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COMPARISON OF SOME TIME-DOMAIN-SYSTEM IDENTIFICATION TECHNIQUES USING APPROXIMATE DATA CORRELATIONS

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ABSTRACT—Most time-domain methods used for modal analysis perform a curve fit to impulse-response data and use the least-squares method as an integral part of their formulations. It is well known that when the data are corrupted, least squares leads to biased parameter estimates, with the damping values being especially sensitive. A number of noniterative techniques that attempt to eliminate the bias—instrumental matrix with delayed observations, double least squares, correlation fit and total least squares—are compared with least squares in terms of the approximate data autocorrelations used in the curve fit. The theoretical comparison is complemented by statistical comparisons upon simple simulated systems. As well as the ability of the methods to reduce the bias on damping estimates, ease of implementation and computational requirements are also investigated.

List of Symbols

a_j = difference equation coefficient
e_j = error upon measurement value
i = $\sqrt{(-1)}$
j,k = integer counters
M = number of system modes
N = number of data points
NA = number of autocorrelation rows used
p = number of rows missed out
$R_{yy}(k) = R_k$ = output autocorrelation at lag k
$R_{y\epsilon}(k)$ = crosscorrelation of output and residual at lag k
$\underline{r}$ = vector of autocorrelations
s = time delay for IMDO method
$y(j\Delta t), y_j$ = output value at time $j\Delta t$
$\underline{y}$ = vector of output values
Δt = sampling interval
ϵ_j = residual value
$\underline{\epsilon}$ = vector of residuals
ζ = damping ratio
ϕ = matrix of output values
ϕ = matrix of time shifted output values
$\underline{\theta}$ = vector of a_i coefficients
λ = eigenvalues of TLS method
μ = roots of characteristic polynomial
χ = matrix of output autocorrelations
$\underline{\Omega}$ = vector of residual crosscorrelations
Ψ = delayed observation matrix
ω = natural frequency
ω_d = damped natural frequency

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Final manuscript received: December 23, 1989.

Introduction

Recently there has been a lot of interest in finding structural modal parameters using time-domain methods rather than the more traditional frequency-domain approaches. The majority of time-domain techniques perform a curve fit to impulse-response data, though free-decay response data could be used as well. It can be shown that the techniques are effectively finding the coefficients of an autoregressive difference equation, from which the modal parameters are calculated.

The least-squares method is an integral part in the formulation of a large number of techniques. It is well known that when the data are corrupted, least squares leads to biased parameter estimates, with the damping values being especially sensitive [1]. Bias is defined as the statistical error on the estimates. To counteract the bias, overspecification of the model order is usually used; however this leads to spurious modes that have to be distinguished from the system modes.

Various other methods have been developed in order to eliminate the bias in the parameter estimates. Those developed in the system field tend to try and model the noise and are iterative in nature. Often they have been considered too computationally intensive to employ on the multi-input/multi-ouptut (MIMO) data encountered in modal testing. A number of noniterative techniques exist which attempt to avoid the terms that give rise to the bias. These methods will be considered in this work.

In this paper the curve fit in terms of output data auto-correlation points will be investigated. Using this approach it is possible to illustrate how the bias in the least-squares method originates and how the other techniques attempt to avoid the bias. The methods will also be compared statistically on simulated data corrupted with measurement noise in order to corroborate the theoretical findings. Only the damping values will be considered in the comparison as they are particularly sensitive to noise.

1. Mathematical Model

The model will be developed in terms of single input/single output (SISO) although it can be easily extended for multiple responses. The decaying response of a damped M degree of freedom system at a particular point can be written as the summation of M exponentially damped sinusoids. An alternative representation to this is the $2M$th-order auto-regressive equation:

$$y_j = -a_1 y_{j-1} - a_2 y_{j-2} - \ldots - a_{2M} y_{j-2M} \quad (1)$$

whose coefficients are related to the modal parameters via the roots of the characteristic polynomial

$$\mu^{2M} + a_1 \mu^{2M-1} + \ldots + a_{2M-1}\mu + a_{2M} = 0 \quad (2)$$

For the underdamped case considered here the roots of the above polynomial occur in M complex conjugate pairs μ_j and μ_j^* where for each mode $\mu = e^{\alpha \Delta t}$ with $\alpha = -\zeta\omega + i\omega_d$. By finding the coefficients of the difference equation it is possible to calculate the system frequencies and dampings. The amplitude and phase terms follow from a separate procedure that will not be considered here. Some methods find a system matrix whose eigenvalues lead to frequencies and dampings and whose eigenvectors lead to the mode shapes.

In this work the crosscorrelation between the data sequences y_j and ϵ_j will be taken as

$$R_{y\epsilon}(k) = \frac{1}{N-k} \sum_{j=1}^{N-k} y_j \epsilon_{j+k} \quad (3)$$

However slight changes in the limits of summations will be ignored. This is a valid approximation to make when there are a large number of data points. For ease of notation let $R_k = R_{yy}(k)$.

