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Supply Chain Finance Models and Applications

Copula-based Approaches

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Supply Chain Finance Models and Applications

Copula-based Approaches

By

BANGDONG ZHI

School of Management
UNIVERSITY OF BRISTOL

A dissertation submitted to the University of Bristol in accordance with the requirements of the degree of DOCTOR OF PHILOSOPHY in the Faculty of Social Science and Law.

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Abstract

Supply chain finance (SCF) seeks to integrate physical, financial, and information flows along the supply chain. Among SCF practices, inventory financing has played a vital role in helping many capital-constrained businesses survive the COVID-19 crisis through alternative financing schemes and supporting the post-COVID economic recovery. However, inventory financing is particularly risky in a volatile market environment, as fluctuating collateral prices increase the default risk. A proper impawn rate (loan-to-value ratio), interest rate, and collateral portfolio can help an inventory financing provider (IFP) effectively manage the risk from inventory financing and be competitive in the financial market. However, prior studies seldom explore how an IFP improves the performance of inventory financing by managing the impawn, interest rates, and collateral portfolio. This doctoral research explores the use of data-driven copula models to determine these three factors. The analytical results show that a copula model can depict the dependence among a series of collateral prices, and its forecasting performance is ideal for determining impawn, interest rates, and the weights of a collateral portfolio. This research contributes to the SCF literature by presenting an innovative data-driven approach to manage risk in inventory financing. Integrating a predictive analytical approach (i.e., data-driven copula model to predict risk) and a prescriptive analytical model (i.e., impawn and interest rate model) to address a contemporary issue such as determining the appropriate impawn rates and interest rates for inventory financing contributes to the emerging business analytics field. This doctoral research presents innovative approaches that can be used by IFPs to control the default risk inherent in inventory financing and gain competitive advantage in financial markets.

Dedication and Acknowledgments

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Author's declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

SIGNED: .... DATE: ....

Statement of Contribution

The contents below were published at *International Journal of Production Economics*.

- The second paragraph in Section 1.5 in Chapter 1 "Introduction"
- The first paragraph in Section 2.3 in Chapter 2 "Literature Review"
- Section 2.4 in Chapter 2 "Literature Review"
- Chapter 3 "Impawn Rate Optimization in Inventory Financing: A Canonical Vine Copula-based Approach"
- Appendix B "Proofs of Chapter 3"
- The second paragraph in Section 6.2 in Chapter 6 "Conclusions and Future Research"

Zhi, B., Wang, X. and Xu, F. 2020, Impawn Rate Optimisation in Inventory Financing: A Canonical Vine Copula-based Approach, *International Journal of Production Economics*, 227, 1-14.

The statement of contributions of the authors is as follows and this declaration is jointly authorized by the signature of the parties below:

Name of the Author	**Contributions**	**Signature**
Bangdong Zhi	Conceptualization, data curation, formal analysis, investigation, methodology, resources, software, validation, visualization, writing original draft, and revision.	[redacted]
Xiaojun Wang	Supervision and editing	[redacted]
Fangming Xu	Supervision and editing	[redacted]

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- The fourth paragraph in Section 1.5 in Chapter 1 "Introduction"
- The second and third paragraphs in Section 2.3 in Chapter 2 "Literature Review"
- Section 2.6 in Chapter 2 "Literature Review"
- Chapter 5 "Portfolio Optimization for Inventory Financing: Copula-based Approaches"
- Appendix D "Proofs of Chapter 5"
- The fourth paragraph in Section 6.2 in Chapter 6 "Conclusions and Future Research"

Zhi, B., Wang, X. and Xu, F. 2021, Portfolio Optimization for Inventory Financing: Copula-based Approaches, *Computers & Operations Research*, 136, 1-11.

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Bangdong Zhi	Conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing original draft, review, revision and visualization	[redacted]
Xiaojun Wang	Supervision and editing	[redacted]
Fangming Xu	Supervision and editing	[redacted]

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List of Abbreviations

The list below describes various abbreviations and acronyms used throughout the thesis. The page on which each abbreviation or acronym is first used or defined is also provided.

Abbreviation	**Meaning**
SCF	Supply Chain Finance
IFP	Inventory financing provider
LME	London Metal Exchange
MVN	Multivariate normal distribution
IIRM	Impawn and interest rate model
VaR	Value-at-Risk
CVaR	Conditional Value-at-Risk

Chapter 1

Introduction

1.1 Background

Since the 2008 financial crisis, small and medium-sized enterprises (SMEs) in supply chains have been more capital constrained, and they have found it hard to get financial support from the financial institution. This has motivated supply chain members to decrease debt ratios, cut interest expenses, and optimize their working capital by adopting various supply chain finance (SCF) instruments (Jia et al. 2020). Typical instruments such as inventory financing, trade financing (Schäfer & Baumann 2014) and factoring have been adopted to reduce the default risk and simplify the process of meeting financing demand (Liebl et al. 2014). Adopting these SCF instruments has not only improved cash-flow and working capital in the supply chain, but it has also integrated the financial, information, and product flows in the supply chain to a greater degree (Wuttke et al. 2013). Among these SCF practices, inventory financing is a popular option for SMEs. Inventory financing can be seen as asset-based lending that allows borrowers to get a revolving line of credit by leveraging their inventory. Such financing is easier to get than more conventional financing services, which often require credit records and fixed assets to guarantee loans. In addition to the traditional financial institutions, such as Bank of China and Barclays, other players like third-party logistics providers are involved in this growing market due to the increasing demand for inventory financing. For example, UPS, the logistics service provider in the U.S., cultivates the inventory financing market by founding financial services unit. This helps their clients purchase new inventory more easily to fill increased orders (UPS 2021). SF Express also provides inventory financing services to retailers, through which retailers with capital constraints pay the supplier in advance[1].

[1] However, products are controlled by SF Express until the retailer sells them and repays the loan with an interest.

For inventory financing providers (IFPs), the impawn rate (also named loan-to-value ratio or "haircut" in the literature on finance and economics) is an important index for default risk control because pledged inventory at the end of the funding period could be below its value at the beginning (Zhi et al. 2020). The impawn rate is the ratio between the loan ascribed to the collateral and the market price of the collateral. It can also be treated as a nominal reduction to the value of an asset when it is used as collateral[2]. Based on the same amount of collateral, the borrower can get more money with a higher impawn rate. Customers want IFPs to provide higher impawn rates, although that increases the default risk. Setting a lower impawn rate can reduce the probability of default, but the inventory financing could be less attractive and the IFP could lose its customers. Therefore, setting an appropriate impawn rate that helps the IFP manage default risk and attract customers is essential for maintaining or improving the performance of its inventory financing business.

The interest rate in inventory financing directly affects the return. It is a percentage charged on the amount borrowed or lent or on the deposit due per period. The total interest paid by the borrower is based on four elements: the interest rate, compounding frequency, principal amount, and the length of the funding period. Although a higher interest rate can help the IFP increase its profit margin, this might make the IFP lose customers. A lower interest rate can help the IFP increase the loyalty of its customers, but it will decrease its profit margin. Therefore, for the IFP, it is also essential to set an appropriate interest rate that simultaneously guarantees the loyalty of customers and the profit margin of its inventory financing service.

Another prudent strategy to mitigate the risk of adverse collateral price movement is to create a balanced portfolio of collateral units that expose the IFP to fewer extreme losses at one time. Focusing too much on specific collateral units can expose it to serious default risk. To illustrate, in the two months between 28/02/2015 and 30/04/2015, the price of zinc increased almost 13.56%. In contrast, the price of tin dropped approximately 10.72% during the same period (LME 2021). In this case, if the IFP allocates an extremely high weight to tin in inventory financing, it will face serious default risk as its customers who use tin as collateral may be unwilling to pay off their debt at the maturity date. However, if the IFP can determine the appropriate weights of both tin and zinc, the risk of adverse price movement would be reduced because the prices of tin and zinc do not move in concert. Thus, allocating appropriate weights to collaterals that have low/inverse correlations can improve the risk profile of the IFP.

In summary, to help the IFP maintain or improve its competitiveness in the financial market, it must set proper impawn rates and interest rates and optimize its portfolio of collateral units. However, the impawn, interest rates and portfolio of collateral units should not be the same in different funding periods because the risk in inventory financing changes as collateral prices fluctuate. For example, in the three months from 31/05/2018 to 31/07/2018, the price of aluminum alloy dropped almost 4.5%. However, its price between 30/09/2008 and 28/11/2008 declined by

[2] https://www.investopedia.com/terms/h/haircut.asp

around 36.38% (LME 2021). If an IFP provided inventory financing for the aluminum alloy from 30/09/2008 to 28/11/2008, it faced serious default risk because its customers who pledged aluminum alloy as collateral might be unwilling to pay back the loan at the maturity date. In contrast, if the IFP provided inventory financing service from 31/05/2018 to 31/07/2018 for the aluminum alloy, it was less likely to face default risks as the level of risk for the aluminum alloy was low in that period. Therefore, to optimize an inventory financing business in different funding cycles, it is also necessary for the IFP to determine impawn rates, interest rates, and a portfolio of collateral units dynamically.

To set the impawn, interest rates, and weights of collateral portfolios dynamically, the IFP first must determine the period for forecasting collateral returns. For a typical transaction cycle, the retailer starts by ordering end-market products, then the supplier purchases raw materials and produces the products. According to the contract, after getting products from the supplier, the retailer may delay the payment until the inventory of the end-market products is sold (Yang & Birge 2018). The commonly identified financing need is the supplier procures and processes the raw materials to fill the order (Babich & Kouvelis 2018, Buzacott & Zhang 2004). This is the reason the funding cycle in inventory financing should not exceed one year (He et al. 2012). With no loss of generality, the funding cycle and the period to update the impawn rate, interest rate, and weights of collateral units do not exceed one year in this doctoral research.

1.2 Research Questions

The example above shows how the impawn rate, interest rate, and portfolio optimization can help an IFP mitigate default risk and gain competitive advantage in inventory financing markets. In fact, the mechanisms of the impawn rate (Brunnermeier & Pedersen 2008, Boissel et al. 2017, Park & Kahn 2018), interest rate (Chan & Thakor 1987, Gorton & Metrick 2012, Spyromitros & Tsintzos 2018), and portfolio management (Markowitz 1952, Alexander & Baptista 2010, Bianchi et al. 2014, Dias 2016, Zhang et al. 2017) have already been extensively investigated in finance and economics literature. However, they have not been well studied in the context of inventory financing. Motivated by this observation, this doctoral research intends to deepen the understanding of how IFPs determine proper impawn rates, interest rates, and the weights of collateral portfolios in inventory financing. Specifically, the following central questions are examined:

Q1: How does an IFP adjust impawn rates to optimize inventory financing in each funding cycle? This central question leads to further, more detailed questions. For example, how can the default risk in inventory financing be effectively evaluated? What factors can affect the impawn rate and what are their roles in this process?

Q2: Besides determining a proper impawn rate, how does an IFP set an appropriate interest rate for inventory financing? Further questions can be derived from this central question. For instance, what is the linkage between an impawn rate and an interest rate, and how is this linkage constructed? Can the riskiest collateral unit be identified by integrating a predictive analytical model and a prescriptive analytical model?

Q3: Can an IFP minimize the risk of adverse collateral price movement in inventory financing by optimizing the portfolio of collateral units? This central question leads to further questions. For example, how can the default risk inherent in inventory financing be mitigated through portfolio optimization? How can the vast amount of collateral price data be incorporated to increase portfolio optimization to manage risk in inventory financing?

1.3 Research Framework

This doctoral research systematically explores three approaches that can be adopted by the IFP to improve the performance of their inventory financing service. The first study illustrates how IFPs adjust impawn rates to maximize the expected profits of their inventory financing business in different funding cycles (see Chapter 3). Based on the theory of option pricing, the second study constructs the linkage between the interest and impawn rates to investigate how both rates are determined in inventory financing (see Chapter 4). The third study shows how an IFP dynamically optimizes its portfolio of collateral units to minimize the risk of adverse collateral price movement in inventory financing (see Chapter 5). The relationship among these three studies are shown in Fig. 1.1. Overall, this doctoral research intends to improve the performance of inventory financing from three aspects: the impawn rate, the interest rate and the portfolio of collateral units.

1.4 Research Methodology

1.4.1 Research Strategy

"Analytics" is defined as *"the scientific process of transforming data into insight for making better decisions"* (INFORMS 2021). There are three categories in business analytics: descriptive analytics, predictive analytics, and prescriptive analytics. The analytics in these three categories intend to answer the following questions: "What is going on?" "What is going to happen?" and "What is the best action we can take?" (Davenport & Harris 2017) Of these three categories, predictive analytics is the most focused. Typical methodologies include machine learning, simulation, forecasting, and data mining. The next big thing could be prescriptive analytics. However, to satisfy the "smarter decisions, better results" promised by business analytics (Davenport et al. 2010), it is necessary to strengthen the linkage between prescriptive and predictive analytics.

There are two ways to strengthen the relationship between prescriptive and predictive analytics. One way is to combine predictive models with optimization and another way is adopting a data-driven predictive analytics approach to generate the prescriptive models (den Hertog

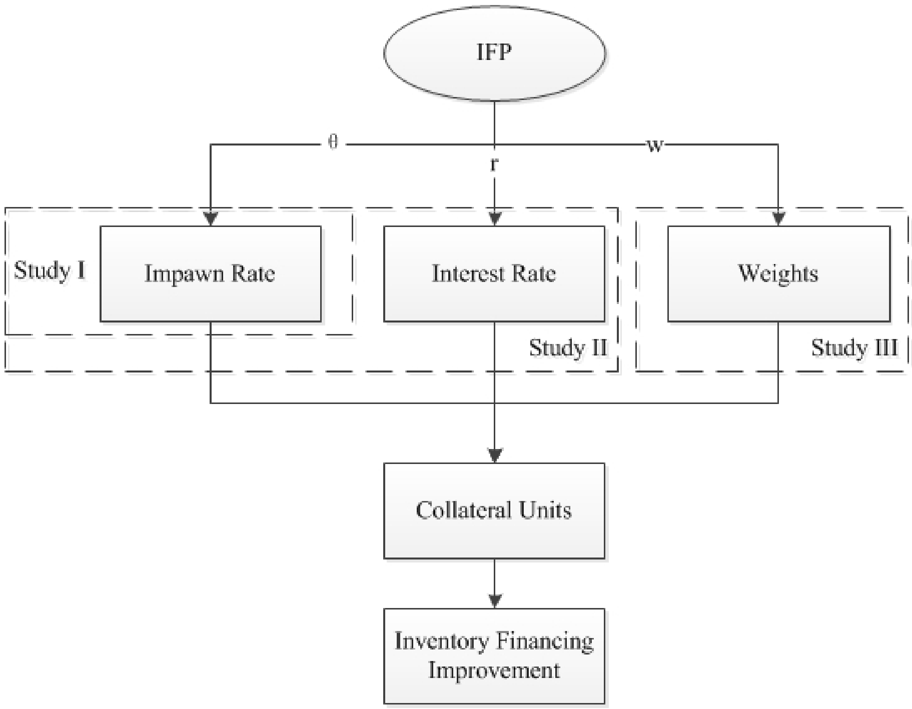

Figure 1.1: Research Framework.

& Postek 2016). The former is about adding value to predictive analytics with optimization. In contrast, the latter is more relevant to how predictive analytics can help build a model for optimization. By using these two approaches, this doctoral research has combined predictive and prescriptive analytics to answer this question: How can the risk associated with inventory financing be managed and how can the performance of inventory financing be improved?

In this dissertation, each study is conducted in the following order: problem investigation, data collection, model construction and validation, model solving, sensitivity analysis, and practical insights (See Figure 1.2). In the problem investigation stage, this doctoral research starts from direct interactions with managers of inventory financing providers and users. Field studies have shown there is consensus among business managers in terms of the difficulties of accessing funds, especially for SMEs. These practitioners agree that investigating an effective data-driven approach to managing risks in inventory financing is necessary and pertinent (See Appendix A). In the model construction and validation stage, the research settings that reflect key characteristics of the external and internal environments are specified. For instance, the objective profit function for inventory financing in study I contains four parts. The first part is the demand function of funding because the impawn rate has a powerful impact on the demand for funding. A high impawn rate increases the demand for funding because the borrower can get more financing support based on one unit of collateral (Ashcraft et al. 2011). In the modeling stage, the predictive analytical approach (i.e., data-driven copula model in each study) and prescriptive analytical model (e.g., the impawn and interest rate model in Study II) are integrated to address a contemporary issue such as how IFPs determine impawn rates, the interest rates, and the weights of a collateral portfolio for inventory financing. In the model-solving stage, optimization theory is used to derive the proper impawn rate and interest rate and the optimal weights of a collateral portfolio. For sensitivity analysis, numerical analysis is used to compare the performance of each inventory financing strategy that can be adopted by an IFP. Finally, based on the analytical results, practical insights are generated to help IFPs understand how they can more effectively and efficiently manage credit, interest rates, and collateral risks in inventory financing.

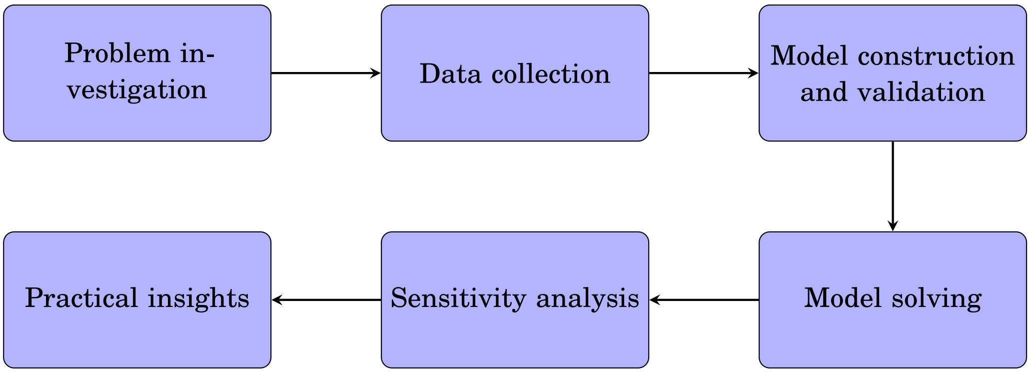

Figure 1.2: The Analytical Process in This Doctoral Research.

1.4.2 Research Design

This section provides an overview of how this doctoral research is designed (See Table 1.1).

Table 1.1: An Overview of Three Studies in the Thesis.

	Study I	**Study II**	**Study III**
Focused area	Impawn rate	Impawn rate Interest rate	Collateral portfolio
Data source	LME		
Predictive model	Canonical vine copula MVN	GARCH-EVT Student T copula R-vine copula	Canonical vine copula MVN
Prescriptive model	IFP's profit function	VaR IIRM	CVaR
Technology	Monte-Carlo simulation Optimization		
Software	R	R and Matlab	R

Note: IIRM is short for impawn and interest rate model. MVN is short for multivariate normal distribution.

Study I considers how impawn rates can be optimized. Specifically, it builds an objective profit function on the multiple factors that an IFP should consider when setting impawn rates. These factors include the default risk, demand for money, interest, and operation-related costs.

Including these factors has identified the linkage between the profits that IFPs can earn and the impawn rate. Based on an iterative evaluation of the default risk, IFPs can dynamically optimize the impawn rate to maximize their profit for each funding period. Primary objectives of the first study include (i) to evaluate the default risk in inventory financing; (ii) to incorporate the default risk in the objective function to maximize the profit of inventory financing with the optimal impawn rate; (iii) to explore the effect of finance- and operation-related factors on setting the impawn rate.

Study II investigates how both impawn rates and interest rates are settled. It constructs an impawn and interest rate model (IIRM) using the option pricing model built by Black & Scholes (1973). The IIRM is developed through the relationship between the interest rate for inventory financing, industrial interest rates, the financing cycle, the variance of the collateral returns, and the impawn rate. To identify a proper approach for incorporating the risk of market volatility into the determination of the impawn rate and interest rate, this study compares three approaches: a historical approach and two GARCH-EVT-Copula-based models. Through a comparison analysis, this study reviews the effectiveness of three approaches for incorporating the market prices of collateral units into setting the impawn rates and interest rates for inventory financing. The analysis is also extended to examine whether the suggested approach can perform continually well throughout the extreme market volatility during the COVID-19 period. In summary, this study addresses the research question of how to incorporate the risks from fluctuating collateral prices effectively when setting appropriate impawn rates and interest rates for inventory financing.

Study III explores the optimization of the collateral portfolio. Multivariate copulas are selected as forecasting models because of their flexibility and accuracy in depicting the dependency structure among time series, and the Conditional Value-at-Risk (CVaR) is selected as the objective function because it is effective to avoid the extreme loss of the portfolio (Aas et al. 2009, Dias 2016, Sahamkhadam et al. 2018). To get the most reliable portfolio strategy, study III compares the performance among nine forecasting models, including eight copula-based forecasting models and multivariate normal (MVN) distribution (the benchmark). Procedures for building appropriate forecasting models also are introduced, especially the process of constructing the most appropriate general vine copula model. Primary objectives of the third study include: (i) to mitigate the default risk by integrating portfolio optimization into inventory financing; (ii) to include a sizeable amount of collateral price data to increase portfolio optimization in managing the risks inherent in inventory financing; (iii) to identify an effective strategy that can well extract the value from a large amount of collateral price data to optimize the portfolio of collateral units.

Studies regarding inventory financing are in infancy stage. How to manage risk and improve the financial performance of inventory financing has not been explored systematically. The effective data-driven approaches provided by this PhD research identify ways to improve inventory financing, which complements to the exisiting SCF literature. In addition, this PhD research deviates from the extant literature on inventory financing (Buzacott & Zhang 2004, He et al.

2012) by employing an integrated approach to set impawn rate, interest rate and weights of collateral units. The combination of predictive and prescriptive models contributes to the business analytics area. Exisiting studies, such as Babich & Kouvelis (2018) and Rockafellar & Uryasev (2002), mainly use a stochastic model to reveal changing external environments such as demand and price in the inventory financing. The application of copulas introduces a new way to simulate various factors, which can help researchers in the inventory financing area improve the precision of analysis.

1.5 Dissertation Organization

The remaining dissertation is organized as follows. Chapter 2 discusses the position of this doctoral research in terms of five strands of the literature: SCF, inventory financing, impawn rate, interest rate, and collateral portfolio.

Chapter 3 presents the first study. It focuses on optimizing impawn rates in inventory financing. In the inventory financing, the optimal impawn rate can help an IFP gain competitive advantage in the financial market. The first study explores how the impawn rate plays its role in improving inventory financing. To set the optimal impawn rate, the first step is to evaluate the default risk and then to incorporate it into the profit function of the IFP. It is assumed that the demand for funding is sensitive to the difference between the individual and industrial impawn rates. Through numerical analysis, the first study demonstrates that the Clayton canonical vine copula has better predictive performance than a multivariate normal distribution (MVN), and thus it can estimate the default risk. Furthermore, it shows that determining specific impawn rate for each collateral unit can make inventory financing more profitable. One interesting note is that, although the optimal impawn rate, the industrial impawn rate and the interest rate have evident impacts on the profits of an inventory financing business, the relationship among them is marginally diminishing[3].

Chapter 4 presents the second study, which provides an innovative data-driven model to set impawn rates and interest rates to improve the competitiveness of IFPs and to manage their risk in financial markets. The linkage between the interest and impawn rates is derived mainly from the option pricing model constructed by Black & Scholes (1973). According to their theory, the inventory financing business can be treated as a European put option, as this kind of option contract restrains the action to a maturity date (Vázquez 1998). This coincides with the fact that IFPs face the risk that borrowers will not pay the money back when the collateral price at the expiration date is lower than the amount of the loan. To identify a proper approach for incorporating the risk of market volatility when determining the impawn and interest rates, the second study compares the performances of three approaches: the historical approach and two GARCH-EVT-Copula-based approaches. The analytical results show that impawn rates estimated

[3]This paragraph is adapted based on the work published in the *International Journal of Production Economics*. The authors' contribution statement has been provided at the start of this dissertation.

by the GARCH-EVT-R-vine-Copula approach can help IFPs extract more value from collateral units because of its ability to capture both the autocorrelations among collateral prices and the dependencies among prices in different collateral units. Interest rates derived from the impawn and interest rate model (IIRM) can reveal the risks of collateral units and serve as a reference for IFPs when drafting inventory financing contracts. GARCH-EVT-Copula's parameterization process can also help IFPs identify the least risky and most predictable collateral unit. Finally, an extended analysis over the COVID-19 period shows that the proposed approach can offer superior performance in a highly volatile market environment.

Chapter 5 presents the third study, which focuses on optimizing portfolios in inventory financing. In asset management, portfolio optimization has been adopted to manage the risks from fluctuating asset prices for a long time. The third study introduces the portfolio management to inventory financing, and investigates how timely collateral prices can be used by IFPs to optimize their collateral portfolios and control default risks. By taking advantage of copulas that can well depict the structure among time series, this study seeks to understand better how the portfolio optimization plays its risk-mitigating in inventory financing service. That is, which copula can forecast future collateral price volatility and how the chosen copulas can be constructed into a portfolio strategy to allocate the weights of collateral in each funding cycle to management the default risk faced by IFPs. A comparison of the predictive performance of the MVN strategy with that of copula strategies shows that the general canonical vine copula can describe the dependence structure among collateral return series, and it has greater predictive performance than the MVN and other copulas. Therefore, the general canonical vine copula can be constructed into portfolio strategies to manage the default risk. To be more specific, two strategies derived from the general canonical vine copula with a normal or skewed Student T marginal and CVaR can make IFPs manage inventory financing risks more effectively. However, their effectiveness relys on the portfolio size. When the number of collateral units is small, the IFP can select the portfolio strategy based on a general canonical vine copula with normal marginal distribution and CVaR. Otherwise, they can select the portfolio strategy derived from a general canonical vine copula with a skewed Student T marginal and CVaR[4].

Chapter 6 summarizes the main findings from the three studies, and it highlights their theoretical and practical contributions. In addition, the research limitations are provided and critically discussed, and future research directions regarding how to address these limitations are pointed out.

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[4]This paragraph is adapted based on the work published in the *Computers & Operations Research*. The authors' contribution statement has been provided at the start of this dissertation.

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Chapter 2

Background Literature

2.1 Introduction

Supply chain finance (SCF) seeks to integrate financial, information, and physical flows in the supply chain. The Global Supply Chain Finance Forum defines SCF as

"the use of financing and risk mitigation practices and techniques to optimize the management of the working capital and liquidity invested in supply chain processes and transactions."

Typical SCF instruments are receivables financing, payables financing, and inventory financing, and inventory financing is playing an increasingly important role because of its flexibility. By making full use of collateral units, the inventory financing provider (IFP) can minimize risk and maximize the profits of the inventory financing service. To explore the role of an IFP in inventory financing, this chapter has five parts: supply chain finance, inventory financing, impawn rate, interest rate, and collateral portfolio.

This review starts by introducing key SCF actors and instruments. Next, Section 2.3 highlights one important SCF instrument, inventory financing and shows how it works in practice. Section 2.4 introduces the notion of the impawn rate, and it details how it is related to the default rate in inventory financing. Previous research on how impawn rates are dynamically determined is discussed in this section. Section 2.5 reviews the progress of the interest rate-related research, and it outlines the relevant research into how the relationship between interest rates and impawn rates can be constructed and how both of them can be dynamically determined in consecutive funding cycles. Section 2.6 treats portfolio optimization and presents how it can be introduced in inventory financing. Section 2.7 introduces copula models and their application in the field of economics and finance. The last section summarizes the research and points out the research opportunities.

2.2 Supply Chain Finance

Based on different dominated SCF actors, SCF models can be divided into four categories: manufacturer-centered SCF (Blackman et al. 2013), bank-centered SCF (Dyckman 2011), e-platform-centered SCF (Jiang et al. 2016), and logistics service provider(LSP)-centered SCF (Hofmann 2009). The primary purpose of all these models is to relieve the financial burden of SMEs in supply chains. In the manufacturer-centered SCF, a manufacturer provides financing support to its small upstream suppliers and downstream retailers. By using the SCF service provided by the manufacturer, the supplier can reduce the length of receivables turnover and the retailer can extend the period of payables turnover (Akgün & Gürünlü 2010). In this SCF model, the manufacturer is usually very familiar with the financial status of firms in the supply chain, and the suppliers and retailers must upload financial data to the platform set up by the manufacturer (Blackman et al. 2013). To maintain competitiveness in financial markets, banks also are developing innovative SCF services (Camerinelli 2015). In bank-centered SCF, capital-constrained companies can use certain collateral, such as receivables, payables, and inventory, to get financing from the bank and improve their liquidity (Dyckman 2011, Caniato et al. 2016). E-commerce platforms bridge the gap between financial institutions and SMEs with low credit. The e-commerce platform can evaluate customers' transaction data and help small suppliers or retailers get financing (Yan et al. 2020). LSP-centered SCF occurs when firms need high levels of inventory or short delivery time to meet the demands of their customers (Zhi et al. 2021). However, after sending out products, firms often are not be paid quickly, and this challenges their financial situation. To fill the need for financing by such firms, LSPs provide inventory financing services by using their familiarity with transactions in the supply chain (Liebl et al. 2014, Martin & Hofmann 2017).

Several SCF instruments have been adopted by these actors, and this has improved the efficiency of transactions and facilitated coordination in supply chains (Hofmann & Belin 2011). Typical instruments include receivables financing, payables financing (Chakuu et al. 2019) and inventory financing (Gelsomino et al. 2016, Jia et al. 2020). Customers often buy goods or services from their suppliers by extending credit, meaning that they do not need to pay until a date in the future. For capital-constrained suppliers, such an arrangement might make it difficult for them to invest in business growth in the immediate term. By enabling payment of invoices before their due date, suppliers using receivables financing can reduce the delay between purchasing raw materials and receiving payment from customers.

In contrast, payables financing is rather like invoice discounting. A buyer asks a financial institution to settle its trade payables, and the buyer either retains a trade payable or debt on its balance sheet. Inventory financing has been popular since the 2008 financial crisis. Owing to a lack of fixed assets, SMEs in supply chains often face capital constraints and find it difficult to get financing from banks (Zhao & Huchzermeier 2019, Jia et al. 2020, Zhi et al. 2020). As an alternative, SMEs use inventory financing provided by IFPs to relieve their capital constraints. Their raw materials, especially industry metals like copper, aluminum alloy, and lead, can function as collateral (Liu & Zhou 2017).

More details about SCF are provided by Gelsomino et al. (2016), Xu et al. (2018), and Chakuu et al. (2019). Gelsomino et al. (2016) classify SCF studies based on their major themes and methods. They find that SCF can be considered from two perspectives: a "finance oriented" perspective and a "supply chain-oriented" perspective. The former concentrates on the financing solution provided by financial institutions, and the latter focuses more on managing cash flow in the supply chain. Xu et al. (2018) adopt bibliometric and network approaches to study SCF literature, and they have traced the development of topical classification over years. In addition, they use content analysis to gain extra insights into the research on SCF. Compared with these two review works, the work done by Chakuu et al. (2019) focus more on how SCF mechanisms, actors, and instruments affect each other. They illustrate how SCF actors, such as retailers, banks, suppliers, and LPSs, use different SCF instruments, such as inventory financing, receivables financing, accounts payable financing, and fixed-asset financing, to develop the supply chain. Promising directions for SCF research are given in all the works reviewed.

2.3 Inventory Financing

In industrial supply chains, the operation of the whole chain is constructed through collaboration and coordination with the focal company, normally recognized as a manufacturer, and other chain members (Nag 2014). To produce industrial goods, the manufacturer (focal company) obtains orders from the market and turns to suppliers, requesting that they provide various materials for manufacturing industrial products. As the volume of industrial goods is relatively large, suppliers need enormous capital to purchase various materials and equipment for manufacturing products. Suppliers, particularly SME suppliers, cannot afford capital for production by themselves, which causes the instability of the whole supply chain operation. To prevent breaks in production, the IFP provides an inventory financing service to capital-constrained suppliers (Liu & Zhou 2017)[1].

Inventory financing allows a business with liquidity problems to use inventory as collateral to get a revolving line of credit. Businesses can use such credit to buy extra inventory and pay many other types of expenses, such as paying employees and purchasing raw materials. As Hofmann (2009) said, "inventory financing" is a short-term financial management problem as inventories are current assets, and carrying circulating capital is a commitment of funds that must be financed. From a borrower's perspective, the choice of financing outlet should be considered first as inventory financing can be provided by different financial services providers, and their interest

[1] This paragraph has been published in the *International Journal of Production Economics*. The authors' contribution statement has been provided at the start of this dissertation.

rates and financing conditions are different (Yang & Birge 2018, Deng et al. 2018, Kouvelis & Zhao 2015). From the perspective of financial services providers, current assets of inventory financing firms are used as collateral or as security for short-term loans. Therefore, IFPs have two goals. First, to avoid credit events that cause defaults, they want to make sure the underlying counterpart risk does not exceed expectations. Second, they demand a risk-adjusted price for the debt capital in the form of interest (Altman et al. 2005). Research on inventory financing considers three aspects: the choice of the financing outlet, the default rate, and the optimization of the credit limit and interest rate.

A large strand of research on operations studies the choice of financing outlets, and it shows that the choice varies based on conditions. For instance, Jing & Seidmann (2014) study the effectiveness of inventory financing in a supply chain comprising a capital-constrained retailer, a bank, and a manufacturer. They show that, when the production costs are relatively high, the inventory financing provided by the bank is more effective in mitigating double marginalization than trade credit provided by a supplier. To compare the financing services provided by suppliers and banks, Yang & Birge (2018) develop an inventory financing system in which one retailer can simultaneously use inventory financing from banks and trade credit from suppliers. They show that the retailer's choice of financial channels depends on their cash level. If the cash level is low, a portfolio of trade and bank loans would finance the retailer's inventory. Rather than considering the trade credit provided by the supplier, Deng et al. (2018) compare bank-led inventory financing with buyer-led financing in a supply chain with one assembler and multiple capital-constrained suppliers that produce heterogeneous components. They have identified the threshold under which the assembler prefers buyer-led financing to inventory financing provided by a bank. Some studies also introduce default risks into the choice of the financing outlet. For example, Kouvelis et al. (2017) develop a newsvendor game theoretical model of a supplier, a bank and a capital-constrained retailer. They find that the supply chain's choice of financing channel is influenced by the supplier's or buyer's credit rating. A low supplier's credit rating motivates the buyer to adopt inventory financing provided by both the supplier and the bank. The default risk is proven to affect the implementation of inventory financing. In a capital-constrained supplier and retailer setting, Kouvelis & Zhao (2015) investigate how a supplier and a retailer coordinate to obtain inventory financing through various contracts (e.g., revenue sharing, buyback, and quantity discount). They find that the effectiveness of these contracts is affected by bankruptcy risks and the costly default of suppliers and retailers within the supply chain system. Another stream of inventory financing literature explores the optimal decision regarding the credit limit and interest rate. For example, Fewings (1992) study how a Markov process can be used to set the upper bounds on the credit limits of supplier-led financing. Buzacott & Zhang (2004) investigate an inventory financing system where a set of independent retailers that have different initial wealth obtain inventory financing from a bank to improve their financial performance. Their research findings indicate that the optimal interest rate is usually above what the bank would actually charge because of the competition among lenders or government regulations [2].

2.4 Impawn Rate

In financial markets, the impawn rate (also called haircut or loan-to-value ratio) refers to a reduction applied to the value of an asset (Ashcraft et al. 2011, He et al. 2012). The settlement of the impawn rate depends on several factors, including the risk (i.e., the volatility of its price) and liquidity (i.e., how easy it is to sell it quickly without a loss of value) of an asset type. An overly high impawn rate or low haircut exposes the lender to risk that is the result of the fluctuating value of collateral. For this reason, the impawn rate tends to decrease during a crisis due to liquidity issues (Brunnermeier & Pedersen 2008, Trebesch & Zabel 2017). This kind of negative correlation between the duration of default and the size of the impawn rate has already been empirically documented by Tobias et al. (2010), Gorton & Metrick (2012), Boissel et al. (2017) and Luo & Wang (2018). However, all of these studies mainly investigate the mechanism of the impawn rate by setting bonds or securities as collateral, rather than in the context in which the IFP sets raw materials as collateral. In actuality, the loan secured by collateral with an impawn rate (inventory financing) has played an important role in facilitating commercial activities in capital-constrained supply chains (He et al. 2012, Liu & Zhou 2017).

For real-world financial institutions, the impawn rate is a key funding constraint. The lender must consider what size buffer is sufficient to cover the risk of not being able to sell the asset at its current value. The reason that Bear Sterns, Lehman and AIG collapsed is that they were unable to meet their margin constraints (Ashcraft et al. 2011). To determine the impawn rate, another important factor that a lender needs to consider is the default rate. Some literature has already investigated the relationship between the impawn rate and borrower default. For example, Boissel et al. (2017) investigate the effect of a impawn rate policy (increase or decrease in the impawn rate) on the sensitivity of repo market rates to sovereign default risk during the Eurozone crisis of 2008-2012. They find that decreasing the impawn rate is ineffective when the sovereign default risk was extremely high in 2011. To study sovereign lending and default, Luo & Wang (2018) construct a model of dynamic contracting with private information that explained the positive correlation between the size of impawn rate and the duration of default. Simultaneously, the evaluation of the default probability has also been underlined by the operations management literature. For example, Shi & Zhang (2010) incorporate default risk into the trade credit offered by a supplier. Wang et al. (2018) describe the probability that a retailer pays on time using an exponential distribution function. Kouvelis & Zhao (2012) examine how trade credit risk affects operating decisions. However, all of these articles evaluate the default probability with a given parameter or a distribution function with fixed parameters, which isolate the default risk from the fluctuating prices of pledged collateral.

[2]The second and third paragraphs in Section 2.3 have been published in the *Computers & Operations Research*. The authors' contribution statement has been provided at the start of this dissertation.

To incorporate the fluctuating prices of collateral in the evaluation of default probability, the first step is to predict the trend of the future prices of collateral. In the finance literature, a classical model that has been used to simulate asset returns is the multivariate normal distribution (MVN). However, its accuracy and efficiency have been questioned as it is unable to capture the asymmetric structure of time series (Low et al. 2013). Fortunately, this situation has been changed since the concept of canonical vine copulas was established by Aas et al. (2009). By using the pair-copula decomposition of a general multivariate distribution, they demonstrate that the canonical vine copula can take advantage of its flexibility in capturing the dependency structure of time series to accurately simulate future returns based on historical time series.

Actually, the impawn rate in inventory financing has already been investigated. For example, by using the AR(1)-GARCH(1,1)-GED formula and the VAR model, He et al. (2014) dynamically set the impawn rate of steel in different risk windows. Through a comparison between the conditional Value-at-Risk (CVaR) and Value-at-Risk (VaR) models, He et al. (2014) further dynamically set the impawn rate of inventory portfolios in various risk windows. However, they assume that the impawn rate does not influence the demand for funding, which is inconsistent with the real business world (Ashcraft et al. 2011). To better control the risk coming from the fluctuation of collateral prices and maintain the competitiveness of IFP on the inventory financing business, it is necessary to dynamically optimize the impawn rate by considering both the default probability and the demand for funding [3].

2.5 Interest Rate

The interest rate is a percentage charged on the amount borrowed, lent, or deposited due per period. The total interest paid by the borrower connects with four elements: the interest rate, compounding frequency, principal sum, and length of the funding period. In summary, the interest rate is what the borrower pays for using another company's money. In the repurchase agreement (repo) market, it is also called the repo rate (Gorton & Metrick 2012). For example, if the value of an asset in the market is \$100 and a financial institution sells it for \$80 and agrees to repurchase it for \$88, then we would say the repo (interest) rate is 10%.

Research has shown that changing the interest rate may affect the behavior of the borrower. For example, Spyromitros & Tsintzos (2018) claim that the increasing demand for credit can be attributed to low interest rates. Cociuba et al. (2016) empirically prove that a low interest rate motivates borrowers to invest in risky assets because a low interest rate increases the attractiveness of riskier assets and decreases the availability of safe bonds in interbank markets. Chan & Thakor (1987) demonstrate that a higher interest rate results in a lower level of effort for borrowers and a higher interest rate reduces the borrowers' payoff when their project succeeds and the loan is repaid.

[3] Section 2.4 has been published in the *International Journal of Production Economics*. The authors' contribution statement has been provided at the start of this dissertation.

