and the job separation rate, have limited capacity in explaining the difference between returns and wage profiles. In our model, it is the failure to upgrade one's human capital while remaining at work that has greater importance in explaining this gap. A counterfactual exercise shows that an improvement in the component of on-the-job learning would close this gap, not only increasing average wage profiles, but also decreasing labour market earnings inequality. Our results, namely that lifecycle wage heterogeneity is due mostly as a failure-to-learn, is related to Lagakos et al. (2018) and suggests that human capital accumulation stories are the ones to be looked at.

3.7 Appendix

A.1 Estimation

We simulate lifecycle employment histories for 5,000 workers that enter the labour market and remain in the market for 35 years. The minimum distance estimator that we use is given by:

$$\hat{\theta} = \underset{\theta}{argmin} \boldsymbol{F}(\theta)' \boldsymbol{I} \boldsymbol{F}(\theta) \tag{A.3}$$

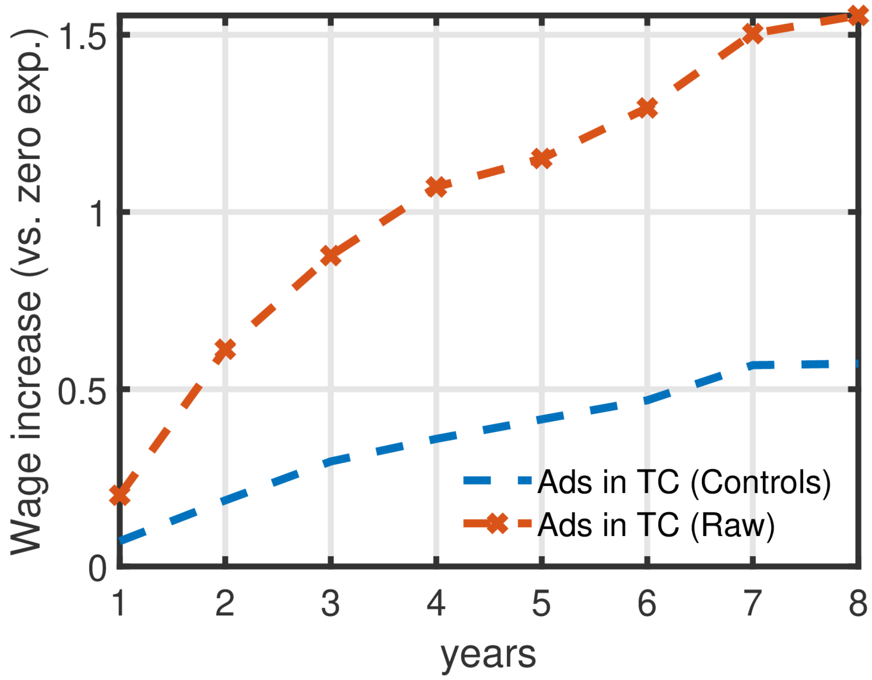
$$\boldsymbol{F}(\theta)_n = \frac{f_n(\theta) - m_n}{\omega_n}, \tag{A.4}$$

where $f_n(\theta)$ is the n^{th} model moment, and m_n is the corresponding n^{th} data moment. Similar to Sanchez and Wellschmied (2020) and Guvenen et al. (2015), we employ moment specific adjustment factors, ω_n. We use these adjustment factors to jointly deal with two issues presented by the data. First, the moments are measured on different scales. For example, employment share (%) is in absolute value about 30 times larger than the E→U hazard. If we had minimized the sum of absolute squared deviations ($\omega_n = 1$), the optimization would not have had put any emphasis on moments with low absolute sizes. At the same time, we have several moments which are close to zero, such as the E→U→E wage loss (%) or the J→J wage gain (%), but fluctuate substantially in relative terms from one age to the next. Hence, if we had minimized the sum of relative squared deviations ($\omega_n = abs(m_n)$), the optimization would have concentrated almost exclusively on these large relative deviations close to zero.

Using moment specific adjustment factors allows us to use absolute deviations but reduce the emphasis on moments with large absolute numbers. Unfortunately, it gives us a degree of discretion. We choose the adjustment factors in an iterative fashion such that the implied loss function is consistent with the model fit we observe in Figures 3.2 and 3.A4.

