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Observing Change In Students' Attitudes Towards Mathematics: Contrasting Quantitative And Qualitative Approaches

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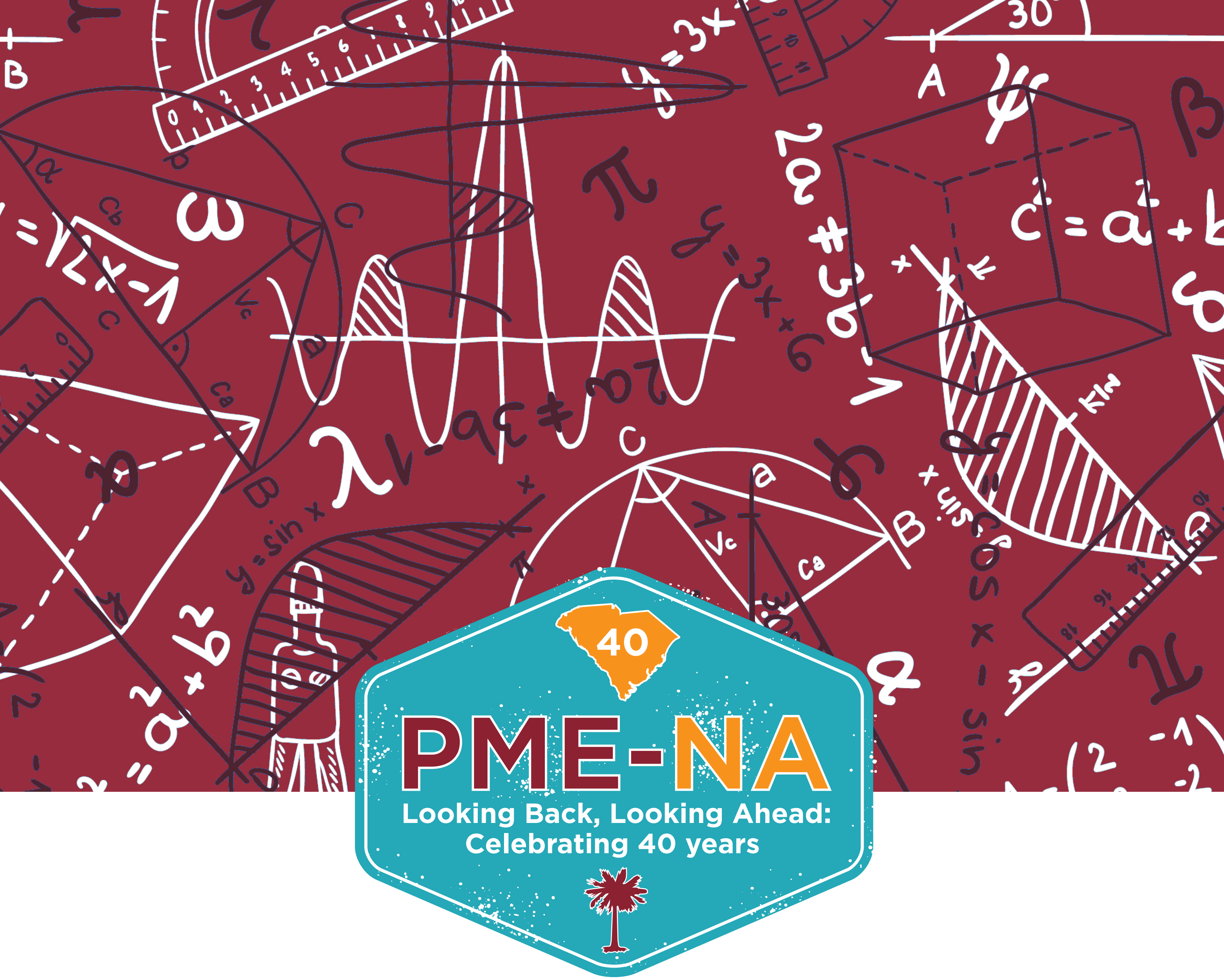

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Looking Back, Looking Ahead

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OBSERVING CHANGE IN STUDENTS' ATTITUDES TOWARDS MATHEMATICS: CONTRASTING QUANTITATIVE AND QUALITATIVE APPROACHES

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A student's attitude towards mathematics affects how they learn and perform in mathematics. What exactly is meant by attitude and how this interacts with mathematics education is a current debate in the mathematics education research community. Regardless, practitioners often acknowledge a consideration of improving students' attitudes towards mathematics in their course design. This creates an impetus to study attitudes towards mathematics in a way that lends itself to observing changes over a course in mathematics. The current study draws on two approaches to observing and measuring attitudes towards mathematics in an effort to contrast disparate approaches and deepen an investigation of students' changes in attitudes. Results indicate there is no superior approach; that the multi-dimensional nature of attitude defies a succinct description, but methods exist to allow us to get a handle on this construct, nonetheless.