2. Correlations Being Fitted by Each Method

It is usual to model any corruption occuring on the data by the residual sequence ϵ_j [2] so that

$$y_j = -a_1 y_{j-1} - \ldots - a_{2M} y_{j-2M} + \epsilon_j \quad (4)$$

or, in terms of approximate correlations,

$$R_{yy}(k) + a_1 R_{yy}(k-1) + \ldots + a_{2M} R_{yy}(k-2M) = R_{y\epsilon}(k) \quad (5)$$

Biased estimates are obtained when the correlation curve fit includes points containing errors from the residual sequence. For the case where the noise sequence is 'white' it can be shown that the output autocorrelations are all uncorrupted except for R_0 which has an error value of $R_{\epsilon\epsilon}(0)$ added to it.

A single degree of freedom (DOF) response will be taken as an example in order to compare which correlation lag values each method uses for the curve fit. From eq (1) it can be seen that a second-order difference equation model is required.

(a) Least Squares (LS)

Extending eq (4) in a column-wise direction,

$$\begin{bmatrix} y_3 \\ y_4 \\ \cdot \\ \cdot \\ y_N \end{bmatrix} = \begin{bmatrix} y_2 & y_1 \\ y_3 & y_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ y_{N-1} & y_{N-2} \end{bmatrix} \begin{bmatrix} -a_1 \\ -a_2 \end{bmatrix} + \begin{bmatrix} \epsilon_3 \\ \epsilon_4 \\ \cdot \\ \cdot \\ \epsilon_N \end{bmatrix}$$

or

$$\underline{y} = \phi\, \underline{\theta} + \underline{\epsilon} \quad (6)$$

Least squares [2] minimizes the function $\underline{\epsilon}^T \underline{\epsilon}$ which gives a solution for the system parameters as

$$\underline{\theta} = (\phi^T \phi)^{-1} \phi^T \underline{y} \quad (7)$$

In terms of approximate correlations this can be written as

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} R_0 & R_{-1} \\ R_1 & R_0 \end{bmatrix} \underline{\theta} \quad (8)$$

so it is seen that the number of correlations used is dependent upon the order of the mathematical model rather than the number of data points used. A collocation fit is performed, as the minimum number of points needed is being used, so the curve goes through every point and no smoothing occurs. Any corruption on the correlation values, for instance a 'spike' at R_0, will result in erroneous parameter estimates. It can also be noted that if the sampling interval is very small, the curve fit will be more sensitive to corruption. This is why decimating the data [3] (increasing Δt) provides better estimates.

(b) Instrumental Matrix with Delayed Observations (IMDO)

There are a number of instrumental-variables [2] methods which work by essentially trying to find a sequence that is uncorrelated with the residual sequence. Most of the techniques are iterative in nature. However the IMDO [4] approach considered here is not. Instead of attempting to generate the sequence of 'instrumental variables', observation values at a delayed time $s\Delta t$ are taken.

Taking the system equation and multiplying by matrix Ψ^T where

$$\Psi^T = \begin{bmatrix} y_{2-s} & y_{3-s} & \ldots & y_{N-1-s} \\ y_{1-s} & y_{2-s} & \ldots & y_{N-2-s} \end{bmatrix}$$

then

$$\Psi^T \underline{y} = \Psi^T \phi\, \underline{\theta} + \Psi^T \underline{\epsilon} \quad (9)$$

and if the expected value $E\{\Psi^T \underline{\epsilon}\} = 0$ then unbiased answers will be obtained. In terms of the correlations being used, the equation

$$\begin{bmatrix} R_{s+1} \\ R_{s+2} \end{bmatrix} = \begin{bmatrix} R_s & R_{s-1} \\ R_{s+1} & R_s \end{bmatrix} \underline{\theta} \qquad (10)$$

is being solved. Once again a collocation fit of four correlation points is used. However the points are shifted away from those containing the corruption provided that delay s is large enough. Least squares is the special case when $s = 0$.

(c) Correlation Fit (CF)

The correlation-fit method was developed [3, 5] with the philosophy of trying to improve upon the problems that the least-squares method incurs in terms of a fit to the correlations. Considering the same system as above, the difference eq (5) relating the correlations is extended columnwise however the first p rows are ignored assuming that they contain corrupted values. The matrix equation

$$\begin{bmatrix} R_{p+1} \\ R_{p+2} \\ \cdot \\ \cdot \\ R_{NA+p} \end{bmatrix} = \begin{bmatrix} R_p & R_{p-1} \\ R_{p+1} & R_p \\ \cdot & \cdot \\ \cdot & \cdot \\ R_{NA+p-1} & R_{NA+p-2} \end{bmatrix} \begin{bmatrix} -a_1 \\ -a_2 \end{bmatrix} + \begin{bmatrix} R_{y\epsilon}(p+1) \\ R_{y\epsilon}(p+2) \\ \cdot \\ \cdot \\ R_{y\epsilon}(NA+p) \end{bmatrix}$$