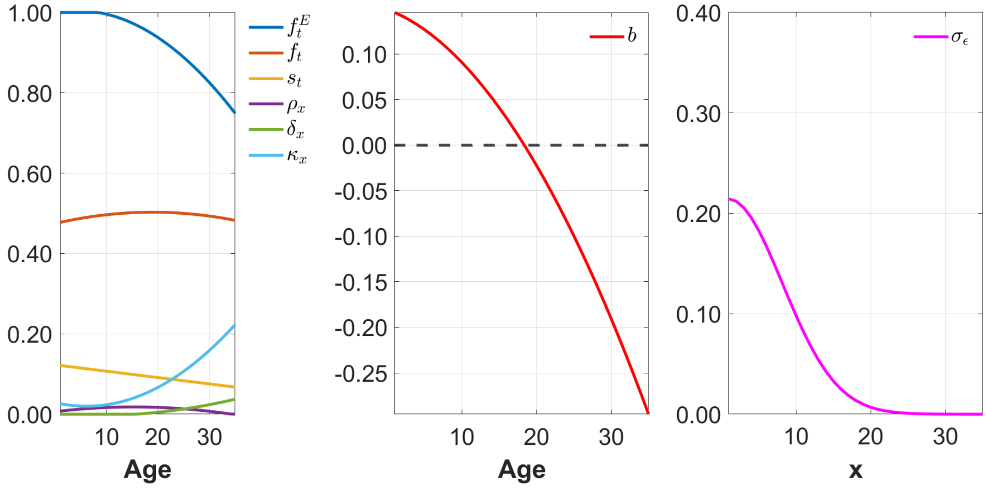
At the estimation, we first obtain reasonable starting values by experimenting with different combinations of parameters. We tested different global minimum algorithms and a pattern search algorithm performed best in finding a minimum. Provided the optimal parameters, we compare the minimum to (possibly) other minima where we start the algorithm from different starting points. We find that the pattern search algorithm, in general, is able to converge to the same minimum from different starting points.

A.2 Additional Figures



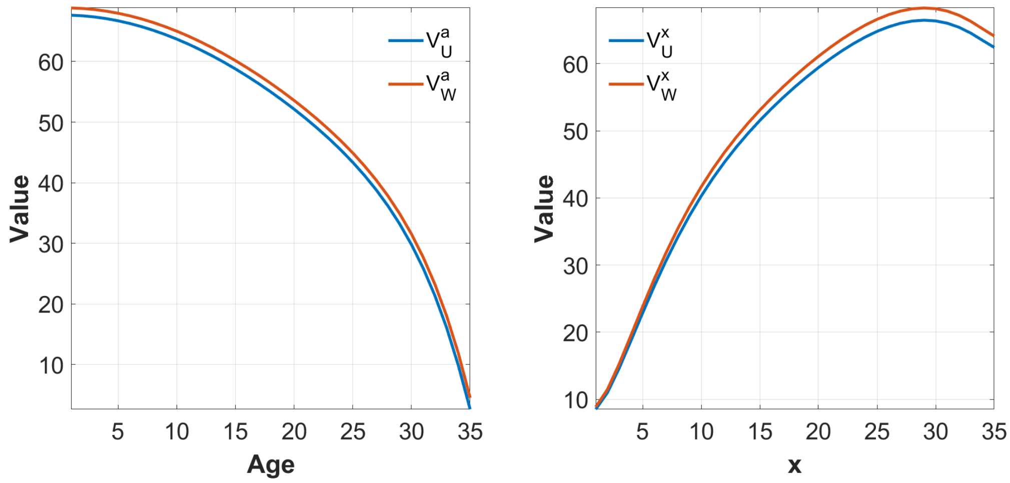
Notes: The red dashed line displays the ads' average posted wages increase (%) for the first 8 years of required experience from the raw TC data. The blue dash line displays the ads' residual average posted wages increase (%) after controlling by firms and job title fixed effects.

Figure 3.A1: Wage Controls



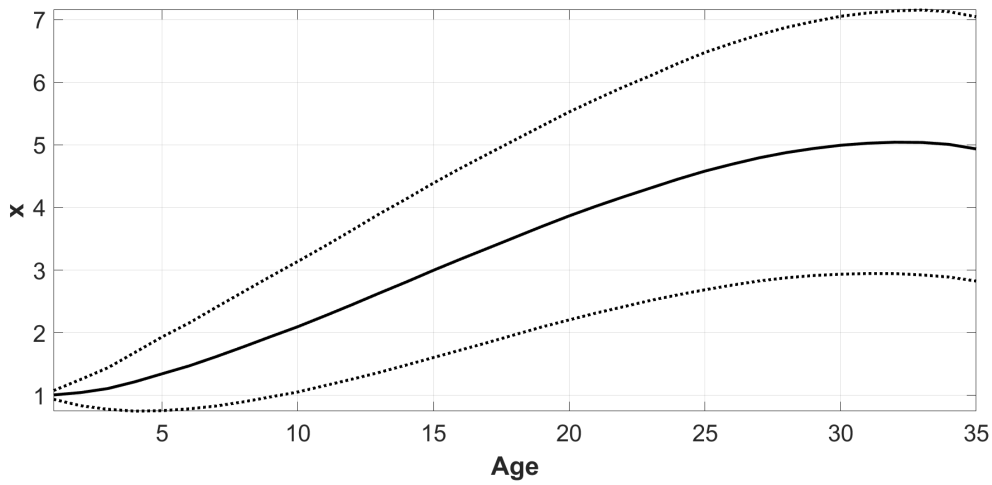
Notes: The figure displays model features from our baseline estimation.