Keywords: Attitudes towards mathematics, undergraduate mathematics education

Introduction and Literature Review

Education practitioners often lament their students' attitudes towards mathematics, and this is employed as an explanation of their poor performance (Di Martino and Zan, 2010). But what is meant by attitude? How might this construct be operationalized, and with a clear articulation of "attitude" in hand, how might students' attitudes be transformed through education and these transformations be observed? Despite decades of research on affect in mathematics education, these questions are only now being addressed in any substantial way (Pepin and Roesken-Winter, 2015; Goldin, et al., 2016).

The most current perspective on attitude is that it is a multi-dimensional construct, with affective, social, emotional, temporal, and other components (Hannula, 2012). Attitude can co-emerge with the setting, and depends on social and individual aspects. In brief – attitude's messy.

Three-dimensional Model for Attitudes (TMA)

In an effort to improve parsimony of the attitude construct, Di Martino and Zan (2010) introduce the Three-dimensional Model of Attitudes (TMA) framework, which conceives of attitudes towards mathematics having i) Emotional, ii) Vision of Mathematics, and iii) Perceived Competence components. These three form stable dimensions of the attitude construct, being based on ~1,600 student essays describing their relationships with mathematics, but each dimension could be further refined in a multitude of ways. Vision of Mathematics could be split into instrumental and relational views following Skemp (1976), for example, or it could be understood in terms of Sfard's structural/relational model (1991). Di Martino and Zan (2010) provide a way to parse the otherwise difficult to grasp attitude construct.

However, theirs is only one such way; indeed, the origins of attitudinal work in mathematics education are in the creation and validation of Likert-type scales; see, for example, (Fenema and Sherman, 1976). Di Martino and Zan (2010) level a valid criticism against much of these origins, in particular against the "measurement era" of creating Likert-type instruments. In particular, Di Martino and Zan (2010) characterize much of the literature as defining "attitude" a posteriori through the instruments used to measure it, creating a researcher's tautology: we chose our research foci because the outcomes of our research indicate they are important and problematic.

It is necessary, then, to have a relatively clear articulation of "attitude" before rigorous research on it can proceed.

Other criticisms towards instruments intended to measure attitudes are presented in Di Martino and Zan (2010). Among these are, i) items run the risk of being chosen in a way that may not be relevant to the survey/instrument responders – responders may be tasked only to give opinions on the items and their underlying structures that are of value to the researchers; ii) how are the scores for grading the items determined? This question is especially relevant to Likert-type instruments, where researchers sometimes take the, totally erroneous, approach of assigning numbers to the Likert responses, calculate subsequent means, for example, and use these for comparison; iii) the act of measuring with a pre-defined scale necessarily compresses or projects the multi-dimensionality of attitude onto smaller dimensions, and thereby loses nuance. They mention further that the enterprise of measuring attitude assumes it can be measured (read: quantified) and can therefore be related to other quantifiable variables present in education. These are valid and powerful criticisms and highlight to this author a need to investigate further the interactions between various measurement approaches present in the literature and, for example, the TMA framework proposed by Di Martino and Zan (2010) to further explicate the attitude construct.

Mathematics Attitudes and Perceptions Survey

Not all attitudinal survey instruments have been created in the same way: at least one, the Mathematics Attitudes and Perceptions Survey (MAPS; Code, et al., 2016) is grounded in empirical data, with the items and scales originating from students' comments about learning in STEM fields, and used not as an objective measure of students' attitudes, but rather as a measure of students' attitudes relative to mathematicians'.

The MAPS items were initially adapted from the Colorado Learning Attitudes About Science Survey (CLASS; Adams, et al., 2006), which were in turn emergent from student interviews. Items were subsequently refined, added or dropped based on student and faculty interviews. The survey then was completed by 3,411 students in differential, integral, and multi-variable calculus, and introduction to proof. An exploratory factor analysis was performed on a subset of this data, with a subsequent confirmatory factor analysis on another. From this cyclical development process, seven categories emerged: growth mindset, a view of the relevance of mathematics to the real world (shortened: real world), confidence, interest, persistence, a drive to make sense of mathematical answers (shortened: sense making), and a view on the nature of mathematical answers (shortened: answers). Though these descriptors are largely self-explanatory, the reader is referred to (Code, et al., 2016) for further elaboration of the categories.