From the perspective of lenders, two important factors can influence interest rates: the default rate and the level of over-collateralization. Aguiar & Gopinath (2006) and Wu et al. (2017) demonstrate that the relationship between interest and default rates is positive. Eren (2014) proves that over-collateralization decreases the interest rate in the repo market. Furthermore, the interest rate can be indirectly affected by the impawn rate, as the latter one can influence the level of over-collateralization and the probability of borrower default (Bakoush et al. 2019). Eren (2014) claims that over-collateralization increases with low impawn rates. Hence, repos with low impawn rates have low interest rates. The relationship between impawn rates and interest rates has been revealed in studies of the repo market. For instance, Ashcraft et al. (2011) state that a central bank's policy of setting a low impawn rate decreases the required returns on collateral (i.e., it decreases the interest rate). This effect is even clearer during times of economic crisis. Like the repo market, when the impawn rate for inventory financing is high, an IFP faces the challenge of over-collateralization and accepts more risk. Thus, when setting the interest rate, IFPs must consider its relationship with the impawn rate. However, research on inventory financing has not captured the linkage between impawn rates and interest rates in a rigorous way (Buzacott & Zhang 2004). Therefore, study II adopts the theory of European call option pricing (Black & Scholes 1973) to construct that relationship.

2.6 Collateral Portfolio

Portfolio optimization has two stages: predicting future returns of assets and assigning weights to the assets in the portfolio using a utility function (Markowitz 1952). There has been abundant research into asset forecasts and the choice of the utility function. For instance, to improve the minimum tracking error variance (TEV) in selecting a portfolio, Alexander & Baptista (2010) propose a new minimum TEV considering various levels of ex-ante alpha. They find that the sensible choice of ex-ante alpha makes selecting portfolios less risky. Zhang et al. (2017) develop a robust strategy that optimizes the portfolio choice when there are estimation errors in forecasts of returns and transaction costs. They show that the influence of estimation errors on portfolio performance can be minimized by a robust strategy. Bianchi et al. (2014) employ a five-factor asset pricing model to investigate long-term returns from 1927 to 2010 from public infrastructure asset portfolios. From a mean-variance and mean-CVaR perspective, they identify that the U.S. listed infrastructure index cannot be seen as a separate asset class. Using the subadditivity of CVaR, Dias (2016) combines mean-variance and CVaR to control large losses. She finds that this combination outperforms the mean-variance strategy during and after the 2008 financial crisis. Jarrow & Zhao (2006) compare the performance of the mean-lower partial moment (M-LPM) and mean-variance (M-V) strategy in the choice of equity portfolios. They conclude that the performance of M-V and M-LPM portfolios are significantly different when asset returns have large left tails and are non-normal.

Although the importance of return forecasts and the choice of the utility function has been recognized in equity portfolio optimization, the investigation of them is still in its infancy in the inventory financing setting. When discussing future research regarding inventory financing, He et al. (2012) call for using portfolio optimization to mitigate the inventory financing risks coming from fluctuating collateral prices. Recently, although Seifert et al. (2016) have introduced the portfolio optimization theory into managing products in various life cycle stages, the effectiveness of portfolio optimization in inventory financing is still unexplored. Like managing the portfolio of securities, the most challenging part of applying portfolio optimization to inventory financing is finding out the most suitable model to forecast collateral price volatility.

Traditionally, the asset price volatility is predicted by the MVN function (Low et al. 2013). However, its preciseness is limited due to the inaccurate description of the dependent structure among series. The emergence of copulas provides a new opportunity to accurately capture the dependent structure among series (Sklar 1973). Copula models can create multivariate distributions that have the flexibility required of risk management models and overcome the limitations of the traditional multivariate models (Patton 2012). Compared with the joint distribution function, copulas can better reveal the data structure due to its high flexibility (See Patton (2012) for an overview of copula models in finance and econometrics). He et al. (2012) also suggest that the copula theory could be used in the inventory financing setting due to its effectiveness in describing the dependence structure between two or more random variables[4].

2.7 Copula Models

Exisiting multivariate distribution functions describe both the marginal behaviour of individual variables and their dependency structure (Aas et al. 2009). The limited choice of these functions constrains the degree of flexibility in specifying and estimating the model. Fortunately, the development of copula-based multivariate models provides an opportunity to specify the models for marginal distributions separately from the depdence structure that links these distributions to constrcut a joint distribution (Patton 2012). For example, we have a random vector $\boldsymbol{X} = (X_1, X_2, ..., X_m)^T$ with cumulative distribution function F, and make F_j represent the marginal distribution of X_j for $j \in 1, 2, ..., m$. Then, for all $\boldsymbol{x} = (x_1, x_2, ..., x_m)$, we have $C\{F_1(x_1), ..., F_m(x_m)\} = F(\boldsymbol{x})$ where C is a copula with uniformly distributed marginals $U(0,1)$ on $[0,1]$ (Sklar 1959).

The flexibility of copula frees the researcher from considering only exisiting multivariate distributions. Due to this, the copula has gained its popularity in the field of economics and finance. Specific areas that adopt copulas include risk management, pricing of derivative contracts, portfolio management. Risk management is the first area that adopts copulas. Typical application is combining copulas with VaR or other measures to evaluate the probability of large lossess (Embrechts & Höing 2006). For example, by using copulas, Rosenberg & Schuermann (2006)

[4]An earlier version of Section 2.6 has been published in the *Computers & Operations Research*. The authors' contribution statement has been provided at the start of this dissertation.

simultanesouly consider market, credit and operational risks in the risk management problems. Another typical application of copula models is pricing credit derivatives, such as collaterized debt obligations (CDOs) and credit default swaps (CDSs), which often involve multiple sources of risk. For example, using Normal Inverse Gaussian (NIG) copula, Kalemanova et al. (2007) improve the application of Large Homogeneous Portfolio (LHP) approach in the pricing CDOs. Compared with the HLP using a one factor Gaussian copula or a Student T copula, the HLP with NIG copula not only makes the description of depdence structure more flexible but also significantly improves the computational efficiency in pricing CDOs. Copula models have also been weidely used in the portfolio management. Portfolio management requires the portfolio manager to optimize the portfolio weights for multiple assets based on certain metrics, such as CVaR and VaR and variance. Therefore, portfolio management needs predictive multivariate distributions for multiple time series. Copula models can play the role in more accurately describing the dependency structure among multiple time series, which improves the accuracy of assigning weights to the collateral portfolio (Low et al. 2013). Considering copula models have a big family (See Appendix D.1) and their advantage in describing the dependency structure, this doctoral research also explores the application of copula models in the inventory financing, especially their role in determing the impawn rate, interest rate and weights of collateral portfolio.

2.8 Summary

The gaps in prior research involve three topics: the impawn rate, the interest rate, and the collateral portfolio. Each of these topics is further discussed in the literature sections of Chapters 3-5.

First, prior research assumes that the demand for funding is not affected by the impawn rate, which contradicts business practice (Ashcraft et al. 2011). To control the risk from fluctuating collateral prices more effectively and to help the IFP gain competitiveness in the financial market, it is essential to optimize impawn rates dynamically by considering both the default risk and the demand for funding. Therefore, adopting canonical vine copulas, the first study intends to measure the default risk in inventory financing dynamically for each funding period. Using the estimated default probability and the demand function of money, impawn rates are dynamically optimized to maximize the profits of the inventory financing service[5].

Second, based on prior research on the repo market, the interest and impawn rates have strong connections (Ashcraft et al. 2011). This means that the interest rate should be adjusted based on the changing impawn rate. Owing to the similarity of inventory financing and the repo market, if the impawn rate is high, an IFP also faces the problem of over-collateralization and must undertake more risks. Therefore, when setting the interest rate, an IFP must consider the linkage between the interest rate and the impawn rate. However, research on inventory financing

[5]This paragraph is adapted based on the work published in the *International Journal of Production Economics*. The authors' contribution statement has been provided at the start of this dissertation.

does not construct the relationship between interest rates and impawn rates in a rigorous way (Buzacott & Zhang 2004). To fill this gap, the second study intends to describe the relationship between impawn rates and interest rates using an impawn and interest rate model (IIRM). This model is derived from the option pricing model built by Black & Scholes (1973), which allows the interest rate to be evaluated dynamically according to the changing impawn rate.

Third, despite increasing interest among practitioners and academics in the use of portfolio optimization in other fields such as managing products in various life cycle stages by Kumar & Park (2019), the effectiveness of portfolio optimization in inventory financing is still unexplored. The third study, therefore, intends to close that gap in the literature by identifying effective copula-based strategies that can optimize the collateral portfolio in inventory financing. Traditionally, the use of copula models is limited to standard Archimedean copulas that are bivariate or trivariate. However, one kind of multivariate copula, the vine copula, is suggested by Bedford & Cooke (2002) and popularized by Aas et al. (2009). It can capture different dependency structures of multidimensional data using different pair copulas (Fan & Patton 2014). For example, using four different pair copulas (i.e., Clayton, Gumbel, Student T, and Gaussian copulas), a five-dimensional vine copula can simultaneously present four kinds of dependence strength on the tails of the bivariate distribution (Okhrin et al. 2013). Therefore, study III mainly compares the performance of the vine copula with other predictive models[6].

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[6]This paragraph is adapted based on the work published in the *Computers & Operations Research*. The authors' contribution statement has been provided at the start of this dissertation.

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CHAPTER 3

IMPAWN RATE OPTIMIZATION IN INVENTORY FINANCING: A CANONICAL VINE COPULA-BASED APPROACH

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In the inventory financing business, an optimal impawn rate (loan-to-value ratio) can help the inventory financing providers (IFPs) maintain competitiveness in the inventory financing market. However, the literature has been silent on how IFPs can optimise the business through the optimisation of the impawn rate. This study examines the role of the optimal impawn rate in the inventory financing business. The key to setting the optimal impawn rate is first evaluating default probability and then incorporating this into the profit function. A data-driven approach is used to explore the copula model in setting the optimal impawn rate. Through numerical analysis, it has been found that the Clayton canonical vine copula has a better performance for the prediction of default probability than the multivariate normal distribution (MVN) and can thus be used to evaluate default probability. In addition, it has been uncovered that setting multiple impawn rates for different collaterals allows inventory financing to yield a higher profit. Further, although the interest rate, industrial impawn rate, and optimal impawn rate have strong effects on inventory financing profit, interestingly, the relationship between them is marginally diminishing.

3.1 Introduction

In inventory financing, inventory is used as a pledge for the purpose of risk aversion. Compared with conventional financing, inventory financing plays an essential role in alleviating the capital constraints for small and medium-size enterprises (SMEs) by allowing them to improve cash flow and the ability to fulfil customer orders (Buzacott & Zhang 2004). Traditionally, inventory financing is often provided by banks and other financial institutions, such as Commercial Capital LLC and Crossroads Financial. Interestingly, more firms, including third-party logistics providers (TPLs), are engaging in inventory financing service (Capgemini 2016, Mayer 2013) as the profit margin generated from traditional logistics operations decreases (Hofmann 2009, Liu & Zhou 2017, Li & Chen 2018). For instance, UPS founded UPS Capital to provide in-transit inventory financing services, and Schneider Logistics Inc. collaborated with U.S. banks to provide better financial solutions to its capital-constrained clients [1].

Because pledged inventory is not guaranteed to maintain its initial value (He et al. 2012, 2014, Wu et al. 2019), when providing inventory financing services, an IFP sets an impawn rate to control the financing risk. The impawn rate is the ratio between the loan ascribed to the collateral and the market value of the collateral. A high impawn rate means the borrower (he) could receive more money based on his collateral, and a low impawn rate indicates that he could receive less money. Currently, the settlement of the impawn rate is diverse and based on the industrial experience of IFPs. For example, Commercial Capital LLC advances up to 80% of the appraised value of collateral. UPS Capital claims that the impawn rate can be up to 100%. Obviously, an impawn rate determined in this way does not reveal the fluctuating value of inventory, as the market is dynamic and the corresponding likelihood that the customer will default changes in different funding cycles. To illustrate, from 30/9/2008 to 28/11/2008, the price of aluminum alloy dropped approximately 36.38% (from \$ 2,130 /ton to \$ 1,355 /ton). From 5/31/2018 to 7/31/2018, the price only dropped approximately 4.5% in three months (from \$ 1,855 /ton to \$ 1,775 /ton) [2]. If an IFP sets an optimal impawn rate from 30/9/2008 to 28/11/2008 based on previous experience, it will face a serious risk of default by customers who use aluminum alloy as a collateral, since they may not be able to return the money at the end of the funding cycle. In contrast, if the IFP allocates a low impawn rate to aluminum alloy from 31/5/2018 to 31/7/2018, although the default risk is lower, the low rate would not increase competitiveness in the inventory financing market. Therefore, evaluating the default probability based on changing collateral value and setting the corresponding optimal impawn rate would be helpful to optimize the inventory financing business.

Moreover, impawn rate optimization has been extensively investigated, and many beneficial explorations on the volatility and risk management of pledges have already been made (Ni et al. 2016, Zhang et al. 2017, Wang et al. 2018). However, a majority of the literature assumes that the

[1] UPS Capital: `https://upscapital.com/`; Schneider Logistics: `https://schneider.com/`

[2] London Metal Exchange (LME): `https://www.lme.com/`

default probability in financial services has no relationship with the changing value of collateral and can be set as a fixed parameter. In reality, the default probability of the borrower is not static and is strongly linked with the fluctuating value of collateral (He et al. 2012, 2014). Motivated by the practical and theoretical cases above, the focus of this study is thus on how IFPs dynamically evaluate the default probability based on the changing value of collateral and set a corresponding impawn rate to optimize their inventory financing business. Specifically, the following research questions are examined. (1) How does an IFP effectively evaluate default probability in the inventory financing business? (2) How can the default probability be incorporated into the objective profit function to set the optimal impawn rate for inventory financing? (3) How do other factors, including the willingness to take risks, the liquidity risk of collateral, the interest rate, and the industrial impawn rate, affect how an IFP sets the optimal impawn rate?

To address the research questions above, this study constructs an objective profit function consisting of demand for money, default probability, interest, and the supervision cost of collateral, showing the relationship between the impawn rate and the profit that an IFP can gain from the inventory financing business. By iteratively evaluating the default probability in the objective profit function, the optimal impawn rate for each funding cycle can be dynamically calculated.

The remainder of this study is organized in the following manner. Section 3.2 discusses prior research in related areas. Section 3.3 presents the setup of the model. Section 3.4 briefly describes the source data and shows the results for the optimal impawn rate. Section 3.5 further extends the original business model to consider the factor of the borrower in the settlement of the optimal impawn rate. Section 3.6 concludes this study. The proof of the propositions is shown in Appendix B.

3.2 Related Literature

In inventory financing, the IFP can make full use of its control of collateral and set a dynamical impawn rate to maximize the profit for its inventory financing business. Some studies have investigated inventory financing from a qualitative perspective. For example, Hofmann (2009) develope the concept of inventory financing, offering initial insights into the significance of the field. He demonstrates that the value and amount of goods have a strong effect on the profit yielded by the inventory financing business. Recently, Li & Chen (2018) adopt a multiple case study approach to identify how the IFP takes advantage of the financial service to generate sustainable competitive advantage.

In contrast, some studies have explored how the IFP plays a role in the financial service from the quantitative perspective. Most of the research has investigated inventory financing service from the perspective of banks. For example, Hwan Lee & Rhee (2010) study how inventory financing costs affect supply chain coordination under four coordination mechanisms: an all-unit quantity discount, buyback, two-part tariffs, and revenue sharing. The authors demonstrate that positive inventory financing costs make revenue sharing less profitable than other mechanisms. Buzacott & Zhang (2004) construct a multi-period inventory control model to investigate the interplay between inventory decisions and asset-based financing. They conclude that asset-based financing can help retailers improve profit. In addition, a number of studies have examined how TPLs provide financing service. For example, Chen & Cai (2011) develop an extended supply chain model with a supplier, a budget-constrained retailer, a bank, and a TPL, comparing different roles of the TPL in providing financial services. They identify that the whole supply chain performs better in the control role model in which the TPL integrates logistics and financial services.

Although the above-mentioned studies on inventory financing business are significant and promising, they are silent regarding how the IFP manages the impawn rate to optimize its inventory financing business. This research intends to fill the gap in the existing literature by dynamically parameterizing the function of default probability in inventory financing for each funding cycle using canonical vine copulas. Based on the parameterized function of expected profit, the impawn rate can be dynamically optimized to maximize the profit of the inventory financing business.

3.3 Model

The IFP has n collaterals and provides corresponding funding for them (See Fig. 3.1). The funding demand for each collateral unit is $M_{i,0}$. θ_i is the impawn rate used by the IFP to manage the risk of each collateral unit. $\theta_i p_{i,0} q_{i,0} = M_{i,0}$. $p_{i,0}$ is the initial price of i^{th} collateral unit and $q_{i,0}$ is the initial quantity of i^{th} collateral unit. The funding demand for each collateral unit is influenced by the settlement of the impawn rate (i.e. $M_{i,0}(\theta_i) = \alpha_i + \beta_i(\theta_i - \bar{\theta})$). A similar linear model for the demand for money has been widely used in economics (Christoffersen & Musto 2002, Suntory & Disciplines 2007, Ashcraft et al. 2011). $M_{i,0}(0) = 0$ and $\beta_i > 0$, which means a higher impawn rate will attract more collateral, and there is no funding demand when the impawn rate is set as 0. The demand function, although quite general, is assumed to be linear in the impawn rate. The linearity assumed here does not affect the analysis in which primary interest focuses on the evaluation of default probability and how the optimal impawn rate depends on relevant factors. The interest rate is r, and the length of interval is k. $M_{i,j} = M_{i,0}\exp(kjr)$. When $M_{i,j} = \theta_i p_{i,j} q_{i,j-1}$, the IFP does not need to call the margin. When $M_{i,j} < \theta_i p_{i,j} q_{i,j-1}$, the borrower can take back extra collateral. When $M_{i,j} > \theta_i p_{i,j} q_{i,j-1}$, the borrower is required to bring more collateral to the IFP until $M_{i,j} = \theta_i p_{i,j} q_{i,j}$. Some risk does exist because if the borrower does not fulfil the contract, the IFP must take time to deal with the collateral. However, it is very likely that the difference between the initial market value and the value realized after liquidation is greater than 0. Therefore, when dealing with collateral, the loss suffered by the IFP is $M_{i,j} - (1-\rho_i) p_{i,j} q_{i,j-1}$, and ρ_i is the level of liquidity risk of i^{th} collateral unit. See Table 3.1 for a summary of the notations.

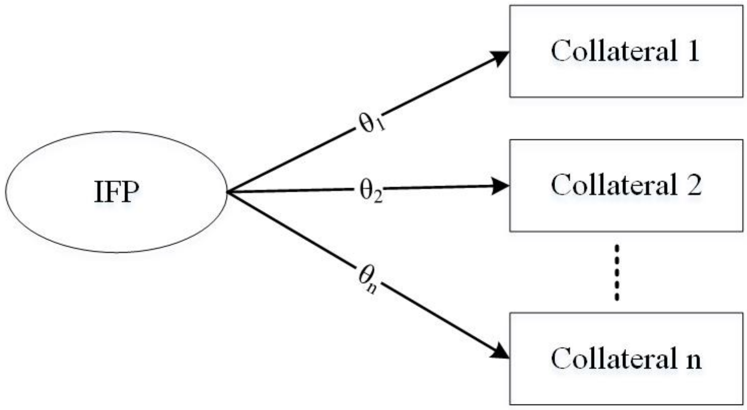


Figure 3.1: Optimal Impawn Rate for Each Collateral Unit.

Table 3.1: Summary of Notation and Assumptions.

Symbol	Description	Assumption
k	The length of the interval	k is nonnegative integer
m	The number of intervals	m is nonnegative integer
n	The number of collateral units	n is nonnegative integer
$M_{i,0}$	The amount of the loan for the i^{th} collateral unit	$M_{i,0} \geq 0$
r	The interest rate provided by the IFP	$0 < r$
$p_{i,j}$	The price of i^{th} collateral at the end of j^{th} interval	$0 < p_{i,j}$
$q_{i,j}$	The quantity of i^{th} collateral at the end of j^{th} interval	$0 \leq q_{i,j}$
θ_i	The impawn rate (decision variable)	$0 < \theta_i < 1$
$\bar{\theta}$	The industrial impawn rate	$0 < \bar{\theta} < 1$
τ	The level of risk that the IFP is willing to undertake	$0 \leq \tau < 1$
ρ_i	The level of liquidity risk of i^{th} collateral unit	$0 \leq \rho_i \leq 1$
α_i	The constant of the correlation between $(\theta_i - \bar{\theta})$ and $M_{i,0}$	$0 < \alpha_i$
β_i	The coefficient of the correlation between $(\theta_i - \bar{\theta})$ and $M_{i,0}$	$0 < \beta_i$
g_i	The supervision fee for one unit of i^{th} collateral in each interval	$0 < g_i$

3.3.1 Objective Function of Expected Profit

Assuming that borrowers who provide the i^{th} collateral unit do not fulfil the contract, the loss suffered by the IFP on the i^{th} collateral unit at the end of the interval is as follows:

$$L_i = [M_{i,j} - p_{i,j} q_{i,j-1}(1-\rho_i)] \exp(-kjr) \tag{3.1}$$

Because $M_{i,j} = M_{i,0}\exp(kjr)$, Eq. (3.1) can thus be further transformed into Eq. (3.2).

$$L_i = M_{i,0} - p_{i,j} q_{i,j-1}(1-\rho_i) \exp(-kjr) \tag{3.2}$$

Assuming that the IFP can tolerate the loss $\bar{M}_{i,0}$, $\bar{M}_{i,0} = \tau M_{i,0}$ (τ is the level that the IFP can tolerate). The probability of loss for the i^{th} collateral unit can be further calculated. For the IFP, the probability of loss in the j^{th} interval is as follows:

$$\mathbb{P}(\bar{M}_{i,0} \leq L_i) = \mathbb{P}(\tau M_{i,0} \leq M_{i,0} - p_{i,j} q_{i,j-1}(1-\rho_i)\exp(-kjr)) \tag{3.3}$$

Because

$$q_{i,j-1} = \frac{M_{i,0}\exp(k(j-1)r)}{\theta_i p_{i,j-1}} \tag{3.4}$$

Eq. (3.3) can thus be further transformed into:

$$\mathbb{P}(\bar{M}_{i,0} \leq L_i) = \mathbb{P}(\tau \leq 1 - \frac{(1-\rho_i)\frac{p_{i,j}}{p_{i,j-1}}}{\theta_i \exp(kr)}) \tag{3.5}$$

and Eq. (3.5) can be further transformed into:

$$\mathbb{P}(\bar{M}_{i,0} \leq L_i) = \mathbb{P}(\frac{p_{i,j}}{p_{i,j-1}} \leq \frac{\theta_i(1-\tau)\exp(kr)}{1-\rho_i}) \tag{3.6}$$

We set $\frac{p_{i,j}}{p_{i,j-1}} = P_i$ and $\frac{\theta_i(1-\tau)\exp(kr)}{1-\rho_i} = Z$. Then:

$$\mathbb{P}(\bar{M}_{i,0} \leq L_i) = \mathbb{P}(P_i \leq Z) \tag{3.7}$$

If we add 'ln' to both the left and right term in the second part of Eq.(3.7), then we have:

$$\mathbb{P}(\bar{M}_{i,0} \leq L_i) = \mathbb{P}(\ln P_i \leq \ln Z) \tag{3.8}$$

$\ln P_i$ is logarithmic return of i^{th} collateral unit. Therefore, we have Proposition 3.1.

Proposition 3.1. *When the IFP can tolerate only the loss $\bar{M}_{i,0}$, $\bar{M}_{i,0} = \tau M_{i,0}$ (τ is the level of risk that the IFP can tolerate). The probability of loss for the i^{th} collateral unit in the j^{th} interval is $\mathbb{P}(\ln P_i \leq \ln Z)$. $P_i = \frac{p_{i,j}}{p_{i,j-1}}$ and $Z = \frac{\theta_i(1-\tau)\exp(kr)}{1-\rho_i}$.*

Proposition 3.1 describes the probability of loss for the i^{th} collateral unit, which is significantly affected by the impawn rate (θ_i) set by the IFP. With an increase in the impawn rate set by the IFP, the probability of loss would also increase.

When the IFP receives the collateral, it expends energy and takes the time to manage it for each funding cycle. Supervision costs also exist here, which are shown as follows:

$$G_i = \frac{M_{i,0}}{\theta_i p_{i0}} g_i m \tag{3.9}$$

Therefore, the profit that the IFP expects to earn for the i^{th} collateral unit is as follows:

$$\pi_i(\theta_i) = \sum_{j=1}^{m} [\alpha_i + \beta_i(\theta_i - \bar{\theta})][1 - \mathbb{P}(\ln P_i \leq \ln Z)][\exp(kjr) - \exp(k(j-1)r)] - G_i \tag{3.10}$$

$\mathbb{P}(\ln P_i \leq \ln Z)$ is the default probability. $\exp(kjr) - \exp(k(j-1)r)$ is the interest owned by the IFP in the j^{th} funding cycle. Therefore, the expected revenue on the i^{th} collateral unit for the j^{th} funding cycle is $[\alpha_i + \beta_i(\theta_i - \bar{\theta})][1 - \mathbb{P}(\ln P_i \leq \ln Z)][\exp(kjr) - \exp(k(j-1)r)]$. Eq. (3.10) can be further simplified into:

$$\pi_i(\theta_i) = [\alpha_i + \beta_i(\theta_i - \bar{\theta})][1 - \mathbb{P}(\ln P_i \leq \ln Z)][\exp(kjr) - 1] - G_i \tag{3.11}$$

To calculate the optimal impawn rate with Eq. (3.11), we can take its first-order and second-order derivative. Here, we have Proposition 3.2.

Proposition 3.2. *There exists a single optimal impawn rate* θ_i^* *in the interval* $(0, \min(\exp(\delta+\mu-\omega),1))$ *that maximizes the profit of the IFP on* i^{th} *collateral unit and* $\omega = \ln\frac{1-\tau}{1-\rho_i} + kr$. *(The proof is in Appendix B.1).*

In Proposition 3.2, μ and δ are the mean and variance of logarithmic returns of i^{th} collateral unit simulated by the predictive model. From the proof in Appendix B.1, we know that there exists an optimal impawn rate in the interval $(0, \min(\exp(\delta+\mu-\omega),1))$ that maximizes the profit of the IFP for the i^{th} collateral unit. Based on the calculated optimal impawn rate for each collateral unit, the total optimal profit from the inventory financing business is as follows:

$$\Pi(\theta_1^*, \theta_2^*, ..., \theta_n^*) = \sum_{i=1}^{n} [\pi_i(\theta_i^*) - G_i(\theta_i^*)] \tag{3.12}$$

3.3.2 Canonical Vine Copula

To optimize the impawn rate, the returns from collateral should first be simulated to parameterize the function of default probability. In this study, the canonical vine copula is adopted to simulate the returns from collateral, as it can capture the dependence structure among different time series very well (Aas et al. 2009).

Every cumulative joint distribution function (CDF) reveals the marginal behavior of individual values and their dependency structure. However, the CDF can be expressed in another way. Consider a vector $\boldsymbol{X} = (X_1, ..., X_n)$ of random variables with a joint CDF $F(x_1, ..., x_n)$ and marginal distributions $F_i (i = 1, ..., n)$; there exists a copula to describe the dependence structure among the marginal distribution functions based on (Sklar 1973) as follows:

$$F(x_1, ..., x_n) = C[F_1(x_1), ..., F_n(x_n)] \tag{3.13}$$

Using the transformation $F_i(X_i) = U_i$, the copula from Eq. (3.13) has the following expression:

$$F(x_1, ..., x_n) = C[F_1(x_1), ..., F_n(x_n)] = C(u_1, ..., u_n) = \mathbb{P}(U_1 \leq u_1, ..., U_n \leq u_n) \tag{3.14}$$

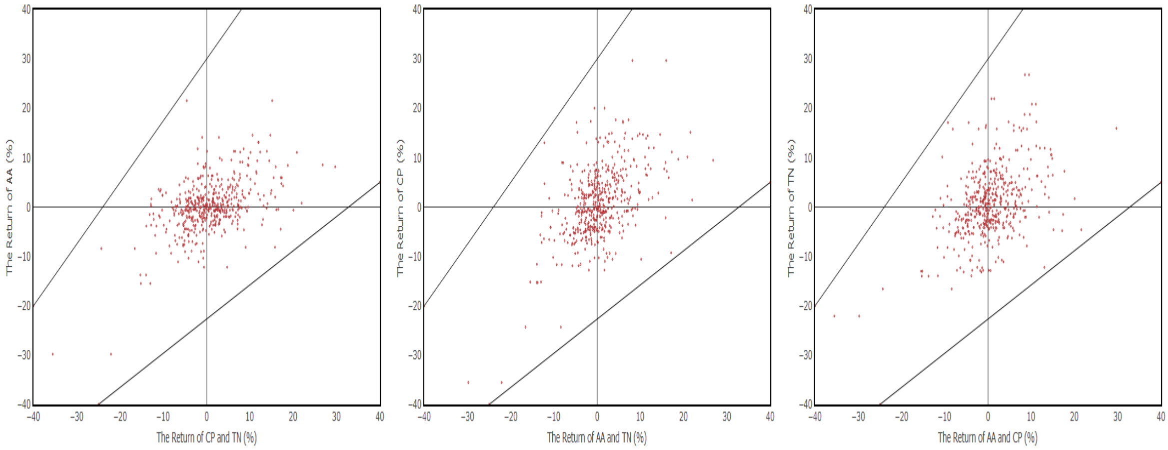
where $C(u_1, ..., u_n)$ is a CDF for a multivariate vector with support in $[0,1]^n$ and uniform margins. If we assume marginal CDF F_i and the copula function C in Eq.(3.14) to be differentiable, the joint density function $f(x_1, ..., x_n)$ and the density of the copula $c(u_1, ..., u_n)$ can be separately defined as:

$$f(x_1, ..., x_n) = c_{1,...,n}[F_1(x_1), ..., F_n(x_n)] \cdot f_1(x_1)...f_n(x_n) \tag{3.15}$$

$$c(u_1, ..., u_n) = \frac{\partial^n C(u_1, ..., u_n)}{\partial u_1, ..., \partial u_n} \tag{3.16}$$

Due to the structure of the lower tail of the time series (See Fig. 3.2), the Clayton copula can be used to capture the dependence structure of the value of collateral (Low et al. 2013).

To illustrate, from Fig. 3.3 and Fig. 3.3d, we can see that compared with other copulas, the density of the Clayton copula has a similar structure as the time series of the sample collateral (See the two lines in each sub-figure of Fig. 3.2). Without a loss of generality, this study takes three raw materials as examples in this study. They are aluminum alloy (AA), copper (CP) and tin (TN), which have a similar dependence structure with other raw materials traded on the LME. However, the Clayton Archimedean copula is characterized by a single parameter, which reduces the accuracy of predictions as its dimensions increase. This weakness makes the Archimedean Clayton copula unlikely to capture varying degrees of dependence structures among multi-variable vectors. To overcome this limitation, the vine copula is introduced into this study. Compared with multivariate Archimedean Clayton copulas, the vine copula is more flexible since it can simultaneously describe varying degrees of dependence structures through iterative conditioning (Aas et al. 2009). The following details how the multivariate vine copula uses pair-copula functions to capture varying degrees of the dependence structure of variable vectors.



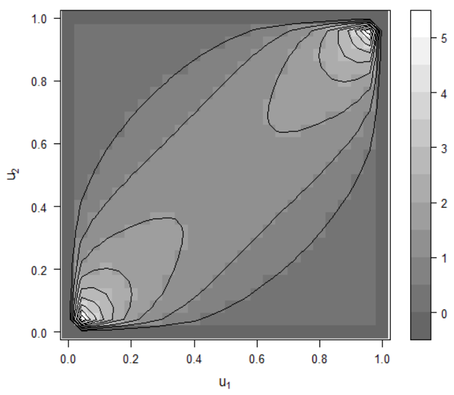
(a) AA on vertical axis (b) CP on vertical axis (c) TN on vertical axis

Figure 3.2: Scatter Plots of Returns.

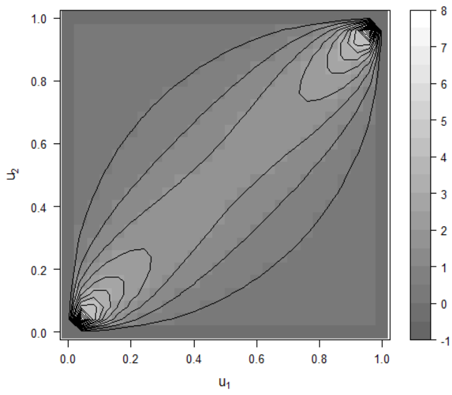
A joint density function $f(x_1,...,x_n)$ can be decomposed without loss of generality by iteratively conditioning as shown in the following:

$$f(x_1,x_2,...,x_n) = f_n(x_n)f(x_{n-1}|x_n)f(x_{n-2}|x_{n-1},x_n)...f(x_1|x_2,...,x_n) \tag{3.17}$$

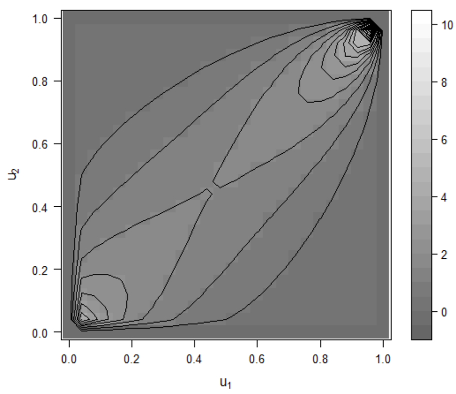
Using conditional copulas, each factor on the right side of Eq.(3.17) can be decomposed further. For example, when $n = 2$, $f(x_1,x_2) = c_{12}[F_1(x_1),F_2(x_2)] \cdot f_1(x_1)f_2(x_2)$. Using $f(x_1,x_2) = f_2(x_2)f(x_1|x_2)$, we can easily obtain $f(x_1|x_2) = c_{12}[F_1(x_1),F_2(x_2)] \cdot f_1(x_1)$. It is now clear that the second factor, $f(x_{n-1}|x_n)$, on the right side of Eq. (3.17) can also be decomposed into the pair-copula $c_{(n-1)n}[F_{n-1}(x_{n-1}),F_n(x_n)]$ and a marginal density $f_{n-1}(x_{n-1})$. Similarly, each term in Eq. (3.17) where v_j is an arbitrarily chosen component of $\boldsymbol{v}$, and $\boldsymbol{v}_{-j}$ is vector $\boldsymbol{v}$ without this component.



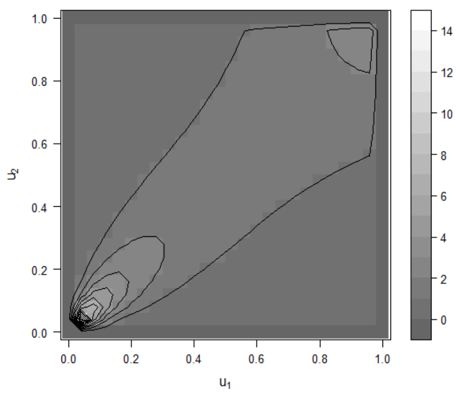
(a) The Density of Gaussian Copula



(b) The Density of Student t Copula



(c) The Density of Gumbel Copula



(d) The Density of Clayton Copula

Figure 3.3: Density of Four Pair Copulas

From Eq.(3.19), we can see that marginal conditional distributions of the form $F(x|v)$ are included in the pair-copula construction. Jose et al. (1996) reveal that for every j:

$$F(x|\boldsymbol{v}) = \frac{\partial C_{x,v_j|\boldsymbol{v}_{-j}}[F(x|\boldsymbol{v}_{-j}),F(v_j|\boldsymbol{v}_{-j})]}{\partial F(v_j|\boldsymbol{v}_{-j})} \tag{3.18}$$

can generally be decomposed into the pair-copula multiplied by conditional marginal density. The general formula for an n-dimensional vector $\boldsymbol{v}$ is as follows:

$$f(x|\boldsymbol{v}) = c_{xv_j|\boldsymbol{v}_{-j}}[F(x|\boldsymbol{v}_{-j}),F(v_j|\boldsymbol{v}_{\boldsymbol{-j}})] \cdot f(x|\boldsymbol{v}_{\boldsymbol{-j}}) \tag{3.19}$$

where $\boldsymbol{v}_{-j}$ is the vector $\boldsymbol{v}$ and excludes the component v_j, and $C_{x,v_j|\boldsymbol{v}_{-j}}$ is a bivariate copula distribution function. If we assume vector $\boldsymbol{v}$ to be one-dimensional, we have

$$F(x|v) = \frac{\partial C_{xv}[F(x),F(v)]}{\partial F(v)} \tag{3.20}$$

When x and v are uniform (i.e., $f(x) = f(v) = 1$, $F(x) = x$ and $F(v) = v$), the conditional distribution function can be represented by the function $h(x, v, \Theta)$, as follows:

$$h(x,v,\Theta) = F(x|v) = \frac{\partial C_{xv}(x,v,\Theta)}{\partial v} \tag{3.21}$$

Parameter v is the conditional variable, and Θ is the parameter for the bivariate copula $C_{x,v}(x,v)$. In the real-world application, $h(x,v,\Theta)$ and the inverse of h-function $h^{-1}(x,v,\Theta)$ are iteratively used for sampling and inference for each pair-copula in the vine (the h-function and its inverse of Clayton copula is in Appendix B.2).

Although there are other vine copulas, such as D-vine and regular vine, here, the canonical vine is selected due to the efficiency of its hierarchical structure (Aas et al. 2009). If the key variable that governs the interactions in the data set is identified during the modelling process, it can be designated as the root of the canonical vine. Consider as an example the joint density of three-dimensional case $\boldsymbol{X} = (X_1, X_2, X_3)$; here, X_1 can be seen as the root of the canonical vine when X_1 governs X_2, and X_3. To build the most accurate canonical vine, we need to choose the right collateral return series as the root. The root is the collateral that has the highest degree of correlation with the other collaterals. Here, the following formula is provided to find the series that has the highest degree of correlation with all the other series:

$$Z_{x_i} = \sum_{j=1}^{N} |\zeta_{ij}|, \quad \text{where} \quad i,j \in N \tag{3.22}$$

ζ_{ij} is an $N \times N$ matrix of the Kendall rank correlation coefficient between each pair of collateral units. The collateral that returns X_i with the highest absolute correlation with all the other collateral units is located as the root of the canonical vine. Once the canonical vine copula is chosen, the dependence structure of a portfolio of n collateral units will be parameterized with $\frac{n(n-1)}{2}$ pairwise Clayton copula parameters. In this research, the number of collateral units is three; thus we need to calculate three copula parameters.

Proposition 3.3. *There exists a PDF $f_{123} = f_1 \cdot f_2 \cdot f_3 \cdot c_{13} \cdot c_{23} \cdot c_{12|3}$ that can be used to simulate the returns of three collateral units. f_n denotes the marginal PDFs and c_n denotes the pairwise copula PDFs (The proof is in Appendix B.3).*

Proposition 3.3 shows the predictive model that can be used to predict the future returns from collateral. By dynamically predicting future returns for three collateral units, we can iteratively parametrise the function of the probability of loss ($\mathbb{P}(\ln P_i \leq \ln Z)$). The constructed predictive model is used to run a Monte Carlo simulation to produce 10,000 returns for each collateral unit, based on which the mean and variance for the objective function are derived.

3.4 Analysis

The analysis section contains three parts that focus on comparing the predictive performance of copulas with that of the MVN, testing the performance of setting a uniform impawn rate with that of setting separate impawn rates, and examining how to determine if willingness to take risks, the liquidity risk of collateral, the interest rate, and the industrial impawn rate affect profit and the settlement of the impawn rate.

3.4.1 Data

The data set contains monthly collateral returns for three raw materials (i.e. aluminum alloy (AA), copper (CP), and tin (TN)), all of which are common types of collateral in the industrial supply chain. In addition, the correlation among these three collateral time serieses are relatively low, which can help improve the robustness of analytical results. The period for these collateral returns extends from 29/1/1999 to 31/12/2018, yielding a total of 720 observations. The first 360 observations from 29/1/1999 to 31/12/2008 are used for estimating parameters, and the remaining 360 monthly return observations served as an out-of-sample set to test the effectiveness of the Clayton canonical vine copula. To determine a reliable predictive model, this study follows DeMiguel et al. (2009), Low et al. (2013) and Sahamkhadam et al. (2018) and use a *"rolling window"* approach to predict future collateral price volatility. In the inventory financing setting, the *"rolling window"* approach is defined as *"using the data within previous funding cycles to parameterize the multivariate probability distribution after the $t = w+1$ funding cycle"*. To dynamically estimate the parameters of copulas, the function (CopulasSimulation3) is first constructed to estimate copulas. Then, 'for' syntax is adopted to update observations and plunge the updated observations into the function to decide the dependency structure and estimate parameters of copulas. The detailed procedure of adopting a rolling window approach is provided in Appendix B.4.