Figure 3.A2: Model Features



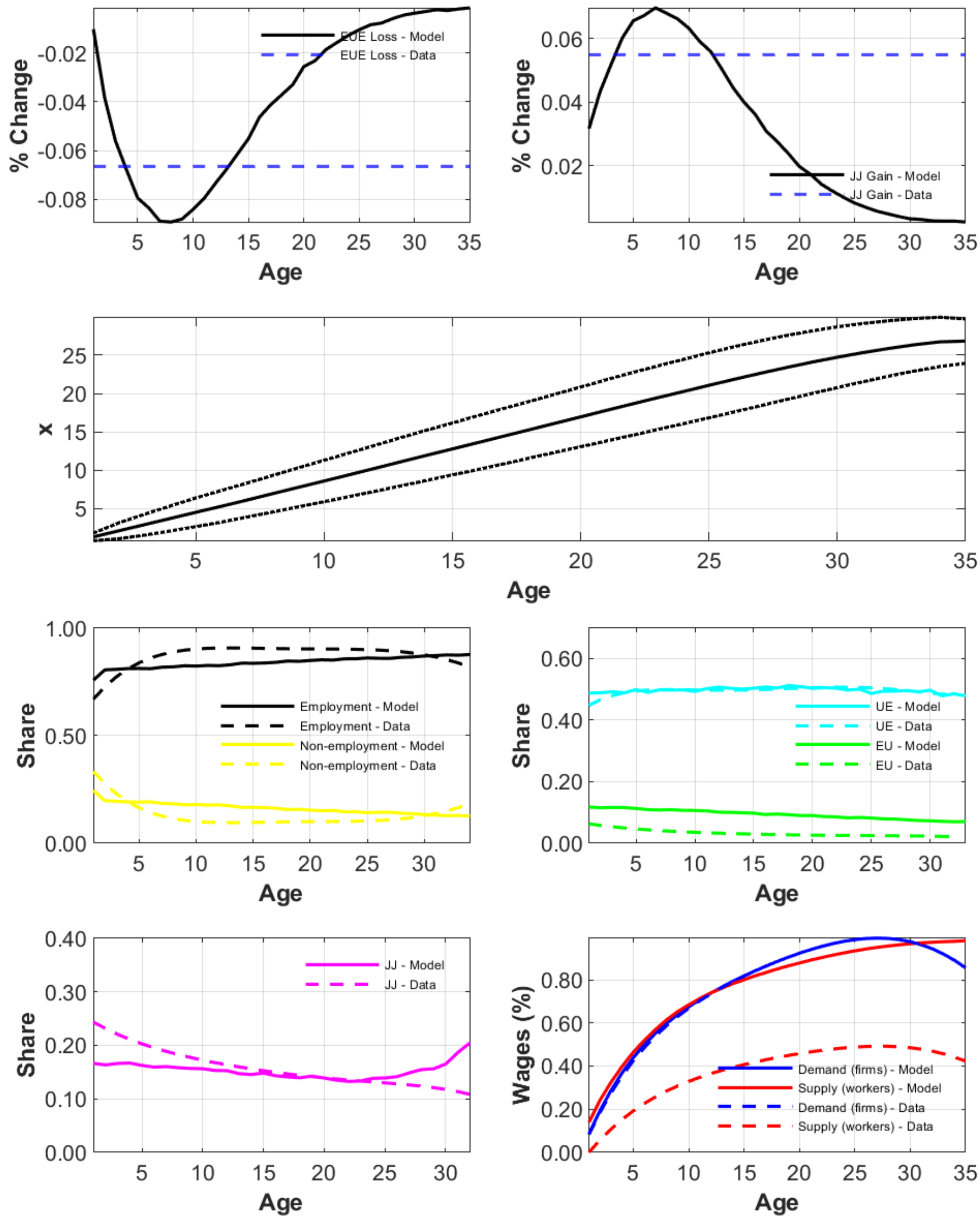
Notes: The figure displays the model's value functions for unemployment and employment over age (left panel) and human capital (right panel) for our baseline estimation.