Importantly, data was also gathered from mathematicians, giving the final MAPS items an expert consensus rating. A student's response to each item is scored relative to the corresponding expert consensus: +1 if in the same "direction" as the mathematician – that is, agree or disagree – and -1 if in the opposite direction. A score of 0 is given for neutral responses.

The intention with this approach was to construct an instrument that could observe movement of students' expert-like views of mathematics. This approach avoids the arbitrariness of other instruments (Fennema and Sherman, 1976) and assesses the students' attitudes, or aspects thereof, relative to the mathematics community's prevailing attitudes. This facilitates a way of understanding how an education in mathematics might bring students into the culture of mathematics, or push them away.

Previous implementations of the MAPS survey have revealed interesting results that corroborate results in the wider STEM literature (Code, et al., 2016): aspects of attitude do correlate with academic achievement, though the directionality of this relationship is unclear; the higher the course in the mathematics sequence, the more expert-like dispositions of the students, though this may be a self-selection effect; and the typical mathematics course experience tends to push students away from more expert-like attitudes towards the field. Disheartening as this, this appears to be the norm; post-secondary education selects those most oriented towards the field, and seldom develops such orientation.

Coordinating approaches to observing attitudes

Given the proliferation of methods and ways of observing, measuring, and understanding attitudes towards mathematics, it seems a worthwhile endeavour to employ multiple methods on a particular group of students in a particular context to explore how each measure might contribute to an overall picture of the students' attitudes towards mathematics. Both the TMA and MAPS have strengths – the TMA provides a way for the students' voices to be heard, and the MAPS lends itself to large-scale educational institutions – and weaknesses – the TMA is applied only to what the student articulates, and the quantitative aspect of MAPS can be construed as feigning "rigor", with educators desiring metrics. It seems that both ought to lead to a more complete picture of students' attitudes towards mathematics.

This study takes this approach with two such methods: the MAPS instrument and a TMA analysis of students' writing about experiences with mathematics. In particular, we attend to the questions: i) is the same attitudinal construct identified by students' free-form writing about their experiences with mathematics identified with a TMA analysis of their writing as that that is identified by their responses to the MAPS instrument? ii) Are the two approaches complementary or supplementary? iii) how might both be used to asses *changes* in attitudes exhibited by students over a mathematics course?

Methods

The data for this study comes from students enrolled in a 5-week Summer pre-university preparation program at San José State University (SJSU). Students in this program were admitted to SJSU, but did not pass an entry-level mathematics test. As a consequence, they were required to enroll in developmental courses during their first year of university. In an effort to get a lead on these courses before the start of the semesters, those students of greatest financial and academic need were invited to attend the Summer program, which is intended to smooth the students' transitions to university and improve their overall chances of success.

The Summer program consisted primarily of courses in elementary mathematics and English, but also included a series of sessions conducted by the university's counseling services that targeted students' attitudes towards mathematics. Specifically, the sessions focused on mindfulness, fostering a positive attitude, self-esteem and confidence in relation to performance, academic skills, stereotype threat, and relaxation. The inclusion of these counseling sessions was intended to target and improve the developmental students' attitudes towards mathematics, as developmental students are known, in general, to have less favourable attitudes towards mathematics than their non-developmental counterparts and that this significantly hinders their progression through university (Maciejewski and Tortora, *under review*). The content of the Summer program is not the focus of the current paper. Rather, I seek to observe changes in students' attitudes towards mathematics as revealed in their writing about mathematics and their responses to a survey instrument on attitudes and dispositions towards mathematics.

Specifically, students in the Summer program were invited at the start and end of the program to write a short response to the prompt:

Tell us about a personal experience you've had with math. Try to write at least 200 words.

This prompt was chosen to be as open as possible and to not narrow responses to be specifically about attitude or approaches to mathematics, etc. The intention here is for the student to recall a memory of their own interactions with mathematics; such memories are known to have associated emotional content, which is often articulated (Maciejewski, 2017).

Prior to writing their response to the above prompt, the students responded to the MAPS instrument. Both the MAPS and the essay responses were completed online, during a class time.

Start-of-program essays ($N = 134$) were matched with end-of-program essays ($N = 134$) and complete MAPS responses (start: $N = 123$; end: $N = 123$) to form the dataset ($N = 116$ start/end matched essay pairs and MAPS responses) for this study. Each essay was scored by the author according to the TMA framework of Di Martino and Zan (2010) following the descriptors in Table 1. As an example of this scoring, consider the following essay.