Based on the historical data, the PDF in Proposition 3.3 is used to simulate the returns of three collateral units (i.e., $n = 3$). The simulated returns are used to parameterize the function of the default probability for each collateral type. Relevant parameters are set as $\bar{\theta} = 0.7$, $\tau = 0.01$, $r = 1.1\%$, $k = 1$ Month, and $m = 6$ Month. Other parameters are for specific collateral units. For the first collateral (AA), $\alpha_1 = \$500,000$, $\rho_1 = 0.2$, and $g_1 = \$10,000$/Million tons/Month. For the second collateral (CP), $\alpha_2 = \$1000,000$, $\rho_2 = 0.2$, and $g_2 = \$10,000$/Million tons/Month. For the third collateral (TN), $\alpha_3 = \$1500,000$, $\rho_3 = 0.2$, and $g_3 = \$10,000$/Million tons/Month.

3.4.2 The Predictive Performance of Copulas

Based on the simulated returns and objective function, the optimal impawn rate is iteratively adjusted for the collateral in each funding cycle. Because the funding cycle interval is set as six months, there is a total of 20 funding cycles. By using Eq. (3.22), we find that the cumulative Kendall value of CP is consistently higher than AA and TN thus can be set as the root of Clayton canonical vine copula in all rolling windows. Interestingly, the cumulative Kendall value has an increasing trend in the first 10 rolling windows and then goes down (See Fig. 3.4). This happens because the positive correlation among three metals is strengthened during the financial crisis.

With the window rolling, more observations during the financial crisis are included, and the cumulative value has an increasing trend before the 10^{th} rolling window. When the cumulative value reaches the summit and the window keeps rolling, the observations during the financial crisis are gradually excluded and more observations in the normal period are included, resulting in the decrease of cumulative Kendall value after the 10^{th} rolling window (See the shadow in Fig. 3.4).

Figure 3.4: Cumulative Kendall Value in Each Rolling Window.

To justify the effectiveness of the Clayton canonical vine copula for capturing the dependency structure of the time series, this study directly uses 20 sets of out-of-sample data to calculate the optimal impawn rate[3]. The Clayton canonical vine copula and the MVN are used to dynamically simulate returns on collateral. Based on the simulated returns, the function of default probability and calculate the optimal impawn rate are parameterized. This study then calculates the difference between the optimal impawn rate produced by the out-of-sample data and that produced by the Clayton canonical vine copula, and the difference between the optimal impawn rate produced by the out-of-sample data and that produced by the MVN. Based on a comparison of the differences, we can evaluate whether the Clayton canonical vine copula performs better than the classic MVN for parameterizing the function of default probability.

Based on the blue and orange bars in Fig. 3.5, we can compare the performance of the MVN and copulas. It is found that the impawn rate produced by the Clayton canonical vine copula is obviously higher than that produced by the MVN. The reason is that the simulated return produced by the canonical vine copula has a lower variance but higher mean compared with the return produced by the MVN. For the first type of collateral (AA), copulas perform better than

[3]The length of each funding cycle is set as six months in this study. There are 120 months from 1/29/2009 to 12/31/2018; thus there are 20 funding cycles in total.

the MVN for 14 funding cycles (See Fig. 3.5a). For the second type of collateral (CP), copulas perform better than the MVN for 17 funding cycles (See Fig. 3.5b). For the third type of collateral (TN), copulas perform better than the MVN for 15 funding cycles (See Fig. 3.5c). It is clear that copulas generally perform better than the MVN for most funding cycles, which means the Clayton canonical vine copula has an advantage when capturing the dependency structure and can thus be used to parameterize the function of the default probability. As Hansen et al. (2010) suggested, a more complex and flexible copula with more than one time-varying parameter may be able to more precisely predict future returns. This finding is similar to the conclusions drawn by Brechmann & Czado (2013) and Low et al. (2013). Brechmann & Czado (2013) claim that highly dimensional vine copulas can accurately capture the characterization of extreme dependency in the equity and bond markets and can thus be used to more accurately manage financial risk. By setting CVaR as an objective function, Low et al. (2013) conclude that the Clayton canonical vine copula can more accurately set weights for a portfolio than the MVN.

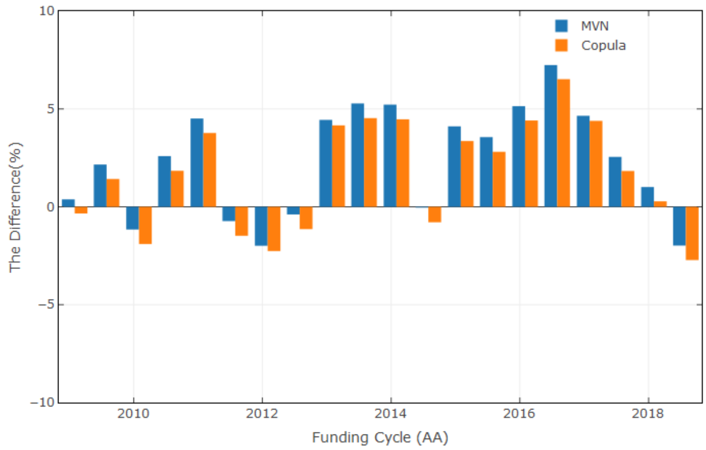
3.4.3 Optimal Impawn Rate

Based on the chosen canonical vine copula-based approach, this study calculates the optimal impawn rate for each type of collateral using Eq.(3.11). To demonstrate the advantage of setting different impawn rates, a uniform impawn rate is also set to maximize the total profit (See Eq. (3.11)). The optimal impawn rates for AA, CP, and TN are 0.7, 0.68, and 0.69, respectively, in the final funding cycle (from 30/6/2018 to 31/12/2018). The impawn rate for AA is higher than CP and TN because the variance of the simulated returns for AA is lower than for the other two types of collateral. The optimal impawn rate for the combination of the three types of collateral is 0.69. The total profit in the case of separation is \$ 179,313.60, and the total profit in the case of combination is \$ 179,020.7. Therefore, setting the impawn rate separately for different collateral units can yield higher profit for the inventory financing business. This conclusion is partially supported by He et al. (2012). Based on the historical prices of steel, the authors demonstrate the benefit of setting impawn rates for specific collateral units.

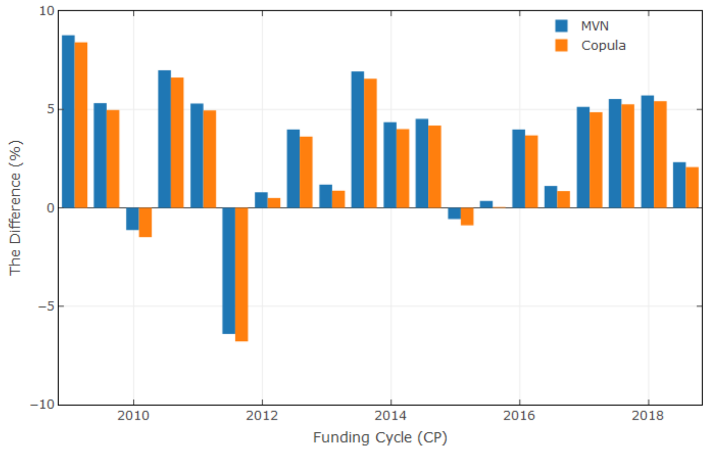
3.4.4 Sensitivity Analysis for Different Collaterals

In the following, four sensitivity analyses are provided to help IFPs identify which factors they should focus on when setting optimal impawn rates for different collateral units.

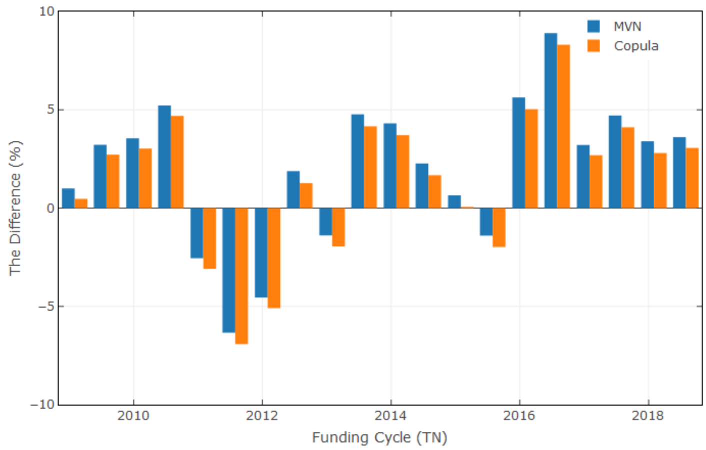
A higher willingness to take risks means the IFP can tolerate more risk that is derived from the default scenario. Fig. 3.7 shows how the IFP's willingness to take risks influences the optimal impawn rate and the expected profit of the inventory financing business. Based on the analytical results, it is found that the optimal impawn rate increases with an increase in the willingness to take risks. Therefore, the IFP should consistently improve its risk-taking ability as this can improve its competitiveness in the inventory financing market (higher optimal impawn rate and expected profit).



(a) Aluminum Alloy

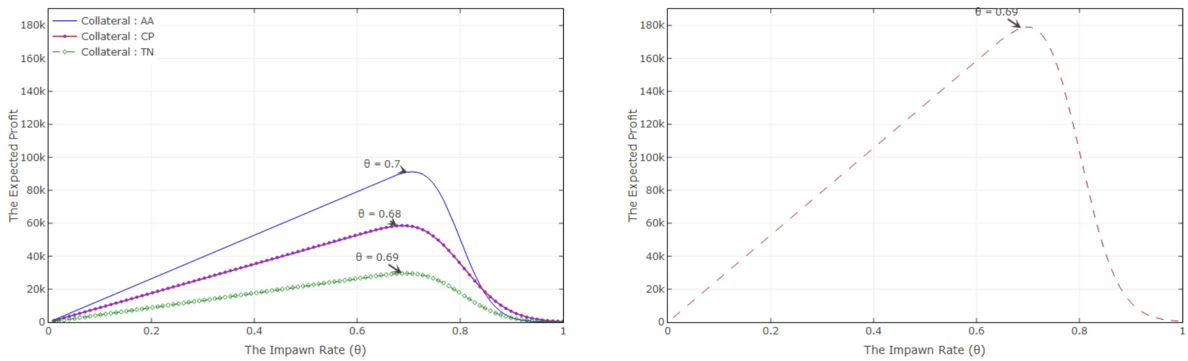


(b) Copper



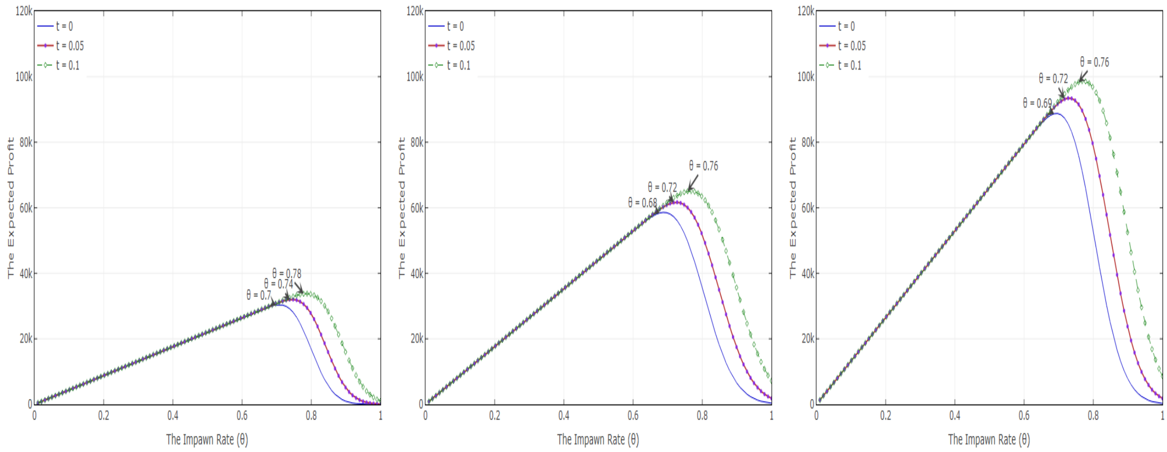
(c) Tin

Figure 3.5: Comparison of the Performance of Copulas and the MVN.



(a) Optimal Impawn Rate for Each Collateral (b) Uniform Impawn Rate for All Collateral

Figure 3.6: Optimize the Profit of Inventory Financing with Impawn Rates

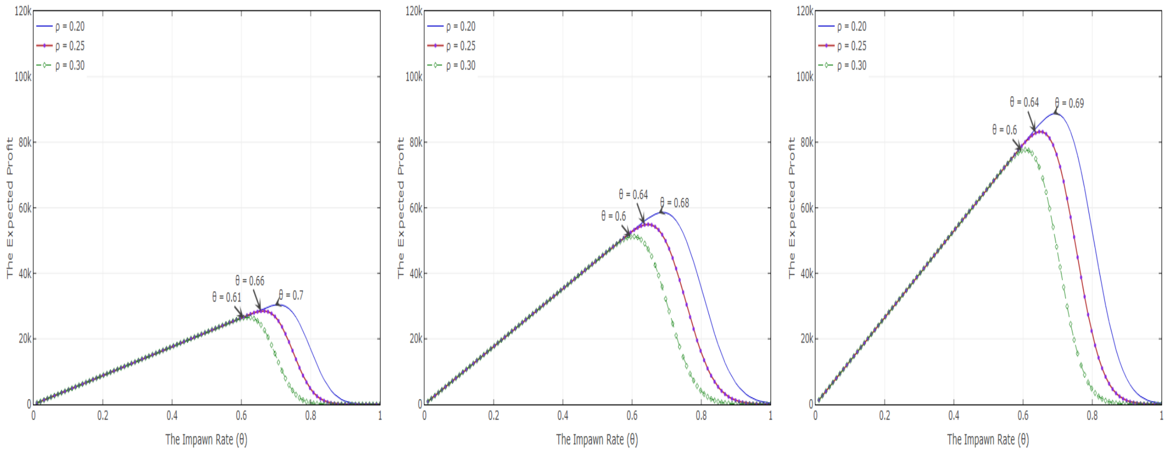


(a) AA (b) CP (c) TN

Figure 3.7: The Effect of Willingness to Take Risks on the Optimal Impawn Rate.

Collateral that is hard to convert into money usually has a high liquidity risk. Based on Fig. 3.8, we can see that the optimal impawn rates decreases with an increase in liquidity risk. Therefore, the IFP should be very selective regarding collateral. If IFPs want to simultaneously improve their competitiveness and increase profit, they should provide more benefits for borrowers whose collateral has a lower liquidity risk. This finding has been empirically demonstrated by Brunnermeier & Pedersen (2008) and Gorton & Metrick (2012). Brunnermeier & Pedersen (2008) construct a model that links an asset's liquidity risk and a trader's funding liquidity, showing that an asset's liquidity risk decreases the impawn rate. Gorton & Metrick (2012) claim that the repo haircut has a strong relationship with confidence in the market. When confidence is low, asset liquidity is low and thus the impawn rate will also be low.

The impawn rate also has a positive relationship with the interest rate but the marginal effect is diminishing. Take AA, for example; when the monthly interest rate increased from 1.0% to 1.1%, the optimal impawn rate increased from 0.70 to 0.74 (See Fig. 3.9a). Initially, an increase

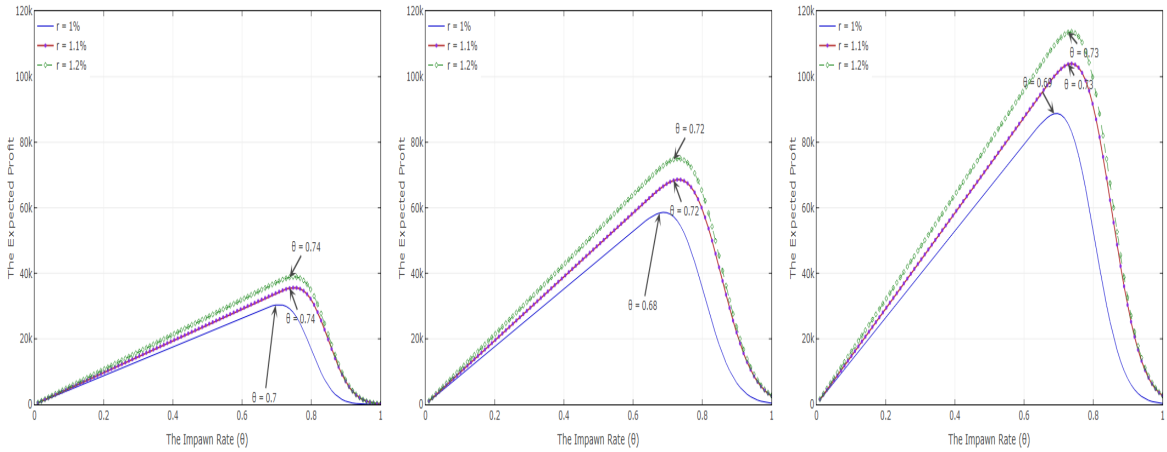


(a) AA (b) CP (c) TN

Figure 3.8: The Effect of Liquidity Risk on the Optimal Impawn Rate.

in the interest rate motivates the IFP to set a higher impawn rate as she is willing to take more risk to earn more money. However, when the interest rate increases from 1.1% to 1.2%, there is little effect on the optimal impawn and it is still 0.74, which shows the marginal effect of the interest rate is decreasing. Although an increase of the interest rate can motivate the borrower to increase the impawn rate to attract more funding demand, the IFP will still take financing risks. An overly high impawn rate will result in the IFP losing the utility gained from the increase in the interest rate. This kind of positive relationship between the interest rate and impawn rate has already been identified by the existing literature. For example, Aguiar & Gopinath (2006) and Wu et al. (2017) empirically justify the positive correlation between the default rate and interest rate. Take the repo market as an example; Eren (2014) claim that over-collateralisation drives down the repo (interest) rate. Within the mechanism of setting the interest rate, the impawn rate can indirectly affect the interest rate by influencing the probability that the borrower defaults (Bakoush et al. 2019) and the level of over-collateralisation (Eren 2014). Eren (2014) claim that over-collateralisation is increased by a higher haircut (lower impawn rate). Hence, repos with a higher haircut (lower impawn rate) receive a lower repo (interest) rate. However, none of these studies identify that the positive relationship between the impawn rate and interest rate is marginally diminishing. In addition, based on the analytical result in Fig. 3.9, we can see that the optimal impawn rate and optimal expected profit increase with an rise in the interest rate. Therefore, in general, if the determined impawn rate does not expose the IFP to more risk, it is easier for its to make more money.

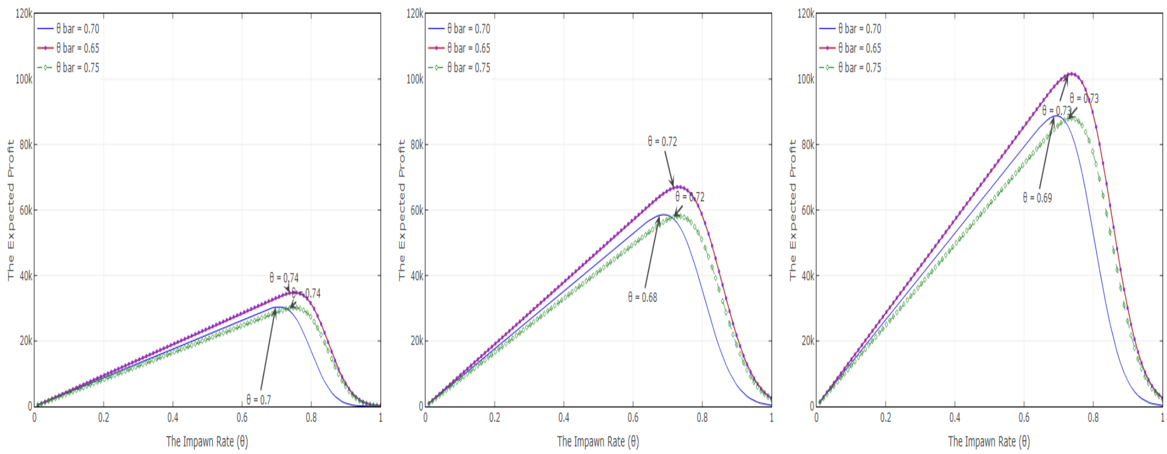
The industrial impawn rate is the standard impawn rate that is widely used in the inventory financing market. Different from previous sensitivity analysis, a strong relationship does not exist between the optimal impawn rate and the industrial impawn rate. However, the industrial impawn rate has a significant effect on expected profit. When the industrial impawn rate is low,



(a) AA (b) CP (c) TN

Figure 3.9: The Effect of the Interest Rate on the Optimal Impawn Rate.

the IFP can gain more profit from the inventory financing business providing the impawn rate is not set too high or too low. Based on the partial line before the optimal impawn rate in Fig. 3.10, it is found that the expected profit increases with the impawn rate increases. However, there is no such relationship on the partial line after the optimal impawn rate, which means the IFP needs to be especially careful to not set the impawn rate too high when the industrial impawn rate is high, as this will result in more uncertainties for the IFP. This finding generates some insights about the existing literature. Although the prevailing studies have already investigated the relationship between the impawn rate, liquidity risk, and the interest rate (Aguiar & Gopinath 2006, Boissel et al. 2017, Luo & Wang 2018, Wu et al. 2017), the investigation of how the industrial impawn rate influences the decision of individual financial service providers remains vague.



(a) AA (b) CP (c) TN

Figure 3.10: The Effect of the Industrial Impawn Rate on the Optimal Impawn Rate.

3.5 An Extended Model

In practice, the default probability of borrowers is influenced by not only the fluctuating collateral prices but also the borrowers' capital status. Therefore, when providing inventory financing services to borrowers, it is worthwhile to take the factor of the borrower into consideration, such as evaluating the capital status of each borrower and then calculating the optimal impawn rate for each collateral unit based on the modified function of default probability (See Fig. 3.11). In short, the extended model takes borrowers' financial status into consideration, along with their collateral portfolios.

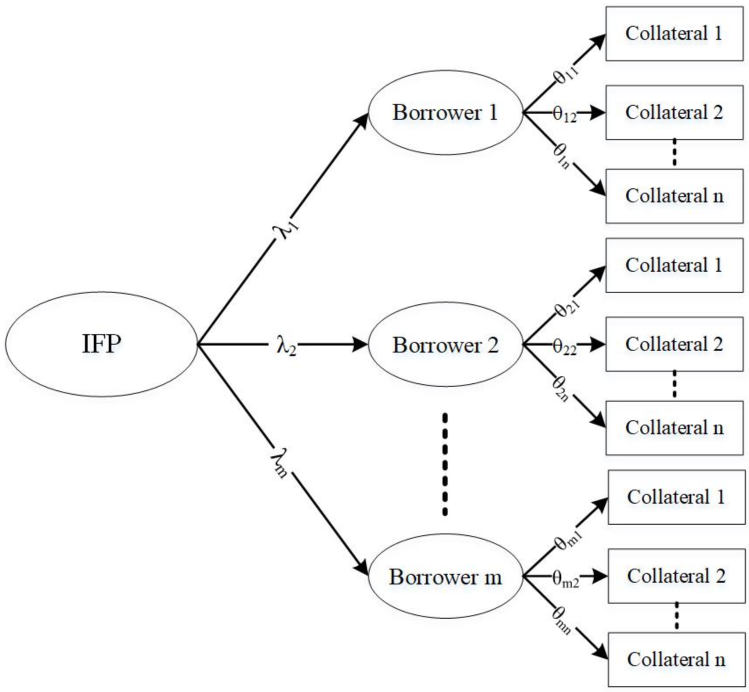


Figure 3.11: Optimal Impawn Rate for Individual Borrowers.

The probability that a borrower defaults is independent of the loan. In this case, the IFP would simultaneously consider the capital status of the borrower and the probability of loss on the i^{th} collateral unit; therefore, we can determine the function of joint probability distribution for the borrower on the i^{th} collateral unit:

$$\mathbb{P}^* = [1 - \exp(-\lambda_m)]\mathbb{P}(\ln P_i \leq \ln Z) \tag{3.23}$$

In Eq. (3.23), the exponential distribution function is used to measure the default probability of the borrower, which is similar to Kouvelis & Zhao (2012) and Wang et al. (2018). However, in contrast to these works, this study also considers the effect of fluctuating collateral prices on default probability, which can be dynamically parametrized with the Clayton canonical vine copula. Using Eq. (3.23), Proposition 3.1 can then further be extended to Proposition 3.4:

Proposition 3.4. *When the IFP can tolerate only the loss $\bar{M}_{i,0}$, $\bar{M}_{i,0} = \tau M_{i,0}$ (τ is the level of risk that the IFP can tolerate). For the m^{th} borrower, the probability of loss on the i^{th} collateral is* $\mathbb{P}^* = [1-\exp(-\lambda_m)]\mathbb{P}(\ln P_i \leq \ln Z)$. $P_i = \frac{p_{i,j}}{p_{i,j-1}}$ *and* $Z = \frac{\theta_i(1-\tau)\exp(kr)}{1-\rho_i}$.

Proposition 3.4 also describes the probability of loss on the i^{th} collateral unit, which is affected by both the impawn rate (θ_i) set by the IFP and capital status (λ_m). With an increase in the impawn rate set by the IFP and the deterioration of capital status, the probability of loss would also increase.

Based on Proposition 3.4, a new profit function for the i^{th} collateral unit of the m^{th} borrower can further designed:

$$\pi_i(\theta_i) = \sum_{j=1}^{m} [\alpha_i + \beta_i(\theta_i - \bar{\theta})](1-\mathbb{P}^*)[\exp(kjr) - \exp(k(j-1)r)] - G_i \tag{3.24}$$

$(1-\mathbb{P}^*)$ is the probability that the borrower who owns the i^{th} collateral unit does not default. Therefore, the new expected revenue for the i^{th} collateral unit in the j^{th} interval is $[\alpha_i + \beta_i(\theta_i - \bar{\theta})](1-\mathbb{P}^*)[\exp(kjr) - \exp(k(j-1)r)]$. Eq. (3.24) can be further simplified into:

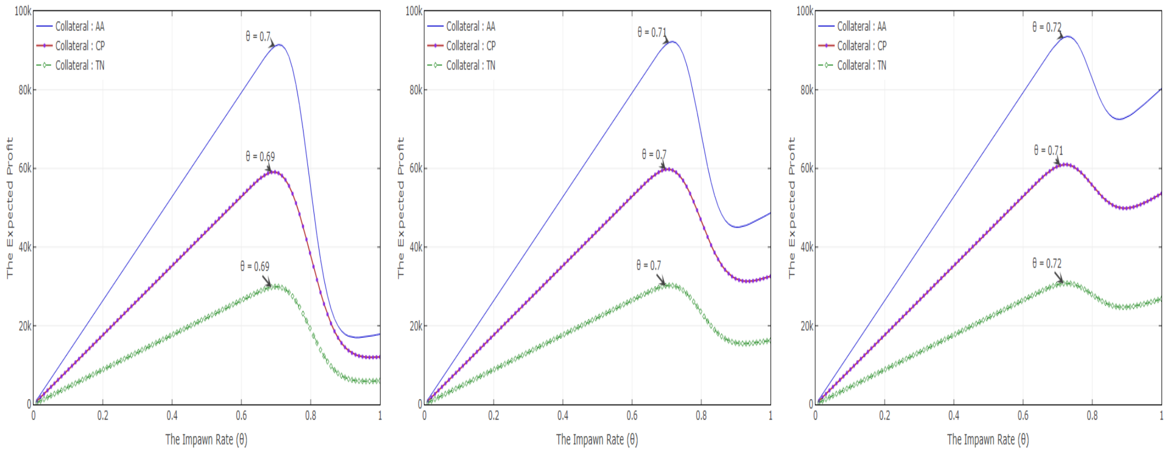
$$\pi_i(\theta_i) = [\alpha_i + \beta_i(\theta_i - \bar{\theta})](1-\mathbb{P}^*)[\exp(kjr) - 1] - G_i \tag{3.25}$$

To calculate the optimal impawn rate with Eq. (3.25), we can take its first-order and second-order derivative. Here, we have Proposition 3.5.

Proposition 3.5. *When* $\bar{F}(\ln\theta_i + \omega)$ *interacts with* $f(\ln\theta_i + \omega) + (1 - \frac{1}{1-\exp(-\lambda_m)})$*, there is a single optimal impawn rate* θ_i^* *in the interval* $(0, \min(\exp(\delta + \mu - \omega), 1))$ *that maximizes the profit of the IFP on the* i^{th} *collateral unit, and* $\omega = \ln\frac{1-\tau}{1-\rho_i} + kr$ *(The proof is in Appendix B.5).*

In Proposition 3.5, μ and δ are the mean and variance of simulated collateral returns. From the proof in B.5, we know that when $\bar{F}(\ln\theta_i + \omega)$ interacts with $f(\ln\theta_i + \omega) + (1 - \frac{1}{1-\exp(-\lambda_m)})$, there is an optimal impawn rate in the interval $(0, \min(\exp(\delta + \mu - \omega), 1))$ that maximizes the profit of the IFP on the i^{th} collateral unit. Otherwise, the profit function tends to be linear. Based on the calculated optimal impawn rate for each collateral unit, we can calculate the total optimal profit of the inventory financing business.

Based on Eq. (3.25), we can calculate the optimal impawn rate for borrowers who have a different capital status. Here, we have three borrowers, each of whom has his own capital status, which can be revealed by λ_m. Good capital status means λ_m is low, and bad capital status means λ_m is high. In the section on the sensitivity analysis, the capital statuses are separately set as 2, 1, and 0.5. The analytical results show that the optimal impawn rate decreases as the capital status of the borrower deteriorates. For the first borrower, the separate optimal impawn rates of AA, CP, and TN are 0.7, 0.69, and 0.69, respectively. For the second borrower, whose capital status is 1, the separate optimal impawn rates for the three kinds of collateral are 0.71, 0.7, and 0.7, respectively. For the third borrower, who has the best capital status, the separate optimal impawn rates for the



(a) The First Borrower (b) The Second Borrower (c) The Third Borrower

Figure 3.12: The Optimal Impawn Rate for Different Borrowers with the Same Collateral.

three kinds of collateral further increase to 0.72, 0.71, and 0.72, respectively. Therefore, borrowers who have good capital status usually have relatively low risk and thus as higher impawn rate can be set (See Fig. 3.12). Interestingly, on the right side of Fig. 3.12, the expected profit increases as the capital status of the borrower improves. Further, on the right side of Fig. 3.12b and Fig. 3.12c, there is an impawn rate that minimizes the expected profit. Therefore, when considering the factor of the borrower, there is a threshold. An impawn rate above that threshold can produce lower expected profit than the profit produced by the highest impawn rate that an IFP can set ($\theta_i = 1$). However, this does make sense. When a borrower has abundant capital ($\mathbb{P}^* \approx 0$), the function of expected profit would be linear (i.e., $\pi(\theta_i) = [\alpha_i + \beta_i(\theta_i - \bar{\theta})][\exp(kjr) - 1] - G_i$. In this case, to maximize the expected profit, the optimal impawn rate for this kind of borrower should be set as 1.

3.6 Conclusion

This research introduces impawn rate optimization into inventory financing from an IFP perspective, thereby helping IFPs improve their competitiveness in the inventory financing business. First, this study compares the predictive performance of MVN and copulas, justifying the advantage of copulas for simulating future returns of collateral. Furthermore, this study compares the expected profit produced by a uniform impawn rate and multi-impawn rates, showing that setting multi-impawn rates can help IFPs gain more profit. Based on the evaluation, a sensitivity analysis is provided to illustrate which factors IFPs should consider when setting multi-optimal impawn rates for different types of collateral. Finally, this study extends the model by providing inventory financing service to borrowers who have different capital statuses. By considering the factor of the borrower, IFPs can further differentiate their inventory financing business by providing customized impawn rates. The main research findings and contributions are as follows:

First, this study provides a new data-driven approach to set the optimal impawn rate in inventory financing, which has not been observed in the existing inventory financing literature (Buzacott & Zhang 2004, Wang et al. 2018, Zhang et al. 2016). To manage the default risk caused by fluctuating collateral prices, this study investigates how an IFP dynamically adjusts the impawn rate in inventory financing and varies from the existing literature in that the impawn rate is the given parameter (Boissel et al. 2017, Buzacott & Zhang 2004). To calculate the optimal impawn rate, this study first constructs the objective profit function of the inventory financing business, which comprises the function of funding demand, the function of default probability, and the revenue from inventory financing. Based on the returns generated by the predictive models for each funding cycle, this study dynamically modifies the parameters of the distribution function and derive the optimal impawn rate for inventory financing during each funding cycle.

Second, this study has identified an effective copula-based approach for the IFPs to dynamically evaluate the default probability for each funding cycle, which differs from the approaches in the existing literature that use the given parameters (e.g., Buzacott & Zhang (2004)) or the distribution function with unchanged parameters (e.g., exponential distribution, as mentioned by Wang et al. (2018)) to evaluate the default risk. To parameterize the distribution function, this study first determine an effective model to predict future returns of collateral. In finance and economics, bivariate copulas have been widely applied due to their effectiveness in capturing the dependence structure between pair time series and forecasting returns for pair assets (Fan & Patton 2014). By adopting the canonical vine copula, bivariate copulas can be further extended to multidimensional copulas to capture the dependence structure among multi-time series and predict returns for multiple assets (Aas et al. 2009). Based on the real data, thi study extends the bivariate Clayton copula to the Clayton canonical vine copula and compare its predictive performance with that of the multivariate normal distribution (MVN), a benchmark model that has been widely used to predict multidimensional returns in the financial field (Low et al. 2013). Through the comparative analysis, this study reveals that the Clayton canonical vine copula can better capture the dependence structure of collateral returns than the MVN and thus help the distribution function more accurately to evaluate the default probability for each funding cycle.

Third, the insights derived from the analysis provide important managerial implications that can help IFPs further improve their inventory financing business. More specifically, the analysis reveals relationships among the critical factors (e.g., impawn rate, interest rate, liquidity risk, risk-taking ability, and industrial impawn rate) and their effects on the financial performance of IFPs. For instance, the marginal effect of the interest rate on the optimal impawn rate gradually decreases; when the interest rate is low, a one-unit increase in the interest rate can greatly motivate the IFP to set a higher impawn rate. However, with the increase of default risk, the motivation for the IFP to set a higher impawn rate decreases. Similarly, the impawn rate and industrial impawn rate have no strong linear relationship. However, the industrial impawn rate has a strong effect on the expected profit. Therefore, with a decrease of the industrial impawn rate, the IFP can gain more profit from its inventory financing business providing she can set the optimal impawn rate. Therefore, when the inventory financing market is depressed and all lenders are inclined to set a low impawn rate, it is easier for the IFP to gain more profit if it can accurately evaluate the default probability.

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CHAPTER 4

DETERMINING IMPAWN AND INTEREST RATES IN INVENTORY FINANCING: THE GARCH-EVT-COPULA-BASED APPROACH

4.1 Introduction

The global economy has been enormously disrupted by the COVID-19 pandemic. Despite the unprecedented economic rescue packages issued by governments, many businesses face significant challenges in surviving the current economic crisis. Government funding has provided a lifeline for many businesses in the short term, however, this is not a long-term solution. Innovative financing approaches are vital to help firms weather economic storms and revive during times of economic recovery. Inventory financing is an emerging financing scheme that has attracted increasing interest among supply chain firms. Through inventory financing, capital-constrained businesses can utilize their high-value collaterals (e.g., industrial metals) to secure commercial loans (He et al. 2012). The fieldwork in the UK and China indicates that many businesses experience difficulty accessing financing, particularly SMEs without a strong credit record. Although new forms of financing schemes are welcomed by business managers, inventory financing can be particularly risky for inventory finance providers (IFPs) in a volatile market environment, as collateral prices fluctuate dramatically during times of crisis, such as that caused by the current global pandemic. At the early stage of the COVID-19 pandemic, the price of zinc dropped 27.26%, decreasing from $2,478.25 on 21/01/2020 to $1,802.75 on 24/03/2020 (LME 2021). Therefore, IFPs must implement appropriate mechanisms to mitigate the default risk caused by bad loans while maintaining financial returns.

Impawn and interest rates are the most commonly adopted mechanism to manage default risk according to the field research with IFPs and experts from the LME, which is the world center for trading industrial metals. Impawn rate is also known as the loan-to-value ratio. Among existing studies on the impawn rate for inventory financing, He et al. (2012) use the autocorrelation of collateral returns to determine the impawn rate for one collateral unit (i.e., steel). Considering the dependency structure of the time series of multiple collateral units, study I evaluates borrowers' default rates and derive the proper impawn rate for multiple collateral units. Furthermore, the increasingly market volatility caused by the COVID-19 pandemic has made it important for IFPs to consider both the autocorrelation and dependency structure of multiple collateral returns in setting impawn rates. While the impawn rate is critical for managing default risk, the interest rate determines the financial gain for IFPs (Alan & Gaur 2018). Therefore, IFPs make joint decisions on the impawn and interest rates to balance the trade-off between risks and financial gains. However, in the studies mentioned above, inventory financing's interest rates are assumed to be exogenous and not linked to impawn rates. To accurately estimate the interest rate, the relationship between interest and impawn rates should be considered.

Motivated by the practical challenge and theoretical research gap, this research proposes an innovative data-driven approach to include the risk of market volatility in determining the impawn and interest rates for different collateral units. The GARCH-EVT-Copula approach is employed as the GARCH model excels at considering autocorrelated collateral prices (Engle 1982, Sahamkhadam et al. 2018) and the dependence structure of collateral prices over time can be captured by the Copula model (Aas et al. 2009, Oh & Patton 2018). Two state-of-the-art copula approaches, including the Student T-Copula and the more flexible R-vine-Copula, are then considered and compared to the benchmark model, in which historical collateral prices are utilized to estimate the impawn rates for different collateral units. Using the Black & Scholes (1973) option pricing theory, this study constructs the impawn and interest rate model (IIRM) to determine the interest rate. According to the option theory, the inventory financing business can be regarded as a European call option, which limits execution to its expiration date (Vázquez 1998). The attribute of the European call option is consistent with the potential risk whereby borrowers may not repay money when the collateral price at the end of the funding period is lower than the loan value. Through a comparative analysis, this study evaluates the effectiveness of the three approaches in incorporating the market prices of collateral units to determine impawn and interest rates for inventory financing. Furthermore, this study extends the analysis of the proposed GARCH-EVT-Copula approaches to include the 2008 financial crisis and COVID-19 pandemic to examine the model performance under volatile market environments.

This study is organized as follows. Section 4.2 discusses relevant studies in the existing literature. Then, Section 4.3 outlines the research methodology, including the adopted descriptive and predictive models, and Section 4.4 justifies the data used for evaluation. Section 4.5 compares the performance of the three proposed approaches. Afterward, the parameterization process for the GARCH-EVT-Copula model is presented in Section 4.6, and the performance of the Copula based models during the COVID-19 period is analyzed in Section 4.7. Finally, Section 4.8 concludes with a discussion of analytical results and research contributions.

4.2 Related Literature

Inventory financing uses materials (e.g., primary alloys, copper, zinc) that have high marketability but low perishability as collateral. Compared with other financial services, the market volatility of collateral prices is the main source of risk for inventory financing (He et al. 2012, Kouvelis et al. 2017) and significant fluctuations in collateral prices expose IFPs to default risks. The existing inventory financing literature outlines several methods of managing this type of risk, including contract design (Kouvelis et al. 2017) and loan limits (Fewings 1992). For example, Kouvelis et al. (2017) compare supply chain contracts regarding their effectiveness in reducing the risk of inventory financing and improving the efficiency of the supply chain. They find that revenue-sharing contracts are flexible and effective in controlling risk through indexing raw material prices and using default penalties. Exploring efficient ways to set borrowers' credit limits, Buzacott & Zhang (2004) adopt multiple programming models to set loan amounts for different kinds of inventories, such as finished, in-progress products, and raw materials. Fewings (1992) studies how upper bounds on credit limits for supplier-led financing can be set by the Markov process.