or

$$\underline{r} = \chi \, \underline{\theta} + \underline{\Omega} \qquad (11)$$

is obtained and minimizing $\underline{\Omega}^T\underline{\Omega}$ in a similar fashion to the usual LS approach gives a CF estimate of

$$\underline{\theta} = (\chi^T\chi)^{-1}\chi^T\underline{r} \qquad (12)$$

which is theoretically unbiased provided p is large enough. Correlation fit therefore improves upon the problems that least squares has by missing out those correlations that are corrupted and also by being able to fit an arbitrary number of correlations in the fit. Least squares is the special case where $p = 0$ and $NA = 2$. A variant of CF has appeared in the literature called the over-extended Yule-Walker (OYW) method [6] which allows an arbitrary number of correlations but does not miss out any of the corrupted equations.

More than the minimum number of points required are used by CF, so a collocation fit is not performed and smoothing occurs in the fit.

(d) Double Least Squares

The approach considered here is a variant of the technique used by Ibrahim [1] and also Juang and Pappa [7] to produce much better parameter estimates, especially for the damping values. The usual LS approach can be shown to always produce a positive bias on the damping estimates for the one degree of freedom case, and this finding is usually borne out in practice on much more complicated systems.

If the system equations are multiplied by a data matrix $\phi^{\wedge T}$ where

$$\phi^{\wedge T} = \begin{bmatrix} y_3 & y_4 & \cdots & y_N \\ y_2 & y_3 & \cdots & y_{N-1} \end{bmatrix}$$

then

$$\phi^{\wedge T}\underline{y} = \phi^{\wedge T}\phi \, \underline{\theta} + \phi^{\wedge T}\underline{\epsilon} \qquad (13)$$

and the matrix is using a time shift of one data point (effectively $s = -1$ in the IMDO method). If the expected value $E\{\phi^{\wedge T}\underline{\epsilon}\} = 0$ then

$$\underline{\theta} = (\phi^{\wedge T}\phi)^{-1}\phi^{\wedge T}\underline{y} \qquad (14)$$

In terms of the correlations used it is found that

$$\begin{bmatrix} R_0 \\ R_1 \end{bmatrix} = \begin{bmatrix} R_{-1} & R_{-2} \\ R_0 & R_{-1} \end{bmatrix} \underline{\theta} \qquad (15)$$

It can be demonstrated that for the one-DOF case the bias upon the damping value is always negative. This generally occurred in the studies mentioned above. The DLS method averages the LS estimate with the estimate just derived with the hope that the bias of the two estimates will cancel out. It is possible to formulate the DLS method so that only a fraction more computation is used than the LS method on its own.

In practice the bias does not disappear as the amount of bias is dependent upon the sampling rate. For a large number of points per cycle the bias will remain negative. Again this has been found in practice as it is usually the lower frequency modes that are investigated. However it is feasible for a positive damping bias to occur.

(e) Total Least Squares (TLS)

The LS method minimizes the square of the error on the measurement vector $\underline{y}$ in the system equation. It therefore assumes that there is no error in the ϕ matrix which is obviously an erroneous assumption as it contains measurement values as well. The TLS method [8] minimizes $\epsilon^T\epsilon$ for the ϕ matrix as well as the $\underline{y}$ vector by letting

$$\underline{\epsilon}^T\underline{\epsilon} = (\underline{y}^T - \underline{\theta}^T\phi^T)(\underline{y} - \phi\,\underline{\theta})$$

$$= [-\underline{\theta}^T \; 1] \begin{bmatrix} \phi^T\phi & \phi^T\underline{y} \\ \underline{y}^T\phi & \underline{y}^T\underline{y} \end{bmatrix} \begin{bmatrix} -\underline{\theta}^T \\ 1 \end{bmatrix} \qquad (16)$$

so the eigenvalue decomposition

$$\begin{bmatrix} \phi^T\phi & \phi^T\underline{y} \\ \underline{y}^T\phi & \underline{y}^T\underline{y} \end{bmatrix} \begin{bmatrix} -\underline{\theta}^T \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} -\underline{\theta}^T \\ 1 \end{bmatrix} \qquad (17)$$

exists. When the smallest eigenvalue is taken, the corresponding eigenvector gives the $\underline{\theta}$ value that minimizes ϵ^T. By setting the solution in such a fashion the columns of the ϕ matrix are minimized as well as the error on the $\underline{y}$ vector. If there is no noise on the system then it is the zero eigenvector of the TLS matrix that is being found.

It is possible to formulate the solution using the singular value decomposition which gives better numerical characteristics to the solution. The TLS approach has been used for the modal-analysis problem in both the time [9] and frequency [10] domains. A method developed by Yang [11] is essentially the same.