Figure 3.A3: Value Functions



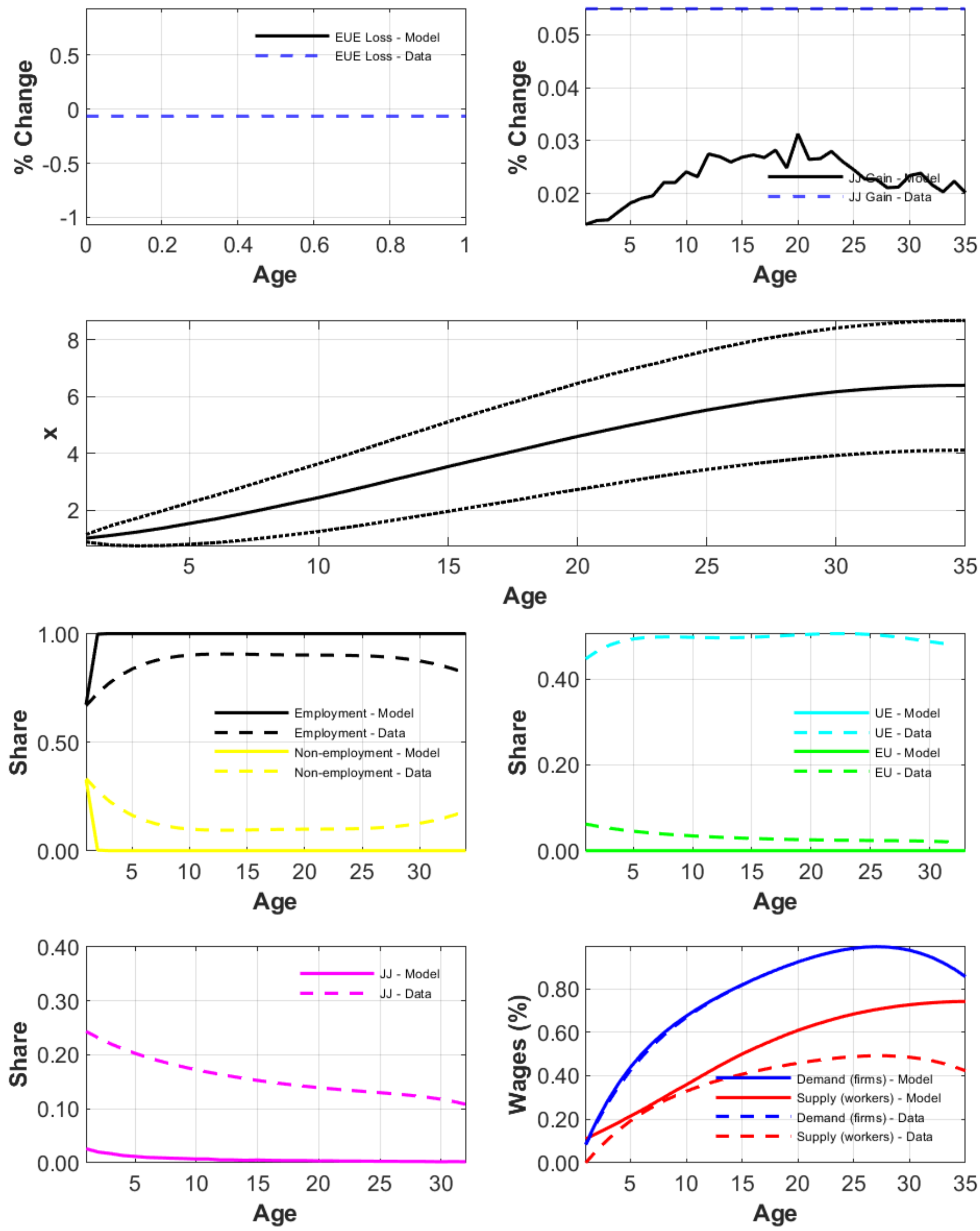
Notes: The bottom panel displays the average human capital (x) over the lifecycle (solid line) and 2 std. deviations (dashed lines).

