8-3-2016

Research Mathematicians & Mathematics Educators: Collaborations For Change

Greg Oates

Wes Maciejewski
San Jose State University, wesley.maciejewski@sjsu.edu

40th PME Annual Conference
Szeged, Hungary, August 3-7, 2016

Mathematics Education:
How to solve it?

Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education

Editors

Csaba Csíkos
Attila Rausch
Judit Szitányi

PME40, Szeged, Hungary, 3–7 August, 2016

Cite as:

Csíkos, C., Rausch, A., & Szitányi, J. (Eds.). *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1. Szeged, Hungary: PME.

Website: http://pme40.hu

The proceedings are also available via http://www.igpme.org

Publisher:

International Group for the Psychology of Mathematics Education

ISSN 0771-100X

ISBN 978-1-365-45506-3

Logo: Lóránt Ragó

Composition of Proceedings: Edit Börcsökné Soós

Printed in Hungary

Innovariant Nyomdaipari Kft., Algyő

www.innovariant.hu

RESEARCH MATHEMATICIANS & MATHEMATICS EDUCATORS: COLLABORATIONS FOR CHANGE

Greg Oates and Wes Maciejewski

MOTIVATION

As a developing university lecturer, Martin-Molina (2016) observes there are many challenges facing young researchers when they finish their Ph.D. and want to embark on a career as teachers at the university level. They often receive little or no training on how to teach, they may face a widely diverse array of students in contrast to their own experience as a mathematics major, they may shift universities and hence student and teaching cultures several times in a few years, and are subject to student evaluations the results of which are critical to their future careers. This is the environment within which many of our mathematics colleagues have developed as instructors. However, the landscape is changing with increasing institutional and student pressures for quality teaching, and a growing number of programmes providing either mentoring or explicit training for new lecturers. Given this context we ask, how can mathematics educators and mathematicians collaborate to develop the instructional practices of current and future teachers of post-secondary mathematics?

The University of Auckland has benefited for some twenty five years of having a mathematics education unit within the mathematics department. This presentation argues that this arrangement has provided unique opportunities for mathematics education and mathematics researchers to collaborate in an examination and development of their teaching practice from a mathematically-focused perspective. The two authors of this paper bring both perspectives to bear, with the second author in particular offering his insights in moving from a Ph.D. in mathematics to a developing career as a mathematics education researcher, alongside his continuing role as a teacher of university mathematics.

OPPORTUNITIES FOR COLLABORATION

The Department of Mathematics at the University of Auckland has four major research groups: Applied Mathematics, Algebra and Combinatorics, Analysis and Topology, and Mathematics Education. One advantage of this grouping is that it enables mathematics education researchers and research mathematicians to work closely together and collaborate on research and development of teaching practice at the tertiary level. It should be noted here that all members of the mathematics education unit maintain roles as university-level mathematics instructors, as well as their research and teaching interests in mathematics education. This presentation will consider several aspects of this blooming community of practice, with examples and data emerging from two nationally-funded research projects that have involved such collaborations.

DATUM Project

The *DATUM* project (Development and Analysis of Teaching in Undergraduate Mathematics) began as a longitudinal project to develop a model for professional development, theoretically grounded in Schoenfeld's (2010) resources, orientations, and goals (ROG) model of teacher action (Barton, Paterson, Oates & Thomas, 2014). A DATUM group includes both mathematicians and mathematics education researchers, to stimulate discussion of both mathematical and pedagogical knowledge. Each member of the group has one of their lectures recorded and from this they select a short (3- to 4-minute) segment for discussion, along with a brief written reflection of their ROGs. The emphasis is on inclusiveness, collegiality, and shared learning and we believe one of the key dynamics of the DATUM groups is that the flow of pedagogical knowledge is not uni-directional from education researcher to mathematician. Participants are encouraged to reflect on and discuss their teaching episodes and thereby develop their practice organically.

The DATUM study has had an enduring impact on teaching practice in our department, with two independent groups of 6-8 colleagues continuing to meet five to six times per year since the initial research project ended in 2012. Positive outcomes of the study have been widely reported (Barton, Paterson, Oates & Thomas, 2014) and DATUM has shed light on undergraduate teaching practices. For example, work emerging from DATUM has revealed lecturers' internal dialogues as both mathematician and teacher when confronted by unplanned problems in class, weighing up pedagogical issues against mathematical values as they make instant decisions as to whether to deviate from their lecture plan (Paterson, Thomas, & Taylor, 2011). Hannah, Stewart and Thomas (2013) consider the role of language and visualisation in teaching linear algebra, while Barton (2011) describes how DATUM discussions led to a consideration of the value of mathematical content from a *pragmatic, epistemic* & *heuristic* perspective, focusing on the interplay between aspects of the "mathematical essence" of the lecture and the "learning culture" in which it is embedded

LUMOS Project

The *Learning in Undergraduate Mathematics Outcomes Spectrum* (LUMOS) initiative, started in 2014, aims to increase our understanding of learning outcomes for undergraduate mathematics. Of course, we expect our students to learn "maths", but what else? What mathematical skills, dispositions, affective outcomes, processes, and knowledge do mathematicians hope their students might develop? A large part of the project has been devoted to developing instruments by which we might observe how these outcomes might be measured. Progress to date includes identifying some potential new orchestrations which helped mediate students' movements towards instrumental genesis when engaged in active-technology tasks (Drijvers et al, 2010 and Artigue, 2001, cited in McMullen, Oates & Thomas, 2015) and an evolving instrument for assessing mathematical communication, trialled with students engaged in Team-Based Learning (TBL) activities (Paterson, Sheryn & Sneddon, 2013). One particular undergraduate learning outcome we wish to highlight here has emerged from a careful analysis of the way mathematicians select and work on mathematical problems