A loan limit based on the value of collaterals is known as the impawn rate. Among the inventory financing studies on impawn rates, He et al. (2012) empirically demonstrate that the value-at-risk (VaR) method is efficient in determining the impawn rate for steel rebar, indicating a promising approach for controlling inventory financing risk. Study I reveals the superior performance of the copula approach over the multivariate normal distribution (MVN) in predicting the impawn rates of collateral units due to the advantage of capturing the dependency structure among prices of collateral units. In contrast to He et al. (2012) and study I, this study combines the GARCH-EVT approach and copula models to consider both the autocorrelations and dependency structure of the prices of different collateral units, ultimately improving the performance of the predictive model.[1]

For IFPs, another important mechanism for controlling inventory financing risk is setting a risk-adjusted price for the debt capital provided as a format of an interest rate (Altman et al. 2005). In the repurchase agreement (repo) market, the interest rate is also referred to as the repo rate (Gorton & Metrick 2012). The existing literature indicates that a change in interest rates influences the behavior of borrowers. For example, low interest rates often contribute to increasing demand for credit (Spyromitros & Tsintzos 2019) and and reduce the availability of safe bonds in interbank markets (Cociuba et al. 2016). Conversely, high interest rates reduce borrowers' payoff when their project succeeds and the loan is repaid, forcing them to reconsider determined interest rates (Chan & Thakor 1987). However, few studies have incorporated the interest rate as a measure for managing inventory financing risk. For instance, both He et al. (2012) and study I

[1] Adopting the AR (1)-GARCH (1,1)-GED model, He et al. (2012) only consider the autocorrelations of collateral prices. Using the copula model, Study I only considers the dependent structure among multiple time series in the process of optimizing the impawn rate.

assume the interest rate to be an exogenous factor rather than an endogenous decision variable. However, Buzacott & Zhang (2004) have proven that the interest rate for inventory financing is not lower than that for banking financing, although they have not investigated its settlement. In contrast to these studies, this study explores how to determine a suitable interest rate for inventory financing, considering the impawn rate.

This study deviates from the extant literature on inventory financing (Buzacott & Zhang 2004, He et al. 2012) by employing an integrated approach to set impawn and interest rates. Moreover, study I only considers the dependent structure among multiple time series using a copula model. To make the predictive model more accurate, this study considers both the autocorrelations of collateral prices and the dependencies among prices of different collateral units by combining the GARCH-EVT approach with two state-of-the-art copula models (Student T- and R-vine-Copula). Finally, the analysis covers the period of the COVID-19 pandemic and thus offers insights into how IFPs can provide inventory financing to businesses in need while managing risks in a volatile market environment. This has not been featured in the first study.

4.3 Model

This section details the models used to estimate the impawn and interest rates for different collateral units. This study adopts the ARMA-GARCH-copulas model to determine the impawn rate. Inventory financing can be regarded as the European call option as it gives borrowers the right to buy (call option) collateral units on a certain date at a specific price (Vázquez 1998). Based on the European call option model, this study derives interest rates by measuring the relationships between the interest and impawn rates, volatility, risk-free interest rates, and funding cycles.

4.3.1 Model Assumption

Considering the fluctuating collateral prices, the IFP must employ effective tools and methods to manage price risk in inventory financing. The direct conversations with IFPs confirm that impawn and interest rates are the most widely used tools for managing the risks associated with inventory financing. However, only borrowers with no significant weaknesses in their balance sheet or daily operations can be involved in the inventory financing service. IFPs check whether a borrower has exhibited fraudulent behaviors, such as using one warehouse receipt to acquire multiple loans. Therefore, in the model setup, this study assumes that the balance sheet and daily operations of the borrower have no significant issues and that the borrower has no intention to use one warehouse receipt to acquire multiple loans.

4.3.2 Setting the Impawn Rate

This section introduces the construction process for the GARCH-EVT-Copula model and the steps involved in deriving the impawn rate. The GARCH-EVT-Copula model is constructed with ARMA-GARCH and copula models. The former model considers the autocorrelations of collateral prices (Sahamkhadam et al. 2018) and the latter describes the dependency structure for the time series of collateral units (Oh & Patton 2018).

4.3.2.1 ARMA-GARCH Model

One part of the predictive model is based on the ARMA-GARCH. The mean equation is derived from a recursive volatility process. The conditional variance $\sigma^2_{j,t}$ is following a GARCH(1,1) process, and the conditional mean μ_{jt} is following an ARMA(1,1) process. The combined ARMA(1,1) and GARCH(1,1) model can be described as:

$$
\begin{cases}
x_{j,t} = \mu_{\mathrm{jt}} + \varphi_j\left(x_{j,t} - \mu_{j,t}\right) + \gamma_j \varepsilon_{j,t-1} + \varepsilon_{j,t} \\
\varepsilon_{j,t} = z_{j,t}\sigma_{j,t} \\
z_{j,t} \approx (i.i.d.) \\
\sigma^2_{j,t} = \omega_j + \alpha_j \varepsilon^2_{j,t-1} + \beta_j \sigma^2_{j,t-1}
\end{cases}
\tag{4.1}
$$

In Eq. (4.1), the actual returns for collateral $j = 1, 2, \ldots, n$ are represented by $x_{j,t}$ and the standardized residuals are represented by $z_{j,t}$. Additionally, the relevant parameters are constrained as follows: $\omega_j > 0$, $\alpha_j \geq 0$, $\beta_j \geq 0$ and $\alpha_j + \beta_j \leq 1$, which is the stationarity condition. Negative parameters cannot be used in the ARMA(1,1) - GARCH(1,1) model (Engle 1982).

4.3.2.2 Tail Behaviour Based on the EVT

For standardized residuals z_{jt}, their tail distributions are depicted by the generalized Pareto distribution (GPD) (Embrechts et al. 2013). The threshold is defined as u, and $y = z_t - u$ is used to define an excess over u. Correspondingly, the conditional probability distribution of y can be defined as:

$$
F_u(y) = P\{Z - u \leq y | Z > u\} = \frac{F(y+u) - F(u)}{1 - F(u)}
\tag{4.2}
$$

Based on EVT, as the threshold u increases, $F_u(y)$ gradually converges to a GPD:

$$
G_{\xi,\beta}(y) = \begin{cases} 1 - (1 + \xi y/\beta)^{-1/\xi} & if\ \xi \neq 0 \\ 1 - exp(-y/\beta) & if\ \xi = 0 \end{cases}
\tag{4.3}
$$

In Eq. (4.3), ξ and β represent shape and scale parameters, respectively. Here, $\xi > 0$, $G_{\xi,\beta}(y)$ corresponds to a fat-tailed distribution, and $\xi < 0$, $G_{\xi,\beta}(y)$ corresponds to a light-tailed distribution. When $\xi = 0$, $G_{\xi,\beta}(y)$ is following gamma or normal distribution.

Based on the above description, the selection of the threshold has strong impact on the shape of the tail distribution. If the threshold is set too high, there will not be sufficient excess data to parameterize the distribution function. When the threshold is set too low, the distribution function may have biases. In this study, a mean excess function is adopted to select an appropriate threshold that has been widely used in existing studies (Sahamkhadam et al. 2018, Karmakar & Paul 2019). After the threshold is determined, the values of the parameters ξ and β are estimated through a maximum likelihood estimation (MLE) method. Here, this study adopts a goodness-of-fit semiparametric estimation method with middle part fitted by the Gaussian distribution and the right and left tails of the distribution fitted by the GPD.

$$F(z) = \begin{cases} (1 - \frac{N_u^L}{N})(1 + \xi^L \frac{z-u^L}{\beta^L})^{-1/\xi^L} & z_j < u^L \\ \Phi(z) & u^L \leq z_j \leq u^R \\ \frac{N_u^R}{N}(1 + \xi^L \frac{u^R - z}{\beta^R})^{-1/\xi^R} & z_j > u^R \end{cases} \tag{4.4}$$

4.3.2.3 Copula

The marginal behavior of individual values and the dependency structure among them can be revealed by the cumulative joint distribution function (CDF). However, the CDF can be alternatively expressed if a vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ of random variables with a joint cumulative distribution function (CDF) $F(x_1, x_2, \ldots, x_n)$ and marginal distributions $F_i\,(i = 1, 2, \ldots, n)$ are considered. Then, the dependence structure among the marginal distribution functions can be described by a copula (Sklar 1959), which is as follows:

$$F(x_1, x_2, \ldots, x_n) = C[F_1(x_1), F_2(x_2), \ldots, F_n(x_n)]. \tag{4.5}$$

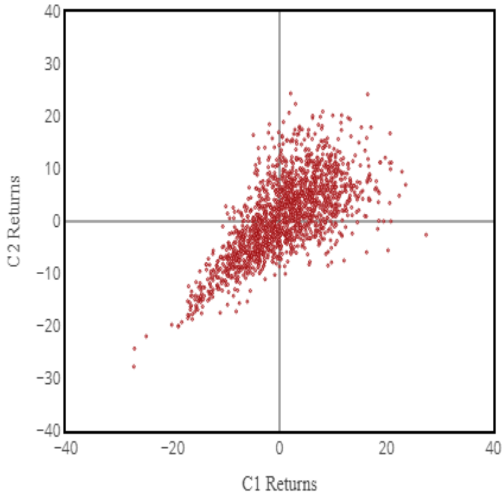
By using transformations $F_i(X_i) = U_i$, the copula from Eq. (4.5) can be expressed as:

$$F(x_1, x_2, \ldots, x_n) = C[F_1(x_1), F_2(x_2), \ldots, F_n(x_n)] = C(u_1, u_2, \ldots, u_n) = P(U_1 \leq u_1, U_2 \leq u_2, \ldots, U_n \leq u_n), \tag{4.6}$$

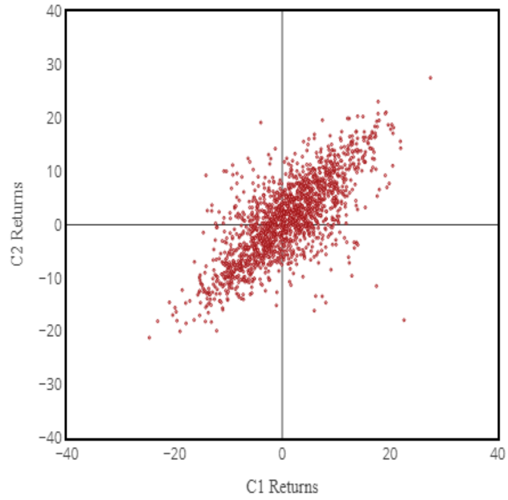
where $C(u_1, u_2, \ldots, u_n)$ is a CDF for a multivariate vector with uniform margins and supported by $[0,1]^n$. If the copula function C and the marginal CDF F_i in Eq. (4.6) are differentiable, then the density of the copula $c(u_1, u_2, \ldots, u_n)$ and the joint density function $f(x_1, x_2, \ldots, x_n)$ are separately defined as:

$$c(u_1, u_2, \ldots, u_n) = \frac{\partial^n C(u_1, u_2, \ldots, u_n)}{\partial u_1, \partial u_2, \ldots, \partial u_n}, \tag{4.7}$$

$$f(x_1, x_2, \ldots, x_n) = c_{1,2,\ldots,n}[F_1(x_1), F_2(x_2), \ldots, F_n(x_n)] \cdot f_1(x_1) \cdot f_2(x_2) \ldots f_n(x_n). \tag{4.8}$$



(a) Returns Simulated by the Clayton Copula ($\rho_{12} = 2$).



(b) Returns Simulated by the Student T Copula ($\rho_{12} = 0.75, \upsilon_{12} = 3$).

Figure 4.1: Simulations Made by the Bivariate Copula.

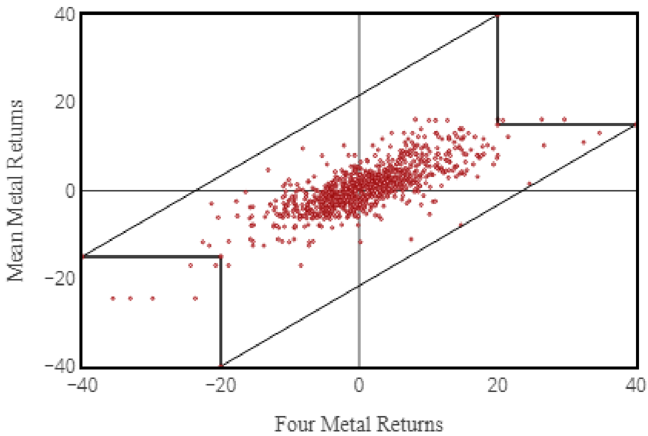
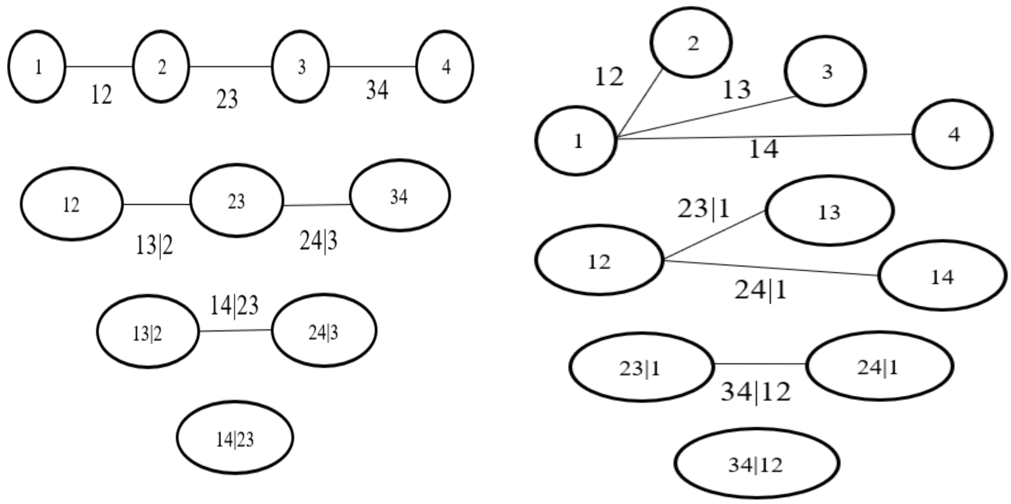


Figure 4.2: Empirical Relationships among Four Collateral Returns.

Collateral units in the financial market have a strong comovement. The Clayton and Student T-Copula describe the low-tail structure (see the left corner of Figure 4.1a and 4.1b). Compared with Student T-Copula, the Clayton copula uses a single parameter to describe the dependency structure among multiple time series, which makes its prediction less accurate as the dimension increases. Therefore, for the multiple copula family, the Student T-Copula is adopted to simulate the dependence structure for collateral returns (its density function is illustrated in Appendix C.2). Alternatively, a more flexible vine copula constructed by bivariate copulas can be adopted as it can depict different levels of dependence structures through iterative conditioning (Dißmann et al., 2013). Aas et al. (2009) popularize two subgroups of regular vines: drawable (D-vines) and canonical vines (C-vines). Both subgroups have limitations regarding their structures, as D-vines have path structures while C-vines possess star structures in their tree sequence (see Figure 4.3). This study adopts the most flexible regular vine copula (R-vine) without limiting the vine copula to certain structures. R-vine has seldomly been used as there exists a huge number of possible R-vine tree sequences. More recent studies have found that it is possible to determine the most suitable R-vine-Copula using an automated strategy to jointly search for an appropriate R-vine tree structure (Dissmann et al. 2013). In this study, both the Student T- and the R-vine-Copula are adopted to identify whether the R-vine-Copula has superior performance in setting impawn rates.



(a) A D-vine with 4 Variables, 3 Trees and 6 Edges.

(b) A C-vine with 4 Variables, 3 Trees and 6 Edges.

Figure 4.3: The Structure of D-vine and C-vine Copulas.

4.3.2.4 VaR, Efficiency and Risk Measurement

VaR is used to measure the risk of loss in the investment. Given a probability, it can gauge how much the investment might lose in a normal market condition within a certain time period. Firms and regulators in the financial industry usually use VaR to estimate the number of assets required to cover possible losses. In this study, a GARCH-EVT-Copula model is constructed to run a Monte Carlo simulation to produce 10,000 returns for each collateral unit. Based on the simulated returns, this study follows Basel III (2019) and adopts VaR at the 99^{th} percentile to estimate the impawn rate. The formula of VaR can be represented as:

$$\text{VaR}_\alpha\left(X_j\right) = -\inf\left\{x_j \in \mathbb{R} : F_{X_j}\left(x_j\right) > \alpha\right\} = F_{Y_j}^{-1}(1-\alpha), \tag{4.9}$$

where X_j is the distribution of loss for the j^{th} collateral unit. Then the smallest number y is the VaR at level $\alpha \in (0,\ 1)$, which makes the probability $Y_j = -X_j$ is at least $1-\alpha$ and does not exceed y_j. Furthermore, $VaR_\alpha(X_j)$ is the $(1-\alpha)$ quartile of Y_j and F_{X_j} is the CDF of the random variable X_j.

4.3.3 Steps for Determining Impawn Rate

In this section, the following parameters are defined: L = window length, T = funding cycle. For the out-of-sample period estimation, this study uses the rolling-window method. Specifically, this study repeats steps 1 to 7 below for each period, i.e., all parameters for each funding cycle are estimated to determine the impawn rate. The entire procedure is summarized below.

Step 1: Use Eq. (4.1)'s ARMA(1,1)-GARCH(1,1) to match collateral returns and estimate the parameters through MLE and obtain standardized residuals:

$$Z_j = (z_{j,t}, z_{j,t+1}, \ldots, z_{j,t+T}), z_j \approx i.i.d.,\ \forall j, t \in [t_0,\ t_0 + L - 1]. \tag{4.10}$$

Step 2: Estimate the parameters in Eq. (4.4) based on estimated standardized residual vector Z_j produced by Step 1. Then, based on the parameterized CDF, the standardized residuals are transformed to uniform variables.

$$u_{jt} = F\left(z_{jt}\right), t \in [t_0,\ t_0 + L - 1],\ u_{jt} \in U(0,1). \tag{4.11}$$

Step 3: Apply Sklar's theorem, insert the estimated uniform variables from Step 2 in the Student T- and R-vine-Copulas, and use MLE to evaluate the parameters of the Student T- and R-vine-Copulas.

Step 4: Use the parameterized Student T- and R-vine-Copulas to generate 10,000 uniform random numbers for each collateral unit. Then, use the inverses of estimated marginal distribution functions generated in Step 2 to convert generated random numbers to standardized residuals.

Step 5: Substitute the new standardized residuals from step 4 into the estimated ARMA(1,1)-GARCH(1,1) forecasting model in Eq. (4.1) and generate 10,000 returns for each collateral unit. Then convert the logarithmic returns into the arithmetic return matrix $R_j = \exp\left(X_j\right) - 1$.

Step 7: Set the confidence level to 99% and use Eq. (4.9) to obtain the VaR, then, use the VaR to obtain the impawn rate $\theta_j = 1 - VaR_{0.99}(X_j)$.

4.3.4 Setting the Interest Rate

Once impawn rates are determined, the next step is to set proper interest rates for collateral units. Relevant notations in this process have been summarized in Table 4.1. According to Black

Table 4.1: Notation.

Symbol	Description	Assumption
M_t	Value of risky mortgage at time t	$M_t > 0$
G_t	Guaranteed value of the collateral unit at time t	$P_t \geq 0$
N_t	Value of the loan without default risk at time t	$N_t > 0$
T	Length of the funding cycle	$0 < T$
N_T	Value of the loan without default risk	$N_T > 0$
r	Interest rate provided by the IFP (decision variable)	$\overline{r} \leq r$
$\overline{r}$	Risk-free interest rate in the financial market	$0 < \overline{r}$
θ	Impawn rate	$0 < \theta < 1$
M_0	Value of loan with risk at the beginning of funding cycle	$M(t) > 0$
V_0	Value of the collateral at the beginning of funding cycle	$V_0 \geq 0$
V_t	Value of collateral obeys geometric Brownian motion	$V(t) \geq 0$
M_T	Sum of the capital and interest rate	$M_T > 0$
σ^2	Variance of collateral returns	$\sigma^2 > 0$

& Scholes (1973), when M_t is the value of a risky loan at time t, G_t is the guaranteed value at time t and N_t is the value of the loan without risk at time t, we have the following equation:

$$G_t = N_t - M_t \tag{4.12}$$

When the length of the funding cycle is T, the sum of the interest and loan at time t is M_t, the interest rate for inventory financing is r, the industrial interest rate is $\overline{r}$, and M_T is the value of the loan with risk at time T, then $M_T \exp(-\overline{r}(T-t))$ is the value of the loan without risk at time t and Eq. (4.12) becomes:

$$G_t = M_T \exp(-\overline{r}(T-t)) - M_t \tag{4.13}$$

Since $M_T \exp(-r(T-t))$ is the value of the loan with risk at time t, the impawn rate (θ) at the beginning of the funding cycle can be represented as:

$$\theta = \frac{M_0}{V_0} = \frac{M_T \exp(-rT)}{V_0} \tag{4.14}$$

The impawn rate equals the ratio between the value of the loan with risk at the beginning of the funding cycle and the value of the collateral at the beginning of the funding cycle V_0. Hence, the guaranteed value of the collateral unit at the beginning of the funding cycle can be expressed as:

$$G_0 = M_T \exp(-\overline{r}T) - M_0 = M_T \exp(-\overline{r}T) - M_T \exp(-rT) \tag{4.15}$$

Based on Eq. (4.14) and Eq. (4.15), the equation for the impawn rate can be converted to:

$$\theta = \frac{M_0}{V_0} = \frac{M_T \exp(-\bar{r}T) - G_0}{V_0} \tag{4.16}$$

When the values of M_T, T, $\bar{r}$ and M_0 are given in Eq. (4.16), the impawn rate θ can be derived as long as G_0 can be calculated (G_0 can be regarded as the value of a European put option). V_t is the value of the collateral at time t, which obeys geometric Brownian motion. Based on the option pricing model constructed by Black & Scholes (1973), the impawn rate can be written as:

$$\theta = \frac{V_0 N(-d_1) + M_T \exp(-\bar{r}T) N(d_2)}{V_0} \tag{4.17}$$

The deduction process is illustrated in Appendix C.1.

Lemma 4.3.1. *Based on the above equations, the interest rate can be derived from the following implicit function:*

$$\exp(-(r-\bar{r})T) = N(-\frac{\sqrt{T}}{\sigma}(r - \left(\bar{r} - \frac{\sigma^2}{2}\right) + \frac{\ln\theta}{T})) \tag{4.18}$$

To derive the interest rate, this study first adopts a predictive model to determine the impawn rate (θ). Here, the GARCH-EVT-Copula models are chosen as they can describe autocorrelative collateral prices and the dependency structure for time series (Sahamkhadam et al. 2018, Karmakar & Paul 2019). The VaR is used to derive the impawn rate (θ) according to collateral returns simulated by GARCH-EVT-Copula models. Based on the impawn rate and other parameters, Eq. (4.18) is used to calculate the interest rate for inventory financing.

4.4 Data

The data set contains daily returns of four high-value metals, including prime aluminum (aluminum), copper, nickel, and zinc. They are selected as the sample collateral units because, based on the interactions with the inventory finance providers, it is known that they are commonly used as collateral for inventory financing due to the nature of industrial commodity and high liquidity. Furthermore, data concerning these metals are easily accessible compared to other collateral units as they are all traded on the LME and the volatility in metal prices since the outbreak of COVID-19 allows us to test the model performance in an extremely volatile environment (see the extended analysis). The period for collateral returns in the initial analysis starts from 01/01/2000 to 31/12/2019, yielding 20,220 observations in total. This study implements a rolling window approach with observations covering the first ten-year period, which is similar to the approach employed by Sahamkhadam et al. (2018). A ten-year observation window provides more information regarding dependence structures among time series, which can increase the accuracy of parameterizing the copula model (Oh & Patton 2018). Moreover, evaluating the parameters of the GPD requires sufficient observations, especially for the lower and upper thresholds (see Eq. (4.3)) (Sahamkhadam et al. 2018). The out-of-sample forecasting is performed from 01/01/2010 to 31/12/2019. In this study, the funding cycle is set to 6 months, creating 20 funding cycles.

Based on the descriptive analysis in Table 4.2, each type of metal has a kurtosis above 3, which indicates that all the return series have serious outlier problems. For the Jarque-Bera test of normality, all null hypotheses are rejected at the 1% level so that, in each case, the skewness and kurtosis could match a normal distribution. The returns of all the collateral units exhibit negative skewness; only the tails for all return series are on the left sides of the distributions. Nickel exhibits both the highest (13.31%) and lowest (-18.36%) daily returns for the sample.

Table 4.2: Descriptive Analysis.

	Max	Min	Mean	Standard deviation	Skewness	Kurtosis	Jarque-Bera
Aluminum	6.4	-8.26	0	0.014	-0.17	5.48	1321.43***
Copper	11.73	-10.36	0.02	0.016	-0.10	7.46	4213.87***
Nickel	13.31	-18.36	0.01	0.023	-0.13	6.59	2742.17***
Zinc	9.95	-11.47	0.01	0.019	-0.16	5.78	1659.69***

Table 4.3 shows the sample Kendall rank correlations. The highest correlation (0.50) is between zinc and copper and the lowest (0.33) between nickel and aluminum. The correlations among collateral units reveal the possibility to improve forecasting accuracy by capturing the dependency structure for the time series (Aas et al. 2009).

Table 4.3: Kendall Correlation.

Collateral Unit	Aluminum	Copper	Nickel	Zinc
Aluminum	1	0.47	**0.33**	0.42
Copper	0.47	1	0.40	0.50
Nickel	0.33	0.40	1	0.36
Zinc	0.42	**0.50**	0.36	1

Based on the data through steps 1 to 6 introduced in Section 4.3.3, an impawn rate and variance for each collateral unit can be obtained, based upon which Eq. (4.18) can be used to calculate the interest rate for inventory financing. In the following sections, the statistical results and analysis are presented.

4.5 Results and Analysis

To determine the interest rates for inventory financing, we first need to estimate the impawn rate for each funding cycle. This study sets the rolling window to 10 years ($L = 10$) and the funding cycle to 6 months ($T = 6$). Then we can iteratively follow Steps 1 to 7 introduced in Section 4.3.3.

4.5.1 Impawn Rate

To measure the performance of a GARCH-EVT-Copula, a straightforward alternative is to use historical data from each collateral unit to estimate the impawn rate (see the first subfigure in Figure 4.4). For example, the historical data has been used to set weights for each asset to measure the performance of predictive models (Low et al. 2013, Sahamkhadam et al. 2018). The interactions with relevant stakeholders further indicate that this approach has been widely used in the industry. In this study, the benchmark model is based on the VaR for 6-month periods of collateral returns over the previous 10 years. The next two subfigures in Figure 4.4 present the impawn rates generated by the GARCH-EVT-Copula models. Again, the rolling window approach is adopted, which rolls the collateral returns from the previous 10 years to estimate the impawn rate for the current funding cycle.

Figure 4.4 shows the different impawn rates that are generated by the three approaches. For the historical approach, the impawn rates of all collateral units are stable throughout the first 18 funding cycles. For the Student T- and R-vine-Copula-based approaches, it is found that only the impawn rate for aluminum is stable, indicating that aluminum is less volatile throughout all funding cycles. Correspondingly, aluminum is considered to be the least risky collateral among the four collateral units. Therefore, it should have the highest impawn rate. Compared with aluminum, the impawn rates for the other collateral units are relatively low and volatile. The impawn rates of copper, in particular, fluctuate more dramatically and produced the lowest impawn rate in the middle period according to the observations of both copula approaches (see the lower subfigure in Figure 4.4). Copper has the lowest estimated impawn rates from the tenth to sixteenth funding cycles. Its impawn rates are extremely low in the 12th and 13th funding cycles (31.04% for Student T-Copula in the 12th funding cycle and 16.5% for R-vine-Copula in the 12th funding cycle), suggesting that an IFP should provide a small amount of loan for one unit of copper. To explore the underlying reason, a regression analysis is implemented for the variance of the simulated returns; impawn rates and regression results are displayed in Table 4.4. The table shows the regression of the variance on the impawn rate for the four collateral units. Under the corresponding regression coefficients, T-statistics are reported in parentheses. Importantly, when the variance of a simulated return changes, the change in the impawn rate is statistically significant, suggesting that a collateral unit with a large variance produces a lower impawn rate. Additionally, the impawn rate for copper had the largest impact on the variance of the simulated returns (coefficient = -4.15 for Student T-Copula and coefficient = -6.54 for R-vine-Copula), indicating that a large variance can lead to an extremely low impawn rate (the variances for the simulated returns are reported in Appendix 4.7). We can see that the coefficient for the variances of four collateral units in Panel B is higher than that in Panel A, indicating that the effect of variance on the impawn rate generated by the R-vine-Copula is more significant than that generated by the Student T-Copula (see Table 4.4).

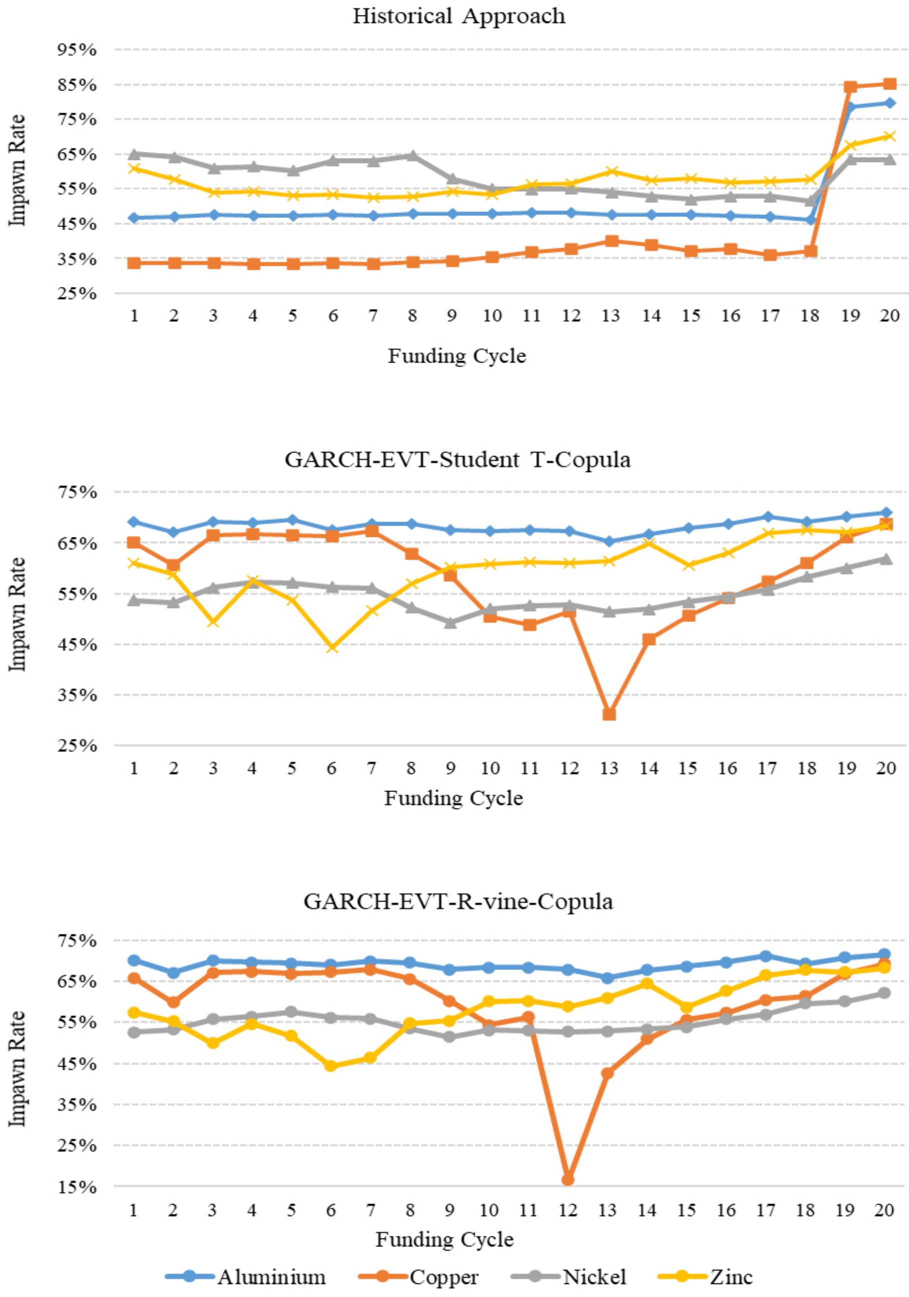


Figure 4.4: Impawn Rate Generated by Three Approaches.

Table 4.4: Regression of the Variance on the Impawn Rate.

	Aluminum	Copper	Nickel	Zinc
Panel A (Student T-Copula)				
	Aluminum	Copper	Nickel	Zinc
Variance	-1.69	**-4.15**	-1.53	-2.5
	(-2.41)	(-7.81)	(-4.08)	(-11.87)
R^2	0.24	0.77	0.48	0.89
Panel B (R-Vine-Copula)				
	Aluminum	Copper	Nickel	Zinc
Variance	-1.93	**-6.54**	-1.77	-2.62
	(-2.97)	(-6.77)	(-6.36)	(-18.46)
R^2	0.32	0.71	0.68	0.95

4.5.2 The Performance of Three Approaches in Terms of Setting the Impawn Rate

To measure which method can better estimate the impawn rates, the efficiency losses for the impawn rates generated by each approach are compared. Eq. (4.19) developed by He et al. (2012) is used to compare the efficiency losses of the methods.

$$EL_j = \frac{V_{j,t+T} - V_{j,t}\theta}{V_{j,t}}, \tag{4.19}$$

where $V_{j,t}$ is the value of the j^{th} collateral unit at time t, $V_{j,t}\theta$ is the amount of funding provided by the IFP at time t, T is the length of the funding cycle, and $V_{j,t+T}$ is the value of the j^{th} collateral unit at the end of the funding cycle. $EL_j < 0$ means that the impawn rate will fail to control the risk of the j^{th} collateral unit. When $EL_j \geq 0$, a lower EL_j value indicates that the corresponding approach is more effective in setting the impawn rate.

It is found that the efficiency losses generated by the three approaches for the four collateral units in all funding cycles to be above 0, indicating that the historical approach and the two copula-based approaches can manage the risks associated with the collateral units. To better measure the performance of each approach, we sum the efficiency losses for the collateral units; the cumulative efficiency losses for each approach across 20 funding cycles are presented in Figure 4.5. Although all three approaches could control the risk involved in inventory financing, the two copula-based approaches can help IFPs to reduce losses in efficiency and extract more value from the collateral. Figure 4.5 shows that the copula-based approaches produce lower efficiency losses than the historical approach in 18 out of 20 funding cycles, indicating that the copula-based approaches are more efficient in setting the impawn rate. This finding is partially consistent with He et al. (2012), who demonstrate that the efficiency loss generated by AR(1)-GARCH(1,1) is less than that of using a fixed impawn rate. A lower efficiency loss results in a higher impawn rate, which also implies that IFPs can manage less collateral for the same loan amount. Therefore, the lower efficiency loss generated by the copula approaches not only improves the competitiveness of IFPs but also reduces managerial costs regarding collateral units. Furthermore, for the copula-based approaches, the R-vine-Copula generates lower efficiency loss than the Student T-Copula in 14 funding cycles (see the grey bar in Figure 4.5), indicating that a more flexible R-vine-Copula can extract more value from collateral units.

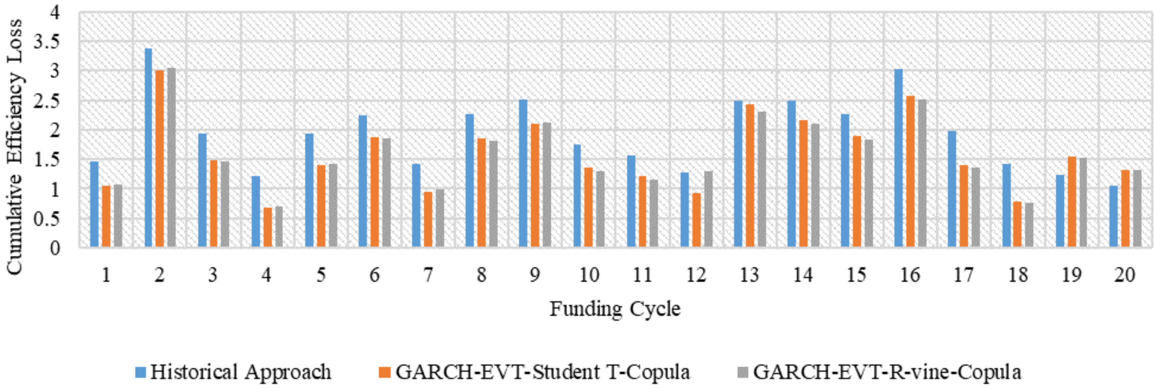
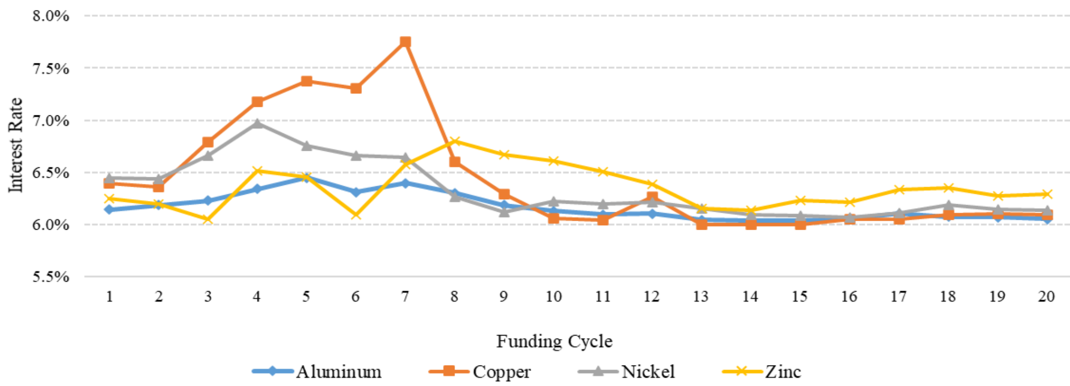


Figure 4.5: The Cumulative Efficiency Loss.

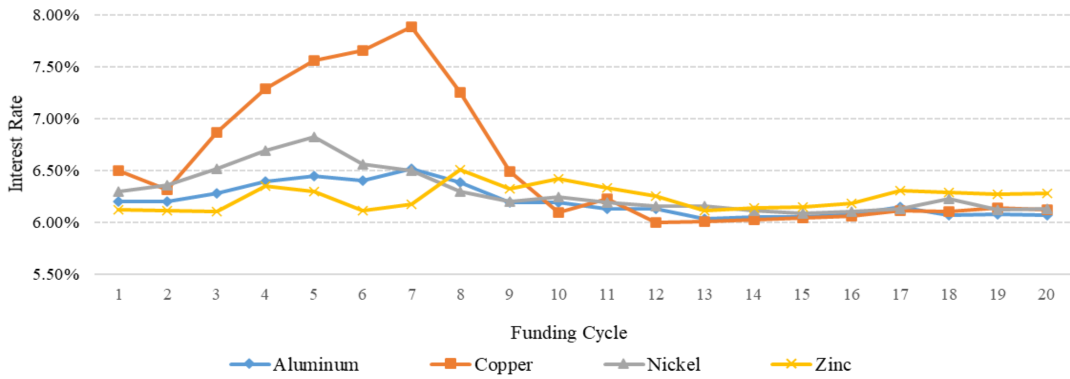
4.5.3 Setting the Interest Rate

Based on the previous analysis, it is found that the GARCH-EVT-Copula models have a lower efficiency loss than the historical approach. Based on the determined impawn rates, Eq. (4.18) is used to derive the corresponding interest rates for inventory financing. In the inventory financing industry, the lowest interest rates range from 4.5% to 10% at the 6-month interval (Zimmermann, 2021). To test the performance of the proposed approach, the industrial interest rate in this study is set to 6% and the funding cycle remains at the 6-month interval ($T = 0.5$). These assumptions do not affect the analytical result. Based on the impawn rates and the variance of the data generated by the GARCH-EVT-Copula models, the interest rates are derived as shown in Figure 4.6.