In terms of correlations the TLS eigenvalues decomposition becomes

$$\begin{bmatrix} (R_0-\lambda) & R_{-1} & R_{-2} \\ R_1 & (R_0-\lambda) & R_{-1} \\ R_2 & R_1 & (R_0-\lambda) \end{bmatrix} \begin{bmatrix} 1 \\ -\underline{\theta} \end{bmatrix} = \underline{0} \qquad (18)$$

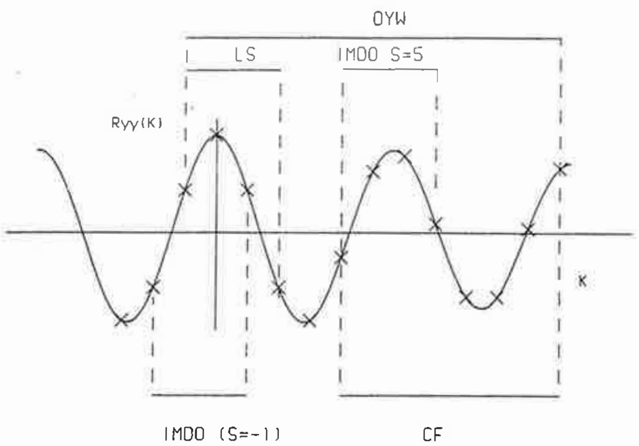

Fig. 1—Autocorrelations fitted by methods for one-DOF system

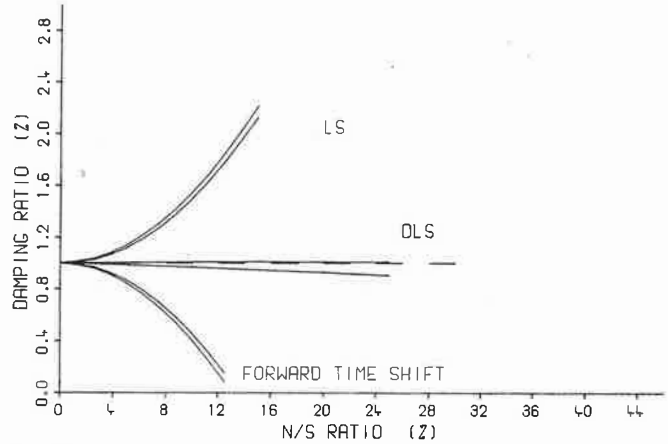

Fig. 2—Damping scatter bands for one-DOF system (10 Hz, one percent), no overspecification. LS and DLS techniques

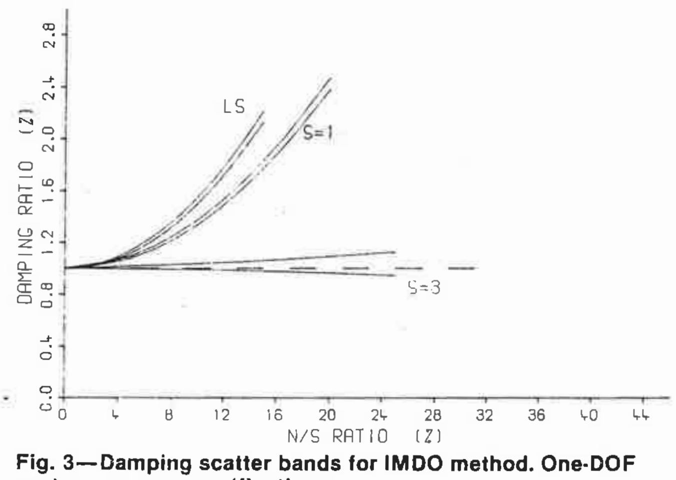

Fig. 3—Damping scatter bands for IMDO method. One-DOF system; no overspecification

so the error spike on R_0 is included in the fit. Thus for white-noise corruption, total least squares will give unbiased estimates. A collocation fit similar to the LS method is being performed but the error spike is fitted through as well. Figure 1 shows how different combinations of autocorrelations are fitted for a single-DOF system by the methods discussed above.

3. Overspecification of the Mathematical Model

All of the above methods can make use of overspecification if required. Two purposes are served by such a procedure: (1) when a real structure is tested the model order is unknown so overspecification is needed to determine the model size and (2) overspecification serves to reduce the bias.

It is essential for the LS-based techniques to use such a measure otherwise very poor estimates are found. Any improvement in the LS values will consequently also be found by DLS estimates. A collocation fit is still used through the correlation values, however more points are employed. The 'spike' is essentially modeled by a combination of spurious modes at various frequencies leaving the rest of the curve to be represented by the desired mode.

Both LS and IMDO use $4*M$ points for the collocation fit of the autocorrelations. Theoretically the CF and IMDO approaches only need overspecification to determine the system modes. However in practice less scatter is obtained upon the estimates when it is employed.

The TLS technique was originally developed assuming that the true model order was known *a priori*. When overspecification is used, the drop in the singular values indicates what the order of the model should be. The technique estimates the 'zero' eigenvalues which each provide a minimization of the cost function. Each of the 'zero' eigenvectors provides a solution, so a problem now arises as to which of the 'zero' eigenvectors should be taken for the estimate. Spurious modes still remain in the solution, though an indication of the rank of the model is given.

