Fall 1983

Pattern and Mathematics: Math Enrichment Activities for Gifted Fourth, Fifth and Sixth Grade Children

Janet K. Zuber

PATTERN AND MATHEMATICS MATH ENRICHMENT ACTIVITIES FOR GIFTED FOURTH, FIFTH AND SIXTH GRADE CHILDREN

A Project
Presented to
The Graduate Faculty
Central Washington University

In Partial Fulfillment
of the Requirements for the Degree
Master of Education

---

by
Janet K. Zuber

October, 1983

TABLE OF CONTENTS

Chapter 1

BACKGROUND OF THE PROJECT

> "Contrary to some popular notions, intellectually superior children are often the most neglected children in the classroom."
>
> California Assembly Interim Commission of Education, 1967

A 184-pound ball shot into space in October of 1957 marked the beginning of the modern era of gifted education and set fire to the necessity for meeting the gifted students' needs. Sputnik caught the attention of the American people and for a moment the flame burned with vigor as "the nation became conscience stricken over its failure to produce sufficient high-level manpower to meet the threat of its idealogical and cold war adversary" (Tannenbaum, 1972, p. 16).

But by the early 1960's, the flame flickered feebly as national sentiment turned to civil rights and concern for the disadvantaged and their plight. Special programs for the gifted child became unnecessary "frills". Gifted students once again became the neglected minority.

Two attitudes concerning the gifted and their needs have been responsible for retarding the growth and acceptance of gifted education.

For many years, gifted and talented children have been thought to "make it on their own".

Some do. Could they have gone farther and have better utilized their abilities with an education designed to meet their talents?

Some do not. Surveys from a number of states report anywhere from 12 to 30 percent of their gifted students are drop outs (Ramos, 1975). Much of the research supports the idea that gifted students are better able to develop their talents with an appropriate educational program.

The second attitude is an outgrowth of our democracy's perennial dilemma over championing excellance and equality simultaneously. It is felt by many that all students should have the same experiences and opportunities. Good education is good for all students. Related to this is the thought that special provisions will tend to develop an educated elite who will engender a kind of closed caste system (Leese and Fliegler, 1961). In actuality, "the programs have not produced arrogant, selfish snobs; special programs have extended a sense of reality, wholesome humility, self-respect, and respect for others" (Clendening and Davies, 1980, p. 6).

Fortunately there was a revival of interest in gifted education in the early 70's sparked by federal legislation resulting in the Marland Report. This landmark document produced a startling and disturbing portrait of neglect and concluded that gifted and talented children reached their potential not because of our schools, but in spite of them. The Office for the Gifted and Talented within the

U.S. Office of Education was established and with increased federal and state funding, new programs for gifted children were created and, more important, many educators and parents became more aware of the special needs of the gifted.

Currently, with the onslaught of federal and state cutbacks, all public education programs are suffering. Gifted education is no exception. But with the abundance of materials being designed for the academically and creatively talented, the increased number of colleges including programs for the training of teachers in this area, and the research which is continuing to seek the manner in which these children think and problem solve, it is hopeful that this time the flame will not be allowed to die.

Rationale for the Project

Typically, the gifted child is advanced several years in knowledge beyond other children in his class. Lack of challenge in a child's gifted field often results in negative behavior, withdrawal or underachievement.

Curriculum for the gifted student emphasizes process; use of higher level thinking skills, approaching problems in more than one way, learning by discovery and creative thinking. The content of curriculum for the gifted child should center around broad based themes or problems.

These two factors, an accelerated base of knowledge and diversified curriculum needs, cause a dilemma for the gifted child's teacher in all academic areas but is of special concern in the field of mathematics.

All around us is evidenced the need for those who have facility with the complexities of mathematics and problem solving. Ours is an age of technology. Computers are becoming household appliances and a knowledge of programming an essential second language.

And yet most mathematics at the elementary level is taught in a sequential manner with major emphasis on computational skills. While a computationally correct solution is important there is need for math activities which emphasize thinking and learning abilities.

There was a short time in the 1960's when "New Math" encouraged the exploring of math processes. Blocks, rods and other manipulative materials were found in use in the elementary classroom. But our "back to basics" philosophy and teachers' and parents' lack of confidence in this method have encouraged most textbooks to resume use of a more traditional method of instruction and little math exploration is found beyond the primary grades.

How are the gifted child's math needs to be met?

Establishing a gifted program may not be the total answer. Though a teacher trained in working with the gifted can provide experiences to challenge a curious and inquiring mind, many gifted programs are pull-out models serving the child only a part of the day. Often the classroom teacher continues to take responsibility for teaching the basic skills and the student still finds himself in the lock-step structure of the traditional math group taught in the traditional manner.

