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CHANGES IN ATTITUDES REVEALED THROUGH STUDENTS' WRITING ABOUT MATHEMATICS

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The ways in which a student relates to mathematics is known to affect how they learn and perform in mathematics: anxiety may be compensated with avoidance; enjoyment with engagement. Therefore, there is a need to understand students' relationships with mathematics and to see how these are affected by mathematics education. This paper presents results from the early stages of a mixed-methods study aimed at assessing changes in students' attitudes towards mathematics as revealed in their writings about mathematics. In contrast to existing survey instruments on attitudes towards mathematics, the methods and discussion presented here have the potential to inform the analysis of more idiosyncratic, personal, and diverse relationships with mathematics in authentic, large-scale educational settings.

INTRODUCTION

Learning and performing in mathematics is seldom strictly about knowledge of mathematics. In particular, a students' beliefs, attitudes, and emotions can affect the way the student (dis)engages with mathematics. However, what constitutes "beliefs, attitudes, and emotions", how best to conceptualize and operationalize these terms, and how these might interact with mathematics learning and performance are evolving, contemporary issues in the mathematics education research literature. A number of thorough literature reviews, monographs, and working groups have emerged over the recent years in an effort to coordinate and clarify the plethora of diverse research and perspectives on beliefs, attitudes, and emotions in relation to mathematics education (Hannula, 2012; Pepin and Roesken-Winter, 2015; Goldin, et al., 2016). The current work considers the issue of observing change in students' attitudes towards mathematics, conceptualized below, over the course of an educational program. The intention with this work is to highlight the need for educational practitioners to assess the effects education has on their students' attitudes towards mathematics and identify challenges in this endeavour.

Attitude towards mathematics

In an effort to clarify constructs in the literature on attitudes towards mathematics, Di Martino and Zan (2010) created the Three-dimensional Model for Attitude (TMA). In this model, the construct *attitude* exists along three dimensions: Emotional, Vision of Mathematics, and Perceived Competence. Each of these dimensions are rich, encompassing arrays of potential student/mathematics relationships. To improve parsimony of the model, the authors suggest the Emotional dimension be conceived as comprising

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positive and negative emotions, the Vision dimension to be, following (Skemp, 1976), comprised of relational and instrumental views of mathematics, and the Competence dimension to be high/low. I follow these suggestions here in an application of the TMA model, being mindful that these dichotomous scales are one *possible* way of refining the TMA model. Even with this simple refinement, the TMA elaborates the attitude construct, realising its true multi-facetness.

Further research on attitude (Hannula, 2012) has related *affect* – a construct that subsumes beliefs, motivation, values, moods, etc., but also in particular, attitudes – to embodied and enactivist theories of learning. In so doing, a *metatheory* of affect is created with i) cognitive, motivational, emotional; ii) ephemeral and stable; and iii) social, psychological, and physiological dimensions. These dimensions emphasize that attitudes are not strictly individual, static traits, but maleable and socially emergent. An implication of this perspective is that attitudes can potentially be changed through education. Though the overall causal nature of attitudes on performance in mathematics has been reported as equivocal – see (Di Martino and Zan, 2010; Goldin, et al., 2016) for reviews of this literature – strongly negative attitudes likely result in poor engagement with mathematics (Maciejewski and Tortora, *under review*) and so "improving" these attitudes ought to be the focus of an education in mathematics, especially for at-risk populations.

The current study utilizes the TMA framework of Di Martino and Zan (2010) as a way of observing changes in students' attitudes towards mathematics over a Summer university preparatory course. Equally as important as the results reported here is the discussion of implementation and feasibility issues that follows the results section.

METHODS

The data for this study comes from students enrolled in a 5-week Summer pre-university preparation program at San José State University (SJSU). Students in this program were admitted to SJSU, but failed an entry-level mathematics test and required to enroll in developmental courses during their first year of university. Students were invited, based on financial and academic need (ie. deficient academic background), to attend the Summer program, which is intended to smooth the students' transitions to university and improve their overall chances of success.