Figure 3.A4: Human capital over the lifecycle



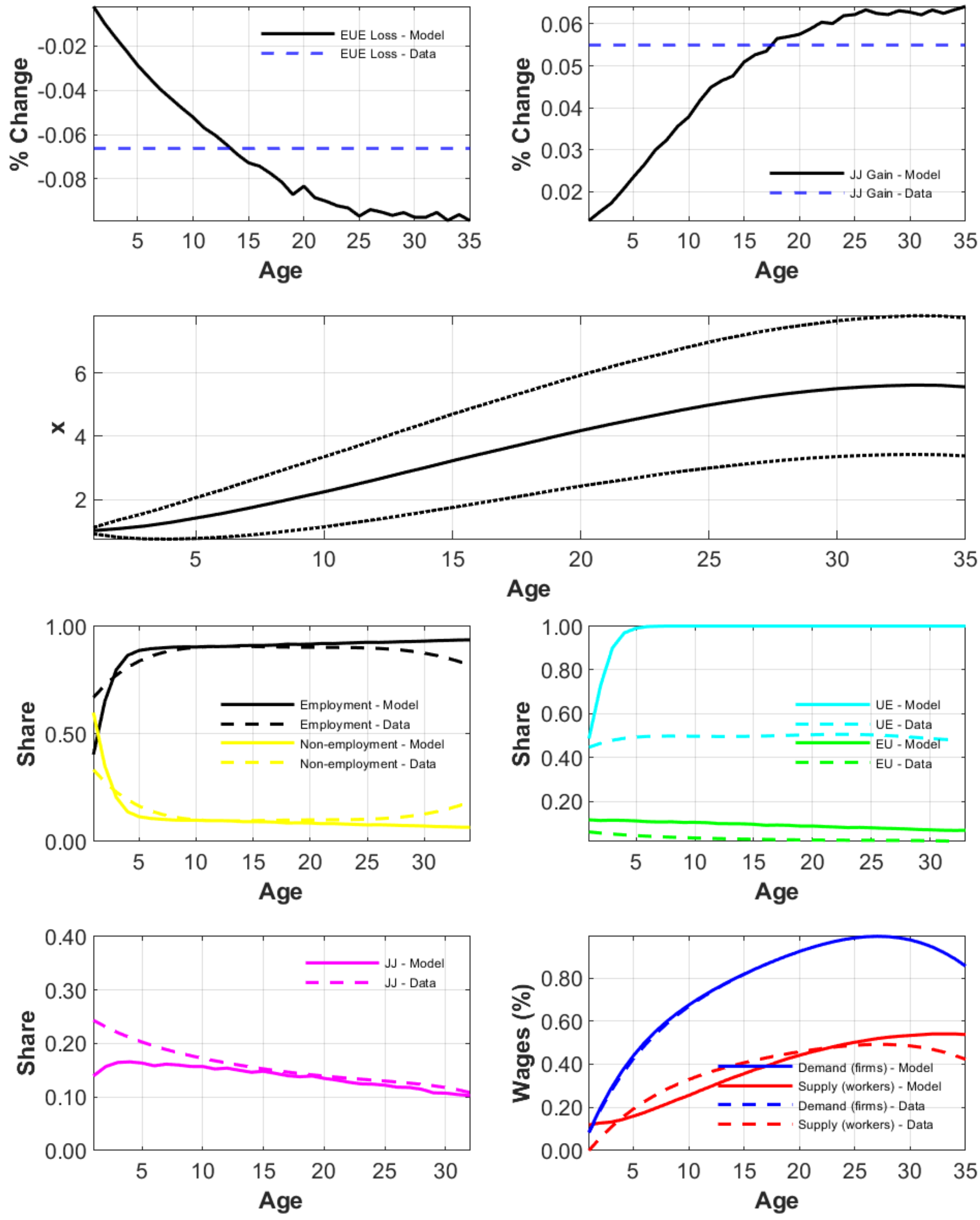
Notes: The figure displays the simulation ($\bar{\rho}_x = 0.10$) model response against the data.

Figure 3.A5: Higher on-the-job learning, $\bar{\rho}_x = 1/12$



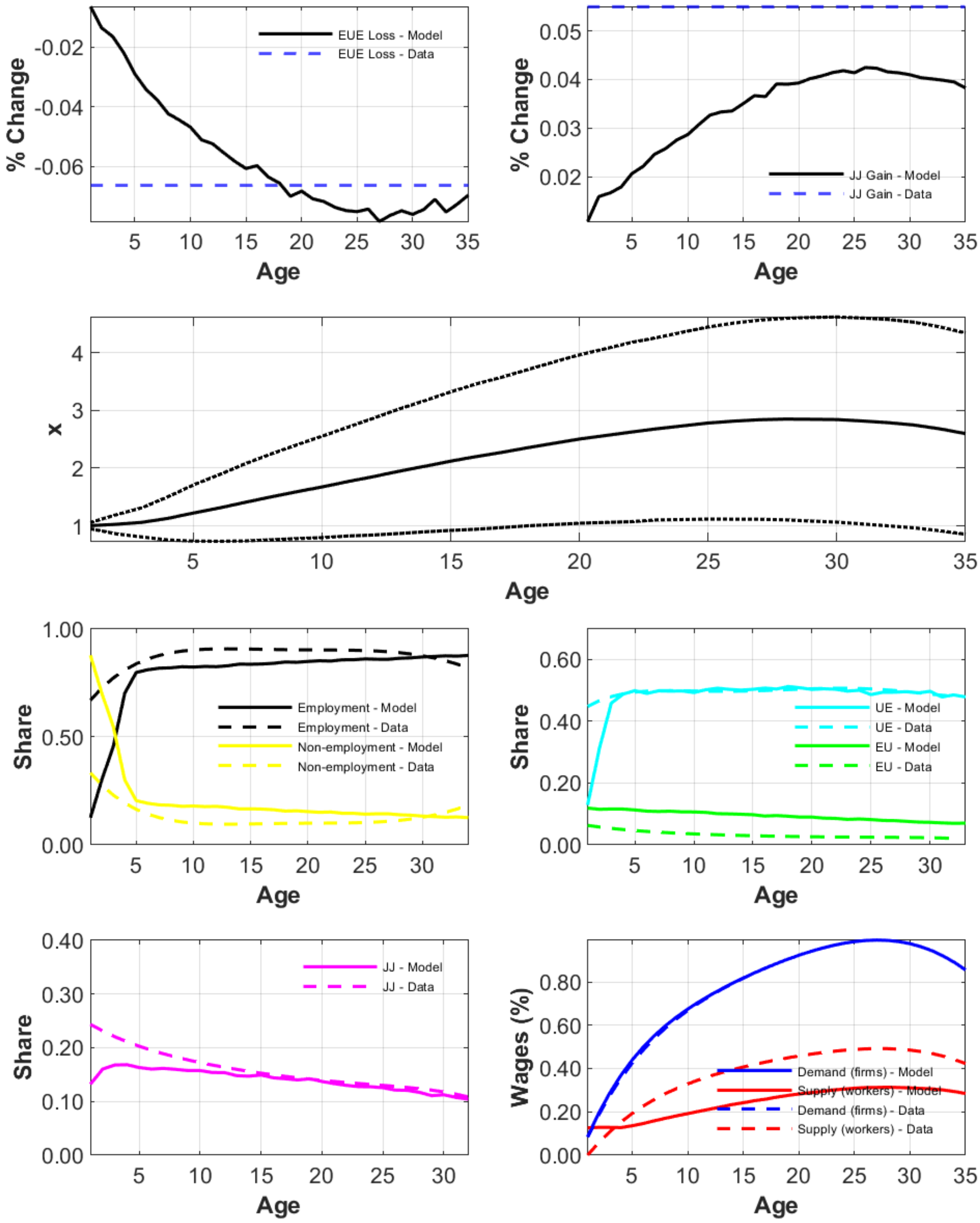
Notes: The figure displays the simulation ($\bar{s} = 0.00$) model response against the data.