It is found that the interest rates for all four collateral units are relatively high and fluctuate in the first eight funding cycles. This pattern of high interest rates for industrial commodity-based inventory financing occurs as a long period is required for the global economy to recover from the 2008 financial crisis before generated returns became more stable. The results in Figure 4.6 also indicate that the determined interest rates for inventory financing are related to both impawn rates and variance. Although the impawn rate for aluminum is the highest among those for the four collateral units (see Figure 4.4), aluminum's interest rate is relatively low (see the blue line in Figure 4.6). This result is attributed to the fact that aluminum has the lowest variance throughout all funding cycles. The low variance indicates a relatively low-risk level for aluminum compared to other collateral units, suggesting a low interest rate (the variances for the collateral units are presented in Figure 4.7. Figure 4.7 presents the variance of returns



(a) GARCH-EVT-Student T-Copula model.



(b) GARCH-EVT-R-vine-Copula model.

Figure 4.6: The Interest Rates Generated by the GARCH-EVT-Copula Models and IIRM.

for different collateral units. From the figure, we can see that aluminum continually produces low variance. The variance for copper and zinc is quite unstable and produces extremely high variance in certain time periods. Specifically, zinc produces extremely high variance from fifth to eighth funding cycles. Copper produces extremely high variance from eleventh to fourteenth funding cycles. Copper's large variance in the first seven funding cycles suggests a high-risk level, indicating that a high interest rate should be set.

As both impawn and interest rates have been derived, the IFP's profit could then be calculated with respect to the three approaches. Profit is calculated using Eq. (4.20), where $i = H,\ S$or R, representing the three methods used to estimate the impawn and interest rates. There are 20 simulations. Theoretically, the ideal approach should be the one that consistently produces higher profit in each funding cycle.

$$\pi^i = \sum_{j=1}^{n} V_{j,t}\theta_{ji}r_{jt} \qquad i = H,\ S \text{ or } R. \tag{4.20}$$

The comparative results are shown in Figure 4.8. It can be observed that the combination of

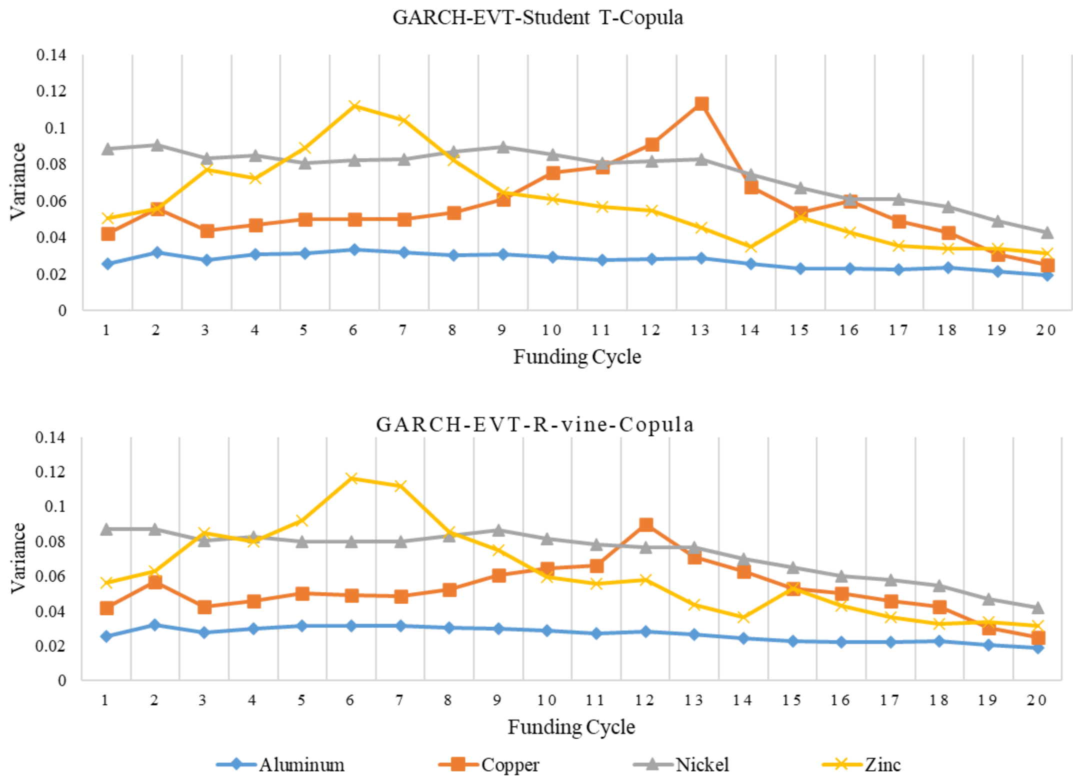


Figure 4.7: The Variance of Returns Simulated by the GARCH-EVT-Copula Approaches.

Copulas and IIRM can improve the financial performance of IFPs across 17 funding cycles. It can also be found that the copula-based approaches perform better for both volatile (e.g., copper) and stable (e.g., aluminum) collateral units (see Figure 4.9). Specifically, the copula-based approaches have superior performance in 17 funding cycles for copper and 18 funding cycles for aluminum. Additionally, as shown in Figure 7, it is found that the R-vine-Copula generates larger profits than the Student T-Copula in 14 funding cycles.

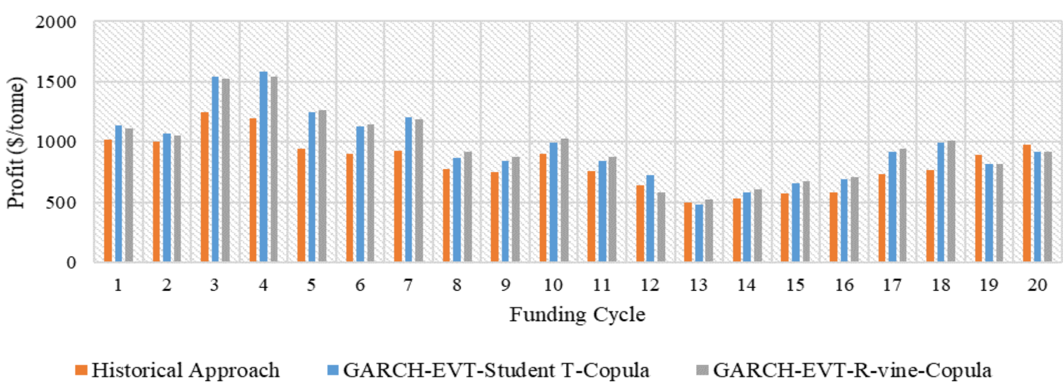
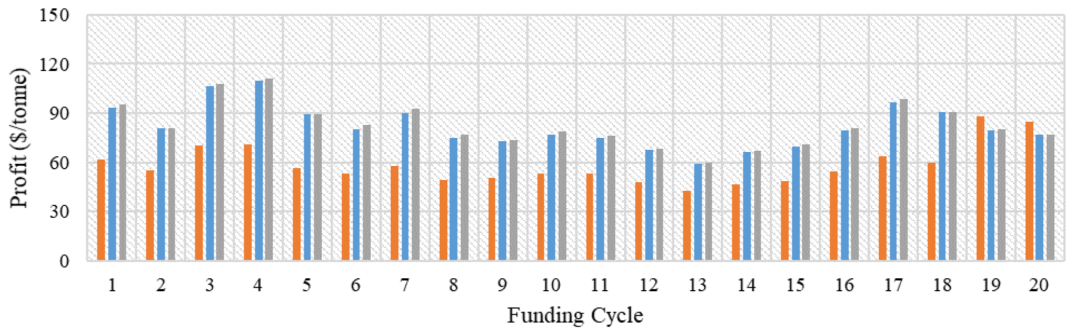
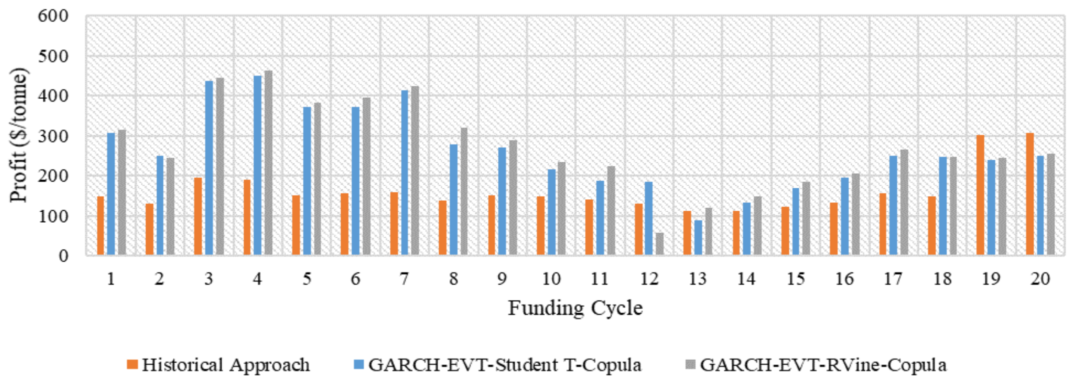


Figure 4.8: The Comparison of the Performances of the Two Approaches.



(a) Aluminum.



(b) Copper.

Figure 4.9: Profit Comparison for Aluminum and Copper.

4.5.4 Model Comparison Results

The previous analysis indicates that the R-vine-Copula has superior performance than both the historical and the Student T-Copula-based approach. To further measure the reliability of these approaches, this section shows the log-likelihood regarding the estimated parameters. In addition, the Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are also reported. Log-likelihood, AIC, and BIC can measure the goodness of fit of a statistical model (Oh & Patton 2018). Based on each of these criteria, the preferred model is indicated in bold. Table 4.5 shows that the more flexible R-vine-Copula has better performance than the Student T-Copula, with an average gain of approximately 80 points in the log-likelihood. Additionally, the GARCH-EVT-R-vine-Copula consistently generates lower AIC and BIC, indicating that the data supports a more flexible R-vine-Copula. The results from the previous sections and Table 4.5 confirm that the GARCH-EVT-R-vine-Copula is the preferred predictive model.

Table 4.5: Model Comparison Results.

	$LogL$	AIC	BIC
GARCH-EVT-Student T-Copula	1429.90	-2845.80	-2804.95
GARCH-EVT-R-vine-Copula	**1507.99**	**-2992.93**	**-2925.71**

4.6 The Effects of Parameters on the Model Performance

This section examines the effects of parameters on the GARCH-EVT-Copula's performance. The parameterization process is detailed for two reasons. First, although the GARCH-EVT-Copula performs better than the historical approach, its performance varies across the four collateral units. The investigation of the parameterization process can improve understanding of the underlying causes of the differences observed. Second, the information in the parameterization process could provide important practical insights for IFPs.

4.6.1 The Parameters of the ARMA-GARCH Model

This study uses the ARMA(1,1) to depict the mean part of the fluctuating returns for the collateral units. To express volatility, this study adopts the GARCH(1,1) model, which has three relevant parameters: the coefficient for the difference between the last daily return ($x_{j,\ t-1}$) and constant mean (μ_j) for the j^{th} collateral (ϕ_j), the coefficient for the difference between the last daily return ($x_{j,\ t-1}$) and the last daily mean $u_{j,t-1}$ (γ_j), and the constant mean for the j^{th} collateral (μ_j).

The first two subfigures in Figure 4.10 show the patterns for the ϕ and γ values across all funding cycles. It can be observed that the fluctuations in the coefficients for aluminum and copper (see the blue and orange lines) are less severe than those for nickel and zinc (see the grey and yellow lines). The stability may increase the predictability of aluminum and copper prices, which could explain why the predicted impawn rates for aluminum and copper exhibit better performances. Furthermore, the third subfigure shows the constant means for the four collateral units across all funding cycles. There are two important observations to report. First, the patterns for the means of returns are similar, which indicates that trends in collateral prices are generally linked. Second, the constant mean increases in the first seven funding cycles and decreases between the 7th and 13th funding cycles. From the 13^{th} to 20^{th} funding cycle, the constant means remain relatively stable. The first seven funding cycles falling between 01/01/2010 and 31/06/2016, close to the 2008 financial crisis, indicate high average returns of collateral units following a serious economic crash. This result complements the study of Sahamkhadam et al. (2018) and demonstrates that patterns of the constant mean change across periods. When the funding cycle occurs over a period further away from the financial crisis, the constant means gradually decrease and eventually normalize (see lines from the 12^{th} to 20^{th} funding cycles). There are two practical implications here for IFPs: the risk indicators for collateral units should be modified dynamically and after the economy has recovered from a financial crisis, they can

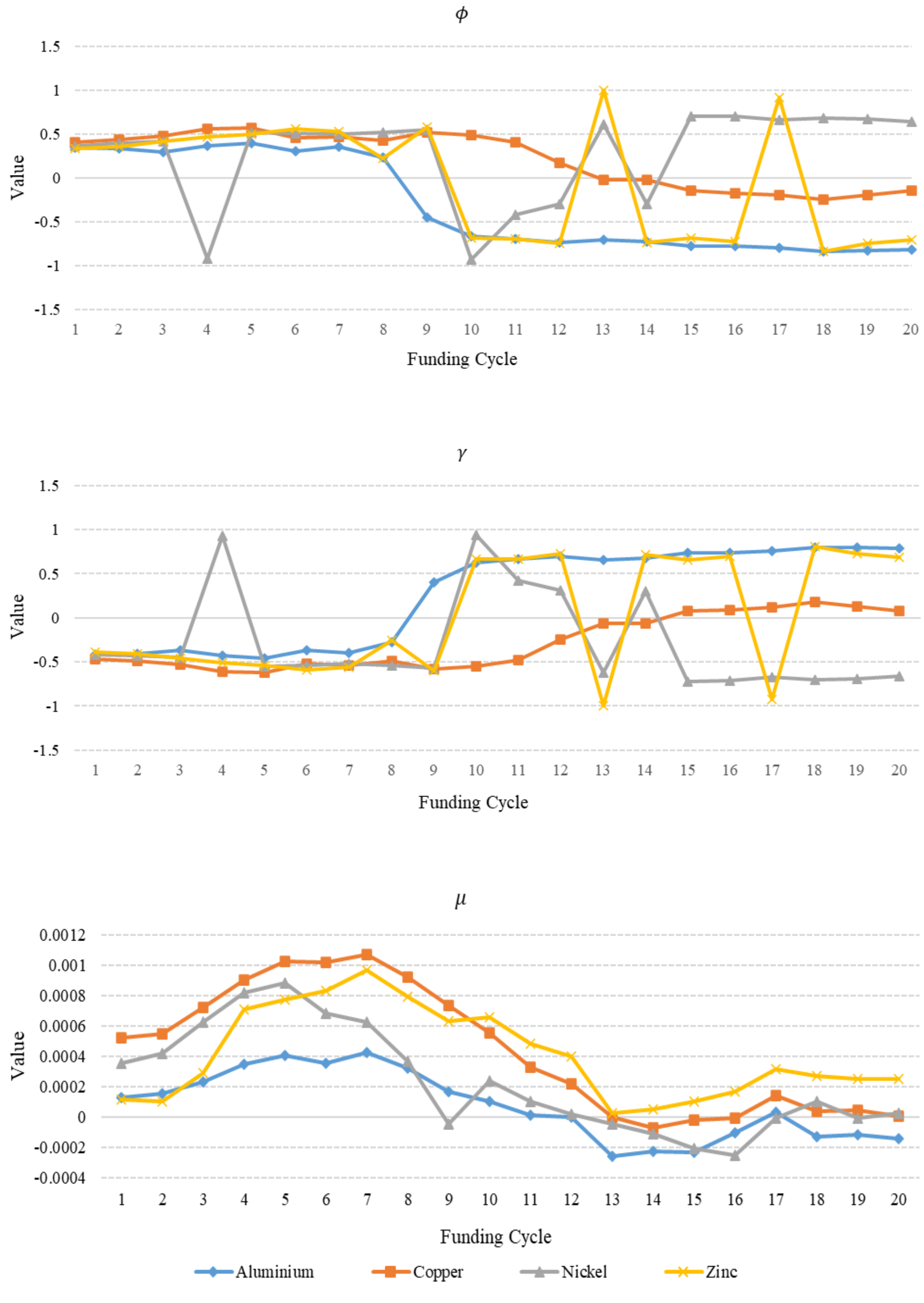


Figure 4.10: The Parameters for the ARMA Model.

ease their risk control and be more aggressive in cultivating clients.

The volatility of returns for each collateral unit is primarily described by $GARCH(1,1)$. Here, ω_j is the constant parameter, $\alpha_{j,1}$ is the coefficient for the difference between the last daily return ($x_{j,\,t-1}$) and last daily mean $u_{j,t-1}$, and $\beta_{j,1}$ is the coefficient for the variance of the previous collateral returns. Figure 4.11 shows the values of these parameters. In the first subfigure, it

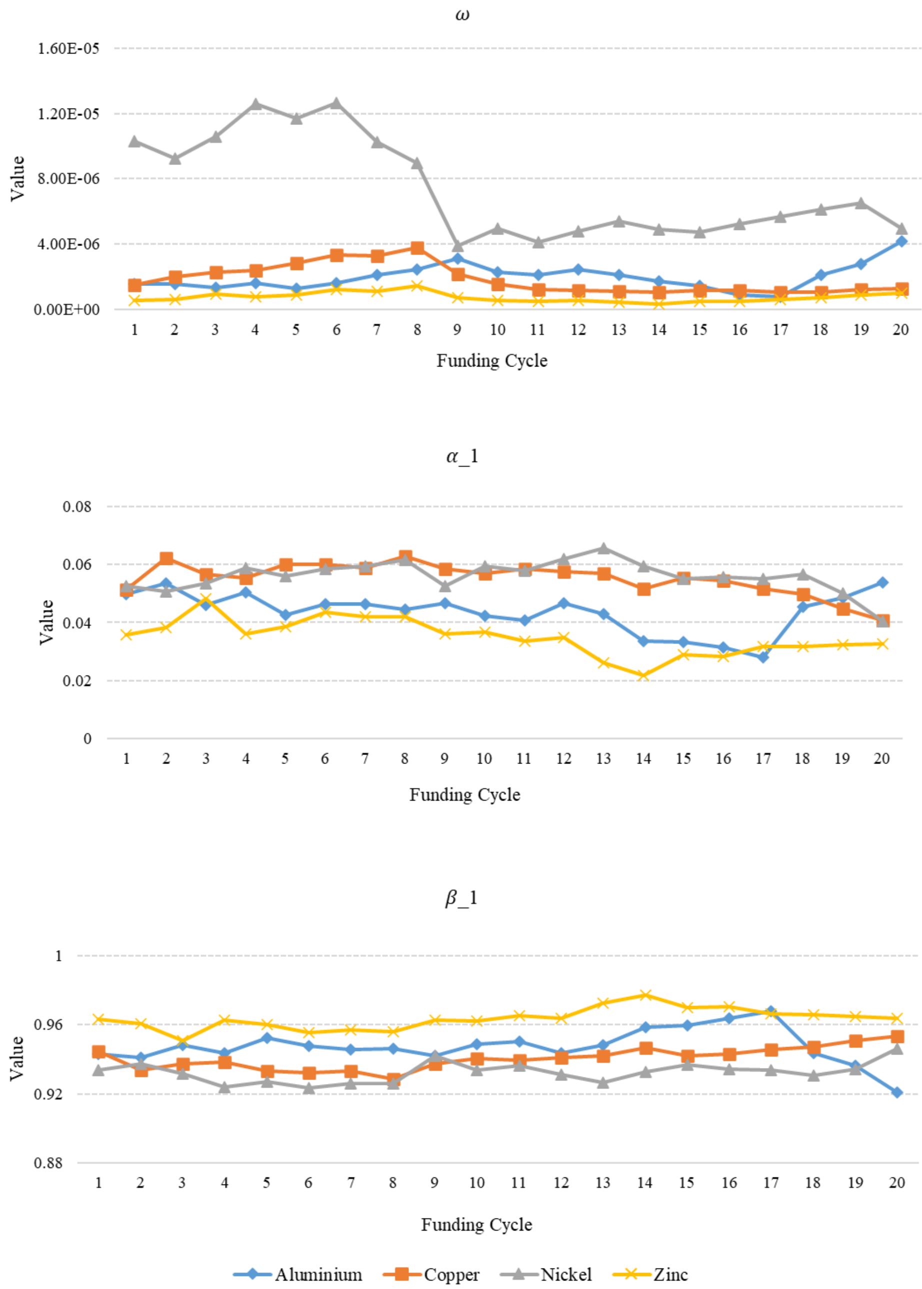


Figure 4.11: The Parameters for the GARCH Model.

can be observed that the constant parameter for nickel is consistently higher than those for the other collateral units, indicating that nickel is less predictable than other collateral units. This unpredictability may explain why the worst performance for the GARCH-EVT-Copula approach is concerning nickel. The second and third subfigures show the values for $\alpha_{j,1}$ and $\beta_{j,1}$, respectively. The sum of $\alpha_{j,1}$ and $\beta_{j,1}$ is less than 1, which is in line with He et al. (2012) and Sahamkhadam et al. (2018) and indicates that the time series for the daily log-returns of all collateral units are stationary. Regarding α_1, the parameters for nickel and copper are relatively higher than those for aluminum and zinc, indicating that the volatilities of returns for nickel and copper have strong relationships with the predictive performances for the last time period. Poor predictive performances for nickel and copper in previous time periods may have made nickel and copper less predictable in subsequent funding cycles. Therefore, when IFPs provide inventory financing services for nickel and copper, they must consider the predictive performances of nickel and copper from previous funding cycles. If the predictive performances in previous funding cycles are poor, they should impose further funding constraints on nickel and copper by decreasing the impawn rate or increasing the interest rate. With respect to β_1, zinc's parameter is relatively higher than those of the other collateral units, indicating that the volatility of returns for zinc has a strong relationship with volatility from previous periods. The high volatility of zinc in previous funding cycles may make it more volatile in the current funding cycle. Therefore, when IFPs fund zinc, they must consider its previous volatility. If the volatility in the last funding cycle was low, the funding constraints for zinc in the current funding cycle may be relaxed.

4.6.2 The Parameters for the GPD

Before implementing the Monte Carlo simulation, this study first transforms the $z_{j,t}$ into their uniform versions using Eq. (4.3) for the Student T-Copula. To adopt the GPD, this study uses the peaks-over-threshold (POT) approach, where all data above a designated high threshold μ^R and below a low threshold μ^L are used. The selection of thresholds is important for the accuracy of the GPD. If μ^L is too large or μ^R is too small, the GPD may not provide a good fit for the model. If μ^L is too small or μ^R too larger, there may be too few excesses to adequately fit the GPD parameters. Similar to other studies, including Karmakar & Paul (2019) and Sahamkhadam et al. (2018), this study adopts an excess function to determine the lower threshold. Owing to the impawn rate focuses on the negative returns of a collateral unit, this study mainly compares u^L, ε^L and β^L for different collateral units. Here, ε^L is a shape paramter and the larger the ε^L, the heavier the tailing of the GPD, which indicates that the probability of producing extemely low returns increases with ε^L. Figure 4.12 presents the estimated paramaters for all funding cycles.

From the first subfigure, it can be observed that the lower threshold for aluminum remains relatively stable and high compared to those for the other collateral units (see the blue line), indicating that the price of aluminum is less likely to decline sharply. Compared with aluminum, there are serious instabilities of copper and nickel in the middle periods, which explains why the impawn rates for nickel and copper are consistently low and fluctuate between the 8^{th} funding cycle to the 13^{th} funding cycle. The second subfigure shows the pattern for ϵ^L. It should be noted that the ϵ^L for these collateral units is higher than that for the stock index (Sahamkhadam et al. 2018), which indicates that the probability of producing extremely low returns for a collateral unit is relatively high. From the second subfigure, it is found that the trendlines for nickel

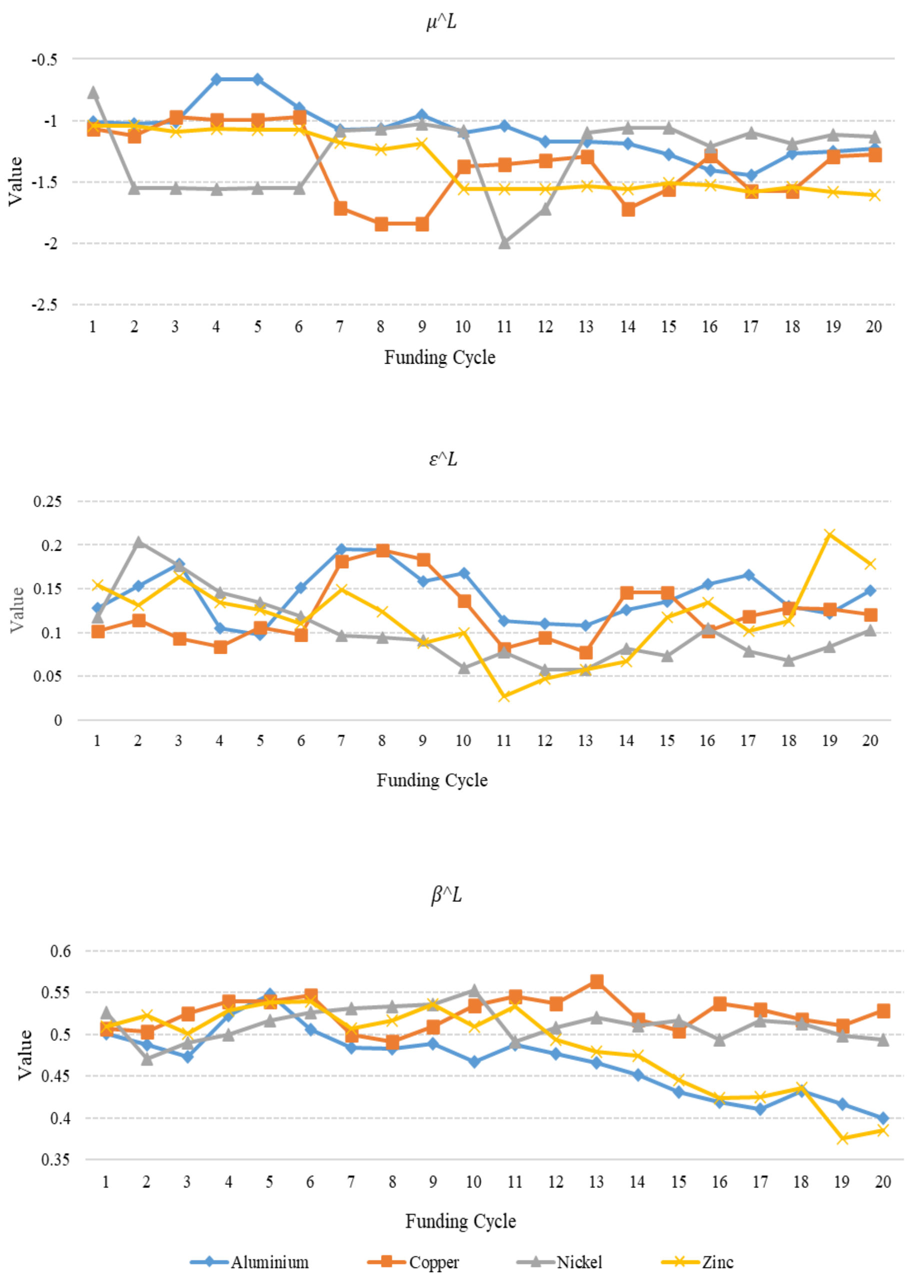


Figure 4.12: The Parameters for the GPD Model.

and zinc decrease in the first 10 funding cycles and increase in the later 10 funding cycles (see the yellow and grey lines). However, the trends for aluminum and copper do not follow this pattern. Following the financial crisis, nickel and zinc may have had a period during which their volatilities were low, which would explain why the GRACH-EVT-Copula and historical approaches yield relatively low efficiency losses in the middle funding cycles. The third subfigure shows the trend for the scale parameter. In the third subfigure, compared with those for copper and nickel, the scale parameters for aluminum and zinc decrease over the last 10 funding cycles (from 01/01/2015 to 12/31/2019), indicating that aluminum and zinc tend to produce relatively lower standard residuals and may be safer during a normal period. Therefore, when volatility in collateral prices stabilizes following a crisis, IFPs may relax financing constraints for aluminum and zinc by increasing the impawn rate or decreasing the interest rate.

4.7 Extended Analysis: The Performance of the GARCH-EVT-Copula Model in the Risky Period

Collateral prices are highly affected by extreme events such as the 2008 financial crisis and COVID-19 pandemic. Two subsamples are adopted to test the performance of the predictive model in relation to these two events. The first subsample starts the period from January 2007 to December 2009, to include the period of the 2008 financial crisis. The second subsample consists of data from January 2020 to February 2021, to include data on the commodity market crash of March 2020. For example, the price of aluminum dropped dramatically from \$1814.50 on 21/01/2020 to \$1425.85 on 08/04/2020, a decrease of 21.42%. To evaluate the performance of proposed approaches, this study establishes a hit function to measure the accuracy of forecasting long-term extreme price risk (Kupiec 1995, Aloui et al. 2011). The exceptions in which the actual value of the collateral is lower than the expected value of the collateral could also be observed:

$$H_j = \begin{cases} 1 & \text{if } r_{jt} < VaR_{jt|jt-1}(\alpha) \\ 0 & \text{or else} \end{cases} \tag{4.21}$$

For the 2008 financial crisis, 759 scenarios are generated to evaluate the performance of the GARCH-EVT-Copula models. It is found that the GARCH-EVT-Copula models are not satisfactory in controlling the risk in inventory financing during the 2008 financial crisis. For example, for the GARCH-EVT-Student T-Copula, the failure rate could be up to 15.8%. In 759 scenarios, the expected value of copper to be lower than the actual value of collateral 120 times. Interestingly, the GARCH-EVT-Copula models perform excellently for the COVID-19 period. The impawn rates derived from the GARCH-EVT-Copula models could adequately manage the risk of all collateral units. In 295 scenarios, the GARCH-EVT-Copula models only fail to control the risk for one collateral in one scenario, which is much higher than the 99^{th} percentile proposed by Basel III (2019).

Furthermore, it is found that the GARCH-EVT-R-vine-Copula produces slightly higher impawn and interest rates than the GARCH-EVT-Student T-Copula (see Table 4.6). This indicates that by using the GARCH-EVT-R-vine-Copula, IFPs can produce lower efficiency losses and generate more profit from one collateral unit.

Table 4.6: The Performance of Copula Based Models During the COVID-19 Pandemic.

GARCH-EVT-Student T-Copula				
Collateral	Aluminum	Copper	Nickel	Zinc
$E(\theta)$	71.89%	70.14%	62.58%	69.75%
$\sum_{i=1}^{295} H_{ij}$	0	0	1	0
$E(r)$	6.02%	6.06%	6.09%	6.20%
$E(\pi)$	73.22$	243.30$	522.09$	98.04$
GARCH-EVT-R-vine-Copula				
Collateral	Aluminum	Copper	Nickel	Zinc
$E(\theta)$	71.94%	70.69%	63.28%	70.01%
$\sum_{i=1}^{295} H_{ij}$	0	0	1	0
$E(r)$	6.02%	6.08%	6.11%	6.20%
$E(\pi)$	73.26$	245.71$	529.11$	98.53$

To identify why the GARCH-EVT-Copula models have better performance during the pandemic, this study compare risks brought forward by the 2008 financial crisis and the Covid-19 pandemic. Specifically, the ARMR-GARCH model is first adopted to compare the volatility of collateral prices before both shocks. Basically, α_1 is approximately 0.05 in a relatively stable market. GARCH volatilities with relatively high α_1 are "spikier" than those with relatively low α_1. Table 4.7 shows that the value of α_1 3 years prior to the 2008 financial crisis is slightly higher than 0.05, indicating the crash of industrial metal prices brought by the 2008 financial crisis is more sudden than that caused by the pandemic. It is also confirmed by the shape parameters as they are consistently lower before the 2008 finanical crisis than those before the COVID-19 pandemic. This is explainable as major events, such as Brexit and the trade war between the US and China, had already contributed to the volatility of the industrial metal market. Additionally, the initial outbreak of the pandemic in China had already caused the disruption in global supply chains before the global pandemic was declared by the World Health Organisation. Compared with the financial crisis resulted by the COVID-19 pandemic, the risk of plummeting collateral prices caused by the 2008 financial crisis is less predictable as there was little sign of metal market volatility prior to the event.

Table 4.7: Model Comparison Results for the 2008 Financial Crisis and the COVID-19.

Parameters	**5 years prior**		**4 years prior**		**3 years prior**	
	14/07/2008	21/01/2020	14/07/2008	21/01/2020	14/07/2008	21/01/2020
ar1	0.35	-0.13	0.19	0.38	0.11	0.14
ma1	-0.42	-0.23	-0.15	0.09	-0.4	-0.17
α_1	0.05	0.05	0.05	0.04	**0.06**	0.04
β_1	0.94	0.89	0.94	0.92	0.93	0.92
shape	6.29	**10.25**	6.93	**11.32**	9.52	**12.03**

Table 4.8 compares the log-likelihood, AIC, and BIC for the copula-based approaches using

data 5, 4, and 3 years prior to both the 2008 financial crisis and COVID-19 pandemic. Through comparison, it is found that the copula-based approaches more consistently produce lower log-likelihood and higher AIC and BIC before the COVID-19 pandemic than before the 2008 financial crisis. For instance, the Student T-Copula performs better for the financial crisis 5 years prior to 2008 compared to when the log-likelihood is over 492 points 5 years prior to COVID-19. The R-vine-Copula performs worse for 5 years prior to COVID-19 than for 5 years prior to the 2008 financial crisis, with the loss in log-likelihood being over 496 points. The extremely high goodness of fit indicates that the industrial metal market is relatively calm before the 2008 financial crisis. Correspondingly, the unexpected shock caused by the 2008 financial crisis increases the failure rate of the predictive model in controlling the risk. Conversely, before the COVID-19 pandemic, the volatile collateral market caused by events such as Brexit, the trade war between the US and China, and supply chain disruptions caused by the initial outbreak of coronavirus in China, enables the GARCH-EVT-Copulas to capture the risk in the parameterizing process, making it a more effective approach in managing the risk of inventory financing. Furthermore, the higher goodness of fit generated by the GARCH-EVT-R-vine-Copula shows that it can better depict the market volatility and, thus, generate superior performance than the GARCH-EVT-Student T-Copula.

Table 4.8: Model Measurement.

		5 years prior		
	14/07/2008		**21/01/2020**	
Model	Student T	R-vine	Student T	R-vine
LogL	1190.18	1198.85	698.07	701.53
AIC	-2366.36	-2379.7	-1382.15	-1385.05
BIC	-2330.37	-2333.43	-1346.15	-1338.77
		4 years prior		
	14/07/2008		**21/01/2020**	
Model	Student T	R-vine	Student T	R-vine
LogL	950.46	961.18	480.22	482.86
AIC	-1886.92	-1904.37	-946.45	-947.72
BIC	-1852.51	-1860.12	-912.02	-903.45
		3 years prior		
	14/07/2008		**21/01/2020**	
Model	Student T	R-vine	Student T	R-vine
LogL	709.61	719.12	325.93	329.27
AIC	-1405.23	-1420.25	-637.87	-640.54
BIC	-1372.83	-1378.59	-605.46	-598.88

4.8 Conclusion

This study proposes new data-driven analytical models aiming to help IFPs make informed decisions regarding impawn and interest rates to improve the performance of their inventory financing businesses. The option theory is used to combine the well-known ARMR-GARCH, copula, and impawn and interest rate models to dynamically set the impawn and interest rates for industrial commodity-based inventory financing. Through a comprehensive analysis of four commonly used collateral units, it is found that the two GARCH-EVT-Copula approaches perform better than the historical approach. Between them, the GARCH-EVT-R-vine-Copula approach effectively incorporates market volatility in setting impawn and interest rates and, therefore, helps IFPs to extract the most value from collateral units while managing risk. Additionally, the GARCH-EVT parameterization process can help IFPs to identify the least risky and most predictable collateral unit. Finally, an extended analysis covering the COVID-19 pandemic and 2008 financial crisis demonstrates that the GARCH-EVT-R-vine-Copula offers superior performance in a volatile market environment.

This study makes the following key contributions. First, this study complements the inventory financing literature by presenting an innovative data-driven approach of setting impawn and interest rates. In contrast to the existing literature (e.g., He et al. (2012)) that only focuses on impawn rates with interest rates as a given parameter, this study explores methods of setting appropriate impawn and interest rates through the combination of the IIRM and GARCH-EVT-Copula model, which is more consistent with industrial practice. Second, from a methodological perspective, the integration of the predictive analytical approach (i.e., using the data-driven Copula model to predict risk) and prescriptive analytical model (i.e., IIRM) for addressing a contemporary issue such as determining appropriate impawn and interest rates for inventory financing contributes to the emerging business analytics field (Baker et al. 2012, He et al. 2012). The integration of the GARCH-EVT approach and state-of-the-art copula models (i.e., Student T and R-vine-Copula) makes the predictive model more accurate in setting the impawn and interest rates for inventory financing. The improved predictive model could also be included in the trade strategy to manage the risks in stock or bond market. Third, in a practical sense, through an extended analysis of the 2008 financial crisis and COVID-19 periods, this study also contributes to a new stream of operations research concerning the COVID-19 pandemic (Choi 2020, Ivanov 2020, Mehrotra et al. 2020). Inventory financing can play a critical role in helping many capital-constrained businesses to survive crises through alternative financing schemes and thus support post-COVID-19 economic recovery. The insights derived from the analytical models are beneficial for IFPs, allowing them to control the default risks inherent in inventory financing and gain competitiveness in the financing market.

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CHAPTER 5

PORTFOLIO OPTIMIZATION FOR INVENTORY FINANCING: COPULA-BASED APPROACHES

An earlier version of this chapter has been accepted for publication as below. The authors' contribution statement has been provided at the start of this dissertation.

Zhi, B., Wang, X. and Xu, F. 2021, Portfolio Optimization for Inventory Financing: Copula-based Approaches, Computers & Operations Research, 136, 1-11. DOI: 10.1016/j.cor.2021.105481

Portfolio optimization has long been used in asset management to mitigate risks of fluctuating asset prices. This study uses copula models and portfolio optimization to investigate how inventory financing providers (IFP) can utilize the timely market information of collaterals to optimize their portfolios of collaterals to mitigate default risks. Through comparing the predictive performance of copula strategies with that of the multivariate normal distribution (MVN) strategy, it has been found that the general canonical vine copula can characterize the dependence structure among collateral return series and has superior predictive performance over the MVN and other copulas. The analytical findings suggest that the general canonical vine copula can be constructed into portfolio strategies that can be adopted by the IFP to mitigate default risks and improve their risk profile.

5.1 Introduction

Inventory financing, a form of asset-based lending, is a popular financing option for small to medium-sized (SME) firms. This type of financing allows borrowers to leverage their inventory to obtain a revolving line of credit. It is relatively easier to obtain than more conventional financing options that often require fixed assets to secure the loan. Besides the traditional financial intermediaries, other players also are entering this growing market due to its increasing popularity. For instance, the American logistics service provider, UPS, offers in-transit inventory financing solutions through its financial service unit UPS Capital, thereby allowing their clients to more easily buy new inventory to fulfill new orders.

From lenders' point of view, inventory financing has its risk. Although inventory serves as collateral for the loan, lenders may not be able to reclaim their loans by selling the pawned inventory. Therefore, customized items that are hard to liquidate, such as specialty pigments used to manufacture paint and work-in-progress goods, are often excluded from the collateral used to determine the borrowing base. In fact, it is much easier for borrowers to secure inventory financing for commodities, e.g., metals from the inventory financing providers (IFP) (Hofmann 2009). However, overly focusing on specific collaterals (e.g., metal commodities) can also expose IFP to the periodically highly volatile markets. For instance, the price of tin dropped approximately 10.72 % from 28/02/2015 to 30/04/2015 (from 17,950 $/ton to 16,025 $/ton), whereas the price of zinc increased almost 13.56 % in the same period (from 2,065 $/ton to 2,345 $/ton).

One logical instinct for reducing the price volatility is to create a basket of collaterals with appropriate weights for the collaterals. According to the seminal work of Markowitz (1952), portfolio optimization comprises two stages: (1) forecasting the asset returns; (2) determining the portfolio weights. The optimal weights of assets are derived from the optimization of investors' utility function for the predicted portfolio asset returns (Low et al. 2013). Based on the 2-stage process, the existing literature of portfolio optimization mainly focuses on two research streams: (1) improving the preciseness of portfolio asset return forecasts (e.g., Low et al. (2013) and DeMiguel et al. (2009)); (2) determining the most reliable utility function (e.g., Jarrow & Zhao (2006), Zhang et al. (2017), Babich & Kouvelis (2018) and Chen et al. (2017)). Although risks associated with fluctuating raw material prices also have attracted growing attention, such as how to manage fluctuating raw material prices using supply contracts (Kouvelis et al. 2017) and how to hedge risks through the adjustment of impawn rate (He et al. 2012), very little research has been done regarding how to manage risks using the portfolio optimization. Although portfolio optimization is a heated topic in the finance literature (e.g., Jarrow & Zhao (2006) and Kouvelis et al. (2017)), it has not been explored in the inventory financing setting. Furthermore, a range of market trade-related data such as commodity prices and market volume is widely available to financial institutions and businesses. Facing the sheer amount of data and greater complexity, it is critical for IFP to develop a data-driven mechanism of incorporating portfolio optimization in inventory financing.