The Summer program consisted primarily of courses in elementary mathematics and English, but also included a series of sessions conducted by the university's counseling services that targeted students' attitudes towards mathematics. Specifically, the sessions focused on mindfulness, fostering a positive attitude, self-esteem and confidence in relation to performance, academic skills, stereotype threat, and relaxation. The inclusion of these counseling sessions was intended to target and improve the developmental students' attitudes towards mathematics, as developmental students are known, in general, to have less favourable attitudes towards mathematics than their non-developmental counterparts and that this significantly hinders their progression through university (Maciejewski and Tortora, *under review*). The content of the

Summer program is not the focus of the current paper. Rather, I seek to observe changes in students' attitudes towards mathematics as revealed in their writing about mathematics.

Specifically, students in the Summer program were invited at the start and end of the program to write a short response to the prompt:

> Tell us about a personal experience you've had with math. Try to write at least 200 words.

This prompt was chosen to be as open as possible and to not narrow responses to be specifically about attitude or approaches to mathematics, etc. The intention here is for the student to recall a memory of their own interactions with mathematics; such memories are known to have associated emotional content, which is often articulated (Maciejewski, 2017).

Start-of-program essays (*N* = 134) were matched with end-of-program essays (*N* = 134) to form the dataset (*N* = 116 start/end matched essay pairs) for this study. Each essay was scored by the author according to the TMA framework of Di Martino and Zan (2010) according to the following chart:

TMA Dimension	Possible Score		
Emotional Disposition	N/A	Positive (+)	Negative (-)
Vision of Mathematics	N/A	Relational (r)	Instrumental (i)
Perceived Competence	N/A	High (h)	Low (l)

Table 1: TMA dimensions and accompanying scores, in parentheses.

Scores for each essay in each category were assigned by the author – acting as educator/researcher – with the specific criteria for the categories within the TMA framework emerging through the reading of the essays. The criteria are as follows.

- Emotional: explicit mention of emotional states or feelings towards mathematics or mathematical activity. Specific emotional words or phrases are:
    - Positive: feel good, love math, proud, favourite subject, enjoyment, etc.
    - Negative: upset, fear, nervous, scared, afraid, frustration, hate, etc.
- Vision of Mathematics:
    - Instrumental: equating understanding in mathematics with assessment outcome ("I understand math because I got a B"); memorization without understanding; practicing formulas.
    - Relational: more than one way to do a problem; interconnectedness; focus on understanding rather than correct answers.
- Competence: indication of the student's perceived ability to perform in mathematics. High/low competence was determined as:
    - High: explicit admission of high ability; attributing successful performance or ability in mathematics to ones' self; a recognition that performance and ability can improve through effort.

- Low: explicit admission of low ability; attributing performance in mathematics to a teacher or an external entity; mathematical ability as fixed.

Considering the essay prompt was general, a student's response may not contain writing related to any one of these dimensions, which would warrant an N/A score on that dimension.

By way of a sample scoring, consider the following essay.

> I've never been good with math. If I do learn something, I usually forget it not too long after. I almost always have a hard time understanding math or even just the point to all the extra formulas or ideas about it. Also since I get really mad and irritated easily when I don't understand a problem it doesn't help me or anyone else.

This was assigned a "negative" on the Emotional scale (for the text "...I get really mad and irritated..."), an "instrumental" on the Vision scale ("...all the extra formulas..."), and a "low" on the competence scale ("I've never been good with math").

After each essay was scored, the aggregate scores were assessed using χ^2 and z tests to test for statistically-significant differences between start- and end-of-term essays. As will be discussed, there is not a singular best way to analyse the aggregate score data. An analysis is then performed on individual-level essays.

RESULTS

In aggregate, end-of-program essays revealed more positive and less negative emotions associated with mathematics, a greater relational and lower instrumental understanding of mathematics, and higher student competence; see Table 2. Note that the N/A scores were higher in the start-of-program essays than those at the end, which left open the possibility of increased positive/relational/high attitude score counts without a corresponding decrease in negative/instrumental/low attitude scores. However, this was not observed.

At this stage of the analysis, a question emerges which deserves to be presented in itself as a result of this study:

> How best to determine if a change of attitudes occurred?