Most classroom teachers are generalists who lack the specialized preparation needed to create challenging math activities for the gifted child. This, coupled with increasing class sizes and the multiple needs of a heterogenous classroom establishes the need for a direction in planning a math program for the gifted student and lessons and materials for his use.

Purpose of the Project

The purpose of this project is to provide assistance to the elementary math teacher in meeting the needs of the student gifted in the area of math.

A collection of activities is provided to use with gifted intermediate students and should serve as an example of the type of activities appropriate for the gifted student.

These activities would be most appropriately used with gifted students in grades four through six. The activities would be of greatest benefit if students were grouped in a homogeneous manner with one teacher taking responsibility for their math needs. Homogeneous grouping would allow for faster-paced instruction and stimulating interaction among the students as well as relieving other teachers of this responsibility and providing continuity to the program.

The activities encourage use of higher level thinking and exploration with a variety of materials. The student is encouraged to think and work as a mathematician does, dealing with both inductive and deductive reasoning.

Focus and Limitations

This project is intended for use with intermediate students, grades four through six. This is a time when many gifted students become bored and disenchanted with math. More paper and pencil, computational work is common in the basal textbooks with limited manipulative and discovery projects.

These students have often mastered the core of basic skills and find repetitive computational work tedious. They can often complete the mathematical process in their head and balk at having to write this process on paper, having already discovered the answer.

In addition, there are fewer appropriate materials which encourage higher level thinking and exploration and expansion of math topics available for the intermediate gifted student.

This project is designed for children identified gifted in the area of math and the students should be working at least one to two years above grade level.

They should be working in a homogeneous ability group which may be multi-graded. A group of no more than twenty students would allow group interaction and discussion but still give the teacher an opportunity to spend time with individuals.

The activities are intended to enrich skills taught at the intermediate grade levels. Therefore, this class would be in place of the regular math program and would meet four to five hours a week.

One teacher, with an interest in math and a willingness to go beyond repetitive computation and simple problem solving to the exploration of mathematics, would act as facilitator.

The activities in the project have been planned around books and materials available in the Enrichment Center housed at Bennett Elementary, Bellevue School District.

Definition of Terms

Gifted Child

Numerous conceptions and countless definitions of giftedness have been put forth over the years. In recent years the following definition stated by the U.S. Office of Education has grown in popularity:

> "Gifted and talented children are those who by virtue of outstanding abilities are capable of high performance. Children capable of high performance include those who have demonstrated any of the following abilities or aptitudes, singly or in combination:
> 1. general intellectual ability
> 2. specific academic aptitude
> 3. creative or productive thinking
> 4. leadership ability
> 5. visual and performing arts aptitude"
>
> (Clendening and Davies, 1980, p. 4).

For the purpose of this project, a more restrictive definition is necessary. A gifted child is one with a specific aptitude in the area of mathematics. The enrichment activities have been designed for intermediate students, two years above grade-level in mathematical comprehension.

<u>Math Enrichment Activities</u>

In general, enrichment refers to a practice whereby the conventional material is supplemented by additional material which broadens and deepens the understanding of the regular work.

Enrichment, specific to the gifted child, would focus on the student's ability to see relationships, patterns, and structures of mathematical systems. Also, enrichment would refer to those activities that stimulate productive and evaluative thinking.

Summary

The balance of this report contains three additional chapters, a bibliography of references and an appendix.

Chapter 2 reviews recent literature on gifted children, their general educational needs and, more specifically, their arithmetic needs at the elementary level.

Chapter 3 reviews the writer's procedure in compiling materials and activities for this project.

Chapter 4 is composed of the project's summary, conclusions and recommendations.

The project document follows as an appendix to the report.

Chapter 2

REVIEW OF LITERATURE

In order to understand a special child, the child with advanced math abilities, it is important to have an understanding of the larger group of which he is a part. Thus, this review of literature has been divided into three parts; The Gifted Child, Educational Needs of the Gifted Child, and Math and the Gifted Child.

The first section of the chapter includes definitions of giftedness, methods of identification, characteristics and learning styles.

The second section deals with the gifted child's educational needs. Guidelines for developing curriculum and programs are included as well as qualities desirable in teachers working with bright children.

The last section of the chapter deals with the more specific field of math and the gifted child. Included in this section is research relating to the characteristics of the gifted math student, special problems and curriculum and program needs specific to this area.