4. Statistical Comparison of Methods

The approach used for the comparison was that used in Ref. 5. Responses for systems with known modal parameters were generated and then corrupted by sequences of Gaussian random noise. The noise to signal ratio was taken as the ratio of the rms of the noise sequence to the rms of the noise

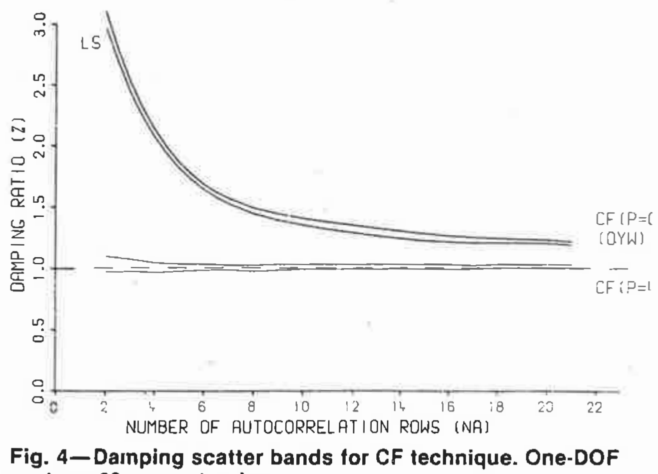

Fig. 4—Damping scatter bands for CF technique. One-DOF system; 20-percent noise

free response expressed as a percentage. Thirty different noisy responses were analyzed for each test case in order to examine the statistical behavior of the estimates. The sample mean and standard deviation of the 30 damping results were calculated and significance bands to a given level of confidence centered on the mean found. The results from a particular case can be considered to be unbiased to a certain level of confidence (95 percent in this paper) if the true damping value lies within the probability band. Although the scatter of the results will not be emphasized here, a greater width of a confidence band infers a greater scatter in the estimate.

A 256-point response for a single-mode system with a natural frequency of 10 Hz and critical damping of one percent was generated with a sampling interval of 0.02 seconds. This simple system is used to illustrate the characteristics of the various techniques. To provide a more difficult test for the methods, a 256-point response of a five-mode system with natural frequencies of 8, 10, 12, 14 and 16 Hz and critical damping values of one percent was generated in a similar fashion with the sampling interval also set at 0.02 seconds.

The N/S ratios considered here, especially for the single-DOF case, are higher than would be usually expected to occur in practice. However the extreme conditions not only show the robustness of the methods, but also serve to illustrate more clearly the behavior of the methods.

(a) One Degree of Freedom Results

The effect of increasing the amount of noise on the LS estimates with no overspecification is shown in Fig. 2. Whereas the bias upon the LS damping estimate is positive and increases very rapidly, the alternative approach suggested in section 2(d) produces a negative bias which also quickly gains in magnitude. However by combining the two estimates the DLS estimate can be seen to give much better damping values, though they do tend to give a slight negative bias.

The number of correlations fitted by the IMDO approach remains the same as LS, but correlations at different lag values are used. Figure 3 shows how increasing the value of s initially decreases the bias and eliminates it when $s = 3$. The scatter increases as more correlations are missed out and also there is a tendency for bias to reappear if the value of s is set too high.

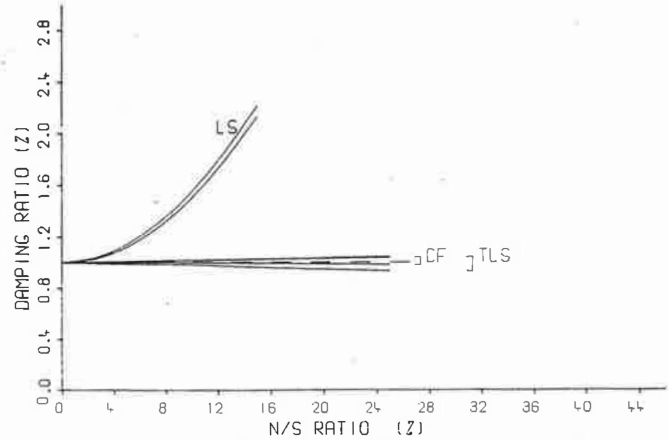

Fig. 5—Damping scatter bands for LS, CF and TLS methods. One-DOF system, no overspecification

When the number of correlations used in the fit is increased, the effect on the damping bias is quite marked. Starting with the LS estimate in Fig. 4, the OYW method reduces the bias by using more correlations. It can be seen however that some bias still remains even when a large number of correlations are used. By removing the correlations containing corruption from the calculation the CF method obtains unbiased answers. The amount of scatter on the results reduces initially as more correlation lag values are used for the fit, though when a large number are involved in the fit, the scatter tends to increase slightly.

Figure 5 shows that it is possible for both the CF and TLS methods to give unbiased estimates up to quite high levels of noise. It can be noted that the damping band for the TLS method is about twice as wide as that of the CF technique.