A naïve application of a statistical test – for example, a χ^2 test – on the start/end positive/relational/high or the negative/instrumental/low scores reveals no statistically significant differences at the 0.05 level for all categories.

However, as noted above, the N/A scores are different for start and end, so a more appropriate measure of aggregate attitude change may be to compare the proportion of positive/relational/high to total number of non-N/A scores using a z-test. This yields a statistically significant difference in start/end Competence ($p < 0.01$) and Emotional scores ($p = 0.04$), but non-significant results for the Vision category.

	Pos./rel./high		Neg./ins./low		N/A	
	Start	End	Start	End	Start	End
Emotional	30	44	49	41	37	31
Vision	10	12	63	53	43	51
Competence	32	53	76	56	8	7

Table 2: Results of a TMA analysis of the student essays.

Another approach is to consider changes in the proportion of positive/relational/high or negative/instrumental/low scores out of the total number of essays. Again, a z-test reveals a significant change for the Competence and Emotional scores ($p < 0.01$).

As noted in (Di Martino and Zan, 2010), a change in *any* category can be taken as a change in attitude. Therefore – and again at the program level – the net number of students whose essays scored as negative/instrumental/low at start-of-program then scored as positive/relational/high at end-of-term may be of interest. These are 6 for Emotional, 1 for Vision, and 16 for Competence. There were, however, shifts in attitudes from the positive/relational/high categories to negative/instrumental/low from start to end: 9 for Emotional, 3 for Vision, and 13 for Competence.

As a final perspective on how changes in attitude might be assessed, I shift focus to changes exhibited in the writing of individual students.

Changes in individual students' attitudes

Shifting to individual students, but still maintaining a focus on broader trends in the program, the number of essays that received the "least favourable" combination of scores, negative/instrumental/low, went from 30 at the start to 17 at the end. The corresponding numbers of essays receiving the "most favourable" combination of positive/relational/high scores went from 5 at the start to 6 at the end.

In terms of trends at the individual level, no one student had an essay categorized as negative/instrumental/low at the start and a corresponding positive/relational/high at the end. This is likely because relatively few essays had relational scores in the Vision category at either the start or end of program.

In terms of a particular student's change in attitude consider the following. At the start of the program, the student writes:

> My personal experience with math was great at the beginning I loved math but when senior year came around and I took trig it become my worst subject and I was not so great at it anymore.

This was rated negative on Emotion (for the past tense "loved"), N/A for Vision, and low for Competence ("not so great at it..."). At the end of the program, they write:

> The experience I have had with math went from being really good. Freshman year through junior year math was my favorite subject and I was so good at it. Once senior year hit math

> got really difficult and I started to become less interested in math. Coming into this program I got a good review and understanding of what I learned in my past years. I really liked it because I received help from my tutors which was very helpful and convenient. I learned how to solve problems I had a hard time with and finally I don't anymore. I also learned to find math fun again even though it can be challenging at times I can say I like math again.

This was rated positive, N/A, and high. The writing clearly indicates, to this researcher/educator, an "improvement" – a construct to be returned to in the discussion – in the student's attitude.

However, change is not always apparent. Consider the student who writes at the start of term (negative/instrumental/low):

> I usually prepare for a test by doing a pratice test with sample questions. However, I could never get a good grade on a test because when the test comes, my mind freezes. The problem feels completely different and more difficult. Even though sometimes the difference in the problem was just a few numbers. I try to get through the problem by thinking hard about the pratice test and writing all the formulas down.

And the student's essay at the end of term:

> Math is very entertaining! I love solving math problems only when I know it. If you have a good teacher then you will learn to understand math. Sometimes I get angry when I get stuck on a problem, everyone can solve. Overall, I love being good at math. It makes me feel smart when I solved for a problem.

The student's writing is contradictory in spots and is difficult to categorize. Ultimately, it was rated positive/instrumental/high, but the question emerges, did the student's attitude *improve*?