An Example from LUMOS: Mathematical Foresight

Interviews with mathematics colleagues has led to the realisation that mathematicians often anticipate the value/utility/beauty of a problem and chart a likely course to a resolution in advance of embarking on actual rigorous work. This ability is distinct from, but not unrelated to, intuition, strategic thinking, and aesthetic sense and has been termed *mathematical foresight* to highlight its similarities to future-thinking behaviour in other domains (Maciejewski & Barton, 2015). Mathematicians have identified this ability as central to their mathematical work and we ask, should instructors strive to develop this in their undergraduate students? How might we judge the success of such an effort?

Since the development of the initial mathematical foresight model, two further studies of students' mathematical foresight have been conducted. The first (Maciejewski & Barton, 2016), characterises students' problem-planning behaviour through a mathematical foresight lens. The second, presented at this conference (Maciejewski, Roberts, & Addis, 2016), draws analogies between foresight in mathematics and in general daily experience. While acknowledging that an undergraduate education in mathematics is not always or necessarily about producing mathematicians, we believe that an examination of mathematicians' practices can nevertheless lend insight into the implicit/hidden mathematics curriculum.

DISCUSSION

We suggest that mathematicians hail from a strong teaching culture with a long history, emerging from mathematicians' evolving perceptions of the nature of mathematics. This culture is pervasive; many mathematicians have a clear idea of what constitutes a good education in mathematics. These ideas, however, may or may not align with those of a mathematics educator. This leads us to ask, how might we invite mathematicians into a conversation about education, especially in a way that is respectful of their teaching culture and is informed by contemporary mathematics education literature? We propose that there must be a willingness of both parties: both must exhibit a "willing suspension of disbelief" by engaging with practices and literature with different standards and forms of conviction to their respective fields. We view the onus as being on the mathematics educator here: they are the facilitators and simultaneously must not be critical of the mathematicians' approaches and be the champions of change. We have incorporated these perspectives into both our DATUM and LUMOS work.

In DATUM, the conversation is started explicitly – both mathematics educator and mathematician discuss the participant's teaching practice as it unfolds. LUMOS takes a different approach: we look for issues that resonate with mathematicians, on topics they can identify in their own practice, as an invitation into the world of mathematics educators. In both projects, authentic undergraduate educational situations are brought to the fore and made accessible to mathematicians and mathematics educators alike.

Both projects rest on a strong theoretical basis, with an emphasis on authenticity and practical relevance which appeals to the practice of the mathematicians and feeds back to the theoretical work of the mathematics education researcher. This is, in our view, a productive and effective collaboration for change.

References

Barton, B. (2011). Growing understanding of undergraduate mathematics: a good frame produces better tomatoes. *International Journal of Mathematical Education in Science and Technology*, *42*(7), 963–974.

Barton, B., Oates, G., Paterson, J., & Thomas, M. (2014). A marriage of continuance: Professional development for mathematics lecturers. *Mathematics Education Research Journal*, *27*(2), 147-164.

Hannah, J., Stewart, S., & Thomas, M.O.J. (2013). Emphasizing language and visualization in teaching linear algebra. *International Journal of Mathematical Education in Science and Technology*, *44*(4), 475–489.

Maciejewski, W., & Barton, B. (2016). A framework for undergraduate students' mathematical foresight. *Proceedings of ICME 13.* Hamburg, Germany: ICME.

Maciejewski, W., & Barton, B. (2015). Mathematical foresight: an integral part of authentic mathematical activity In Beswick , K., Muir, T., & Wells, J. (Eds.). *Proceedings of 39th Psychology of Mathematics Education conference*, Vol. 1, p. 185. Hobart, Australia: PME.

Maciejewski, W., Roberts, R., & Addis, D.R. (2016). Episodic future thinking in mathematical situations. *PME 40.* Szeged, Hungary: PME

Martin-Molina, V. (2016). The challenge of being a mathematics teacher. In *Proceedings of First Conference of the International Network for Didactic Research in University Mathematics (INDRUM), Montpellier, France* (pp. 195-196): sciencesconf.org:indrum2016:84315

McMullen, S., Oates, G., & Thomas, M.O.J. (2015). An integrated technology course at university: Orchestration and mediation. In K. Beswick, T. Muir, & J. Wells (Eds.), *PME 39*, Vol. 1, (pp. 249-257). Hobart, Australia: PME.

Paterson, J., Sheryn, L., & Sneddon, J. (2013). Student responses to team-based learning in tertiary Mathematics courses. *Proceedings of 15th Annual Conference on Research in Undergraduate Mathematics Education* S. Brown, G. Karakok, M. Oehrtman Eds February 23, 2013 Vol 2 (pp. 619–626).

Paterson, J., Thomas, M.O.J., & Taylor, S. (2011). Decisions, decisions, decisions: what determines the path taken in lectures? *International Journal of Mathematical Education in Science and Technology, 42*(7), 985–996.

Schoenfeld, A. H. (2010). How we think. A theory of goal-oriented decision making and its educational applications. New York: Routledge.