The Gifted Child

Who are the gifted? There have been numerous conceptions and countless definitions of giftedness put forth over the years (Renzulli, 1978).

We find very specific definitions. "The gifted are in the top one per cent of the juvenile population in general intelligence" (Hollingsworth, 1942, p. 298). Lewis Terman (1926, p. 43) defines giftedness as "the top 1% level in general intellectual ability, as measured by the Stanford-Binet Intelligence Scale or a comparable instrument."

"The academically talented student is one who receives scores of about 115 or over on a Stanford-Binet Intelligence Test or falls above a similar point on one of the Wechsler Intelligence Scales" (American Psychology and Guidance Association, 1961).

At the other end of the continuum, we find more liberal definitions. Paul Witty's (1958, p. 62) definition of gifted is but one example:

> There are children whose outstanding potentialities in art, in writing, or in social leadership can be recognized largely by their performance. Hence, we have recommended that the definition of giftedness can be expanded and that we consider any child gifted whose performance, in a potentially valuable line of human activity, is consistently remarkable.

The American Association for Gifted Children (1951) defines an academically gifted student as one who consistently excels in one or more of socially useful endeavors.

Robert DeHaan and Robert Havighurst (1961) consider as gifted, a child whose superiority in any field would enable him to contribute significantly to cultural progress.

Since 1972, the definition being adopted by many states and school districts is the definition set forth by the U.S. Office of Education. This definition is a result of extensive Congressional investigation originating with the Marland Report.

> Gifted and talented children are those who by virtue of outstanding abilities are capable of high performance. These children require differentiated educational programs and/or services beyond those normally provided by the regular school program in order to realize their contribution to self and society.
>
> Children capable of high performance include those who have demonstrated any of the following abilities or aptitudes, singly or in combination: 1) general intellectual ability, 2) specific academic aptitude, 3) creative or productive thinking, 4) leadership ability, 5) visual and performing arts aptitude, 6) psychomotor ability (S.P. Marland, Education of the Gifted and Talented, 1972).

Most recently Joseph Renzulli (1978) has offered a new, research-based definition of the gifted and talented. He acknowledges above-average ability and creativity as two ingredients of giftedness. He adds a "non-intellective" cluster of traits which his research shows to be found consistently in creative/productive persons, a refined or focused form of motivation known as 'task commitment' (Renzulli, 1978).

A definition which summarizes and generalizes his research is:

> Giftedness consists of an interaction among three basic clusters of human traits---these clusters being above-average general abilities, high levels of task commitments, and high levels of creativity. Gifted and talented children are those possessing or capable of developing this composite set of traits and applying them to any potentially valuable area of human performance. Children who manifest or are capable of developing an interaction among the three clusters require a wide variety of educational opportunities and services that are not ordinarily provided through regular instructional programs.(Renzulli, 1978, p. 261).

It soon becomes apparent, in studying the gifted, that there has been a major problem of agreement among educators as to just what is the definition of gifted.

Gallagher (1975) among others, points out that the definition of "giftedness" is culture bound. Giftedness reflects those dimensions that the culture values. As the values of the society change, the views toward the nature of excellence also undergo subtle change.

Thus, much like the population it describes, the definition of giftedness is fluent, flexible and dynamic.

<u>Identification of the Gifted</u>

The definition and identification of the gifted are closely associated. Says Gallagher (1975, p. 42):

> No matter how elaborate and all-encompassing the definition of giftedness, the specific identification tools that are actually used are often the real determiners and the real definer of giftedness in a school setting."

And like the definition of giftedness, the subject of identifying giftedness has not been altogether settled. On the one hand, we have the problem of deciding what qualities of human beings can be categorized as gifted and on the other hand we have the problem of deciding the extent to which they are measurable.

The intelligence quotient, which shows the relationship of mental and chronological age, has long been the primary standard of finding the gifted student (Love, 1972, p. 37).

The Stanford-Binet Intelligence Test and the Wechsler Intelligence Scale for Children are considered to be among the best instruments currently available for identifying children with high general intellectual ability on an individual basis (Nazarro, 1979).

Group-administered tests include the California Test of Mental Maturity, the Raven's Progressive Matrices, and the Goodenough-Harris Draw-A-Man test as well as tests that subscribe to the measurement of many intellectual abilities, as for example, the Primary Mental Abilities, Differential Aptitude Test, and the Multiple Aptitude Tests.

Another school screening device for detecting giftedness is the achievement test. This test measures the amount of learning that has occurred in subjects covered in school (Durr, 1964). Among the standardized achievement measures used in the United States for the purpose of screening gifted children have been the Stanford Achievement Test, California Achievement Test, and Metropolitan Achievement Test.