> I usually prepare for a test by doing a practice test with sample questions. However, I could never get a good grade one a test because when the test comes, my mind freezes. The problem feels completely different and more difficult. Even though sometimes the difference in the problem was just a few numbers. I try to get through the problem by thinking hard about the practice test and writing all the formulas down.

This essay was assigned a "-" in the Emotional category for the language around the problem feeling "different and more difficult"; an "i" in the Vision category for the view of mathematics as formulas, and a "l" in Competence, because of their admission of not being able to attain a good grade.

Table 1: TMA dimensions and accompanying scores, in parentheses.

TMA Dimension	Possible Score		
Emotional Disposition	N/A	Positive (+)	Negative (-)
Vision of Mathematics	N/A	Relational (r)	Instrumental (i)
Perceived Competence	N/A	High (h)	Low (l)

After each essay was scored, the aggregate scores were assessed using χ^2 and z tests to test for statistically-significant differences between start- and end-of-term essay response categorizations. As will be discussed, there is not a singular best way to analyse the aggregate score data. However, the dichotomous parsing of the TMA categories employed here is sufficient to observe appreciable differences in start and end of term essay responses.

The MAPS responses were scored by the author according to the instructions in the MAPS literature (Code, et al., 2016). The start and end of term MAPS results were compared on each category using Kruskal-Wallace tests.

Both TMA and MAPS data were then compared with standard logistic regression methods. This was intended to articulate the relationships between the MAPS and TMA categories .

Results

The results presented below reveal some aspects of the participating students' attitudes towards mathematics changing over the course of the Summer program. However, that is not the focus of the results; the reader is encouraged to attend to i) how the start and end of term TMA data are compared – the TMA framework has not been used to assess changes in students' attitudes over the duration of a course or program, as far as this author is aware; ii) the contrasting MAPS results; iii) relationships between MAPS and TMA categories present in the data.

In all, the underlying theme of the results takes the form of a question, that deserves being presented as a result in and of itself:

How best to determine if a change of attitudes occurred?

TMA

As reported elsewhere (Maciejewski, *under review*), there was a statistically-significant change in the TMA scores, taken as a proportion of positive/relational/high to total number of non-N/A scores ($p < 0.01$). A summary of the raw counts for each TMA category is in Table 2. The main observation here is that a TMA analysis of the student essays reveals a change in their attitudes towards mathematics.

Table 2: Results of a TMA analysis of the student essays.

	Pos./rel./high		Neg./ins./low		N/A	
	Start	End	Start	End	Start	End
Emotional	30	44	49	41	37	31
Vision	10	12	63	53	43	51
Competence	32	53	76	56	8	7

Changes in individual students' attitudes towards mathematics were observed by comparing their start and end of term essays. However, as is presented in (Maciejewski, *under review*), these are not necessarily *improvements*, which would be a normative assessment of the student writing. Indeed, the writing was often sufficiently rich to defy such a normative assessment.

MAPS

A comparison of the start and end-of-program MAPS scores indicate only one category with a statistically-significant change: Interest decreased ($p = 0.03$). On the surface, this seems to be a null, or negative, result. However, results from previous studies conducted with the MAPS instrument typically indicate a decrease in *all* MAPS categories over the duration of a program or course (Code, et al., 2016). The maxim, *if you can do no good, at least do no harm*, seems apropos.

Interactions Between MAPS and TMA scales

In an effort to understand better the seemingly disparate results from the TMA essay analysis and the MAPS results, I seek statistical relationships between the two. That is, this subsection of the results answers the question, do high or low MAPS category scores correspond to either of the values of the TMA category variables?

To this end, a logistic regression analysis is performed between each of the, non-zero, TMA variables and the MAPS variables; the N/A values in the TMA categories are not included in this analysis, to reflect the practice of excluding neutral responses in the MAPS analysis.

The results of the logistic regression analysis are summarized in Table 3. This analysis reveals that there are significant interactions between each TMA variable and multiple MAPS categories.

Table 3: Logistic regression model results for MAPS and TMA data. Only significant ($p < 0.05$) results are reported. Odds ratios are reported with χ^2 and p-values in parentheses.

