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Cover Page Footnote

The authors would like to thank Brittenee, Jakayla, Kevin, Matt, Sydney, Vicki, Sloan, Sarah, Spencer, and Javier for ongoing input, ideas, and fruitful discussions.

Revisiting Prejudiced Polygons: Adapting a Familiar Activity During a Time of Unknowns

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This article describes the design process behind various iterations of Prejudiced Polygons, a Math Circles activity about segregation. In particular, we frame our discussion around two guiding principles from User Experience (UX) Design in thinking about the interconnected components of a Math Circles session, which include all the people, the physical or virtual setting, the technology, and the world context. Additionally, we describe how we think about developing a "low floor" and "high ceiling" for math content, social issues content, as well as technology and access.

Keywords: Activity Design, Low Floor High Ceiling, Math Circles, Segregation, Social Issues

1 Introduction

Two defining moments of 2020 were the murder of George Floyd and the start of the COVID-19 pandemic. While students and educators were facing new complexities in learning and teaching due to the pandemic, many were also grappling with grief and trauma. Promoting social engagement and connection was particularly important during this difficult time, but some disciplines, like math, do not have a tradition of providing the vocabulary nor tools to process these concerns.

During this year when racial injustice was at the forefront of many people's minds, we decided to revisit the Math Circles activity that we call Prejudiced

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Polygons. We first launched this activity in 2016, and it is based on the interactive website by Vi Hart and Nicky Case [12]. The activity starts with a "game" and simple rules, which provide a familiar terrain for participants to engage in. During the activity, participants discover a model of segregation [32] and have opportunities to talk about social issues with their math communities. In our first implementations, we saw participants productively discuss racial segregation, gender bias, and other related issues simply because they were prompted to think about the topics by playing this "game." Thus, when asked to present for a Math Circles event during this time of unknowns, we felt it was time to reinvent the activity and to incorporate new elements of both content and design. This article presents a detailed account of Prejudiced Polygons and its various iterations.

This analysis of the Prejudiced Polygons activity for the *Math Circles in Times of Physical Distancing* special issue serves as a general lesson plan intended to support leaders of Math Circles sessions. However, Prejudiced Polygons is easily adapted for use in other outreach programs or for professional development. Prejudiced Polygons is also versatile enough for use with participants of various ages and backgrounds with demonstrated success implemented in either online or in-person presentation format. Here, we seek to support the *Journal of Math Circles'* larger goal of disseminating local knowledge to the broader math community and showcase Prejudiced Polygons as a resource for math practitioners and educators interested in community responsive programs and/or outreach. Specifically, the implementation of a "low floor, high ceiling" program design allows for multiple entry points to participate in the activity, thus extending its benefits for math education and the application of math as a means to better understanding of important social issues including segregation, racial and economic justice, and other topics. In addition, we demonstrate the ways in which Prejudiced Polygons promotes the development of problem-solving skills and building communities of mathematical thinkers. Lastly, Prejudiced Polygons illuminates the ways in which math and math education can be connected to everyday, real-world issues and problem solving. Many people perceive math as irrelevant to their everyday lives, but this activity demonstrates how math can model very real social issues.

2 Who is This for?

We are aware that there are many individuals with personal or professional experience with social justice math. We know there is an extensive body of literature on the topic, and there are multiple professional societies built

on the foundation of equity in math. This article is not primarily intended for those with extensive personal or professional experience in social justice math. Instead, we are writing for math educators and interdisciplinary teams who want to learn the insights and guiding principles that we followed when modifying Prejudiced Polygons over time. We hope this is a resource for those interested in creating and continually refining their own social justice math activities. For those who are interested in additional resources, we cannot provide an exhaustive list here, but we recommend starting with TODOS: Mathematics for ALL [38], the websites by Harper [11] and Ince [17], and our list in Appendix B.

3 Underlying Principles

Beneath the design of all versions of this activity are the ideas of providing the familiar when there are many unknowns, which we think of as a kind of scaffolding. In our latest version of Prejudiced Polygons, we had to consider additional technological challenges with a virtual event, so we looked at the principles of People-Centered Design from the field of User Experience (UX) Design. These general principles can be retroactively applied to describe our decisions in earlier versions, so we will weave these ideas throughout our chronology.