Figure 3.A6: Lower job separation rate, $\bar{s} = 0.00$



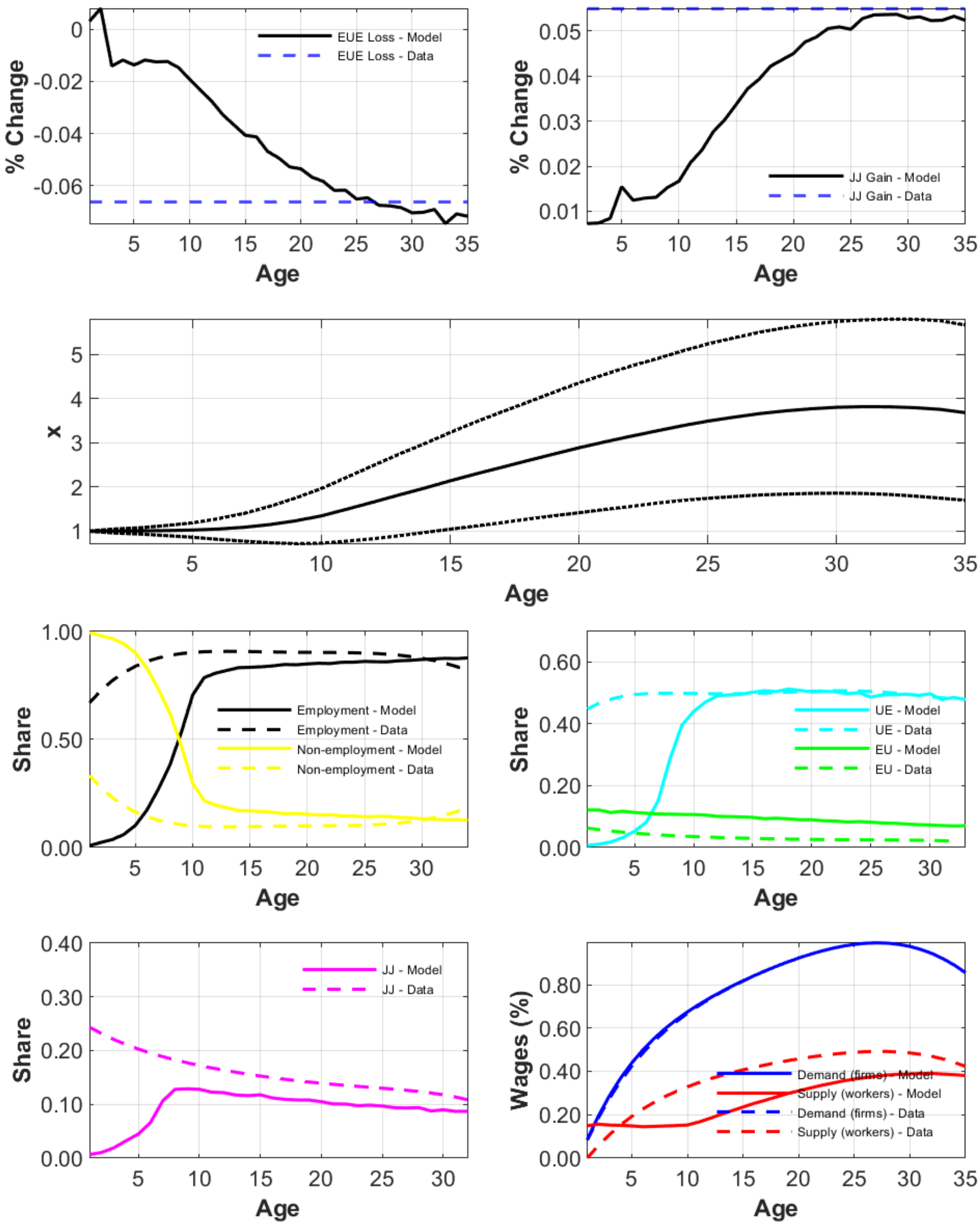
Notes: The figure displays the simulation ($\bar{f} = 1.00$) model response against the data.