	TMA Dimension		
MAPS Category	Emotion	Vision	Competence
Growth Mindset			
Real World	10.45 ($\chi^2(1) = 12.48$, $p < 0.01$)		
Confidence	10.81 ($\chi^2(1) = 7.46$, $p < 0.01$)	14.61 ($\chi^2(1) = 11.11$, $p < 0.01$)	15.76 ($\chi^2(1) = 19.76$, $p < 0.01$)
Interest	10.04 ($\chi^2(1) = 12.5$, $p < 0.01$)	4.88 ($\chi^2(1) = 4.93$, $p = 0.03$)	3.28 ($\chi^2(1) = 4.77$, $p = 0.03$)
Persistence			
Sense Making			
Answers			5.66 ($\chi^2(1) = 6.35$, $p = 0.01$)

Many of these relationships ought to be expected – confidence and interest cut across all TMA categories, for example. It is a reasonable expectation that these two do lead to more positive emotions towards mathematics and are related in some way to competence, though the analysis here cannot discern the direction of the relationship; perhaps improved competence leads to both greater confidence and interest. Persistence not being related to any of the TMA categories is also a plausible result – people persist for any number of reasons, whether they are competent or like it or not.

Some of the relationships are not expected, both in terms of relationships present and missing. For example, it is not clear why the Real World category ought to be related to emotions in mathematics – perhaps an appreciation of the pervasiveness of mathematics in one's life results in greater comfort or otherwise positive emotions. The lack of a statistically-significant relationship between the MAPS Answers and the TMA Vision of Mathematics categories is also unexpected, as these ought to be two different perspectives on the same underlying phenomena. However, the p-value ($p = 0.08$) in the logistic regression analysis between these variables was not far from the arbitrary significance cut-off of 0.05. The students in this study had a fairly uniform instrumental vision of mathematics, and the dichotomous parsing of the Vision category may have overlooked some of the nuance present in their true views of the nature of mathematics.

Discussion

This work concerned observing change in students' attitudes towards mathematics through two different methods and analyses. The first, an application of the TMA framework (Di Martino and Zan, 2010) to student essay responses, revealed significant positive changes in student attitudes towards mathematics over the duration of the program. The second, a start and end-of-program administration of the MAPS instrument (Code, et al., 2016), revealed only one significant change: student interest decreased. How might we understand these seemingly disparate results?

Returning to the research questions posed near the start of this paper, I address (iii) first. Changes in students' attitudes towards mathematics are difficult to identify and describe. The MAPS categories, by their quantitative nature, give the illusion of readily being able to describe change. However, changes do not always occur in terms of moving towards or away to experts. It is possible for a student to have a "horizontal" shift in their attitudes – they may develop a love for solving rich modeling problems and increasingly despise computation, the net effect of which is that the student is no closer to the experts in that aspect of attitude towards mathematics. The MAPS is also restricted to revealing those aspects of the students' attitudes that are asked about by the MAPS items. Though the MAPS categories tend to be identified in the research

As revealed through logistic regression analyses, there are significant interactions between the TMA essay analyses and MAPS results. This lends support to the following hypothesis, phrased in terms of a mathematical analogy.

A student's attitude towards mathematics is a multi/high-dimensional object. As articulated in the literature (Goldin, et al., 2016; Pepin and Roesken-Winter, 2015; Hannula, 2012), *attitude* consists of psychological, social, emotional, and temporal states. Any attempt to observe, describe, and otherwise measure the attitude object necessarily results in a reduction of dimension. A student writing about their relationship with mathematics reveals only some features of attitude, and a subsequent TMA analysis slices through those features. A MAPS instrument implementation reveals sections of a students' attitude object, that they may or may not have written about, and attempts to quantify these, metrizing the sections along the MAPS categories/dimensions. The MAPS sections may, as in the data reported here and revealed through the predictive variables in the logistic regression, intersect with the TMA slices, or be parallel.

The key with this mathematical analogy is that a high-dimensional object is difficult to comprehend by humans embedded in a spatially three-dimensional world. This difficulty is not an impossibility, since we have ways of bringing high-dimensional objects into three-dimensions, but this dimensional reduction necessarily results in a loss of information. In our efforts to understand that which is not readily graspable, we lose features of the object. Bringing the object into our world in one way may reveal the smooth curves of one side, and leave the sharp corners and divots of another hidden.

Taking this view of attitude towards mathematics as a high dimensional object avoids privileging of one method to understand that attitude over another. That is, a qualitative analysis of a student's writing about their relationship with mathematics is neither better nor worse than the student's responses to a Likert-based questionnaire on attitudes towards mathematics, or vice-versa. Both approaches reveal only restricted features of attitudes, and subsequent analyses further reduce those features. Both taken together can reveal more features of overall attitude than either one alone.

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