DISCUSSION AND FUTURE DIRECTIONS

The TMA framework was originally intended to aid in clarifying and strengthening theoretical constructs in the mathematics education literature around students' attitudes. The framework was used here to assess and categorise students' attitudes towards mathematics as revealed through their writing about mathematics. Generally, it was useful in helping parse the essays, but was limited in its ability to reveal changes in students' attitudes, for at least two reasons. The first stems from the elaboration of the three TMA categories in (di Martino and Zan, 2010). For example, emotions are not always conveniently placed in a dichotomy – a student may love proving and despise computation, or enjoy mathematical modelling and fear examinations. It would seem, then, that "emotional disposition" toward mathematics is too broad a category in which simple, dichotomous changes can be observed. As for "Views of Mathematics," the instrumental/relational dichotomy is only one of many possible ways of conceptualizing a view of mathematics; Sfard (1991) proposes, for example, the structural/operational conceptualization and argues that this is not a dichotomous view of mathematics.

Issues with the TMA framework aside, once a satisfactory – to the researcher/educator – categorization of the essays was reached a second major unresolved issue emerged: how might the application of the TMA framework reveal changes in students' attitudes towards mathematics? The analysis performed here explored this question using a variety of quantitative methods. These methods and the ensuing results should not be interpreted as authoritative. Rather, the message of this research is in the methods and analysis: even with a rigorous framework for the conceptualization of students' attitudes towards mathematics, the issue of how best to observe change – or even to define "change" – in these attitudes remains. This is especially true in larger-scale educational settings where individual interviews or essay analyses are impractical. For example, the program in this study had a total enrollment of 134, and these students fed into a program of enrollment approximately 970. There is a tension: we as researcher-educators desire to understand students' idiosynchratic relationships with mathematics and simultaneously desire they transform productively through the education we offer, yet there are often so many students that meaningful one-on-one interactions are impractical.

Another, inescapable issue related to the above is the notion of "improvement" in students' attitudes. It is a plausible expectation that educators desire improvements in students' attitudes towards mathematics, but what might constitute an "improvement", and who determines that? In the final sample essay given in the results section, the student writes that they "love being good at math" but also that they "get angry when [they] get stuck on a problem…". Having a "positive" emotion (love) associated with performance and a "negative" emotion (anger) associated with a natural state of being stuck on math is likely not the type of emotional relationship with mathematics an educator desires for their students. One approach to this issue, and working towards an operationalization of the notion of improvement, is to refine and narrow the categories in the TMA and scales, positive/negative or otherwise, to each. However, this is akin to the development of the surveys that initiated research in this domain, which are not without their shortcomings.

In terms of survey instruments designed to assess aspects of attitudes towards mathematics, Di Martino and Zan (2010) raise genuine concern over their development and continued use. In particular: how are survey items chosen, their scales assigned, and can they describe something as multidimensional as attitude? However, some researchers have considered these specific issues in the design, construction, and refinement of their Likert-based instruments. For example, the Mathematics Attitudes and Perceptions Survey (MAPS; Code, et al., 2016) traces the genesis of its survey items in students' authentic writing and talking about learning and performing in STEM fields. Moreover, the MAPS instrument avoids the issue of creating normalized scales for each category it reports on by assigning scores relative to expert consensus. This, in a sense, makes for a more versatile, holistic instrument – there isn't a universal definition of "interest", for example, but mathematics experts have a consensus on "interest" which can act as a relative, community-defined datum for the students' “interest”. Further, studies with MAPS have analysed interactions between the MAPS categories (Code, et al., 2016). The criticisms of Di Martino and Zan (2010) remain quite valid, however, and a future study will compare a TMA analysis of students’ attitudes towards mathematics present in their writings with their attitudes as revealed by a multi-dimensional survey instrument, such as the MAPS (Code, et al., 2016). This can further address the challenge of scale, mentioned above.

One of the important points of this current work is that observing changes in attitudes is not a straight-forward endeavor. Assessing individual students for attitudinal changes, through interviews or through their writing, can lend insights into that particular student’s relationship with mathematics. However, educators are often faced with the task of evaluating the effectiveness of an entire program, which often enrolls large numbers of students, in transforming students’ atitudes. The TMA framework (Di Martino and Zan, 2010) clarifies what is meant by “attitude” towards mathematics; the next step is to clarify “change” and “improvement” in attitudes.

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