Figure 3.A7: Higher job finding rate from unemployment, $\bar{f} = 1.00$



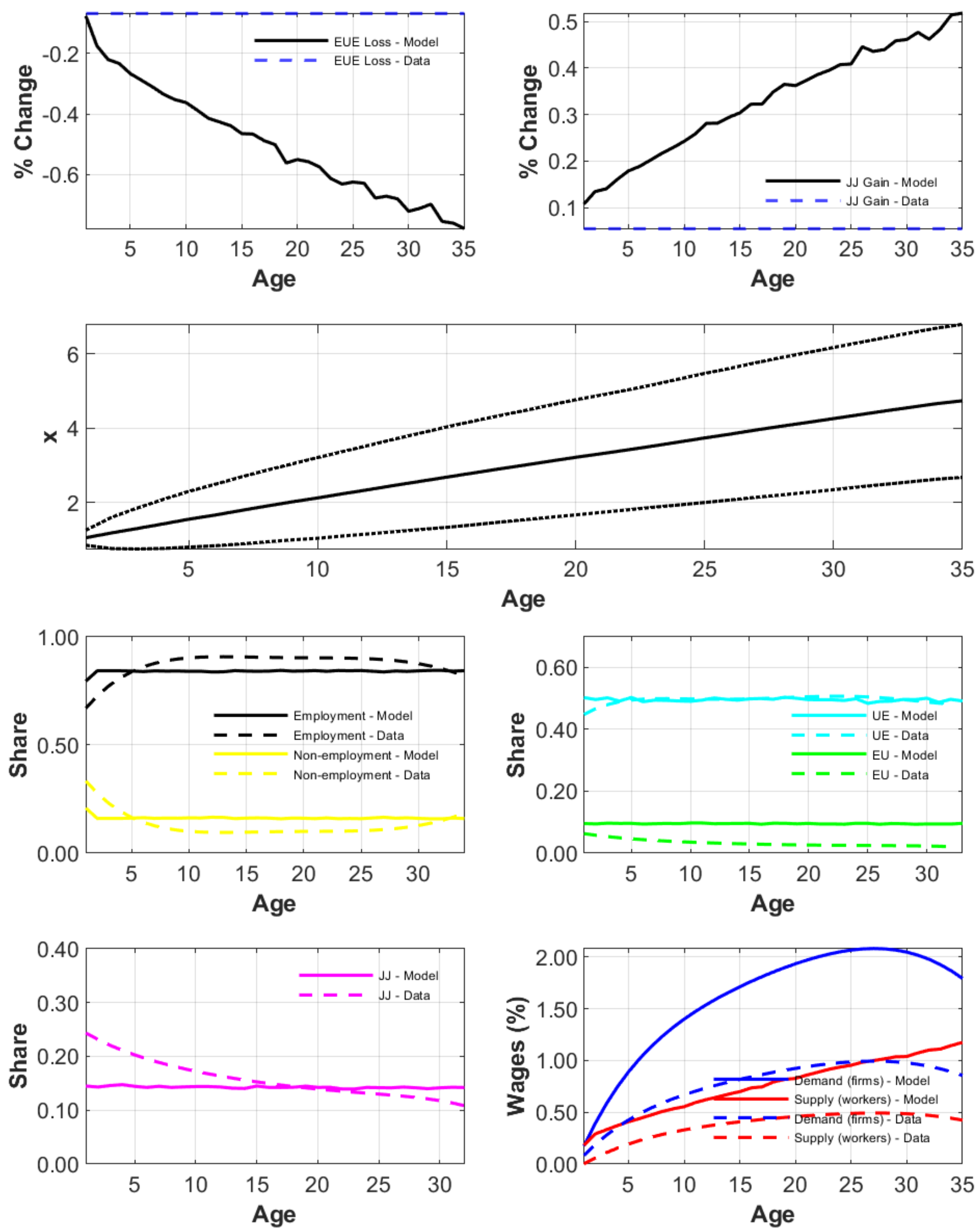
Notes: The figure displays the simulation ($\bar{\delta}_x = 0.10$) model response against the data.