Motivated by the empirical evidence and the literature gap discussed above, this study intends to deepen the understanding of the role of portfolio optimization in mitigating inventory financing risks. Specifically, this study examines the following questions: (1) How can portfolio optimization be integrated into inventory financing to mitigate default risks? (2) How to incorporate the vast amount of asset price data to improve the efficacy and accuracy of portfolio optimization in inventory financing?

To address the above questions, this study investigates how to improve the precision of collateral return forecasts and find the appropriate utility function. Multivariate copulas are chosen as the predictive model due to their precision and flexibility in describing the dependency structure of tails (Aas et al. 2009), and Conditional Value-at-Risk (CVaR) model as an optimization function due to its effectiveness in avoiding extreme losses of the portfolio (Dias 2016, Sahamkhadam et al. 2018). To obtain the most reliable portfolio strategy, this study makes a comparison of the performance among nine predictive models, namely, the multivariate normal (MVN) distribution (treated as the benchmark) and eight other copula-based predictive models. This study also introduces the procedures for constructing suitable predictive models, in which the constructing process of the most suitable canonical vine copula strategy is illustrated. Through the comparative analysis, the analytical results reveals that copulas outperform the MVN in capturing the dependence structure of collateral returns, which helps the IFP more accurately allocate weights to the collaterals within the portfolio in each funding cycle. More specifically, two portfolio strategies based on a general canonical vine copula with normal or skewed Student T marginal and CVaR can help the IFP better manage inventory financing risks. In addition, the results suggest that the effectiveness of these models is sensitive to the portfolio size. When the number of collaterals is small, the IFP can choose the portfolio strategy constructed by a general canonical vine copula with normal marginal and CVaR; otherwise by a general canonical vine copula with skewed Student T marginal and CVaR.

The rest of this study is organized as follows. Section 5.2 discusses prior research in related areas. Section 5.3 briefly describes the source data used in this study. Section 5.4 lays out the model of prediction and optimization. Section 5.5 examines the performance of different composite models. Section 5.6 includes this study.

5.2 Related Literature

For inventory financing, the most prominent one source of risks comes from the fluctuating raw materials prices (Kouvelis et al. 2017). The significant variation of collateral prices can make the IFP face default risks. Significant drops of collateral values will make borrowers have less motivation to return the money back at the end of funding period, which make the inventory financing very risky. Existing studies seldom consider the effect of fluctuating collateral prices on the performance of inventory financing. Although existing research, such as He et al. (2012), has examined how an IFP periodically reduces the impawn rate to mitigate default risks in inventory financing, the reduction of the impawn rate would make the inventory financing provider less competitive. To simultaneously maintain the competitiveness of the IFP and manage default risks, it is necessary introduce dynamic portfolio optimization into inventory financing.

By introducing dynamic portfolio optimization, the IFP can iteratively adjust the weights of collaterals to reduce inventory financing risks based on the consistently fluctuating collateral prices. Recently, although Seifert et al. (2016) have introduced the portfolio optimization theory into managing products in various life cycle stages, the effectiveness of portfolio optimization in inventory financing is still unexplored.

This study intends to fill the gap in the exisiting literature by exploring the most effective strategy to optimize the portfolio of collateralls in inventory financing. More specifically, this study combines copula models with two marginals, namely, the marginal normal distribution and the marginal skewed Student T distribution. The skewed Student T marginal distribution itself can also capture the dependent structure of series data (Hansen 1994), but the normal distribution marginal cannot. Therefore, by including the marginal skewed Student T distribution, any asymmetry found in the dependence structure truly reflects dependence and cannot be attributed to the poor modelling of the marginals. In second step, this study uses CVaR rather than the VaR or mean-variance model as the utility function because compared with VaR and Mean-Variance, CVaR is more reliable regarding the control of extreme losses. As Rockafellar & Stanislav (2000), Rockafellar & Uryasev (2002) suggested, the portfolio that has low CVaR will also have low VaR. To help the IFP avoid extreme losses as much as possible, such as Chollete et al. (2009), Boubaker & Sghaier (2013), and Karmakar & Paul (2019), CVaR is chosen as the optimization function. Like managing the portfolio of securities, the most challenging part of applying portfolio optimization to inventory financing is finding out the most suitable model to forecast collateral price volatility. This study explores the most effective copulas strategy to optimize the portfolio of collaterals in inventory financing.

5.3 Data

The data set contains monthly collateral returns on seven raw materials (i.e., prime aluminium (PA), aluminium alloy (AA), copper (CP), lead (LD), nickel (NK), tin (TN) and zinc (ZN)). The reason these industry metals are chosen as the collaterals is that the loss severity in inventory financing is also influenced by the liquidity of collaterals (Caselli et al. 2008). The liquidity risk is less for industry metals than for semi-finished goods and finished goods as raw materials, such as steel and other similar industry metals, are rather "liquid" and can be traded on metal exchange (Buzacott & Zhang 2004, Hofmann 2009). The period of these collateral returns extends from 31/01/1998 to 31/12/2017, yielding 1,680 observations in total. The first 840 observations from 31/01/1998 to 31/12/2007 are used for estimating parameters, and remaining 840 monthly returns serve as out-of-sample to test the effectiveness of the established model.

Strategies are implemented in portfolios of three, five and seven constituents as shown in Table 5.1. No material exhibits excess kurtosis, which indicates no return series has any serious outlier problem. All month collateral returns except NK reject the null hypotheses for the Jarque-

Bera test of normality at the 1% level, which means only the skewness and kurtosis of NK return series do not match a normal distribution. The returns of PA, AA, CP, LD and ZN exhibit negative skewness and the returns of NK and TN show positive skewness, showing only the tail of NK and TN return series is on the right side of the distribution. NK and CP separately exhibit the highest (34.75%) and the lowest (-35.55%) returns for the sample.

Table 5.1: Descriptive Analysis of Collateral Returns.

Raw Material	Mean	St.Deviation	Skewness	Kurtosis	Min	Max	Jarque-Bera
PA	0.28	0.051	-0.181	1.381	-18.82	16.34	20.37*
AA	0.25	0.052	-0.345	5.684	-29.81	21.55	327.79*
CP	0.87	0.073	-0.177	3.091	**-35.55**	29.68	96.77*
LD	0.96	0.082	-0.069	1.137	-27.23	25.78	13.12*
NK	0.81	0.101	0.224	0.122	-23.66	**34.75**	2.15
TN	0.78	0.07	0.392	1.33	-22.09	26.8	23.84*
ZN	0.74	0.075	-0.139	1.675	-33.04	26.73	28.81*

* indicates statistical significance at the 1% level.

In addition, the sample Kendall rank correlations are reported in Table 5.2. The highest correlation arises between ZN and CP (0.54) while the lowest (0.21) occurs between TN and PA, and TN and AA, suggesting that there exists space to diversify the risks from fluctuating collateral prices.

Table 5.2: Kendall Rank Correlation.

	PA	AA	CP	LD	NK	TN	ZN
PA	1						
AA	0.35	1					
CP	0.3	0.36	1				
LD	0.22	0.23	0.4	1			
NK	0.23	0.27	0.39	0.29	1		
TN	**0.21**	**0.21**	0.3	0.26	0.31	1	
ZN	0.29	0.3	**0.54**	0.45	0.37	0.28	1

5.4 Model

To guarantee the preciseness of return forecasts, here this study follows DeMiguel et al. (2009), Low et al. (2013) and Sahamkhadam et al. (2018) and use a *"rolling window"* approach to predict future collateral price volatility. In the inventory financing setting, the *"rolling window"* approach is defined as *"using the data within previous funding cycles to parameterize the multivariate probability distribution after the $t = w + 1$ funding cycle"*. Based on the parameterized multivariate probability distribution and utility function, this study iteratively adjusts weights for the collaterals within the portfolio in each funding cycle.

5.4.1 Multivariate Copula

The marginal behavior of individual values and their dependency structure can be revealed in every cumulative joint distribution function (CDF). However, there is another way to represent the CDF. Let $\boldsymbol{X} = (X_1,...,X_n)$ to be a random vector with a CDF $F(x_1,...,x_n)$. Let $F_i(i = 1,...,n)$ denote marginal distributions. Then there exists a copula to depict the dependence structure among marginal distribution functions (Sklar 1973).

$$C[F_1(x_1),...,F_n(x_n)] = F(x_1,...,x_n) \tag{5.1}$$

The copula from Eq.(5.1) has the following expression with the transformations $F_i(X_i) = U_i$.

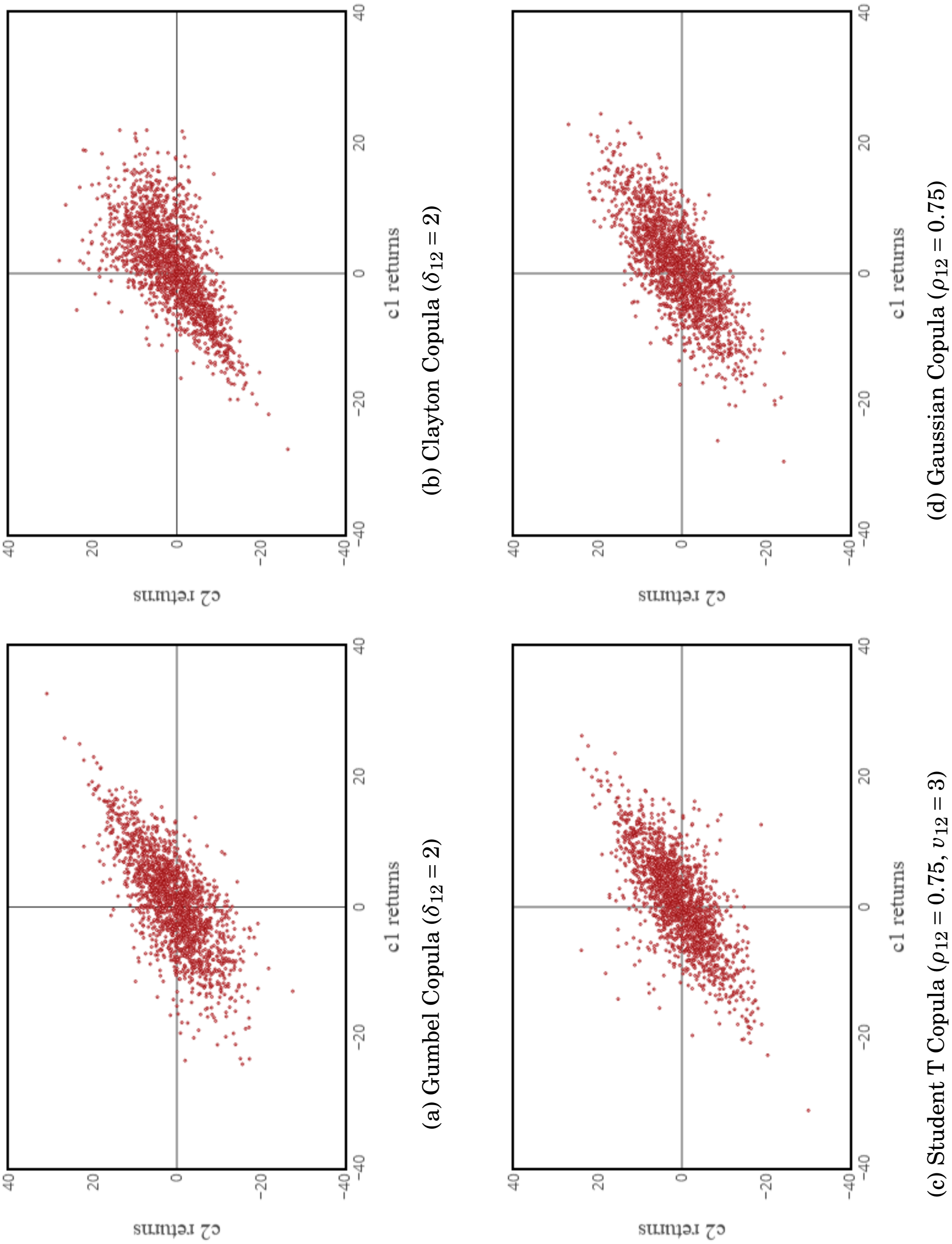
$$F(x_1,...,x_n) = C[F_1(x_1),...,F_n(x_n)] = C(u_1,...,u_n) = P(U_1 \leq u_1,...,U_n \leq u_n) \tag{5.2}$$

A copula $C(u_1,...,u_n)$ is a function that maps marginal CDFs F_i to the joint distribution F. The multivariate vector is supported with $[0,1]^n$. Assuming marginal CDF F_i and the copula function C in Eq.(5.2) are differentiable, the density of the copula $c(u_1,...,u_n)$ and the joint density function $f(x_1,...,x_n)$ can be defined as:

$$c(u_1,...,u_n) = \frac{\partial^n C(u_1,...,u_n)}{\partial u_1,...,\partial u_n} \tag{5.3}$$

$$f(x_1,...,x_n) = c_{1,...,n}[F_1(x_1),...,F_n(x_n)] \cdot f_1(x_1)...f_n(x_n) \tag{5.4}$$

To determine the appropriate multivariate copulas, four scatter plots produced by different bivariate copulas (i.e., Gumbel, Clayton, Student T and Gaussian copula) are first evaluated (for detail, see Figure 5.1). The produced scatter plots have different strength of dependence on the tails of the bivariate distribution. Figure 5.1 shows the dependence structure of four copulas in the bivariate case, suggesting that the first two copulas have a simple closed form. The Gumbel copula is upper-tail dependent (See Figure 5.1a), and Clayton copula is lower-tail dependent (See Figure 5.1b). The latter two copulas are characterized by normal mixture distributions (Aas et al. 2009). The Student T copula is both lower- and upper-tail dependent (See Figure 5.1c), while the Gaussian is neither lower- nor upper-tail dependent (See Figure 5.1d). To select copulas that can be used to do return forecasts, this study further makes a comparison between the scatter plots in Figure 5.1 and the empirical returns (Figure 5.2). Based on scatter plots, the copulas that can not capture the tail structure of empirical returns will be excluded in the following analysis.



(a) Gumbel Copula ($\delta_{12} = 2$)

(b) Clayton Copula ($\delta_{12} = 2$)

(c) Student T Copula ($\rho_{12} = 0.75$, $v_{12} = 3$)

(d) Gaussian Copula ($\rho_{12} = 0.75$)

Figure 5.1: Scatter Plots Produced by Four Classical Copulas.

Based on the scatter of diagram of seven collateral returns, it is found that the dependence structure of collateral returns is asymmetric and shows a lower-dependence structure (See the right and left square frame in Figure 5.2. In Figure 5.1c and Figure 5.1d, both the Student T and Gaussian copula show a symmetrical structure and capture a partial structure of real collateral returns (tail dependence structure for Student T Copula and non-tail dependence structure for Gaussian Copula). In contrast, although the Gumbel copula shows an asymmetric structure, the structure of scatters is upper-dependent, which is contrary to the structure of sample returns. Thus, Gumbel copula would not be set as a predictive model in this research. Finally, Gaussian, Student T and Clayton copula are set as predictive models since they can at least capture the partial dependence structure of sample returns. In a multivariate setting, the relationships among series can only be described by one copula. However, by introducing the general canonical vine copula, different time series' dependent structure can be described by different bivariate copulas (Aas et al. 2009). In the following analysis, this study would further compare the predictive performance of the Gaussian, Student T, Clayton copula and general canonical vine copula with MVN as the benchmark.

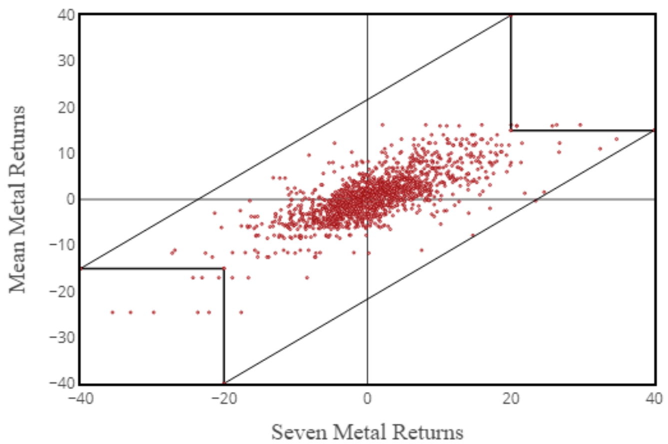


Figure 5.2: Empirical Relationships among Seven Collateral Returns.

5.4.2 Canonical Vine Copula

Compared with the traditional multivariate copulas, the vine copula constructed by bivariate copulas is more flexible since it can simultaneously describe varying degrees of dependence structures through iterative conditioning (Aas et al. 2009). The following details how the multivariate vine copula uses pair-copula functions to capture varying degrees of the dependence structure of variables vectors.

Without loss of generality, by iteratively conditioning, a joint density function $f(x_1, x_2, ..., x_n)$ can be decomposed as follows:

$$f(x_1, x_2, ..., x_n) = f_n(x_n) f(x_{n-1}|x_n) f(x_{n-2}|x_{n-1}, x_n) ... f(x_1|x_2, ..., x_n) \tag{5.5}$$

Using conditional copulas, we can decompose each factor in the right side of Eq.(3.5) further. For instance, when $n = 2$, $f(x_1,x_2) = c_{12}[F_1(x_1),F_2(x_2)] \cdot f_1(x_1)f_2(x_2)$. With $f(x_1,x_2) = f_2(x_2)f(x_1|x_2)$, $f(x_1|x_2) = c_{12}[F_1(x_1),F_2(x_2)] \cdot f_1(x_1)$ can be easily obtained. Obviously, we can decompose the second factor, $f(x_{n-1}|x_n)$, in the right side of Eq.(3.5) into the a marginal density $f_{n-1}(x_{n-1})$ and pair-copula $c_{(n-1)n}[F_{n-1}(x_{n-1}),F_n(x_n)]$. It is now clear that each term in Eq. (5.5) can be decomposed into the conditional marginal density times pair-copula. A n-dimensional vector $\boldsymbol{v}$ can be represented with general formula.

$$f(x|\boldsymbol{v}) = c_{xv_j|\boldsymbol{v}_{-j}}[F(x|\boldsymbol{v}_{-j}),F(v_j|\boldsymbol{v}_{-\boldsymbol{j}})] \cdot f(x|\boldsymbol{v}_{-\boldsymbol{j}}) \tag{5.6}$$

where $\boldsymbol{v}_{-j}$ is the vector $\boldsymbol{v}$ without one arbitrarily chosen component v_j.

Eq.(5.6) shows that the pair-copula construction includes marginal conditional distributions of the form $F(x|v)$. Jose et al. (1996) identify that, for every j as follows:

$$F(x|\boldsymbol{v}) = \frac{\partial C_{x,v_j|\boldsymbol{v}_{-j}}[F(x|\boldsymbol{v}_{-j}),F(v_j|\boldsymbol{v}_{-j})]}{\partial F(v_j|\boldsymbol{v}_{-j})} \tag{5.7}$$

where $C_{x,v_j|\boldsymbol{v}_{-j}}$ is a bivariate copula distribution function and $\boldsymbol{v}_{-j}$ is the vector $\boldsymbol{v}$ that excludes the component v_j. Assuming the vector $\boldsymbol{v}$ has one dimension, we have

$$F(x|v) = \frac{\partial C_{xv}[F(x),F(v)]}{\partial F(v)} \tag{5.8}$$

The conditional distribution function can be represented by the function $h(x,v,\Theta)$ if x and v are uniform, i.e., $f(x) = f(v) = 1$, $F(x) = x$ and $F(v) = v$. Then we have

$$h(x,v,\Theta) = F(x|v) = \frac{\partial C_{xv}(x,v,\Theta)}{\partial v} \tag{5.9}$$

Θ is the parameter for the bivariate copula $C_{x,v}(x,v)$ and v is the conditioning variable. The $h(x,v,\Theta)$ and its inverse function $h^{-1}(x,v,\Theta)$ are iteratively used to sample and inference for each pair-copula in the vine (the h-function and its inverse of Clayton copula is in Appendix D.1).

There exists other vine copulas, such as regular vine and D-vine. However, the canonical vine is selected in this study because it is efficient in depicting the hierarchical structure (Aas et al. 2009). During the modelling process, when a variable can govern interactions in the data set, it can be treated as the root of the canonical vine. For example, let $\boldsymbol{X} = (X_1,X_2,X_3)$ be a three-dimensional vector. X_2 can be treated as the root of canonical vine when X_2 governs X_1 and X_3. To get the most accurate canonical vine, it is essential to have the right time series as the root. The collateral unit that has the highest degree of correlation with the other collateral units can be treated as the root. Eq.(5.10) is used in this study to find the root.

$$\Theta_{x_i} = \sum_{j=1}^{N} |\theta_{ij}|, \quad \text{where} \quad i,j \in N \tag{5.10}$$

θ_{ij} is an $N \times N$ matrix of the Kendall rank correlation coefficient between each pair of time series. The collateral return series X_i that has the highest absolute correlation with all the other collaterals will be located as the root of the canonical vine. Similar criterion has also been applied by Low et al. (2013). However, different from Low et al. (2013), the criterion provided by this study is more robust regarding selecting the highest correlation because the absolute of correlation can also help to consider the negative correlation.

Once the canonical vine copula is chosen, the dependence structure of a portfolio of n collaterals will be parameterized with $\frac{n(n-1)}{2}$ pairwise copula parameters. In this research, the size of the portfolio is set as three, five and seven. As a result, multivariate probability distributions for all cases are as shown in Eq.(5.11), Eq.(5.12) and Eq.(5.13), where f_n denotes the marginal PDFs and c_n denotes the pairwise copula PDFs as follows (the detail for seven-dimensional pair-copula decompositions is provided in Appendix D.2):

$$f_{123} = f_1 \cdot f_2 \cdot f_3 \cdot c_{13} \cdot c_{23} \cdot c_{12|3} \tag{5.11}$$

$$f_{12345} = f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_5 \cdot c_{12} \cdot c_{13} \cdot c_{14} \cdot c_{15} \cdot c_{23|1} \cdot c_{24|1} \cdot c_{25|1} \cdot c_{34|12} \cdot c_{35|12} \cdot c_{45|123} \tag{5.12}$$

$$\begin{aligned} f_{1234567} = {} & f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_5 \cdot f_6 \cdot f_7 \cdot c_{12} \cdot c_{13} \cdot c_{14} \cdot c_{15} \cdot c_{16} \cdot c_{17} \cdot c_{23|1} \cdot c_{24|1} \cdot c_{25|1} \cdot c_{26|1} \cdot \\ & c_{27|1} \cdot c_{34|12} \cdot c_{35|12} \cdot c_{36|12} \cdot c_{37|12} \cdot c_{45|123} \cdot c_{46|123} \cdot c_{47|123} \cdot c_{56|1234} \cdot c_{57|1234} \cdot c_{67|12345} \end{aligned} \tag{5.13}$$

For each tree edge of the general canonical vine copula, the bivariate copulas can be selected by it include Student T copula, Gaussian copula, and Clayton copula without/with rotations. Similar to Boubaker & Sghaier (2013), for each edge, the bivariate copula that has minimum Akaike information criterion (AIC) is selected.

5.4.3 Marginals Modelling

To avoid the bias of the chosen marginal, this study models the marginal using two alternative methods. A normal marginal is chosen as a baseline. Then, the marginal distribution is modelled using highly flexible univariate skewed Student T distribution with the four parameters. The skewed Student T marginal distribution itself can also capture the dependence structure of series data Hansen (1994), but the normal distribution marginal cannot. Therefore, by including the marginal skewed Student T distribution, we are confident that any asymmetry found in the dependence structure truly reflects dependence and cannot be attributed to the poor modelling of the marginals.

The skewed Student T distribution has the PDF as follows:

$$f_{ST}(x; u, \sigma, \lambda, q) = \frac{\Gamma(\frac{1}{2} + q)}{v\sigma(\pi q)^{\frac{1}{2}} \Gamma(q) (\frac{|x-u+m|^2}{q(v\sigma)^2(\lambda sign(x-u+m)+1)^2} + 1)^{\frac{1}{2}+q}},$$

where u is the mean, $\sigma > 0$ is the variance, $-1 < \lambda < 1$ controls Skewness and $p,q > 0$ control the Kurtosis. v is the beta function as follows: $v = q^{-\frac{1}{2}}\left[(3\lambda^2+1)\left(\frac{1}{2q-2}\right) - \frac{4\lambda^2}{\pi}\left(\frac{\Gamma(q-\frac{1}{2})}{\Gamma(q)}\right)^2\right]^{-\frac{1}{2}}$. The formula of m is as follows: $m = \frac{2v\sigma\lambda q^{\frac{1}{2}}\Gamma\left(q-\frac{1}{2}\right)}{\pi^{\frac{1}{2}}\Gamma\left(q+\frac{1}{2}\right)}$. As illustrated in this section, the usage of the skewed Student T distribution can avoid the poor modelling of the marginal, thus giving us greater confidence that any asymmetry found in the dependence structure can be precisely captured.

5.4.4 Conditional Value-at-Risk Optimization

Given that the focus of this study is on improving the IFP's ability of managing inventory financing risks and came from the extreme losses of collateral price volatility, it is appropriate to choose an optimization function that has an emphasis on the control of extreme losses. Accordingly, this study chooses to minimize the CVaR that can consistently measure risk (Rockafellar & Stanislav 2000, Rockafellar & Uryasev 2002) as the optimization function. Rockafellar & Stanislav (2000), Rockafellar & Uryasev (2002) show CVaR as follows:

(5.14)
$$\phi_\beta(\boldsymbol{w}) = \frac{1}{1-\beta}\int_{f(\boldsymbol{w},\boldsymbol{r})\geq\alpha_\beta(\boldsymbol{w})} f(\boldsymbol{w},\boldsymbol{r})p(\boldsymbol{r})\,d\boldsymbol{r}$$

$f(\boldsymbol{w},\boldsymbol{r})$ is the loss function with the decision vector $\boldsymbol{w}$ and $p(\boldsymbol{r})$ is the density of $\boldsymbol{r}$. The vector $\boldsymbol{w}$ is a portfolio and the vector $\boldsymbol{r}$ stands for the uncertainties. For each $\boldsymbol{w}$, the loss $f(\boldsymbol{w},\boldsymbol{r})$ is a random variable having a distribution induced by that of $\boldsymbol{r}$. The probability of $f(\boldsymbol{w},\boldsymbol{r})$ not exceeding a threshold $\boldsymbol{\alpha}$ is given by $\Psi(\boldsymbol{w},\alpha) = \int_{f(\boldsymbol{w},\boldsymbol{r})\leq\alpha} p(\boldsymbol{r})d\boldsymbol{r}$. $\Psi(\boldsymbol{w},\alpha)$ is the cumulative distribution for the loss associated with $\boldsymbol{w}$. $\alpha_\beta(\boldsymbol{w}) = min\{\alpha \in \mathscr{R} : \Psi(\boldsymbol{w},\alpha) \geq \beta\}$. $\alpha_\beta(\mathbf{w})$ is the β-VaR that values for the loss random variable associated with $\boldsymbol{w}$ and any specified probability level β in (0, 1). In this case, Eq.(5.15) is a suitable approximation to minimize CVaR for a given level of return as follows:

(5.15)
$$\min_{(\boldsymbol{w},\alpha)} F_\alpha(\boldsymbol{w},\beta) = \alpha + \frac{1}{q(1-\beta)}\sum_{k=1}^{q}[-\boldsymbol{w}^T\boldsymbol{r}_k - \alpha]^+,$$

where

(5.16)
$$\begin{cases} u(\boldsymbol{w}) \leq -R \\ \boldsymbol{w}^T\mathbf{1} = 1 \end{cases}$$

q is the number of sample returns generated by a Monte Carlo simulation, so to guarantee the robustness of the portfolio strategies, this study simulates 10,000 returns using the Monte Carlo method, which is also widely adopted in existing research (Low et al. 2013, Karmakar & Paul 2019). Therefore, here $q = 10{,}000$. α represents CVaR, and $\mathbf{1}$ is a vector of ones. β is the threshold value. This study sets it as 0.99, and $\boldsymbol{r}_k$ is the k^{th} vector of simulated returns. As the IFP is considered averse to the extreme losses of collateral returns, this study sets β to 0.99 – analogous to minimizing the losses at the 1% level of CVaR, which coincides with the standard of Basel III (2019). The vector of portfolio weights $\boldsymbol{w}$ is extracted from the optimization procedure to generate the portfolio that minimises CVaR for a given R.

5.5 Results

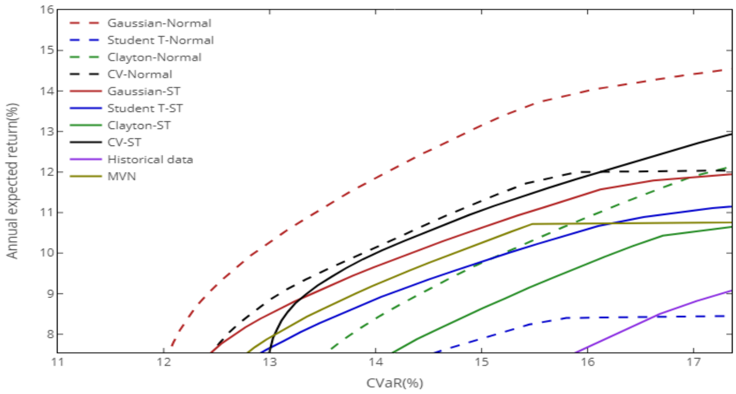
This study compares the predictive performance of different multivariate probability models in the context where the IFP wishes to mitigate inventory financing risks that came from the fluctuating collateral prices. First, the whole period (240 months) returns are used to calculate efficient frontiers from a range of probability models in the case where the size of portfolio is three, five and seven (year from 1998 to 2017). Then, this study performs a twenty-period, out-of-sample study that uses the probability models in the first step to forecast returns and minimize the CVaR of the portfolio. To compare the performance of different probability models and describe the procedures of managing the portfolio of collaterals, the second step will introduce a wider statistical and economic metrics to measure the predictive performance of each portfolio strategy. In this step, rolling period returns from 31/01/1998 to 31/12/2017 are iteratively used to estimate the parameters of copulas and marginals.

5.5.1 Efficient Frontiers

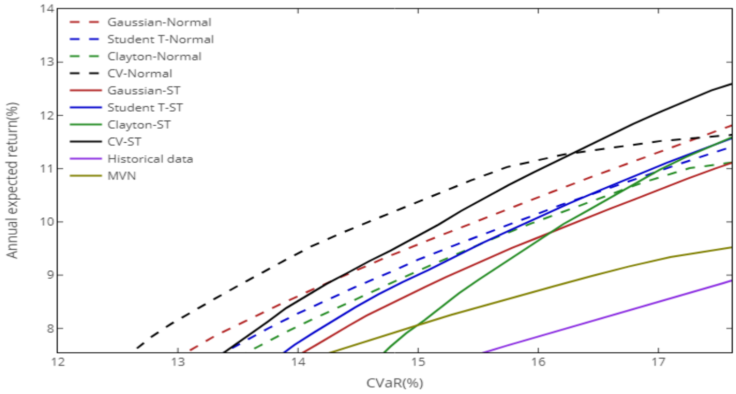
Based on the whole period month returns from 31/01/1998 to 31/12/2017, this study calculates efficient frontiers from a range of probability models for the portfolio where $N = 3$, $N = 5$ and $N = 7$ (see Figure 5.3). In the relative small portfolio (e.g., $N = 3$ and $N = 5$), collateral units that have less relevant prices are included. Such setting can help diverse the risk and test robustness of analytical results. The analysis of efficient frontiers intends to evaluate the relationship between risks and returns produced by different strategies. The copulas used are (1) Student T; (2) Gaussian; (3) Clayton; and (4) Canonical vine copula. Their marginals are modelled by the normal distribution with two parameters and more flexible skewed Student T distribution with four parameters. Here, MVN is introduced as a benchmark to compare the performance of different predictive models more easily. To make comparison consistent, the parameters of introduced MVN are also iteratively updated for each funding cycle. Through the combination of four copulas and two marginals, eight different copula strategies are derived. The shortened form of these strategies is shown in Table 5.3 and will be consistently used in the following analysis.

Table 5.3: Portfolio Strategies based on Copulas.

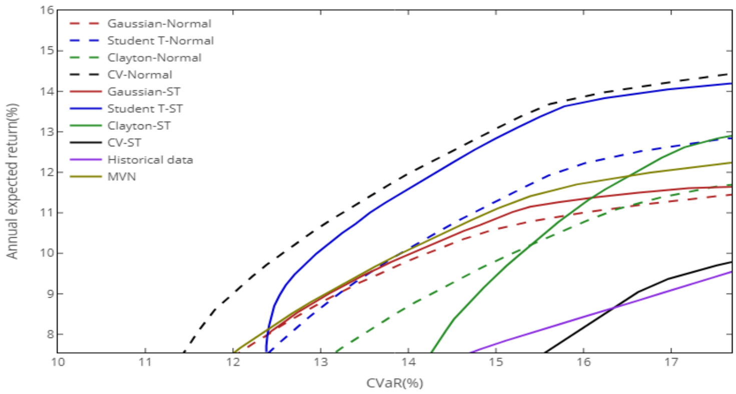
Portfolio Strategy	**Shorted Form**
Student T copula with normal marginals	Student T-Normal
Gaussian copula with normal marginals	Gaussian-Normal
Archimedean Clayton copula with normal marginals	Clayton-Normal
General Canonical vine copula with normal marginals	CV-Normal
Student T copula with skewed Student T marginals	Student T-ST
Gaussian copula with skewed Student T marginals	Gaussian-ST
Archimedean Clayton copula with skewed Student T marginals	Clayton-ST
General Canonical vine copula with skewed Student T marginals	CV-ST



(a) $N = 3$



(b) $N = 5$



(c) $N = 7$

Figure 5.3: Efficient Frontier for Different Sizes of Portfolio.

For the collaterals-3 case, the predictive model's performance of simulating efficient frontiers depends on whether the marginal is normal or skewed Student T. In regard to the position of the efficient frontiers, it is found that the efficient frontiers produced by copulas with the Skewed Student T marginal are closer to the efficient frontier produced by the MVN (benchmark). However, this trend changes as the size of the portfolio increases. For the collaterals-5 and collaterals-7 cases, some of the efficient frontiers produced by the models constructed by copulas with the skewed Student T marginal detach far from the line produced by the MVN (e.g., CV-ST in collaterals-5 case, and CV-ST and Student T-ST in the collaterals-7 case). In contrast, the efficient frontiers produced by the models constructed by Student T, Clayton and Gaussian copulas with the normal marginal cluster together, and are close to the efficient frontier produced by the MVN. This finding is in contrast with the conclusion made by Low et al. (2013). In the securities market and one month returns updated setting, they find that the efficient frontiers produced by copulas with the normal marginal only cluster together when the size of the portfolio is three. As for the aspects of risks and returns, it is found that the efficient frontier produced by the Gaussian-Normal strategy has a higher return and lower risk when the size of the portfolio is three, and the efficient frontier produced by the CV-Normal strategy has a higher return and lower risk when the size of the portfolio is five or seven. Meanwhile, it is found that those efficient frontiers near the efficient frontier produced by historical data have a lower return and higher risk.

5.5.2 Out-of-sample Portfolio Performance

In the first step, it is found that Gaussian-Normal and CV-Normal strategies can produce efficient frontiers that have a higher return and lower risk. In contrast, efficient frontiers produced by the Clayton-ST strategy have a lower return and higher risk. However, the strategies' ability to produce predictive efficient frontiers cannot guarantee their real performance. Therefore, rolling month returns are further used to compare their real performance in the following subsections.

Table 5.4 shows descriptive statistics of the cumulative returns in twenty funding cycles (six months per cycle) for nine portfolio strategies (including the MVN strategy). This study reports mean, standard deviation, skewness, kurtosis, and minimum negative values of cumulative returns in twenty cycles. From Table 5.4, we see that the means of cumulative returns produced by the Gaussian-Normal strategy are consistently larger than the value produced by the MVN strategy, which supports Hafner & Reznikova (2010) and Sahamkhadam et al. (2018). Using the dynamic Gaussian copula, they find the interest of the portfolio constructed by two bivariate stock indexes or ten indexes consistently outperforms their standard competing models. However, for the collaterals-5 and collaterals-7 cases, the means of cumulative returns (2.03 % and 3.07 %) produced by CV-ST strategy are much larger than other strategies (including the MVN strategy). This means that the CV-ST strategy can be used to mitigate default risks faced by the IFP that has a large portfolio of collaterals, as it consistently performs better in term of returns when the size of the portfolio is large. This finding complements the existing literature, which underlines the advantage of other copulas, such as the grouped Student T, in describing the dependence structure among multi-series data (Creal & Tsay 2015).

Table 5.4: Out of Sample Copula-based Portfolio Strategy Performance.

Metric	Size	MVN (Benchmark)	Gaussian -Normal	Student T -Normal	Clayton-Normal	CV-Normal	Gaussian -ST	Student T-ST	Clayton -ST	CV -ST
Mean	3	1.67	**1.9**	**1.98**	1.31	2.05	**1.96**	**1.79**	1.49	1.55
	5	0.89	**0.93**	0.95	0.43	1.19	**1.17**	0.52	**1.64**	**2.03**
	7	1.28	**1.58**	1.06	1.11	0.96	**1.89**	**2.42**	**2.34**	**3.07**
Standard Deviation	3	20	19.96	19.97	19.3	**20.01**	19.99	19.89	19.45	19.8
	5	18.39	18.15	18.03	18.36	18.11	**19.24**	**19.63**	**21.04**	**19.11**
	7	18.69	**18.7**	18.58	18.38	18.74	**19.65**	**19.54**	**21.65**	**19.73**
Kurtosis	3	1.818	**1.881**	**1.974**	**2.414**	**1.976**	1.776	**2.022**	**2.38**	**2.135**
	5	2.31	2.302	**2.418**	**2.523**	2.042	2.213	2.177	1.353	1.886
	7	1.724	1.664	1.714	**2.807**	**2.146**	**2.291**	**1.784**	1.28	0.986
Skewness	3	-0.78	-0.78	-0.83	-0.99	-0.73	-0.76	-0.85	-0.99	-0.67
	5	-0.91	-0.88	-0.9	-0.96	-0.80	-0.97	-0.89	-0.72	-0.8
	7	-0.69	-0.75	-0.79	-1.08	-0.84	-1.17	-1.04	-0.59	-0.63
Minimum Negative	3	-55.72	-55.54	-55.99	-56.56	**-55.37**	-55.21	-56.21	-56.74	**-55.53**
	5	-53.4	-52.43	-52.41	-54.5	**-51.06**	-55.66	-57.02	-57	**-52.8**
	7	-51.4	-51.38	-51.88	-54.88	-53.58	-57.19	-54.47	-56.79	**-49.77**
Omega	3	3.338	**5.615**	6.272	0.977	**7.125**	**6.146**	4.519	2.459	**3.439**
	5	-1.796	**-0.937**	-0.665	-5.849	**1.736**	**0.022**	-6.083	2.055	**7.886**
	7	1.661	**4.243**	-0.575	0.064	-1.434	**4.869**	10.407	7.146	**17**

From Table 5.4. it is also found that the standard deviations produced by most copula strategies are lower than the standard deviation produced by the MVN strategy in the collaterals-3 case. It means that the returns produced by the copula strategies are relatively stable when the size of the portfolio is small. However, this situation changes as the size of the portfolio increases. For the collaterals-5 and collaterals-7 cases, all copulas with skewed Student T marginal strategies produce a higher standard deviation than do the MVN strategies, while most copulas with normal marginal strategies produce a lower standard deviation than MVN strategies, which is contrary to the conclusion made by Okhrin et al. (2013) that the variance of returns produced by copulas with a normal marginal is stronger. This study shows that the cumulative returns produced by copulas with normal marginal strategies are more stable than the returns produced by copulas with skewed Student T strategies. Furthermore, when the portfolio size increases, although the CV-ST strategy produces higher mean cumulative returns, the produced standard deviations are higher than the value produced by the MVN strategy.