Two principles of People-Centered Design include "focusing on people" and "taking a systems point of view" [39]. The founder of The Design Lab and the Nielsen Norman Group further emphasizes that good design considers all people who are involved in the system [24]. In our case, the people involved in a Math Circles session include the participants, the facilitators, and the hosts of the event. Our system consists of all the people, the physical or virtual setting for the Math Circles event, and the current world events that affect how individuals are feeling. Ultimately, our goals are to have our participants use this math activity to engage in fruitful conversations about race and to better navigate related challenges in their classrooms and communities. Thus, before we started working on the math activity, we spent time reflecting on all of these interconnected parts. For instance, we could have individuals who grew up with a "colorblind" mentality of not talking about race, and we could also have those who have personally dealt with bias or injustice because of the color of their skin. For the math content, we knew everyone was likely comfortable with basic fractions, but not everyone was familiar with math modeling or game theory. Lastly, for our pandemic-era virtual version, we knew participants could be joining with a smartphone, with a dual-monitor desktop setup, or with anything in between.

Despite these differences, we wanted to design an activity for everyone with multiple entry points and ample opportunities for open-ended discussions. In other words, we hoped to design a "low floor, high ceiling" social justice math task while taking into account that there are multiple types of floors and ceilings. In the pre-pandemic face-to-face versions of Prejudiced Polygons, we considered floors and ceilings for both the math content and the discussions about social issues. In the pandemic-era virtual versions, we additionally considered technology and access. The specifics for each iteration are detailed below.

4 Designing Versions of Prejudiced Polygons

4.1 Inspiration

We were originally inspired by Hart and Case's "Parable of the Polygons," an interactive website [12] based on Schelling's game theory model of segregation [32]. In the website, a user first sees Triangles and Squares in a grid representing a neighborhood. These Triangles and Squares are "slightly shapist" because they follow one rule,[1] which is that they want to move if fewer than one-third of their neighbors are like them. Polygons are happy if more than or exactly one-third of their neighbors are like them. In an easy-to-navigate interface, the user is prompted to drag and drop polygons until all of the shapes are happy. Inevitably, the collective neighborhood is segregated, a fact that Hart and Case demonstrate with a larger series of simulations and corresponding graphs of segregation over time. Hart and Case explain how a "small individual bias can lead to large collective bias." They utilize additional interactive neighborhoods with corresponding graphs where the user can adjust sliders to modify the conditions on the amount of polygon bias.

If the user keeps scrolling down the page, Hart and Case present an optimistic solution where polygons are demanding some amount of diversity. This changes the original rule of the simulation to one where polygons will move if fewer than 10% or greater than 80% of their neighbors are like them. In this scenario, the graph shows some measure of segregation decreasing over time. In the footnotes, Hart and Case cite a few research articles on models of segregation and institutionalized bias, including gender bias. They also

[1] Technically, there are two rules, but the second one was rarely used, so we will focus on the one-third rule in this article. The second rule stated that polygons would move if they were completely isolated and had no neighbors.

mention that they simplify a real-world scenario to give the model and story a happy ending.

4.2 Prejudiced Polygons Beta

After we saw the power of the experience and message from this interactive website, we wanted to bring the idea to a Math Circles session. In particular, we wanted to slow down and extend the reflection, so that participants internalized how a simple model could imply significant consequences (e.g., segregation is inevitable under certain conditions). We also wanted to explore the nuances that Hart and Case referred to in their footnotes.

Our initial team started with only two math educators (the first and third authors of this article). Knowing that we weren't representative of our entire Math Circles community or system, we sought to find a more diverse team to consider the possibilities of using math to talk about segregation. We invited a small group to pilot our beta version of Prejudiced Polygons,[2] consisting of faculty and students in sociology, math, and math education with a mix of genders and racial/ethnic backgrounds. Our beta version testers gave helpful feedback on facilitation and implementation, including the fact that the activity made the discussions about bias and segregation more accessible for non-experts (defined as those without extensive personal or professional experience in the field of sociology). This was a recurring comment that we would hear in later implementations of the activity.