Much better results are obtained, as expected, by the LS methods when overspecification is allowed. The bias decreases as the model size is increased though, as Fig. 6 shows, a large model does not guarantee unbiased damping estimates. Of course the reduction in bias is achieved at the expense of extra computation and also by having to eliminate the spurious modes. It can be seen that the DLS estimates remain unbiased throughout and that there is a reduction in the amount of scatter. Similarly the TLS results also stay unbiased throughout but the scatter tends to vary.

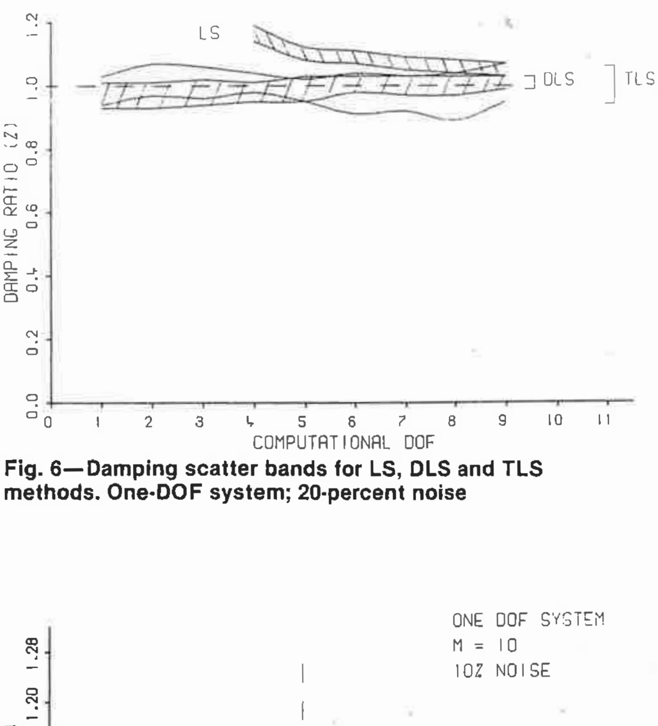

Fig. 6—Damping scatter bands for LS, DLS and TLS methods. One-DOF system; 20-percent noise

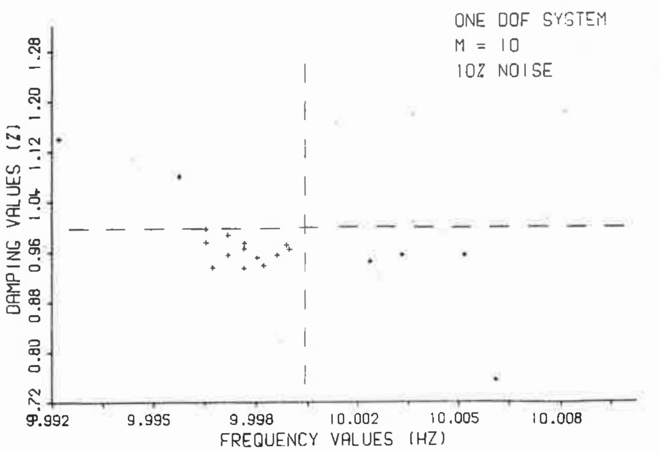

Fig. 7—TLS estimates from zero eigenvectors. One-DOF system; ten-percent noise. Computational order = 10 DOF

It is noted in section 3 that when overspecification is used for the TLS technique there is a number of 'zero' eigenvectors that all theoretically give the correct estimate. Values of the frequency and damping from all of the 'zero' eigenvectors for the ten-percent noise case when the model order is assumed to be 20 are shown in Fig. 7. Whereas the variation in the frequency is less than ±0.06 percent, the variation of the damping values is more marked, values between 0.77 percent and 1.14 percent being found. There is no particular rule as to which of the 'zero' eigenvalue gives the most accurate estimates. For the overspecified case considered in this work, the first 'zero' eigenvector was always taken.

(b) Five Degree of Freedom Results

When applied to the 5-DOF case, the methods tended to behave in a similar way to the single-DOF case. It was found that the dampings were much more sensitive to corruption and thus the results are given only up to a five-percent N/S ratio. In order to save on space, only the results for the 14-Hz mode are given. The behavior of the other modes is similar.

Figure 8 shows the effect of increasing the size of the computational model on the damping estates of the 14-Hz mode.