In the inventory financing setting, the avoidance of extreme collateral returns is preferred, as the occurrence of an extremely low return is easy to cause a default risk. From Table 5.4, it is found that most copula strategies produce a larger kurtosis value than does the MVN strategy in the collaterals-3 case. However, this situation changes as the size of the portfolio increases. For the collaterals-5 and collaterals-7 cases, the Gaussian-Normal, Clayton-ST and CV-ST consistently produce lower kurtosis, which means these strategies have fewer outlier problems. In particular, the CV-ST strategy produces a much lower kurtosis (0.986) than that of the other strategies in the collaterals-7 case. It means that the CV-ST strategy will be less likely to produce extreme returns compared to other strategies when the portfolio size is big. Additionally, this strategy consistently produces a larger skewness value than most other strategies for all portfolio sizes, which means the CV-ST strategy is also less likely to produce extreme negative returns.

Owing to the steep drop in collateral prices being the most critical risky source of inventory financing, the IFP should pay attention to the minimum negative drop between the cumulative returns in sequential cycles. We can see from Table 5.4 that the CV-Normal and CV-ST strategy consistently produce higher minimum negative values than that of most other copulas strategies. It means that the CV-Normal and CV-ST strategies could effectively deal with the extreme situation where there exists a steep drop in successive cycles.

Furthermore, the Omega ratio is used to compare the performance of different strategies. The Omega ratio is a risk-return performance measure. It is calculated by creating a partition in the cumulative return distribution to create an area of losses and an area for gains relative to this threshold. The best performing strategy shows the highest value for each measure. From Table 5.4, it is found that the Gaussian-Normal and Gaussian-ST strategies consistently produce a higher Omega ratio for all portfolio sizes than the values produced by the MVN strategy. Among the different copula strategies, it is found that the value of metrics produced by the CV-ST strategy is much higher than the values produced by other strategies in the collaterals-5 and collaterals-7 cases. It means that the CV-ST strategy has a much better economic performance than the other strategies when the portfolio size is large, suggesting that CV-ST is a good strategy for mitigating default risks faced by the IFP with a large portfolio.

5.5.3 The Analysis of Portfolio Rebalances

The investigation recalculates the desired target weights at the beginning of each funding cycle (6 months per cycle), and the portfolio is rebalanced accordingly. In the twenty funding cycles setting, adjustments to portfolio weights are due to the volatility of out-of-sample collateral returns. Here, this study uses the same funding cycle (6 months) for each portfolio size; the adjustments to the weights of collaterals within the portfolio can capture the varying changes in the decision of portfolio optimization made by each strategy. A large adjustment due to rebalancing means a high operational cost for the IFP; thus, it could undermine the effectiveness of a portfolio strategy. Therefore, those strategies that require a small adjustment could be more desirable due to low operational cost. The index of maximum positive and negative adjustment also exhibits the degree of difficulty to manage the portfolio. When the maximum positive adjustment is too high, or the minimum negative adjustment is too low, the operational cost of managing the portfolio of collaterals will be high.

From Table 5.5, it is found that the advantage of copula strategies regarding operational cost is relatively small compared to that of the MVN strategy. For different portfolio sizes, almost all the variances of weights for twenty funding cycles are consistently larger than those produced by the MVN strategy. Additionally, the maximum positive adjustment values produced by the MVN strategy are lower than the values produced by most copula strategies.

Table 5.5: The Analysis of Portfolio Rebalances.

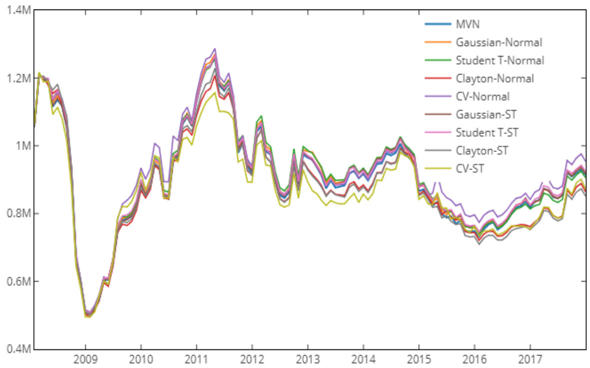
Metric	Size	MVN (Benchmark)	Gaussian-Normal	Student T -Normal	Clayton-Normal	CV-Normal	Gaussian -ST	Student T-ST	Clayton -ST	CV -ST
Variance	3	0.001	0.001	0.002	0.002	0.014	0.002	0.002	0.003	0.016
	5	0.001	0.004	0.003	0.005	0.016	0.001	0.002	0.005	0.001
	7	0.006	0.007	0.01	**0.001**	0.015	**0.003**	0.033	**0.001**	**0.005**
Maximum	3	7.57	**6.67**	8.17	14.37	31.32	10.79	10.55	13.55	37.49
Positive	5	8	29.02	14.32	22.31	25	12.69	9.07	21.19	8.6
Adjustment	7	18.02	29.22	23.64	**8.8**	31	**15.59**	44.13	**14.2**	19.4
Minimum	3	-8.41	**-8.24**	**-8.22**	-16.08	-29.9	-9.58	-9.42	-16.3	-32.47
Negative	5	-9.99	-12.22	-19.48	-27.24	-27.8	**-6.23**	-12.53	-20.57	-11.6
Adjustment	7	-22.4	**-18.75**	**-18.18**	**-9.36**	-49.2	**-15.88**	-44.02	**-8.69**	**-21.6**

For the collaterals-3 and collaterals-5 cases, the variances produced by most copula strategies are higher than the values produced by the MVN strategy. Meanwhile, the MVN strategy has more adjustment advantages regarding the metrics of the maximum positive adjustment and maximum negative adjustment in these two cases. It is found that most copula strategies produce a higher maximum positive adjustment value and lower negative adjustment value than does the MVN strategy. In the collaterals-7 case, although this situation has been changed, the adjustment advantage of copula strategies is still slight. Only the Clayton-Normal, Gaussian-ST and Clayton-ST strategies consistently produce better performance regarding the metrics of variance, maximum positive adjustment and maximum negative adjustment. In particular, the Clayton-Normal strategy produces an extremely low maximum positive adjustment value (8.8%), and the Clayton-ST strategy has a relatively high minimum negative adjustment (-8.69%).

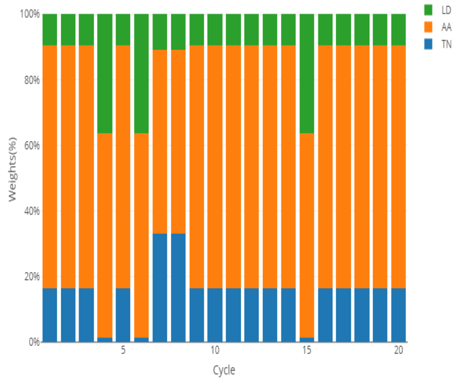
5.5.4 The Performance of Strategies in Twenty Cycles

The former section has analyzed the operational cost of rebalancing the portfolio using nine different strategies. This section further compares the performance of the different strategies in successive twenty cycles. Different from the portfolio optimization in the securities market, the issue that an IFP would be most concerned with is how much does the value of collaterals decline in each funding cycle because the steep drop in collateral value will cause the IFP to face default risks. Assume an IFP has a portfolio of collaterals worth 1 million dollars; by adopting nine different strategies (one MVN strategy and eight copula strategies), the firm can obtain nine different trends of portfolio value in twenty cycles.

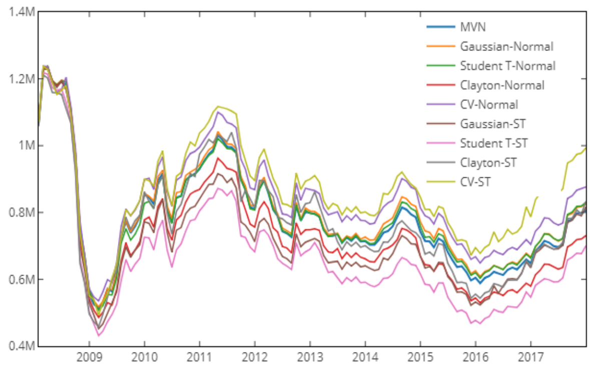
For the collaterals-3 case, it is found that the CV-Normal strategy (See the purple line in Figure 5.4a) generally performs better than the other strategies in twenty cycles. For the collaterals-5 case, although the CV-Normal strategy still has good performance, its advantage is slight compared with the CV-ST strategy (See the purple and golden lines in Figure 5.4c). In this case, the cumulative value produced by the strategy performs much higher than the values



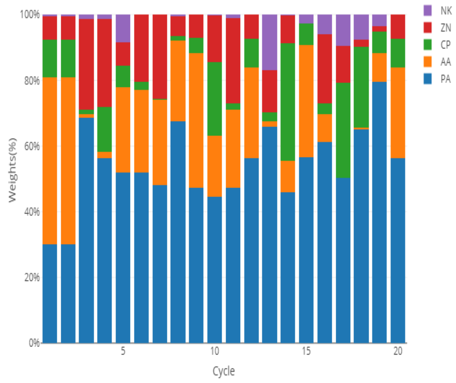
(a) $N = 3$



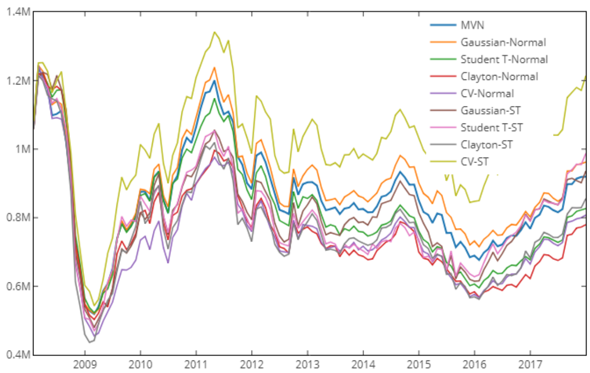
(b) CV-Normal Strategy



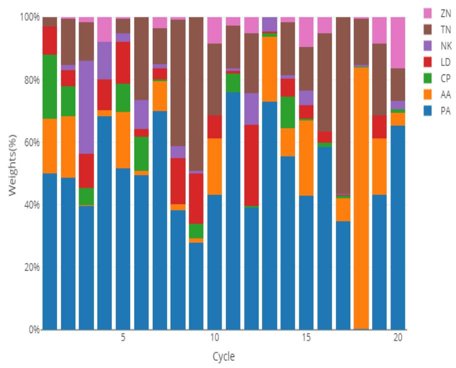
(c) $N = 5$



(d) CV-ST strategy



(e) $N = 7$



(f) CV-ST strategy

Figure 5.4: Cumulative Value of the Collaterals for Different Strategies and Initial Weights Produced by the Optimal Strategy in Twenty Cycles.

produced by the other strategies. This finding supports the conclusion made by Okhrin et al. (2013) and Brechmann & Czado (2013). By constructing a Multi-dimensional portfolio of stocks and comparing the performance of different portfolio strategies, they demonstrate that vine copulas have an advantage in describing the dependence structures among time series and thus can more accurately allocate weights to the portfolio. Furthermore, for the collaterals-7 case, it is found that the performance of the CV-Normal strategy deteriorates. However, the CV-ST strategy still performs much better than do the other strategies (See the golden line in Figure 5.4e). Although it is in the inventory financing setting, this finding supports the idea that the portfolio strategy constructed by canonical vine copulas combined with CVaR dominates the strategy constructed by Gaussian copulas with CVaR (Chollete et al. 2009), and the performance of canonical vine Copulas reveals the importance of accurately describing the dependence structure in portfolio optimization, which has also been underlined by Garcia & Tsafack (2011). The right subgraphs in Figure 5.4 show the assigned weights produced by the strategies with high performance at the beginning of each cycle. From Figure 5.4, we see that there are twenty bars (cycles) in each right subgraph, and the proportion of different colors in each bar shows the distribution of collaterals within the portfolio. From Figure 5.4b, it is found that the proportion of the yellow color is high, suggesting that allocating more weights to AA can help the IFP mitigate default risks. In the case where the size of the portfolio is 5 or 7, PA is allocated with high weight in most funding cycles, which means PA is less risky collateral for the IFP.

5.6 Conclusion

This research introduces portfolio optimization into the inventory financing from an IFP's perspective, thereby exploring the optimal inventory financing strategy based on copulas. Based on the evaluation, an optimal portfolio strategy for mitigating default risks in inventory financing is proposed. The main research findings are as follows.

On the one hand, the effectiveness of the decision on the allocation of collateral weights in each funding cycle depends on which portfolio strategy is chosen. The findings reveal that directly using historical returns to calculate the best weights is not reliable. Based on the analysis in Section 5.5.1, it is found that although the efficient frontiers produced by the Clayton-ST strategy in the Collaterals-3 case, the MVN strategy in the Collaterals-5 case, and the Clayton-Normal strategy in the Collaterals-7 case are close to the efficient frontiers produced by historical returns; they do not perform better than other strategies in the real application. From Figure 5.4, it is found that the cumulative returns produced by the CV-based strategies perform much better than the other strategies. This finding is supported by Sahamkhadam et al. (2018), who have demonstrated that the portfolio of indexes based on the predictive returns produced by copula models performs much better than the portfolio based on historical data. Therefore, to precisely allocate weights to the collaterals within the portfolio, we still need to choose the most reliable generating function to estimate the parameters based on historical returns. Based on newly generated returns, we can then accurately allocate weights for the collaterals within the portfolio. On the other hand, the strategic decision on the allocation of collateral weights in each funding cycle depends on the size of the portfolio. Through analysis, it is found that the general canonical vine copula is an ideal generating function to be used to capture the dependence structure of multi-collateral return series. More specifically, the CV-Normal strategy is the ideal strategy for the small portfolio, and the CV-ST strategy is the ideal strategy for the large portfolio. From Figure 5.4a, it is found that CV-Normal strategy consistently performs better in the collaterals-3 case. From Figure 5.4c and Figure 5.4e, it is found that the CV-ST strategy performs much better than the other strategies in the collaterals-5 and collaterals-7 cases. This finding verifies the hypothesis made by Hafner & Reznikova (2010), assuming that in a high dimensional setting more complex and flexible copulas with time-varying parameters can better capture the dependence structure among multi-dimensional time series.

This study makes the following key contributions. First, this research contributes to the inventory financing literature by introducing the concept of portfolio optimization into inventory financing. It differs from a significant strand of literature that investigates how a bank and a seller optimize their financial decisions and measure the borrower's creditworthiness (Rockafellar & Stanislav 2000, Rockafellar & Uryasev 2002, Aas et al. 2009, Babich & Kouvelis 2018). This study explores how portfolio optimization effectively and efficiently mitigates the risks of fluctuating collateral prices in inventory financing. Such an exploration provides some novel strategies that can be used to reduce default risks in inventory financing, which has not been observed in the existing literature (e.g., Rockafellar & Uryasev (2002), Babich & Kouvelis (2018), and Chen et al. (2017)). Second, this study adopts a data-driven analytical approach to incorporate timely market data on collateral prices to update dependence parameters of predictive models and collateral weights in the inventory financing, which provide new insights on how to improve the accuracy of the analysis in operation research. Existing operations research literature that studies inventory financing mainly uses a stochastic model to reveal changing external environments such as the demand and the price (Rockafellar & Uryasev 2002, Babich & Kouvelis 2018). The introduction of copulas provides a new way to simulate changing factors, which can help operation researchers in the inventory financing field improve the preciseness of their analysis.

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Chapter 6

Conclusions and Future Research

6.1 Introduction

The final chapter reflects on the applications of copulas in SCF. Section 6.2 summarizes the main research findings from the three studies in this doctoral research. It is followed by a discussion of theoretical contributions in Section 6.3, showing how this research contributes to the knowledge of SCF. Section 6.4 presents managerial implications to show how IFPs can learn from the insights generated by this research. Finally, Section 6.5 discusses the limitations of this research and the paths to follow in the future.

6.2 Research Findings

This doctoral research explores the applications of copulas in SCF. Specifically, it examines how they improve the performance of inventory financing by managing the impawn rate, interest rate, and portfolio of collateral units. The main research findings for each study are summarized as follows.

In the first study (**Chapter 3**), the research examines the performance of copulas in optimizing impawn rate for the inventory financing. By comparing the predictive performance of the MVN and the Clayton canonical vine copula, it finds that the latter one can evaluate default probability very well and thus be used to help the IFP parameterize the objective profit function of the inventory financing business. Although the existing literature seldom investigates the Clayton canonical vine copula's ability to manage default risk, its ability to predict future returns more precisely has already been underlined (Brechmann & Czado 2013, Low et al. 2013). Furthermore, based on the chosen copula, the first study calculates and compares the expected profit with a uniform impawn rate and multiple impawn rates. It is found that setting multiple impawn rates can help IFPs gain more profit and thus be applied in the inventory financing business. This analytical result is intuitive; the settlement of multiple optimal impawn rates is based on the optimization of each objective profit function. However, the settlement of a single optimal impawn rate is based on the sum of three objective profit functions, which induces triple marginalization (Chen et al. 2018). Similarly, Buzacott & Zhang (2004) investigate performance when setting heterogeneous interest rates. They demonstrate that financial service providers can optimize asset-based financing by choosing an appropriate interest rate for each borrower. To further take advantage of the Clayton canonical vine copula, which characterizes the dependency structure among time series very well, the original business model can be extended to another business model, namely, setting the optimal rate based on borrowers who have different capital status. Based on the analysis, the first study finds that for borrowers with low default motivation, a higher optimal impawn rate can be set, even considering the effect of fluctuating collateral prices on default probability. With a decrease in default motivation, the expected profit function tends to be linear. In extreme cases, if the borrower has very good credit (the value of λ_m causes $\bar{F}(\ln\theta_i + \omega)$ to not interact with $f(\ln\theta_i + \omega) + (1 - \frac{1}{1-\exp(-\lambda_m)})$, the optimal impawn rate can be set as 1. Thus, when providing inventory financing, IFPs, especially, need to look at the historical credit of borrowers. Based on the evaluated default motivation of borrowers, they can set more accurate impawn rates[1].

In the second study (**Chapter 4**), the research examines the performance of GARCH-EVT-Copula in setting both impawn rates and interest rates for inventory financing. In this study, an impawn and interest rate model (IIRM) is constructed using the option pricing model built by Black & Scholes (1973). According to their theory, the inventory financing business can be treated as a European put option, as this version of the option contract limits execution to its expiration date (Vázquez 1998). This is in line with the practice that IFPs face the risk that borrowers will not pay the money back when the collateral price at the end of the funding period is lower than the amount of the loan. Based on this principle, the IIRM can be developed through the relationship between the interest rate for inventory financing, the industrial interest rate, the financing cycle, the variance of the collateral returns and the impawn rate. To identify a proper approach for incorporating the risk of market volatility when determining impawn and interest rates, this second study compares three approaches: the historical approach, the GARCH-EVT-Student T-Copula approach, and GARCH-EVT-R-vine-Copula approach. The first approach uses historical collateral prices to estimate impawn rates. The GARCH-EVT-Copula is considered an approach for two reasons. First, the collateral prices are autocorrelative and the GARCH model excels at taking this issue into an account (Engle 1982, Sahamkhadam et al. 2018). Second, the collateral prices have strong relationships with each other, and the dependence structure for time series can be described by the Copula model well (Aas et al. 2009, Oh & Patton 2018). Through a comparison analysis, this study evaluates the effectiveness of the three approaches for using the market

[1]This paragraph has been published in the *International Journal of Production Economics*. The authors' contribution statement has been provided at the start of this dissertation.

prices of collateral units to set the impawn rates and interest rates for inventory financing. The analysis is also extended to look at whether the suggested approach can perform well through the extreme market volatility of the COVID-19 period. Compared with the historical approach, impawn rates estimated by the GARCH-EVT-Copula approach can help IFPs extract more value from collateral units. Moreover, the interest rate derived from the IIRM reveals well the risk involved in collateral units. When collateral units have high impawn rates but low volatility or low impawn rates but high volatility, it indicates that a low interest rate should be used. Moreover, the impawn rates and interest rates derived through the GARCH-EVT-Copula approach deliver better financial performance for IFPs. The GARCH-EVT-Copula parameterization process can help IFPs identify the least risky and most predictive collateral unit. Finally, the extended analysis shows that combining the GARCH-EVT-Copula with IIRM models can help to manage the risk of inventory financing in the COVID-19 period.

In the third study (**Chapter 5**), the research examines the performance of copulas in optimizing the portfolio of collateral units for inventory financing. This research introduces portfolio optimization into inventory financing from an IFP perspective, to determine the optimal inventory financing strategy based on copulas. First, it compares the collateral return scatters produced by four bivariate copulas with the empirical returns. Based on the analysis, three bivariate copulas are chosen and further developed into multivariate copulas. Meanwhile, a more flexible general canonical vine copula is used to select different bivariate copulas, including Student T copula, Gaussian copula, and Clayton copula. Four chosen copula models are extended into eight portfolio strategies by introducing the normal and skewed Student T marginal. This study then adopts the CVaR utility function, and it sets the MVN as a benchmark to evaluate the performance of the eight portfolio strategies that are extended. Based on the evaluation, an optimal portfolio strategy for mitigating default risks in inventory financing is proposed. The main research findings are as follows. On the one hand, the effectiveness of the decision on the allocation of collateral weights in each funding cycle depends on which portfolio strategy is chosen. The findings reveal that directly using historical returns to calculate the best weights is not reliable. Based on the analysis in Section 5.5.1, It is found that although the efficient frontiers produced by the Clayton-ST strategy in the Collaterals-3 case, the MVN strategy in the Collaterals-5 case, and the Clayton-Normal strategy in the Collaterals-7 case are closest to the efficient frontiers produced by historical returns; they do not perform better than other strategies in the real application. From Figure 5.4, it can be found that the cumulative returns produced by the CV-based strategies perform much better than the other strategies. This finding is supported by Sahamkhadam et al. (2018), who have demonstrated that the portfolio of indexes based on the predictive returns produced by copula models performs much better than the portfolio based on historical data. Therefore, to precisely allocate weights to the collaterals within the portfolio, it is necessary to choose the most reliable generating function to estimate the parameters based on historical returns. Based on newly generated returns, weights for the collaterals within the portfolio can then be accurately allocated. On the other hand, the strategic decision on the allocation of collateral weights in each funding cycle depends on the size of the portfolio. The analytical results reveal that the general canonical vine copula is an ideal generating function to be used to capture the dependence structure of multi-collateral return series. More specifically, the CV-Normal strategy is the ideal strategy for the small portfolio, and the CV-ST strategy is the ideal strategy for the large portfolio. From Figure 5.4a, it is found that CV-Normal strategy consistently performs better in the collaterals-3 case. From Figure 5.4c and Figure 5.4e, it is found that the CV-ST strategy performs much better than the other strategies in the collaterals-5 and collaterals-7 cases. This finding verifies the hypothesis made by Hafner & Reznikova (2010), assuming that in a high dimensional setting more complex and flexible copulas with time-varying parameters can better capture the dependence structure among multi-dimensional time series[2].

6.3 Contributions

This doctoral research makes the following key contributions. First, from a theoretical perspective, this study complements the literature on SCF. The core aim of SCF instruments is to improve cash flow and the performance of working capital in a supply chain. Among SCF instruments, inventory financing is playing an increasingly important role in helping SMEs relieve their financial burden and improve the performance of their working capital (Chakuu et al. 2019). However, the study regarding inventory financing is only a beginning. How to mitigate risk and improve the profitability of inventory financing has not been studied systematically. This doctoral research intended identify effective data-driven approaches to improve inventory financing, which is seldom investigated in the SCF literature (Buzacott & Zhang 2004, Zhang et al. 2016, Wang et al. 2018). More specifically, to mitigate default risk caused by fluctuating collateral prices, the first study explores how impawn rates in inventory financing can be dynamically adjusted by an IFP. Based on GARCH-EVT-Copulas and IIRM, the second study investigates how both the impawn rates and interest rates are determined for inventory financing. Finally, the third study introduces portfolio management into inventory financing. Specifically, it incorporates timely market data on collateral prices to update the dependent parameters of predictive models and collateral weights in inventory financing. This provides IFPs with novel strategies to identify and manage risky collateral units.

Second, from a methodological viewpoint, combining predictive and prescriptive models also contributes to the field of business analytics. Studies of inventory financing mainly use a stochastic model to reveal changing external environments such as demand and price (Babich & Kouvelis 2018, Rockafellar & Uryasev 2002). Introducing copulas provides a new way to simulate changing factors, which can help researchers of inventory financing increase the precision of their analysis. That is, prior research mainly uses a distribution function with unchanged parameters, such as

[2]This paragraph bas been published in the *Computers & Operations Research*. The authors' contribution statement has been provided at the start of this dissertation.

an exponential distribution (Wang et al. 2018)), or given parameters (Buzacott & Zhang 2004) to evaluate the default risk. However, the first study has identified an effective copula-based strategy to help IFPs estimate the default probability for each funding period dynamically. The second study combines the data-driven copula model and the IIRM to address the contemporary issue of determining impawn rates and interest rates for inventory financing (He et al. 2012, Yang & Birge 2018). Combining the IIRM and GARCH-EVT-Copula models provides some novel strategies to help IFPs control default risk in inventory financing and gain competitiveness in the financing market. The third study differs from a significant strand of literature that investigates how a bank and a seller optimize their financial decisions and measure the borrower's creditworthiness (Aas et al. 2009, Rockafellar & Stanislav 2000, Rockafellar & Uryasev 2002, Babich & Kouvelis 2018). It explores how portfolio optimization effectively and efficiently mitigates the risks of fluctuating collateral prices in inventory financing. That exploration provides some novel strategies that can reduce default risk in inventory financing. However, this has not been fully examined in the SCF literature (e.g., Rockafellar & Uryasev (2002), Chen et al. (2017), and Babich & Kouvelis (2018)).

Third, this doctoral research also makes critical practical contributions. Direct interactions with practitioners of inventory financing (See Appendix A) have indicated that the risks in inventory financing come mainly from three aspects: credit, collateral, and interest rates. IFPs manage these risks mainly by referring to historical collateral prices and industrial practices, but this often leads to severe losses of efficiency in inventory financing. This doctoral research proposes innovative approaches that IFPs can use to manage the credit, collateral, and interest rate risks, reduce losses of efficiency in inventory financing, and improve their competitiveness in the financial market. Specifically, to manage credit risk, the first study shows how copula can be adopted to evaluate the default probability of the borrower and how important factors in inventory financing, such as the industrial impawn rate, risk-taking ability, liquidity risk, and interest rate, affect this process. The first study generates insights into how these factors are coordinated to mitigate the credit risk in inventory financing. Collateral risk comes mainly from fluctuating collateral prices. The first two studies have also shown how risk can be mitigated by setting proper impawn rates. Collateral risk can also be mitigated through portfolio management. Using portfolio optimization theory, the third study shows how risky collateral units are identified and how weights of the collateral portfolio are optimized to reduce collateral risk in changing funding cycles. For the interest rate risk, using the option pricing model constructed by Black & Scholes (1973), the second study develops IIRM to reveal the nonlinear relationship between the impawn rates and interest rates. This can help IFPs set proper interest rates for different collateral units.

6.4 Managerial Implications

This doctoral research has significant managerial implications that IFPs can use to improve their performance in inventory financing. It presents how impawn rates, interest rates, and the weights of a collateral portfolio can be dynamically managed to reduce risks and losses of efficiency. For this type of financing, the primary source of risk is fluctuating collateral prices. In the same funding cycles, the risks for different collateral units are often different. For the same collateral units, the risk level could also change in different funding cycles. Only if IFPs capture these changing factors can they manage their inventory financing businesses proactively and more likely gain competitiveness. To arrive at this target, a natural way is to capture more information from the relationships between the changing prices of different collateral units. This doctoral research shows that, by using copulas, IFPs can describe dependence structures of multiple time series of collateral prices and more effectively manage credit risk, interest rates, and collateral risks in inventory financing for different funding cycles. More specifically:

The research findings in the first study generate practical insights in terms of how IFPs dynamically evaluate the probability that borrowers will default and optimize impawn rates to balance credit risk and collateral risk to profit in inventory financing. IFPs are recommended to address fluctuating collateral prices actively by settling different impawn rates in different funding cycles. In addition, the factors identified in the sensitivity analysis also provide IFPs with some insights on how to set impawn rates. Specifically, when the interest rate is high, IFPs must take special care to not set an overly high impawn rate as this can increase the probability of default and decrease the profits that the IFP can gain. In addition, when most IFPs in the inventory financing market are risk-averse and inclined to set a low impawn rate, an IFP can set a higher impawn rate (if the default probability function suggests the IFP should do so). This can help the IFP gain a competitive advantage. Setting multiple impawn rates for different collateral units can help IFPs gain more profit from inventory financing[3].

The research findings in the second study benefit IFPs who would like to control risks by dynamically determining both impawn rates and interest rates. With the GARCH-EVT-R-vine-Copula model, IFPs can set more appropriate impawn rates without hedging their risk. The conversation with IFPs shows that, although hedging services can help IFPs manage risk, they incur extra operation costs. If the proposed approach can directly set an impawn rate that can manage risk, the reduced expenditure on hedging services can help IFPs improve their market competitiveness. In addition, the nonlinear relationship between impawn rates and interest rates has not been specified in industrial practice. The model used in this study to determine interest rates in inventory financing can provide a reference for IFPs when they draft inventory financing contracts for borrowers. The proposed method in the second study can also help IFPs reduce losses of efficiency and improve the performance of their inventory financing businesses. For

[3]This paragraph is adapted based on the work published in the *International Journal of Production Economics*. The authors' contribution statement has been provided at the start of this dissertation.

instance, setting impawn rates based on historical returns may reduce the efficiency of IFPs' inventory financing businesses. Considering the impawn rates, IFPs can set interest rates for different collateral units to manage the risk of their inventory financing businesses. Furthermore, the model constructed by ARMA(1,1) and GARCH(1,1) can help IFPs identify less risky collateral units. This can help IFPs that are more vulnerable to default focus on more predictable collateral units (e.g., aluminum in study II). Finally, the flexible GARCH-EVT-R-vine-Copula can adequately manage risk in a volatile market environment. IFPs can use this approach to manage default risk or bad loans of inventory financing business during periods of extreme market volatility. Specifically, IFPs can refer to the impawn rate derived from the proposed approach to determine a reduced amount to avoid unnecessary losses of efficiency.

The research findings in the third study are beneficial to IFPs that would like to mitigate credit and collateral risks by optimizing the structure of a collateral portfolio. That study demonstrates that copulas-based inventory financing strategies have an advantage in capturing the dependence structure of multi-collateral return series. The IFP can thus allocate weights for collaterals within the portfolio accurately to mitigate inventory financing risks. Exploring portfolio optimization in inventory financing provides IFP managers with a new way to mitigate financing risks by making full use of their control of the collaterals. Specifically, by introducing copulas, especially the general canonical vine copulas, into portfolio optimization, the third study illustrates how newly updated collateral prices can be used iteratively to produce effective portfolio strategies in each funding cycle. By making full use of this, an IFP can mitigate default risk in inventory financing without the need to reduce the impawn rate or increase the interest rate, and this would further improve their competitiveness in the inventory financing market[4].

6.5 Limitations and Future Research

This doctoral research is the first attempt to explore how IFPs employ copulas to manage the impawn rate, interest rate, and portfolio of collateral units. Effective strategies have been identified to help IFPs control the risks and reduce efficiency loss in their inventory financing business. However, for this doctoral research, there are still some limitations, and several extensions can be made to address these limitations.

First, competition between IFPs could also affect how impawn rates and interest rates are determined. Future studies can consider the competitive relationships between IFPs and include this element when constructing expected profit functions for individual IFPs. This could help them make more informed decisions on setting impawn rates and interest rates. One solution is to combine a game-theoretical model with copulas. Game theoretical models can study how multiple competing players gamble with each other and make their decisions (Chen & Cai 2011, Wang

[4]This paragraph has been published in the *Computers & Operations Research*. The authors' contribution statement has been provided at the start of this dissertation.

et al. 2018). Adopting copula models can help players estimate the probability that competing players will choose certain strategies, and this can make their analysis more accurate.

Second, for analytical purposes, this research incorporates only seven industry metals. The collateral units in the study can be further extended to include more raw materials. Thus, future researchers can expand the dimension of copulas by introducing other collateral units into a portfolio and then test the effectiveness of the strategies recommended in this research. In addition, the models proposed by this doctoral research are flexible. This means they can also be applied in the financial area. One extension could use the models proposed in this study to optimize the financing of pawned and stock funds.

Third, the clearest advantage of copulas is their precision in depicting the dependence structure among time series (Patton 2012). To make full use of this advantage, more operational factors, such as the relationships between the demands for different products, can be included in the model, which can make the analysis more accurate. One promising extension would be to introduce copulas into operational decision models, such as the economic production quantity model, newsvendor, and the economic order quantity model. The operations literature that focuses on the control of fluctuating material costs still uses the traditional stochastic model (e.g., Kouvelis et al. (2017)). The combination of copulas and operational decision models may generate more practical insights, especially when companies have more interfaces in the growing global supply chain [5].

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[5]The third and fourth paragraphs in Section 6.5 are adapted based on the work published in the *International Journal of Production Economics* and *Computers & Operations Research*. The authors' contribution statements have been provided at the start of this dissertation.

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Appendix A

Documentation for Interactions with Relevant Constituents

A.1 Summary of Interactions with Relevant Constituents

This section documents the interactions with relevant constituents, including the Vice President of Sales (VP sales) of company A, Commodity Financing Managers of several inventory financing providers, i.e., company B, company C and company D, and practitioners including the Purchasing Manager in company E and managers of company E's suppliers. The nature of conversations with these inventory financing providers and users includes formal interviews and field visits. The detailed conversations are categorized into separate sections: research motivation, model assumptions and setup, and insights. Examples of interview questions and the organization profiles of these main constituents are also included.

A.2 Research Motivation

This research is inspired by the interactions with business managers and inventory financing service providers. The Purchasing Manager of company E, a subsidiary of Telecom Corporation China are interviewed. To relieve the financial burden of their suppliers, company E founded a supply chain finance platform to help SMEs gain extra financial support. The Purchasing Manager of company E also indicated that inventory financing had gained more attention.

"The disadvantage of existing supply chain finance models is the implementation does not achieve risk reduction, but merely transfers the banks' risk to the guarantors. Many firms are not willing to guarantee bank loans for other firms since they do not see any benefit but risk arising

from it."

The conservations with the Trade Finance Manager of company B and company C and the VP sales of company A further confirm that inventory financing plays an vital role in relieving the financial burden of SMEs. It is also learnt from conversation that receivable financing, payable financing, and inventory financing have attracted increasing interest among the existing supply chain finance schemes. Inventory financing is growing rapidly – more than 20% per year prior to the pandemic. Through conversations, it is known that IFPs are actively searching for effective approaches to manage the risk associated with inventory financing. The Commodity Finance Manager of company D said:

"Hedging risk will create extra operating cost. We are looking forward to identifying more accurate approach to set proper haircut for different collateral."

From the conversation with the VP sales of company A and the Trade Finance Manager of company B, it is known that the pandemic has made the inventory financing business riskier. Most inventory financing providers said they had further decreased the impawn rate during the pandemic. For example, the manager of Trade Finance in company C said:

"At the beginning of the COVID-19 pandemic, the headquarter issued an urgent message to decrease the impawn rate.... It is difficult to predict future price of collateral. So, the impawn rate for some commodities, such as iron bar, copper, has been decreased to less than 30%."

It's also known from the conversation that it became more difficult for the inventory financing providers to manage risks during the pandemic. Therefore, this doctoral research examines the effectiveness of the proposed approaches for determining the impawn and interest rates during the pandemic.

A.3 Assumptions and Model Setup

The conversations with the VP sales of company A, the manager of Trade Finance in company B, and the manager of Commodity Finance in company D help the research team develop the theoretical model and make relevant assumptions. Based on the conversations and relevant literature, three main types of risks in inventory financing are identified: credit risk, collateral risk, and interest rate risk. The assumptions and model setup are informed by these three risks. How each risk is considered will be further explained in the following.

A.3.1 Credit Risk

Credit risk is more about the company itself. If there is a significant credit issue in the company, the borrower cannot obtain funding from the inventory financing provider even using collaterals. As the manager of Trade Finance in company B said:

"To avoid the credit risk in the inventory financing, the manager first needs to check the balance sheet of the borrower and make sure their operation is healthy."

The inventory financing provider also carefully checks the warehouse receipt to prevent the borrower from acquiring multiple loans with one collateral unit. As the VP sales of company A said:

"The lender needs to make sure the borrower is not using one warehouse receipt to gain multiple loans."

Therefore, regarding the credit risk, it is assumed that the borrower has no significant issue in their daily operation and is not motivated to gain multiple loans with one warehouse receipt.

A.3.2 Collateral Risk

Collateral risk originates from the fluctuating value of collaterals. As the VP sales of company A said:

"Collateral risk describes the risk that arises from accepting an asset – whose value can move up and down – as collateral to guarantee the value of the loan."

From the conversations, it is also known that the most important risk management tool in inventory financing is the impawn rate. As the manager of Trade Finance in company C and company B said:

"To manage collateral risk, we reduce the amount we will lend compared to the notional value of the collateral. For the collateral that is valuable, standardized or easy to trade, we will give relatively high impawn rate."

It's known that the historical prices of collateral are used by inventory service providers to determine impawn rate. As the manager of Commodity Financing in company D said:

"When doing inventory financing, we always check the excel sheet that includes historical price of the commodity. We look at the historical price of collateral. If the historical price is very

flatulating, we would set a high haircut."

Therefore, study I and study II intend to identify an effective approach to determine the impawn rate. To evaluate the effectiveness of the proposed approach, study II also uses the historical approach used by the industry as the benchmark.

A.3.3 Interest Rate Risk

Interest rate risk can be explained as the opportunity cost of lenders to provide the credit to their borrowers. It is linked to the industrial interest rate and risk. As the VP sales in company A said:

"The interest rate in the inventory financing is pretty like the mortgage. It is the opportunity cost of the lender to provide credit to certain borrowers. The inventory financing provider will set up higher interest rate compared with other lower risk business. ... Higher impawn rate means higher risk. If the borrower would like to negotiate with higher impawn rate, we will increase the interest rate to compensate the increased risk. However, their relationship is not linear."

Based on the option theory, the second study estimates the interest rate by considering the relationship between the impawn rate and industry interest rate.

A.4 Insights

Insights attained from direct interactions with relevant constituents help to make better sense of the analytical results, which can inform the industrial practices. First, the IFP can use innovative data-driven approaches like the GARCH-EVT-R-vine-Copula model instead of directly estimating the impawn rate with the historical price. With the GARCH-EVT-R-vine-Copula model, the IFP can determine a more appropriate impawn rate, which has the potential to reduce the cost of managing the collateral and hedging the risk. Second, the analytical result shows that the relationship between the interest and impawn rates is nonlinear, which in line with the argument made by the VP sales of company A. However, the nonlinear relationship between the impawn and interest rates has not been specified in the industrial practice. In this study, the model used for determining the interest rate in inventory financing can provide a reference for the IFPs when negotiating interest and impawn rates with the borrower. Third, analytical results have demonstrated the effectiveness of the proposed approach during the period with extreme market volatility. When reducing the impawn rate in an extremely volatile environment, the IFP can refer to the proposed approach to determine a reduced amount to avoid unnecessary efficiency losses.

A.5 Interview Questions

1. Is inventory financing an important revenue stream for your company? How do you describe your role in inventory financing?
2. Which collateral units are preferred in the inventory financing? Is marketability an important factor to be considered?
3. What are the main risks in the inventory financing business?
4. How are these risks managed in the inventory financing? (In addition to the company's profile, what are other factors are being considered to manage the risk in the inventory financing?) What tools or models are mainly used in the industry?
5. How does your organization manage the risk of inventory financing from the fluctuating collateral prices?
6. In response to price risk, how to determine the impawn and interest rates in the inventory financing?
7. Which tools are used to determine impawn and interest rates? Are people considering using different tools to set impawn and interest rates?
8. How does the COVID-19 pandemic affect inventory financing? Especially, how the impawn and interest rates are affected?

A.6 The Organization Profile of Relevant Constituents

A.6.1 Company A

Company A was founded in 1877, which is the global largest market in terms of options, futures contracts, standardized forward contracts on base metals. The metals trade on company A, such as nickel, aluminum and copper, are particularly welcomed in the inventory financing due to high marketability.