4.3 Prejudiced Polygons 1.0

In December of 2016, we launched what we now refer to as version 1.0 of Prejudiced Polygons [14]. Participants were given a physical board with a 10×10 grid and a predetermined neighborhood of happy and unhappy polygons (see Figure 1). These examples were generated from Hart and Case's interactive website [12], and the detailed facilitator notes can be found on the first author's website [13]. Each group had a different configuration, so that participants could later compare the results across boards. Each polygon piece was two-sided with one side being happy and one side being unhappy. This way, participants could easily flip a polygon around to indicate the change in happiness level as the polygons moved around in a neighborhood. We started with the same rule as the original website where polygons would move if fewer

[2]This activity was originally called "Triangles, Squares, and Segregation" because we wanted to be explicit in our topic, and we didn't want to surprise any participants. After a recommendation from the Math Teachers' Circles Network, we renamed the activity to the shorter "Prejudiced Polygons," which still encapsulates the same meaning.

than one-third of their neighbors were like them. As a math lesson, this became an activity on comparing fractions.

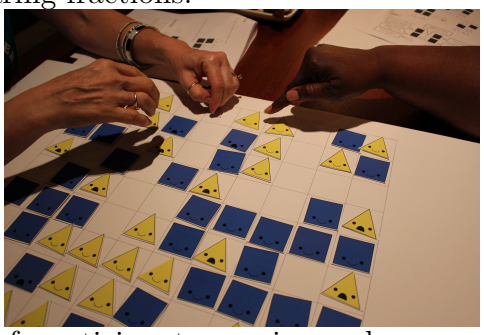

Figure 1. Photo of participants moving polygons around in Prejudiced Polygons version 1.0.

We asked participants to work in small groups of four to six people, and they were all in-service and pre-service math teachers from schools near the coast of South Carolina. Our teachers were of a range of ages and genders, and identified as white or Black. We thought that the physical board and "game" format better captured the seemingly harmless nature of the one-third rule. Additionally, we wanted participants to experience physically moving the polygons, a process that took on more deliberation when the participants realized that they were segregating the neighborhood.

Regarding the math content, the floor of the activity was knowledge in comparing fractions [22, CCSS.MATH.CONTENT.3.NF.A.3]. At the time, we did not have explicit whole-group guidance for discussions about math models. However, small-group discussions and subsequent iterations of Prejudiced Polygons better align with the math modeling process in the Guidelines for Assessment and Instruction in Mathematical Modeling Education [10, p. 12] in discussions about modeling, assumptions, and connections to the real world.

As for the social issues content, we looked at the Southern Poverty Law Center's Teaching Tolerance project and Social Justice Standards [33]. This publication provides an anti-bias framework for K-12 students and includes standards as well as age-appropriate outcomes. In our case, for the participants who were more comfortable with the floor of comparing fractions, we found that they could still engage in discussions of segregation by using the vocabulary of segregated polygons. In many cases, these participants moved from talking about the polygons to talking about students at their schools, which aligns with the anchor standard of "analyz[ing] the harmful impact of bias and injustice on the world, historically and today [33, Justice Anchor Standard 13]. Some other participants were ready to explore the ceiling in conversations about "collective action against bias and injustice" as

well as assessments of effectiveness of strategies [Action Anchor Standard 20]. The floors and ceilings are summarized in Table 1.

Table 1

Floors and ceilings for math and social issues content.

	Math Content	Social Issues Content
Ceiling	Analyzing and critiquing a math model; Considering assumptions of a math model; Modifying the model	Assessing strategies to address issues of bias and segregation; Considering actions and solutions
Floor	Comparing simple fractions	Recognizing the issues of bias and segregation

4.4 Prejudiced Polygons 1.X

After we launched our version, others utilized their own takes on Prejudiced Polygons. We know of versions that were hosted at regional Math Teachers' Circles, a county professional development event for teachers, and various classes from sixth grade to university. We call these versions 1.X given the reported variations in local discussions. Based on the needs and experiences of their own communities, some chose to talk more about assumptions of the activity and modifications to the model [22, "High School: Modeling"]. Others discussed local data regarding segregation by using the Racial Dot Map [5]. Additionally, some participants reflected about their own experiences with segregation and bias. In our own facilitation, we saw grade school children as well as experienced teachers reflect about the challenges in their communities and the positive benefits of diversity [33, Diversity Anchor Standards].