Research has revealed that one method should not dictate the identification procedure. J.P. Guilford (1967) feels strongly that a single test score, such as that for mental age used in computing I.Q., cannot accurately measure all types of human intelligence, which comprises too many components to be summarized in one number on a scale.

This concern is reflected in the variety of data used by most programs as indicators of giftedness. Project for Education of the Gifted, headed by Virgil Ward (1962) found the following seven types of data being used:

1. group intelligence test
2. teacher judgment
3. school records, including achievement test scores, and teacher grades.
4. individual intelligence test administered by a qualified person.
5. appraisal of social and emotional maturity and adjustment.
6. parent interviews.
7. pupil ambition and drive.

Several recommendations accompanied this list. Ward's group suggested that teacher judgments may be useful but that teachers have a tendency to recognize as gifted the child who is attractive, well-behaved, and conforming. In using school records, the Ward group cautioned that poor teacher marks should not be used in excluding a child from programs for gifted children, if the child shows signs of giftedness on the basis of intelligence tests, standardized achievement tests, and the like. They also recommended that individual intelligence tests be used to check on the validity of group intelligence tests.

Abraham (1958) and DeHaan (1957) recognize still another means of identification---that of relationship in the peer group. Because they have intimate contacts quite removed from the classroom, children can provide information that is ordinarily unavailable to the teacher. Sociometric techniques point to leadership, intellectual ability, mechanical, and physical aptitude.

With the recognition and inclusion of the creative child among the gifted, new methods of identification have become necessary. Says Goldberg (1958, p. 40) "When we use conventional identification procedures only, young people able to produce novelty in the learning process as well as remembrance of course content will be missed." To this Torrance (1968, pp. 67-68) adds:

> If one uses only an intelligence test and thereby identifies the upper twenty percent as gifted, he would miss seventy percent of those who would be identified as falling in the top twenty percent on tests of creative thinking ability."

Torrance and Guilford have been leaders in the field of creativity testing. Their tests include tests for original thought, fluency and flexibility.

Characteristics of the Gifted Child

Whenever describing the characteristics of a population one is describing a compilation of many and distills this to a "typical" member. This is even more true when trying to give a description of the gifted child. Strang (1960, p. 215) reports:

> Individual differences are as widespread among the gifted as they are in any population. The IQs of children considered superior, range from 120 to more than 200 . . . Thus the gifted as a group are less homogeneous than are children with only average or lower IQs.

Therefore, it must be kept in mind that for whatever general statements are made, numerous exceptions can be found.

The largest proportion of research has been directed toward the high-performance gifted child. Classic studies were begun by Terman and Odin (1951) in 1921. From a study of 1500 gifted pupils ranging over a 25-year period, their study touched on just about every aspect of the gifted child's life. Terman and Oden (1951, p. 23-24) reported these data:

(a) the gifted child is a slightly better physical specimen,
(b) he is healthier,
(c) he is accelerated in grade placement,
(d) he is not equally as high in all school subjects,
(e) he learns to read quite early and he likes to read,
(f) he has many hobbies,
(g) his knowledge of play and games is two to three years advanced,

(h) he grows up to be well adjusted
(i) he tends to marry someone with high mental ability.

James Gallagher (1959) enumerates the outstanding intellectual abilities of gifted children related to their reasoning abilities. The intellectual abilities enjoyed by these children include the association of basic concepts, the critical evaluation of facts and arguments, the creation of new ideas, and the ability to analyze complex problems through an understanding of other situations, other times, and other persons.

Because he is better balanced emotionally, by and large, than the child of average or low ability, the gifted child is able to respond more appropriately to social situations (Strang, 1960). The gifted child has a built-in level of confidence, based on the reality of school grades and other comparison with peers (Drews, 1957). Strang (1960) notes, too, that highly intelligent children are usually endowed with more versatility, creativity, logic, empathy, and curiosity and are possessed of a broader range of interests than are their contemporaries.

The gifted respond to socialization pressures and accept adult values readily. They generally have good work habits and are highly competitive. Barbe (1963) and Smith (1962) too, describe their behavior as being more dominant, forceful, independent, and competitive than the average child and yet studies by a number of researchers

(Grace and Booth, 1958), (Grupe, 1961), (Johnson and Kirk, 1950) and (Miller, 1956) support the fact that they are popular with their peers and are able to identify correctly the social status of others and themselves better than the average child. Studies also agree that intellectually gifted children seek out companions who are somewhat older and closer in intellectual maturity to them than are their own contemporaries (Gold, 1965).