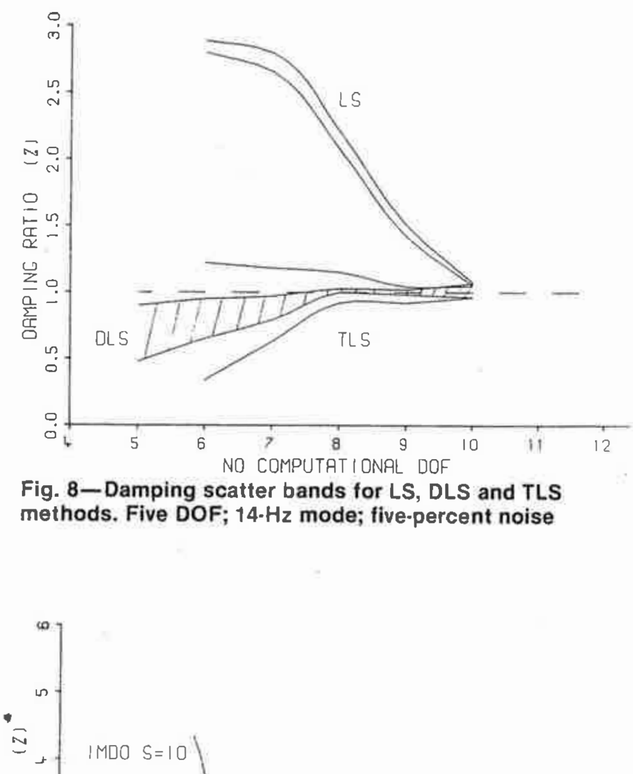

Fig. 8—Damping scatter bands for LS, DLS and TLS methods. Five DOF; 14-Hz mode; five-percent noise

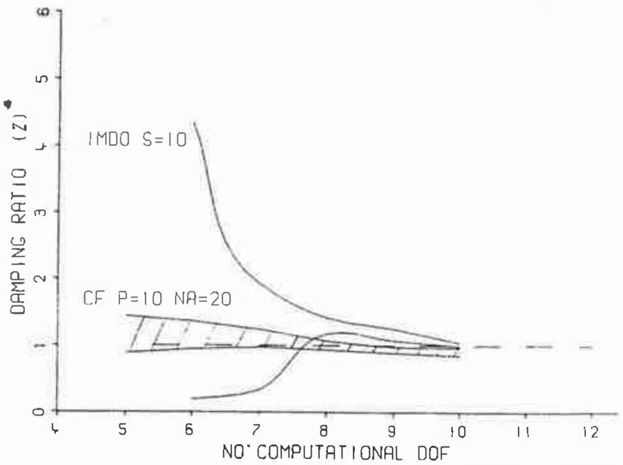

Fig. 9—Damping scatter bands for IMDO and CF methods. Five DOF; 14-Hz mode; five-percent noise

As with the single-mode case considered above, the LS estimates are very poor until a substantial amount of overspecification is applied. Meaningless results were obtained when the LS method was used without any overspecification. There is a dramatic decrease in the amount of bias when the model order was increased. The TLS technique similarly gave very poor answers for a model size of five DOF, but with an increase in the model order unbiased estimates of the damping values were found. The DLS approach gave much better estimates for a model size of five DOF which became unbiased with an increase in model size. It can be seen that the scatter in the TLS estimates is greater than that of the other two methods.

In Fig. 9 it can be seen that the CF method with $P = 10$ and $NA = 20$ gave unbiased estimates without any overspecification. When the model order was increased the amount of scatter was reduced, however a slight amount of bias occurred when the model size was set at ten DOF. The IMDO technique with s set at ten ($P = 10$) produced meaningless estimates with a five-DOF model. And although the six-DOF estimate is unbiased, there is a very large amount of scatter. The scatter reduces as the order is increased but then bias occurs. Only by the time that the model is twice the system size has the bias been virtually eliminated.

Finally, Fig. 10 shows the time that each method takes to perform 30 different test cases for increasing model orders. The IMDO method takes the same amount of computation as the LS technique and the OYW approach uses the same amount of computation as the CF method when the same number of correlations are used in the curve fit. It can be seen that the LS method takes the smallest amount of time; although due to the way that the DLS technique was implemented it only uses fractionally more computation. As the model order increases the TLS method becomes increasingly more time consuming as it is dependent upon an eigensolver. Although the CF technique is more expensive than the LS method, it must be remembered that to get a corresponding amount of accuracy the LS method must use a much larger model than CF and hence more computation.

5. Discussion

When considering the autocorrelation curve fits that are performed, the methods generally behaved as they were expected to. Although the single-mode case provides a good illustration of the behavior of the methods, when multi-DOF

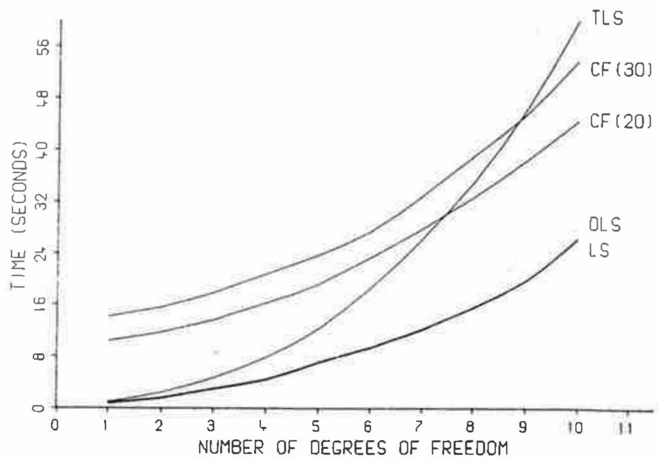

Fig. 10—Timings for 30 tests for the various methods

systems are considered the behavior of the modes is not so clear cut. Some of the modes may be unbiased for one particular condition whereas others may still be biased.