4.5 Prejudiced Polygons 2.0

Because issues of racial inequities were at the center of national conversations in 2020, we felt it was time to revisit Prejudiced Polygons with a more robust change. We knew we needed a sociologist in the redesign team for version 2.0 [15]. Just like before, we hoped that this Math Circles activity could help facilitate important conversations about bias and segregation that some math educators and students felt under-prepared to engage in. This is how the second author joined the team.

In line with previous iterations of Prejudiced Polygons, we aimed to create a low floor, high ceiling approach to the extended sociological contribution of version 2.0. Both the unequal impacts of COVID-19 and the massive racial

justice protests of 2020 meant that some exposure to issues of racial bias would have been hard to avoid, no matter a person's previous experience, interest, or social location. Implicit bias, police violence, racial health disparities and other topics were frequently included in media coverage from a variety of sources and perspectives. While we assumed Math Teachers' Circles participants would have some *exposure* to these issues, we made no assumptions about the conclusions participants might have drawn regarding these current events. Therefore, the floor and ceiling for social issues remained the same as in previous versions of the activity. That is, we felt we could reasonably assume most participants would come into the presentation at least at "floor" level: recognizing the issues of bias and segregation. Then, through the course of the exercise, we would reach for the "ceiling:" assessing strategies to address issues of bias and segregation, and considering actions and solutions (see details of facilitator questions in Appendix A).

After participants completed the game and engaged in a brief discussion about the strategies and outcomes that surfaced as they worked to place each polygon in their "happy place," we transitioned to the sociological segment of the activity. This was guided by the three main conclusions offered by Hart and Case for their original game: 1) small individual biases lead to large collective biases, 2) the past haunts the present, and 3) it is up to us to demand diversity [12]. We encouraged audience participation with open-ended discussion questions and then connected current sociological data on segregation and racial bias to illustrate each of Hart and Case's own conclusions.

The math-based portion of the activity generated significant and thoughtful discussion about segregation on its own. As people realized their strategies for keeping the polygons "happy" were creating segregated spaces, most participants sought (and sometimes struggled) to develop an alternative. Many participants shared these challenges after completing the game. When transitioning to the sociological perspective, participants were eager to discuss, ask questions, and learn how sociological data about people and real-world segregation connected to a "game" about polygons. To transition from fractions and polygons into social issues, we first posed a series of general questions asking participants to reflect on what their neighborhoods were like growing up, whether they felt most people in those neighborhoods were similar to each other, and how their childhood neighborhood experience shaped their view of the world. We also posed a series of questions about participants' current neighborhoods. The virtual presentation format in Version 2.0 allowed participants to attend the session from across the US and around the world, and this contributed to the richness of discussion around

the social issues content. Participants from various races, ethnicities, and geographies generously and enthusiastically shared information about their experiences with both segregated and integrated neighborhoods.

We then moved the discussion to center around Hart and Case's three conclusions of their work, connecting these conclusions both to the game and to ongoing segregation in the United States. The first conclusion of Hart and Case is that small individual biases lead to large collective biases. When asked to discuss how this was evidenced in the game, participants could easily identify that the Triangles and Squares didn't harbor ill will toward each other–they just felt more comfortable around shapes like themselves. We then discussed this in the context of social research demonstrating that segregation persists, in part because of small biases that seemingly have nothing to do with race or racism: the desire to live in a neighborhood in which one feels "safe" or seeking a home in a good school district, for example [3, 26, 37]. These aspirations, in and of themselves, are not necessarily indicative of racial bias. However, the likelihood that a neighborhood is safe (or perceived to be so) or has a good school system, is connected to historical and political legacies that are very much racialized. Thus, seemingly innocuous "colorblind" criteria for desirable neighborhoods nevertheless has the effect of perpetuating racial segregation.