A.6.2 Company B

Company B was founded on February 5, 1912 and its headquarter is in Beijing, China. It is one of the four biggest state-owned commercial banks in China. With branches in 61 countries, it is also the most international bank in China. Its service segments include treasury operations, personal banking, corporate banking, investment banking, insurance, etc. Specifically, the corporate banking mainly services government authorities, financial institutions and corporate customers such as derivative products, credit facilities, foreign currency, trade related products, custody, loans, overdrafts, deposits and current accounts. The trade financing service includes the inventory financing, forfaiting, two-factor import factoring, two-factor export factoring, etc.

A.6.3 Company D

Company D is a financial services company and British multinational investment bank. Its headquarter is in London, England. Company D is organized into four core segments: investment management, wealth management, corporate banking and personal banking. For the business of corporate banking, they provide trade financing to the businesses. The standardized, valuable and marketable raw materials are particularly welcomed in their inventory financing business.

A.6.4 Company E

Company E is a branch of Telecom Corporation China. In 2008, it was acquired by Telecom Corporation China. Currently, it is the key telecommunication service provider in the Southwest part of China, which employs more than 21,000 staff. Its services are comprehensive, including customized information communication solutions, internet broadband, mobile phones and telephone landlines. Company E founded a supply chain finance platform to help small and SMEs gain extra financial support.

APPENDIX B

PROOFS OF CHAPTER 3

B.1 Proof of Proposition 3.2

Based on Eq. (3.11), taking the first-order and the second-order derivative of $\pi_i(\theta_i)$ with the respect to θ_i, we have

$$\frac{d\pi_i}{d\theta_i} = [\exp(kjr)-1]\beta_i[1-F(\ln\theta_i+\omega)-f(\ln\theta_i+\omega)] \tag{B.1}$$

and

$$\frac{d^2\pi_i}{d\theta_i^2} = -\frac{\beta_i}{\theta_i}[\exp(kjr)-1][f(\ln\theta_i+\omega)+f'(\ln\theta_i+\omega))] \tag{B.2}$$

$\omega = \ln\frac{1-\tau}{1-\rho_i}+kr$. Because the $F(x)$ and $f(x)$ are the CDF and PDF of normal distribution, by using the transformation $F(x)=\Phi(\frac{x-\mu}{\sigma})=\int_{-\infty}^{x}\phi(\frac{t-\mu}{\sigma})dt$ and $f(x)=\phi(\frac{x-\mu}{\sigma})$ ($\Phi(\cdot)$ and $\phi(\cdot)$ are CDF and PDF of standard normal distribution. μ and σ are mean and variance of logarithmic returns of i^{th} collateral unit. Then we further have:

$$\frac{d\pi_i}{d\theta_i} = \beta_i[\exp(kjr)-1][1-\Phi(\frac{\ln\theta_i+\omega-\mu}{\sigma})-\phi(\frac{\ln\theta_i+\omega-\mu}{\sigma})] \tag{B.3}$$

and

$$\frac{d^2\pi_i}{d\theta_i^2} = -\frac{\beta_i}{\theta_i}[\exp(kjr)-1][\phi(\frac{\ln\theta_i+\omega-\mu}{\sigma})+\phi'(\frac{\ln\theta_i+\omega-\mu}{\sigma})] \tag{B.4}$$

Because $\phi(x)=\frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})$ and $\phi'(x)=\frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})(-x)$, Eq. (B.4) can be further wrote into

(B.5)
$$\frac{d^2\pi_i}{d\theta_i^2} = -\frac{\beta_i}{\theta_i\sqrt{2\pi}}[\exp(kjr)-1]\exp(-\frac{(\frac{\ln\theta_i+\omega-\mu}{\sigma})^2}{2})(1-\frac{\ln\theta_i+\omega-\mu}{\sigma})$$

If $\frac{d^2\pi_i}{d\theta_i^2} < 0$, then $0 < \theta_i < \exp(\sigma+\mu-\omega)$. Because $0 < \theta_i < 1$, therefore, here we have two cases:

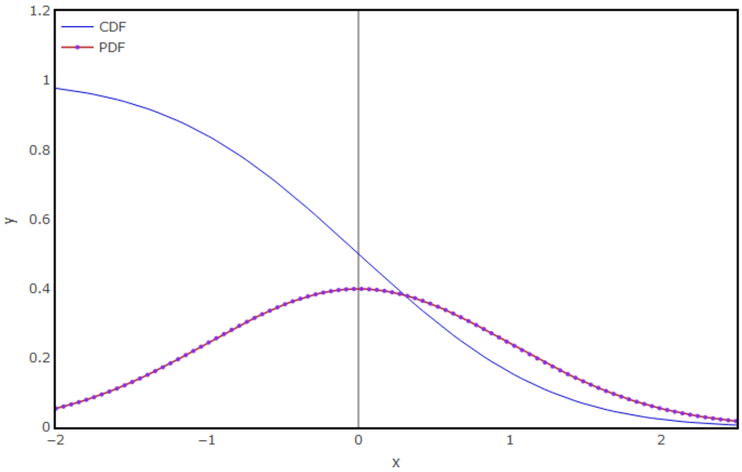


Figure B.1: The Relationship between the Standard Normal Distribution and Its CDF.

(i) When $\sigma+\mu-\omega < 0$, $0 < \theta_i < \exp(\sigma+\mu-\omega)$.

(B.6)
$$\frac{d\pi_i}{d\theta_i}(\exp(\sigma+\mu-\omega)) = [\exp(kjr)-1]\beta_i[\bar{\Phi}(1)-\phi(1)]$$

Because $\frac{\ln\theta_i+\omega-\mu}{\sigma}$ is monotonically increasing for θ_i, there exists θ_i to make $\frac{d\pi_i}{d\theta_i} = 0$ (See Fig. B.1). Therefore, there exists a single optimal impawn rate θ_i^* in the interval $(0, \exp(\sigma+\mu-\omega))$ to maximize π_i.

(ii) When $\sigma+\mu-\omega \geq 0$, $0 < \theta_i < 1$.

(B.7)
$$\frac{d\pi_i}{d\theta_i}(1) = [\exp(kjr)-1]\beta_i[\bar{\Phi}(\frac{\omega-\mu}{\sigma})-\phi(\frac{\omega-\mu}{\sigma})]$$

Because $\frac{\omega-\mu}{\sigma} > 1$, thus there exists a single optimal impawn rate θ_i^* in the interval of $(0,1)$ to maximize π_i (See Fig. B.1).

In summary, there exists a single optimal impawn rate θ_i^* in the interval $(0, min(\exp(\sigma+\mu-\omega),1))$ to maximize π_i.

B.2 The Bivariate Clayton Copula

The density of the bivariate Clayton copula is as follows:

$$c(u_1,u_2) = (1+\delta_{12})(u_1u_2)^{-1-\delta_{12}} \times (u_1^{\delta_{12}} + u_2^{-\delta_{12}} - 1)^{-1/\delta_{12}-2} \tag{B.8}$$

where $0 < \delta_{12} < \infty$ controls the dependence. $\delta_{12} \to \infty$ implies perfect independence, while $\delta_{12} \to 0$ means independence.

$$h(u_1,u_2,\delta_{12}) = u_2^{-\delta_{12}-1}(u_1^{-\delta_{12}} + u_2^{-\delta_{12}} - 1)^{-1-1/\delta_{12}} \tag{B.9}$$

and the inverse of the h-function is shown as:

$$h_{12}^{-1}(u_1,u_2,\delta_{12}) = \left\{(u_1 \cdot u_2^{\delta_{12}+1})^{-\frac{\delta_{12}}{\delta_{12}+1}} + 1 - v^{-\delta_{12}}\right\}^{-1/\delta_{12}} \tag{B.10}$$

B.3 Proof of Proposition 3.3

When $n = 3$, the joint density of the three-dimensional case, $f(x_1,x_2,x_3)$, can be represented by a function of the bivariate condition copulas as follows:

$$f(x_1,x_2,x_3) = f_1(x_1) \cdot f(x_2|x_1) \cdot f(x_3|x_1,x_2) \tag{B.11}$$

The second factor in the right side of Eq. (B.11) can be decomposed into the pair-copula and a marginal density. We have

$$f(x_2|x_1) = c_{12}[F_1(x_1),F_2(x_2)] \cdot f_2(x_2) \tag{B.12}$$

The third factor in the right side of Eq. (B.12) can be decomposed into

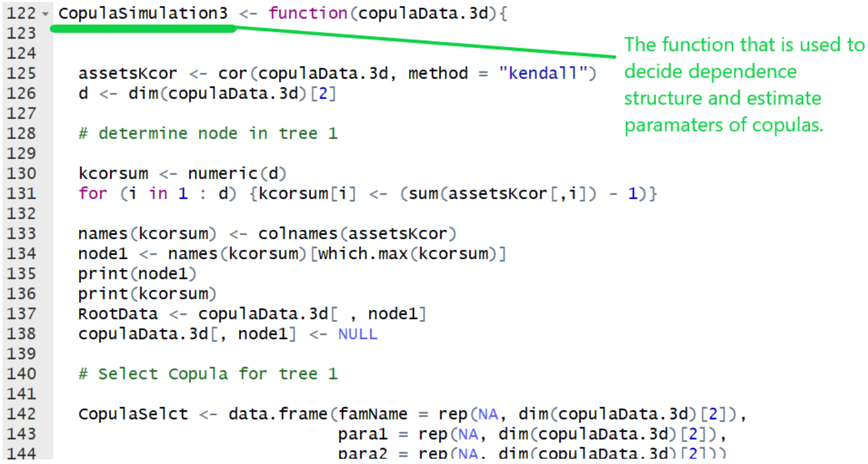
$$\begin{aligned} f(x_3|x_1,x_2) = \frac{f(x_2,x_3|x_1)}{f(x_2|x_1)} &= \frac{c_{23|1}[F(x_2|x_1),F(x_3|x_1)] \cdot f(x_2|x_1) \cdot f(x_3|x_1)}{f(x_2|x_1)} \\ &= c_{23|1}[F(x_2|x_1),F(x_3|x_1)] \cdot f(x_3|x_1) \\ &= c_{23|1}[F(x_2|x_1),F(x_3|x_1)] \cdot c_{13}[F_1(x_1),F_3(x_3)] \cdot f_3(x_3) \end{aligned} \tag{B.13}$$

B.4 The Procedure of Adopting Rolling Window Approach

B.5 Proof of Proposition 3.5

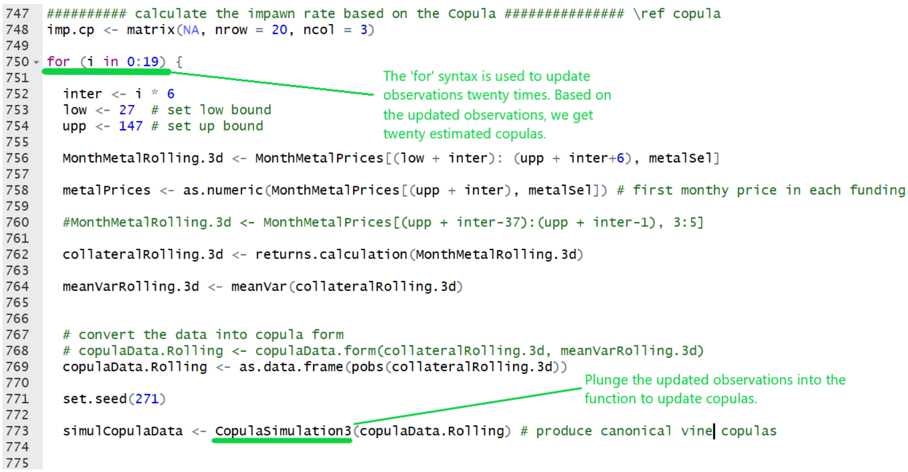
Based on Eq. (3.25), taking the first-order and the second-order derivative of $\pi_i(\theta_i)$ with the respect to θ_i, we have

$$\frac{d\pi_i}{d\theta_i} = [\exp(kjr)-1]\beta_i\{1-[1-\exp(-\lambda_m)]F(\ln\theta_i+\omega) - [1-\exp(-\lambda_m)]f(\ln\theta_i+\omega)\} \tag{B.14}$$



```r
CopulaSimulation3 <- function(copulaData.3d){


  assetsKcor <- cor(copulaData.3d, method = "kendall")
  d <- dim(copulaData.3d)[2]

  # determine node in tree 1

  kcorsum <- numeric(d)
  for (i in 1 : d) {kcorsum[i] <- (sum(assetsKcor[,i]) - 1)}

  names(kcorsum) <- colnames(assetsKcor)
  node1 <- names(kcorsum)[which.max(kcorsum)]
  print(node1)
  print(kcorsum)
  RootData <- copulaData.3d[ , node1]
  copulaData.3d[, node1] <- NULL

  # Select Copula for tree 1

  CopulaSelct <- data.frame(famName = rep(NA, dim(copulaData.3d)[2]),
                            para1 = rep(NA, dim(copulaData.3d)[2]),
                            para2 = rep(NA, dim(copulaData.3d)[2]))
```


Figure B.2: The Function Used to Construct Copulas.



```r
########## calculate the impawn rate based on the Copula ############### \ref copula
imp.cp <- matrix(NA, nrow = 20, ncol = 3)

for (i in 0:19) {

  inter <- i * 6
  low <- 27  # set low bound
  upp <- 147 # set up bound

  MonthMetalRolling.3d <- MonthMetalPrices[(low + inter): (upp + inter+6), metalSel]

  metalPrices <- as.numeric(MonthMetalPrices[(upp + inter), metalSel]) # first monthy price in each funding

  #MonthMetalRolling.3d <- MonthMetalPrices[(upp + inter-37):(upp + inter-1), 3:5]

  collateralRolling.3d <- returns.calculation(MonthMetalRolling.3d)

  meanVarRolling.3d <- meanVar(collateralRolling.3d)


  # convert the data into copula form
  # copulaData.Rolling <- copulaData.form(collateralRolling.3d, meanVarRolling.3d)
  copulaData.Rolling <- as.data.frame(pobs(collateralRolling.3d))

  set.seed(271)

  simulCopulaData <- CopulaSimulation3(copulaData.Rolling) # produce canonical vine copulas
```


Figure B.3: 'for' Syntax.

and

$$\frac{d^2\pi_i}{d\theta_i^2} = -\frac{\beta_i}{\theta_i}[\exp(kjr)-1][1-\exp(-\lambda_m)][f(\ln\theta_i+\omega)+f'(\ln\theta_i+\omega)] \tag{B.15}$$

$\omega = \ln\frac{1-\tau}{1-\rho_i} + kr$. Because the $F(x)$ and $f(x)$ are the CDF and PDF of normal distribution, by using the transformation $F(x) = \Phi(\frac{x-\mu}{\sigma}) = \int_{-\infty}^{x}\phi(\frac{t-\mu}{\sigma})dt$ and $f(x) = \phi(\frac{x-\mu}{\sigma})$ ($\Phi(\cdot)$ and $\phi(\cdot)$ are CDF and PDF of standard normal distribution), then we further have:

(B.16)

$$\frac{d\pi_i}{d\theta_i} = \beta_i[\exp(kjr)-1]\{1-[1-\exp(-\lambda_m)]\Phi(\frac{\ln\theta_i+\omega-\mu}{\sigma})-[1-\exp(-\lambda_m)]\phi(\frac{\ln\theta_i+\omega-\mu}{\sigma})\}$$

and

$$\frac{d^2\pi_i}{d\theta_i^2} = -\frac{\beta_i}{\theta_i}[\exp(kjr)-1][1-\exp(-\lambda_m)][\phi(\frac{\ln\theta_i+\omega-\mu}{\sigma})+\phi'(\frac{\ln\theta_i+\omega-\mu}{\sigma})] \tag{B.17}$$

Because $\phi(x) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})$ and $\phi'(x) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})(-x)$, Eq. (B.4) can be further wrote into

$$\frac{d^2\pi_i}{d\theta_i^2} = -\frac{\beta_i}{\theta_i\sqrt{2\pi}}[\exp(kjr)-1][1-\exp(-\lambda_m)]\exp(-\frac{(\frac{\ln\theta_i+\omega-\mu}{\sigma})^2}{2})(1-\frac{\ln\theta_i+\omega-\mu}{\sigma}) \tag{B.18}$$

If $\frac{d^2\pi}{d\theta_i^2} < 0$, then $0 < \theta_i < \exp(\sigma+\mu-\omega)$. Because $0 < \theta_i < 1$, thus here we have two cases:
(i) When $\sigma+\mu-\omega < 0$, $0 < \theta_i < \exp(\sigma+\mu-\omega)$.

$$\frac{d\pi_i}{d\theta_i}(\exp(\sigma+\mu-\omega)) = [\exp(kjr)-1]\beta_i[1-\exp(-\lambda_m)][\bar{\Phi}(1)-(\phi(1)-\frac{1}{1-\exp(-\lambda_m)}+1)] \tag{B.19}$$

Because $\frac{\ln\theta_i+\omega-\mu}{\sigma}$ is monotonically increasing for θ_i, when $\bar{F}(\ln\theta_i+\omega)$ interacts with $f(\ln\theta_i+\omega)+(1-\frac{1}{1-\exp(-\lambda_m)})$, there exists θ_i to make $\frac{d\pi_i}{d\theta_i}=0$. Therefore, in this case there exists a single optimal impawn rate θ_i^* in the interval $(0, \exp(\sigma+\mu-\omega))$ to maximize π_i (See Fig. B.1).

(ii) When $\sigma+\mu-\omega \geq 0$, $0 < \theta_i < 1$.

$$\frac{d\pi_i}{d\theta_i}(1) = [\exp(kjr)-1]\beta_i[1-\exp(-\lambda_m)]\{\bar{\Phi}(\frac{\omega-\mu}{\sigma})-[\phi(\frac{\omega-\mu}{\sigma})-\frac{1}{1-\exp(-\lambda_m)}+1]\} \tag{B.20}$$

Because $\frac{\omega-\mu}{\sigma} > 1$, therefore, when $\bar{F}(\ln\theta_i+\omega)$ interacts with $f(\ln\theta_i+\omega)+(1-\frac{1}{1-\exp(-\lambda_m)})$, there exists a single optimal impawn rate θ_i^* in the interval of $(0,1)$ to maximize π_i (See Fig. B.1).

In summary, when $\bar{F}(\ln\theta_i+\omega)$ interacts with $f(\ln\theta_i+\omega)+(1-\frac{1}{1-\exp(-\lambda_m)})$, there exists a single optimal impawn rate θ_i^* in the interval $(0, min(\exp(\sigma+\mu-\omega),1))$ to maximize π_i.

APPENDIX C

PROOFS OF CHAPTER 4

C.1 Proofs of IIRM

Based on the option pricing model developed by Black and Scholes (1973), we have the following equations:

(C.1)
$$\frac{dV}{V} = \mu dt + \sigma dz$$

(C.2)
$$dz = \phi\sqrt{dt}$$

(C.3)
$$G(V,t) = -V_t N(-d_1) + M_T \exp(-\bar{r}(T-t))N(-d_2)$$

in which,

(C.4)
$$d_1 = \frac{\ln\frac{V_t}{M_T} + (\bar{r} + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

(C.5)
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

(C.6)
$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{\mu^2}{2}} d\mu$$

Taking the first order of $d_1 = \frac{\ln\frac{V_0}{M_T}+(\bar{r}+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$ regarding M_T, we have $\frac{\partial d_1}{\partial M_T} = \frac{\partial d_2}{\partial M_T} = -\frac{1}{M_T\sigma\sqrt{T}}$. Taking the first order of $N(d_1)$ regarding d_1 , we have:

$$N^{'}(d_1) = -\frac{1}{M_T\sigma\sqrt{T}}\exp(-\frac{d_1^2}{2}). \tag{C.7}$$

Taking the first order of $N(d_2)$ regarding d_2, we have:

$$N^{'}(d_2) = -\frac{1}{M_T\sigma\sqrt{T}}\exp(-\frac{d_1^2+\sigma^2T-2\sigma d_1\sqrt{T}}{2}). \tag{C.8}$$

When $t = 0$,

$$d_1 = \frac{\ln\frac{V_0}{M_T}+(\bar{r}+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}. \tag{C.9}$$

By taking Eq. (C.9) into Eq. (C.8), we have:

$$N^{'}(d_2) = -\frac{1}{M_T\sigma\sqrt{T}}\exp(-\frac{d_1^2}{2}+(\ln\frac{V_0}{M_T}+\bar{r}T)). \tag{C.10}$$

Then the relationship between $N^{'}(d_1)$ and $N^{'}(d_2)$ can be represented as:

$$N^{'}(d_1) = N^{'}(d_2)\frac{M_T}{V_0}\exp(-\bar{r}T). \tag{C.11}$$

Taking the first order of $\theta = \frac{V_0N(-d_1)+M_T\exp(-\bar{r}T)N(d_2)}{V_0}$ regarding M_T, we have

$$\frac{d\theta}{dM_T} = \frac{V_0N^{'}(-d_1)\frac{1}{M_T\sigma\sqrt{T}}+N^{'}(d_2)\left(-\frac{1}{M_T\sigma\sqrt{T}}\right)\exp(-\bar{r}T)M_T+N(d_2)\exp(-\bar{r}T)}{V_0}. \tag{C.12}$$

By taking Eq. (C.11) into Eq. (C.12), we have:

$$\frac{d\theta}{dM_T} = \frac{N(d_2)\exp(-\bar{r}T)}{V_0} \tag{C.13}$$

By taking the first order of Eq. (4.14), we have:

$$\frac{d\theta}{dM_T} = \frac{\exp(-rT)}{V_0} \tag{C.14}$$

Owing to $\frac{N(d_2)\exp(-\bar{r}T)}{V_0} = \frac{\exp(-rT)}{V_0}$, M_T can be represented as:

$$M_T = V_0 \exp[\left(\bar{r} - \frac{\sigma^2}{2}\right) T - \sigma\sqrt{T} N^{-1}(\exp(\bar{r}T - rT))] \tag{C.15}$$

Because the first order of $\theta = \frac{M_T \exp(-rT)}{V_0}$ with respect to M_T equals the first order of $\theta = \frac{V_o N(-d_1) + M_T \exp(-\bar{r}T) N(d_2) N(d_2)}{V_0}$ with respect to M_T, the relationship between the impawn rate and the interest rate can be expressed as follows

$$\theta = \exp[\left(\bar{r} - \frac{\sigma^2}{2} - r\right) T - \sigma\sqrt{T} N^{-1}(\exp(\bar{r} - r)T)] \tag{C.16}$$

Based on Eq. (C.16), the implicit function regarding the interest rate r and the impawn rate θ can be derived.

C.2 The Bivariate Student T Copula

The density of the bivariate Student copula can be represented by

$$c(u_1, u_2) = \frac{\frac{\Gamma(\frac{v_{12}+2}{2})}{\Gamma(\frac{v_{12}}{2})}}{v_{12}\pi dt(x_1, v_{12}) dt(x_2, v_{12})\sqrt{1-\rho_{12}^2}} \times \{1 + \frac{x_1^2 + x_2^2 - 2\rho_{12}x_1x_2}{v_{12}(1-\rho_{12}^2)}\}^{-\frac{v_{12}+1}{2}},$$

where v_{12} and ρ_{12} are two parameters of the copula, $x_1 = t_{v_{12}}^{-1}(u_1)$, $x_2 = t_{v_{12}}^{-1}(u_2)$, $dt(\bullet, v_{12})$ is the probability density and $t_{v_{12}}^{-1}(\bullet)$ is the quantile function. Degrees of freedom, expectation and variance of the standard univariate Student t distribution are respectively represented by v_{12}, 0 and $\frac{v_{12}}{v_{12}-2}$.

APPENDIX D

PROOFS OF CHAPTER 5

D.1 Pair-copula

D.1.1 The Bivariate Gumbel Copula

The density of the bivariate Gumbel copula is as follows:

$$c(u_1,u_2,) = \exp\left[-\left\{(-\log u_1)^{\delta_{12}} + (-\log u_2)^{1/\delta_{12}}\right\}^{1/\delta_{12}}\right](u_1u_2)^{-1} \times \left\{(-\log u_1)^{\delta_{12}} + (-\log u_2)^{\delta_{12}}\right\}^{-2+2/\delta_{12}}$$
$$\times (\log u_1 \log u_2)^{\delta_{12}-1} \times \left\{1 + (\delta_{12}-1)[(-\log u_1)^{\delta_{12}} + (-\log u_2)^{\delta_{12}}]^{-1/\delta_{12}}\right\}$$

where $\delta_{12} \geq 1$ controls the dependence. $\delta_{12} \to \infty$ implies perfect dependence, while $\delta_{12} = 1$ means independence.

D.1.2 The Bivariate Clayton Copula

The density of the birariate Clayton copula is as follows:

$$c(u_1,u_2) = (1+\delta_{12})(u_1u_2)^{-1-\delta_{12}} \times (u_1^{\delta_{12}} + u_2^{-\delta_{12}} - 1)^{-1/\delta_{12}-2}$$

where $0 < \delta_{12} < \infty$ controls the dependence. $\delta_{12} \to \infty$ implies perfect independence, while $\delta_{12} \to 0$ means independence.

The h-function of Clayton copula is as follows:

$$h(u_1,u_2,\delta_{12}) = u_2^{-\delta_{12}-1}(u_1^{-\delta_{12}} + u_2^{-\delta_{12}} - 1)^{-1-1/\delta_{12}}$$

and the inverse of the h-function is shown as:

$$h_{12}^{-1}(u_1,u_2,\delta_{12}) = \left\{(u_1 \cdot u_2^{\delta_{12}+1})^{-\frac{\delta_{12}}{\delta_{12}+1}} + 1 - v^{-\delta_{12}}\right\}^{-1/\delta_{12}}$$

D.1.3 The Bivariate Student T Copula

The density of the bivariate Student T copula can be represented by

$$c(u_1,u_2) = \frac{\Gamma(\frac{\nu_{12}+2}{2})/\Gamma(\frac{\nu_{12}}{2})}{\nu_{12}\pi dt(x_1,\nu_{12})dt(x_2,\nu_{12})\sqrt{1-\rho_{12}^2})} \times \left\{1+\frac{x_1^2+x_2^2-2\rho_{12}x_1x_2}{\nu_{12}(1-\rho_{12}^2)}\right\}^{-\frac{\nu_{12}+1}{2}}$$

where ν_{12} and ρ_{12} are two parameters of the copula, $x_1 = t_{\nu_{12}}^{-1}(u_1), x_2 = t_{\nu_{12}}^{-1}(u_2)$, $dt(\cdot,\nu_{12})$ is the probability density and $t_{\nu_{12}}^{-1}(\cdot)$ is the quantile function. Degrees of freedom, expectation and variance of the standard univariate Student t distribution are respectively represented by ν_{12}, 0 and $\frac{\nu_{12}}{\nu_{12}-2}$.

D.1.4 The Bivariate Gaussian Copula

The density of the bivariate Gaussian copula is shown as

$$c(u_1,u_2) = \frac{1}{\sqrt{1-\rho_{12}^2}}\exp\left\{-\frac{\rho_{12}^2(x_1^2+x_2^2)-2\rho_{12}x_1x_2}{2(1-\rho_{12}^2)}\right\}$$

where ρ_{12} is the parameter of the copula, $x_1 = \Phi^{-1}(u_1)$, $x_2 = \Phi^{-1}(u_2)$ and $\Phi^{-1}(\cdot)$ is the inverse of the standard univariate normal (Gaussian) distribution function.

D.2 Seven-dimensional Pair-copula Decompositions

When $n = 7$, the joint density of the seven-dimensional case, $f(x_1,x_2,x_3,x_4,x_5,x_6,x_7)$, can be represented by a function of the bivariate condition copulas as follows:

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5,x_6,x_7) = {} & f_1(x_1)\cdot f(x_2|x_1)\cdot f(x_3|x_1,x_2)\cdot f(x_4|x_1,x_2,x_3)\cdot f(x_5|x_1,x_2,x_3,x_4) \\ & \cdot f(x_6|x_1,x_2,x_3,x_4,x_5)\cdot f(x_7|x_1,x_2,x_3,x_4,x_5,x_6) \end{aligned} \tag{D.1}$$

The second factor in the right side of Eq. (D.1) can be decomposed into the pair-copula and a marginal density. We have

$$f(x_2|x_1) = c_{12}[F_1(x_1),F_2(x_2)]\cdot f_2(x_2) \tag{D.2}$$

The third factor in the right side of Eq. (D.1) can be decomposed into

$$\begin{aligned} f(x_3|x_1,x_2) = \frac{f(x_2,x_3|x_1)}{f(x_2|x_1)} &= \frac{c_{23|1}[F(x_2|x_1),F(x_3|x_1)]\cdot f(x_2|x_1)\cdot f(x_3|x_1)}{f(x_2|x_1)} \\ &= c_{23|1}[F(x_2|x_1),F(x_3|x_1)]\cdot f(x_3|x_1) \\ &= c_{23|1}[F(x_2|x_1),F(x_3|x_1)]\cdot c_{13}[F_1(x_1),F_3(x_3)]\cdot f_3(x_3) \end{aligned} \tag{D.3}$$

The fourth factor in the right side of Eq. (D.1) can be decomposed into

$$
\begin{aligned}
f(x_4|x_1,x_2,x_3) = \frac{f(x_3,x_4|x_1,x_2)}{f(x_3|x_1,x_2)} &= \frac{c_{34|12}[F(x_3|x_1,x_2),F(x_4|x_1,x_2)]\cdot f(x_3|x_1,x_2)\cdot f(x_4|x_1,x_2)}{f(x_3|x_1,x_2)} \\
&= c_{34|12}[F(x_3|x_1,x_2),F(x_4|x_1,x_2)]\cdot \frac{f(x_2,x_4|x_1)}{f(x_2|x_1)} \\
&= c_{34|12}[F(x_3|x_1,x_2),F(x_4|x_1,x_2)]\cdot \frac{c_{24|1}[F(x_2|x_1),F(x_4|x_1)]\cdot f(x_2|x_1)\cdot f(x_4|x_1)}{f(x_2|x_1)} \\
&= c_{34|12}[F(x_3|x_1,x_2),F(x_4|x_1,x_2)]\cdot c_{24|1}[F(x_2|x_1),F(x_4|x_1)]\cdot f(x_4|x_1) \\
&= c_{34|12}[F(x_3|x_1,x_2),F(x_4|x_1,x_2)]\cdot c_{24|1}[F(x_2|x_1),F(x_4|x_1)]\cdot c_{14}[F_1(x_1),F_4(x_4)]\cdot f_4(x_4)
\end{aligned}
\tag{D.4}
$$

Similarly, $f(x_5|x_1,x_2,x_3)$ can be decomposed into

$$
\begin{aligned}
f(x_5|x_1,x_2,x_3) = \frac{f(x_3,x_5|x_1,x_2)}{f(x_3|x_1,x_2)} &= \frac{c_{35|12}[F(x_3|x_1,x_2),F(x_5|x_1,x_2)]\cdot f(x_3|x_1,x_2)\cdot f(x_5|x_1,x_2)}{f(x_3|x_1,x_2)} \\
&= c_{35|12}[F(x_3|x_1,x_2),F(x_5|x_1,x_2)]\cdot \frac{f(x_2,x_5|x_1)}{f(x_2|x_1)} \\
&= c_{35|12}[F(x_3|x_1,x_2),F(x_5|x_1,x_2)]\cdot \frac{c_{25|1}[F(x_2|x_1),F(x_5|x_1)]\cdot f(x_2|x_1)\cdot f(x_5|x_1)}{f(x_2|x_1)} \\
&= c_{35|12}[F(x_3|x_1,x_2),F(x_5|x_1,x_2)]\cdot c_{25|1}[F(x_2|x_1),F(x_5|x_1)]\cdot f(x_5|x_1) \\
&= c_{35|12}[F(x_3|x_1,x_2),F(x_5|x_1,x_2)]\cdot c_{25|1}[F(x_2|x_1),F(x_5|x_1)]\cdot c_{15}[F_1(x_1),F_5(x_5)]\cdot f_5(x_5)
\end{aligned}
\tag{D.5}
$$

The fifth factor in the right side of Eq. (D.1) can be decomposed into

$$
\begin{aligned}
f(x_5|x_1,x_2,x_3,x_4) &= \frac{f(x_4,x_5|x_1,x_2,x_3)}{f(x_4|x_1,x_2,x_3)} \\
&= \frac{c_{45|123}[F(x_4|x_1,x_2,x_3),F(x_5|x_1,x_2,x_3)]\cdot f(x_4|x_1,x_2,x_3)\cdot f(x_5|x_1,x_2,x_3)}{f(x_4|x_1,x_2,x_3)} \\
&= c_{45|123}[F(x_4|x_1,x_2,x_3),F(x_5|x_1,x_2,x_3)]\cdot f(x_5|x_1,x_2,x_3)
\end{aligned}
$$

Inserting Eq. (D.5) into above expression, we have

$$
\begin{aligned}
f(x_5|x_1,x_2,x_3,x_4) = c_{45|123}&[F(x_4|x_1,x_2,x_3),F(x_5|x_1,x_2,x_3)]\cdot c_{35|12}[F(x_3|x_1,x_2),F(x_5|x_1,x_2)] \\
&\cdot c_{25|1}[F(x_2|x_1),F(x_5|x_1)]\cdot c_{15}[F_1(x_1),F_5(x_5)]\cdot f_5(x_5)
\end{aligned}
\tag{D.6}
$$

Similarly, $f(x_6|x_1,x_2,x_3,x_4)$ can be decomposed into

$$
\begin{aligned}
f(x_6|x_1,x_2,x_3,x_4) = c_{46|123}&[F(x_4|x_1,x_2,x_3),F(x_6|x_1,x_2,x_3)]\cdot c_{36|12}[F(x_3|x_1,x_2),F(x_6|x_1,x_2)] \\
&\cdot c_{26|1}[F(x_2|x_1),F(x_6|x_1)]\cdot c_{16}[F_1(x_1),F_6(x_6)]\cdot f_6(x_6)
\end{aligned}
\tag{D.7}
$$

The sixth factor in the right side of Eq. (D.1) can be decomposed into

$$
\begin{aligned}
f(x_6|x_1,x_2,x_3,x_4,x_5) &= \frac{f(x_5,x_6|x_1,x_2,x_3,x_3,x_4)}{f(x_5|x_1,x_2,x_3,x_4)} \\
&= \frac{c_{56|1234}[F(x_5|x_1,x_2,x_3,x_4),F(x_6|x_1,x_2,x_3,x_4)]\cdot f(x_5|x_1,x_2,x_3,x_4)\cdot f(x_6|x_1,x_2,x_3,x_4)}{f(x_5|x_1,x_2,x_3,x_4)} \\
&= c_{56|1234}[F(x_5|x_1,x_2,x_3,x_4),F(x_6|x_1,x_2,x_3,x_4)]\cdot f(x_6|x_1,x_2,x_3,x_4)
\end{aligned}
$$

Inserting Eq. (D.6) in to above expression, we have

$$\begin{aligned} f(x_6|x_1,x_2,x_3,x_4,x_5) = c_{56|1234}[F(x_5|x_1,x_2,x_3,x_4),F(x_6|x_1,x_2,x_3,x_4)] \\ \cdot c_{46|123}[F(x_4|x_1,x_2,x_3),F(x_6|x_1,x_2,x_3)]\cdot c_{36|12}[F(x_3|x_1,x_2),F(x_6|x_1,x_2)] \\ \cdot c_{26|1}[F(x_2|x_1),F(x_6|x_1)]\cdot c_{16}[F_1(x_1),F_6(x_6)]\cdot f_6(x_6) \end{aligned} \tag{D.8}$$

Similarly, $f(x_7|x_1,x_2,x_3,x_4,x_5)$ can be decomposed into

$$\begin{aligned} f(x_7|x_1,x_2,x_3,x_4,x_5) = c_{57|1234}[F(x_5|x_1,x_2,x_3,x_4),F(x_7|x_1,x_2,x_3,x_4)] \\ \cdot c_{47|123}[F(x_4|x_1,x_2,x_3),F(x_7|x_1,x_2,x_3)]\cdot c_{37|12}[F(x_3|x_1,x_2),F(x_7|x_1,x_2)] \\ \cdot c_{27|1}[F(x_2|x_1),F(x_7|x_1)]\cdot c_{17}[F_1(x_1),F_7(x_7)]\cdot f_7(x_7) \end{aligned} \tag{D.9}$$

The last factor in the right side of Eq. (D.1) can be decomposed into

$$\begin{aligned} f(x_7|x_1,x_2,x_3,x_4,x_5,x_6) = \frac{f(x_6,x_7|x_1,x_2,x_3,x_3,x_4,x_5)}{f(x_6|x_1,x_2,x_3,x_4,x_5)} \\ = \frac{c_{67|12345}[F(x_6|x_1,x_2,x_3,x_4,x_5),F(x_7|x_1,x_2,x_3,x_4,x_5)]\cdot f(x_6|x_1,x_2,x_3,x_4,x_5)\cdot f(x_7|x_1,x_2,x_3,x_4,x_5)}{f(x_6|x_1,x_2,x_3,x_4,x_5)} \\ = c_{67|12345}[F(x_6|x_1,x_2,x_3,x_4,x_5),F(x_7|x_1,x_2,x_3,x_4,x_5)]\cdot f(x_7|x_1,x_2,x_3,x_4,x_5) \end{aligned}$$

Inserting Eq. (D.9) in to above expression, we have

$$\begin{aligned} f(x_7|x_1,x_2,x_3,x_4,x_5,x_6) = c_{67|12345}[F(x_6|x_1,x_2,x_3,x_4,x_5),F(x_7|x_1,x_2,x_3,x_4,x_5)] \\ \cdot c_{57|1234}[F(x_5|x_1,x_2,x_3,x_4),F(x_7|x_1,x_2,x_3,x_4)] \\ \cdot c_{47|123}[F(x_4|x_1,x_2,x_3),F(x_7|x_1,x_2,x_3)]\cdot c_{37|12}[F(x_3|x_1,x_2),F(x_7|x_1,x_2)] \\ \cdot c_{27|1}[F(x_2|x_1),F(x_7|x_1)]\cdot c_{17}[F_1(x_1),F_7(x_7)]\cdot f_7(x_7) \end{aligned} \tag{D.10}$$

Inserting Eq. (D.2), Eq. (D.3), Eq. (D.4), Eq. (D.6), Eq. (D.8) and Eq. (D.10) into Eq. (D.1), we have

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5,x_6,x_7) = f_1(x_1)\cdot f_2(x_2)\cdot f_3(x_3)\cdot f_4(x_4)\cdot f_5(x_5)\cdot f_6(x_6)\cdot f_7(x_7) \\ \cdot c_{12}[F_1(x_1),F_2(x_2)]\cdot c_{13}[F_1(x_1),F_3(x_3)]\cdot c_{14}[F_1(x_1),F_4(x_4)]\cdot c_{15}[F_1(x_1),F_5(x_5)] \\ \cdot c_{16}[F_1(x_1),F_6(x_6)]\cdot c_{17}[F_1(x_1),F_7(x_7)]\cdot c_{23|1}[F(x_2|x_1),F(x_3|x_1)] \\ \cdot c_{24|1}[F(x_2|x_1),F(x_4|x_1)]\cdot c_{25|1}[F(x_2|x_1),F(x_5|x_1)]\cdot c_{26|1}[F(x_2|x_1),F(x_6|x_1)] \\ \cdot c_{27|1}[F(x_2|x_1),F(x_7|x_1)]\cdot c_{34|12}[F(x_3|x_1,x_2),F(x_4|x_1,x_2)] \\ \cdot c_{35|12}[F(x_3|x_1,x_2),F(x_5|x_1,x_2)]\cdot c_{36|12}[F(x_3|x_1,x_2),F(x_6|x_1,x_2)] \\ \cdot c_{37|12}[F(x_3|x_1,x_2),F(x_7|x_1,x_2)]\cdot c_{45|123}[F(x_4|x_1,x_2,x_3),F(x_5|x_1,x_2,x_3)] \\ \cdot c_{46|123}[F(x_4|x_1,x_2,x_3),F(x_6|x_1,x_2,x_3)]\cdot c_{47|123}[F(x_4|x_1,x_2,x_3),F(x_7|x_1,x_2,x_3)] \\ \cdot c_{56|1234}[F(x_5|x_1,x_2,x_3,x_4),F(x_6|x_1,x_2,x_3,x_4)] \\ \cdot c_{57|1234}[F(x_5|x_1,x_2,x_3,x_4),F(x_7|x_1,x_2,x_3,x_4)] \\ \cdot c_{67|12345}[F(x_6|x_1,x_2,x_3,x_4,x_5),F(x_7|x_1,x_2,x_3,x_4,x_5)] \end{aligned} \tag{D.11}$$