The LS method showed itself to be particularly sensitive to noise as a fit through only $4*M$ correlation lag points is being performed. Overspecification results in more correlation points being curve fitted which in turn leads to better estimates. The IMDO method fits the same number of collocation points but uses those suffering from less corruption. It can be seen that the technique gave a much greater amount of scatter on the estimates than the LS approach. Although the correlation values at low lag values suffer from corruption to a greater extent than those at higher lags, they contain the most information about the system. So taking out the low lag values will tend to result in more scatter on the estimates. Both LS and IMDO find it difficult to give relatively good estimates unless there is quite a large increase in the model order.

The DLS method gave very good estimates considering that it merely takes the average of two very poor estimates. When overspecification was employed unbiased damping values with small scatter were obtained. An added bonus is that only a very small increase in the amount of computation over that of LS is required.

The results obtained for the CF technique show that it is capable of returning unbiased estimates without the use of overspecification. However there is a tendency for the technique to produce a fair amount of scatter, though nowhere near as much as with the IMDO. However it can also be seen that the use of the method is not simply a case of missing out the first $2*M$ lags and including a lot more correlations in the fit. A compromise needs to be made between including autocorrelations of low-lag values in order to include a lot of information about the system, and taking out enough correlations to avoid most of the corruption. Correlation values for high lag values tend to include little good information about the system so their inclusion leads to some bias.

Although theoretically the CF approach should give the 'best' estimates, it is difficult to achieve in practice as the optimum number of lags needed to be used and missed out is not clear. Multi-DOF data provide particular difficulties as some modes behave differently than others in different test criteria.

The TLS method performed fairly well and gave much better estimates than the LS method which was expected as the error on R_0 is included in the fit. Overspecification was required to give good damping values as only the same number of correlation points are fitted through as used with the LS method. Surprisingly there was quite a lot of scatter in the estimates. Possibly this is due to there being a multiplicity of zero eigenvectors which all give parameter estimates. Whether all these estimates should be examined is unclear, but this would certainly increase the amount of computation required. As the model order increases the time required increases at a much greater rate than the other techniques.

Conclusions

The least-squares, instrumental-matrix with delayed-observations, double-least-squares, correlation-fit and total-least-squares methods were compared in terms of the autocorrelations that are curve fitted. The theoretical comparison was supported by a statistical comparison upon simulated one- and five-DOF systems. It was found that the latter three methods performed best in the presence of noise, however there were some difficulties in implementation. Further investigation of the behavior of these methods is required.

Acknowledgment

The author would like to thank Dr. J.R. Wright of Queen Mary College, University of London for his helpful comments during the preparation of this paper.

References

1. *Ibrahim, S.R., "Double Least Squares Approach For Use In Structural Modal Identification," AIAA. J.*, 24 *(3), 499-503 (March 1986).*

2. *Eykhoff, P., "System Identification," John Wiley and Sons (1974).*

3. *Wright, J.R., "Flutter Test Analysis in the Time Domain Using a Recursive System Representation," J. Aircraft, 774-777 (Dec. 1974).*

4. *Defaye, G., Caralp, L., Jouve, P. and Baucou, R., "Modelisation d'un de Propaniser Comparison de Differentes Methodes Statistiques d'Identification Dynamique," Automatisme,* 22 *(6/7), 177-189 (1977).*

5. *Cooper, J.E. and Wright, J.R., "Comparison of Some Time Domain Methods For Structural System Identification," Proc. 2nd Int. Symp. on Aeroelasticity and Struct. Dynamics, Aachen, W. Germany (1985).*

6. *Stoica, P., Freidlander, B. and Soderstrom, T., "Least Squares, Yule Walker and Overdetermined Yule-Walker Estimation of AR Parameters: A Monte Carlo Analysis," Int. J. Control,* 43 *(1), 13-27 (1986).*

7. *Pappa, R.S. and Juang, J.N., "Galileo Spacecraft Modal Identification Using an Eigensystem Realization Algorithm," J. Astro. Sci.,* 33 *(1), 95-118 (1985).*

8. *Golub, G.H. and Van Loan, C.F., "An Analysis of the Total Least Squares Problem," SIAM J. Num. Anal.,* 17 *(6), 883-893 (1980).*

9. *Zhang, L., Yao, Y. and Lu, M., "An Improved Time Domain Polyreference Method for Modal Identification," Proc. IMAC V, 536-543 (1987).*

10. *Rost, B. and Leuridan, J., "A Comparison of Least Squares and Total Least Squares for Multiple Input Estimation of Frequency Response Functions," ASME 85 - DET - 105 (1985).*

11. *Yang, Y., "A Time Domain Identification Technique: The Oversized Eigenmatrix Method," J. Vib. Acous., Stress and Reliability in Design,* 107, *53-59 (1985).*