This brings us to Hart and Case's second conclusion: the past haunts the present. People of all racial backgrounds seek safe neighborhoods and good schools for their children. Why is it that white families are more likely to live in such places? Math Circles participants easily made it to the "floor" of recognizing the issues of bias and segregation. There was some divergence in whether participants viewed segregation as a structural or systemic problem, or whether this was the outcome of individual choice and behavior. We then connected the dots between historical and persistent income and wealth disparities between racial groups as well as exclusionary practices like "redlining." A primary example is the denial of GI Bill benefits to African American soldier after WWII. These benefits allowed a generation of white Americans to build wealth through home ownership and college education. This wealth could then be passed on to their children [41]. At the same time, governments and banks employed "redlining" tactics that marked non-white neighborhoods unsafe, making it less likely for non-white families to get loans and ensuring that even those who achieved home ownership had less equity in their home than their white counterparts [30]. Like segregation, exclusionary lending practices are illegal today, but investigations show they still happen [20]. That these historical legacies of discrimination persist into the present underscore the ways in which "colorblind" choices can serve to perpetuate

segregation and underscores Hart and Case's point that the past does indeed haunt the present.

Hart and Case's third conclusion is that we must demand diversity. This section relates most clearly to the "ceiling" of the activity: assessing strategies to address issues of bias and segregation. This section of the presentation elicited enthusiastic discussion. Participants put forth a range of potential strategies to address bias and segregation ranging from individually focused actions (e.g. educating oneself on racial justice issues, expanding one's social circle to include more diverse people, choosing to live in a diverse area, etc.) to more systemic solutions like making sure law and policies against discrimination are updated and enforced. Social science research shows employing both individual and systemic approaches is necessary to fully address persistent issues of racial bias and segregation. These include individual level actions like listening and learning from people of color, speaking out when witnessing an injustice, and being an active member of a diverse community (if not one's neighborhood, then perhaps an organization). Systemic approaches include examining the policies and procedures of the communities and organizations to which we belong (neighborhoods, workplaces, schools, etc.), advocating for diversity and inclusion, and seeking policy change.

The breadth and depth of participant discussion reached in Prejudiced Polygons 2.0 was facilitated by the virtual format, which allowed participants to log in from across the country and around the world. This ensured a diverse group of participants in terms of race, ethnicity, gender, income, occupation, immigration history (i.e. whether a person is a recent immigrant to the US), and geography. From this diversity flowed a richness of discussion that drew out additional nuances and connections between a math "game" involving segregated polygons and the very real-world consequences of persistent racial bias and segregation.

4.5.1 Technology and Access

Along with the significant addition of sociological content, we also made technology and access changes to Prejudiced Polygons due to the COVID-19 pandemic bringing about new challenges with a virtual format. We thought about designing a low floor that used the minimum amount of technology, namely, paper and pen. Although the hosts ran the event as a virtual meeting, and we were using slides to present, we explicitly gave instructions for how to engage in the activity with hand-drawn pictures. We also redrew our digital polygons by hand to further emphasize that a low-tech version was

appropriate. We also slightly changed the setup of the initial "game" to better accommodate a paper and pen format and to examine a different assumption about our implicit model of segregation. In this version, we started with a new neighborhood (e.g. a blank grid), and participants were asked to place a certain number of Triangles and Squares while following the same one-third rule about moving as before. Similar to the in-person version, we had participants split into groups of four to six people. Participants worked in virtual breakout rooms and took turns in the "game" while the facilitators moved between virtual rooms to help guide the conversations. In the two times that we facilitated Prejudiced Polygons 2.0, we saw that some participants chose to use a shared whiteboard and virtual annotation tools as well.

See Table 2 for a summary of the floor and ceiling of technology and access.

Table 2

Floor and ceiling for technology and access.

	Technology and Access
Ceiling	Editing a shared virtual whiteboard or document; Using virtual annotation tools for collaboration
Floor	Verbally discussing hand-drawn polygons and grids on paper

5 Reflection and Conclusion

Prejudiced Polygons continues to evolve, and we make a point to keep a record of its significant changes [13]. After our facilitation of version 2.0, Math Communities already made and ran a Desmos Activity version for their Math Mondays series of events [4, 6]. We are also working on our own versions 2.X for different audiences and situations. Because the activity is undergoing continual refinements, we direct the reader to the first author's website [13] for updated facilitation details.