Figure 3.A8: Higher human capital depreciation from unemployment, $\bar{\delta}_x = 1/12$



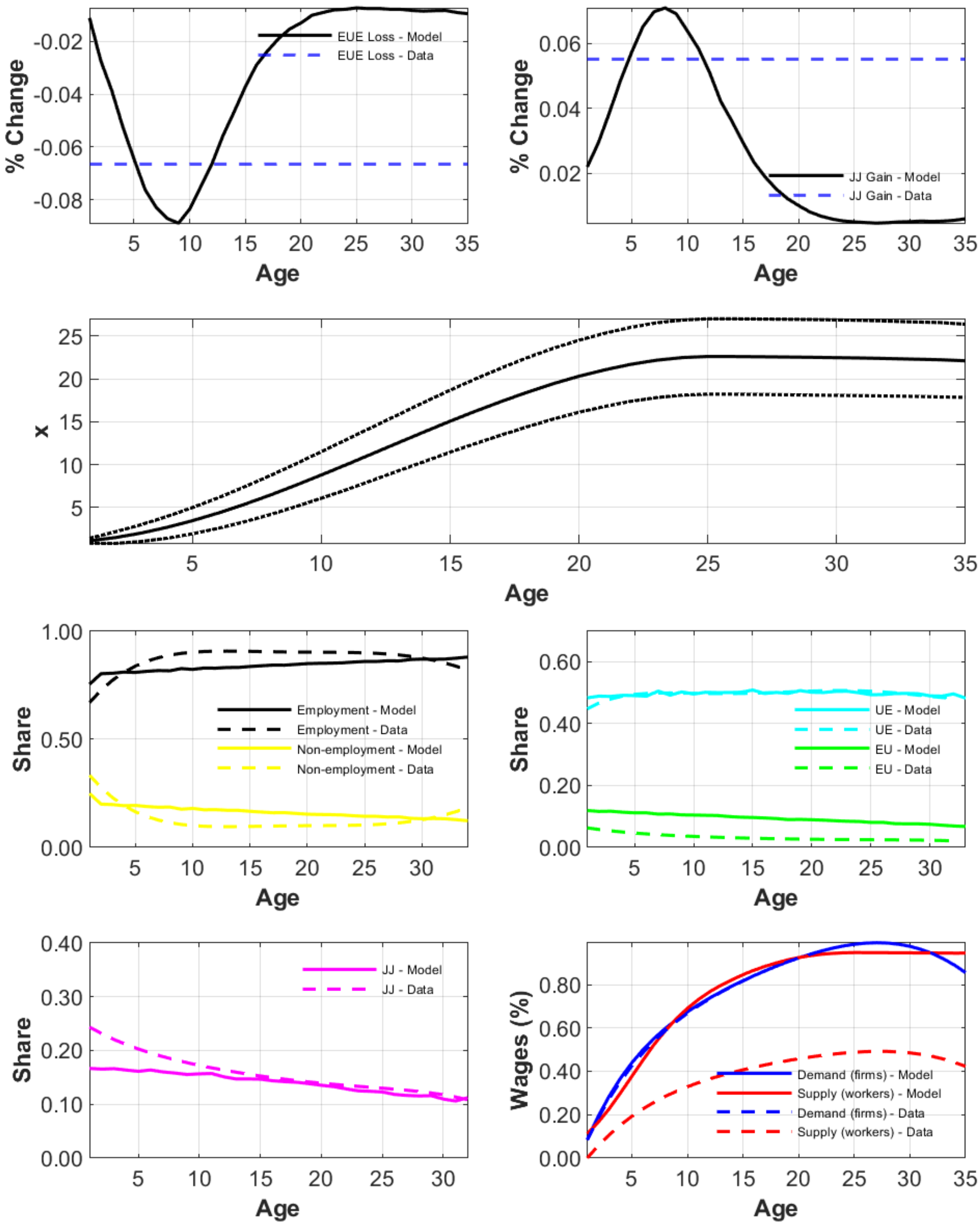
Notes: The figure displays the simulation ($\bar{\kappa}_x = 0.50$) model response against the data.

Figure 3.A9: Higher human capital depreciation from J→J, $\bar{\kappa}_x = 0.50$



Notes: The figure displays the simulation (constant parameters at their lifecycle average values) model response against the data.

Figure 3.A10: Age-unvarying Parameters



Notes: The figure displays the counterfactual ($\rho_x = 0.0544$) of closing the worker vs. ads wage growth gap.

Figure 3.A11: Counterfactual: closing the worker vs. ads wage growth gap

A.3 Additional Tables

Table 3.A1: Parameter Estimates

$\rho_{x,1}$	0.0062	f_1	0.4744
$\rho_{x,2}$	0.0015	f_2	0.0030
$\rho_{x,3}$	−0.0001	f_3	−0.0001
$\kappa_{x,1}$	0.0291	s_1	0.1229
$\kappa_{x,2}$	−0.0030	s_2	−0.0016
$\kappa_{x,3}$	0.0002	s_3	0
$\delta_{x,1}$	0	f_1^E	1.0000
$\delta_{x,2}$	−0.0009	f_2^E	0.0023
$\delta_{x,3}$	0.0001	f_3^E	−0.0003
$\sigma_{\epsilon,1}$	−3.0938	b_1	0.1487
$\sigma_{\epsilon,2}$	0.0330	b_2	−0.0031
$\sigma_{\epsilon,3}$	−0.0188	b_3	−0.0003

Notes: The table displays parameter estimates of the model presented in Section 3.3. We use the SMM estimation method.

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