In planning ahead for future Math Circles events, we look back at the process that we have considered, and we re-examine the two People-Centered Design principles of focusing on all the people and considering the interconnected pieces of the system. This includes the people, the physical or virtual setting, the technology, and the world context. We suggest first working through the following questions:

- Who are all the people?
- Who does this activity benefit?
- What is the physical or virtual setting?

- What is the current world context?
- What might be familiar and unfamiliar about the content or technology?
- How can we establish a low floor and multiple opportunities to explore high ceilings?

We believe that starting with the answers to these questions can improve engagement and provide more equitable access to future Math Circles activities.

The power of Math Circles to inspire broader social change should not be overlooked. Because Math Circles specifically engage students and teachers in hands-on discovery, there exists immense potential for these activities to facilitate critical thinking and real-world connections between math education and current social issues. This would serve a dual purpose in generating interesting and engaging content for the Math Circles programs but also in illuminating the deep connections between mathematics, education, and social issues. Perhaps this paper may inspire additional interdisciplinary approaches to Math Circles. Interdisciplinary, multidisciplinary, and transdisciplinary education has been lauded as a potential path to address some of most pressing issues of our time, like climate change, health inequities, and the digital divide, to name a few [35, 40]. Pedagogies built on social justice education can facilitate problem solving, cultivate diversity and inclusion, and create learning environments in which students learn about their world, how to live in it, understand it, influence it, make it better [35]. As we build more equitable Math Circles communities, there is the potential to create substantive change in the math and education communities and beyond.

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Appendix A: Guiding Questions for Prejudiced Polygons 2.0

Guiding Questions for Math Activity

- What do you want to change about the "game" or model?
- What assumptions are made in this model?
- How might this be applicable to real life scenarios?

Guiding Questions for Sociological Discussion

- Think about the neighborhood you grew up in. Were you surrounded by people who were mostly like you? How do you think this neighborhood shaped your worldview?
- What are some small biases that lead to large collective biases across society?
- What happens when we fail to develop meaningful connections with people different than us?
- In what ways does our past influence our present? How did we get here?
- What do we do now? How do we demand diversity? From whom?

Appendix B: Additional Resources

We provided many of these resources to participants at the end of our facilitation for Prejudiced Polygon 2.0. We also recommend these for future facilitators.

Books: Psychology/Sociology

- *Why are all the Black Kids Sitting Together in the Cafeteria?*, Beverly Daniel Tatum [37]
- Racism without Racists: Color Blind Racism and the Persistence of Racial *Inequality in the US*, Eduardo Bonillo-Silva [3]
- *Color of Law: A Forgotten History of How Our Government Segregated America*, Richard Rothstein [30]
- *Pushout: The Criminalization of Black Girls in Schools*, Monique Morris [21] and associated documentary [1]
- *Punished: Policing the Lives of Black and Latino Boys*, Victor Rios [28] and associated TED talk [29]
- *Race After Technology: Abolitionist Tools for the New Jim Code*, Ruha Benjamin [2]

Articles Citing Academic Research

- "What School Segregation Look Like in the US Today," Erica Frankenburg [8]
- "Schools are Still Segregation and Black Children are Paying the Price," Emma García [9]
- "Attacking the Black-White Opportunity Gap that Comes from Residential Segregation," Kimberly Quick and Richard Kahlenberg [27]

Books: Journalism/Commentary

- *Me and White Supremacy*, Layla Saad [31]
- *So You Want to Talk about Race*, Ijeoma Oluo [25]

Multimedia

- Immigration History, Immigration and Ethnic History Society [16]
- Race: The Power of an Illusion, California Newsreel [23]
- 13th, Directed by Ava Duvernay [7]
- Teach Us All, Directed by Sonia Lowman [19]

Facilitation Tips

- "Facilitating Difficult Race Discussions," Derald Wing Sue [36]
- "Let's Talk: Discussing, Race, Racism and Other Difficult Topics," Teaching Tolerance [34]
- "Navigation Guide for Difficult Conversations about Race in Troubled Times," Joint Federal Financial Agencies Office of Minorities, Women, and Inclusion [18]