Gifted children come from a variety of backgrounds, however some similarities are found.

A disproportionately large number of first-born children appear among the gifted in all studies reported. In a study of bright adolescents done by Drews (1957) there appeared almost twice as many first-born in her research as would be accounted for by chance. Similar findings occurred in the studies of Terman (1925) and Cobb and Hollingsworth (1926).

"Research on the occupational background of parents of the gifted tend to relate high occupational status to the rearing of gifted offspring" (Gold, 1965, p. 27). The National Merit Corporation also "agrees that scholars come from hovels and palaces alike, but . . . finds that at least half of them come from homes which would be classified in the upper business or professional group" (Goldberg, 1958. p. 152). The professional group, comprising

3 per cent of the population supplies 30 per cent of the "scholars" (Holland and Stalnaker, 1958).

In studies relating sex differences to mental ability, researchers (Terman, 1925), (Miles,1958), (Hollingsworth, 1942) found that intellectual superiority is fairly well divided between the sexes, slightly favoring the boys. However, Miles (1954) questions procedure and the effect of secondary factors including the amount of practice or experience with different kinds of subject matter. Gallagher (1975) feels that social pressure and stereotyping may be negatively affecting the girls' performance.

A second group of gifted children have received less attention due to their small population. This is the group of extremely high ability children with IQs exceeding 180. According to test standardization data, such children should appear about once in one million cases. Researchers predicted (Hollingsworth, 1942) and observation confirmed (Terman and Oden, 1947) that social adjustment is one of these youngsters' most difficult tasks, They tend to be poor mixers and solitary children (Gallagher, 1975).

In recent years, a third group of children has received increased notice. This group is composed of highly creative children. The work of Getzels and Jackson (1962, pp 57-58) points out differences in goals and behaviors between the "highly intelligent" and "highly creative" students.

> The high creative group was less concerned with conventional vocation goals and more interest in so-called off-beat vocations (inventor, artist, disc jockey). Neither were they overly concerned with whether or not they possessed the character traits admired by teachers and parents. These highly creative students were more self-reliant and independent, and despite the fact that they scored significantly lower (127) in mean IQ scores than their brilliant classmates (150), they attained the same degree of academic achievement.

<u>Learning Styles of the Gifted Child</u>

In reviewing the research studies describing the characteristics of the gifted student, the rapidity with which they learn as well as their ability to master complex, profound material are faculties frequently cited. Virgil Ward (1961) summarizes other learning characteristics of superior children as follows:

1. accurate perception of social and natural situations, with insight into part-whole relationships;
2. independent learning of fact and principle;
3. superior powers of retention and recall;
4. sensitivity to inferences;
5. spontaneous elevation of immediate observations to higher abstractions planes;
6. analysis and organization of factors;
7. critical thinking about self, other persons, and situations.

In studying the preferred learning style of the gifted student, Rita Dunn and Gary Price (1980, pp. 34-35) found that:

> The gifted subjects preferred a formal design, did not need structure, were less responsible and more persistent than the nongifted group, preferred to learn through their tactile and kinesthetic senses, and indicated less preference than their classmates for using the auditory sense for learning.

Learning characteristics coupled with preferred learning style provide direction for method of instruction for these students. Guidelines suggested by Dunn and Price (1980) follow:

1. Since these gifted students are more persistent than nongifted students, there may be implication for how closely gifted students need to be supervised once they have been given an assignment they understand and are willing to complete.
2. Choices for completing assignments should provide options for creative selections and should help gifted students to understand that it is their uniqueness, rather than a lack of responsibility, that urges them to complete tasks in a variety of ways.
3. The preference of many of the gifted towards learning in a formal, rather than informal design, suggests they may achieve more easily in a traditional classroom than do nongifted students. However, since this preference was not chosen by all of the gifted students, gifted students should have an option of selecting an informal instructional environment.
4. Manipulative materials and active, "real-life" experiences should be available to the gifted students since they prefer to learn more through those modalities. In addition to books, they should be permitted to learn through resources such as learning circles, task cards, selector-boards and body games; lectures, discussions, and tapes are not the preferred methods for teaching gifted students based on this study.

Educational Needs of the Gifted Child

What should a program provide?

> A program for the gifted and talented provides multidimensional and appropriate learning experiences and environments which incorporate the academic, psychological and social needs of these students . . . A program assures each student of alternatives which teach, challenge, and expand his knowledge while simultaneously stressing the development of an independent learner who can continuously question, apply, and generate information.(Kaplan, 1974, p.

Gallagher (1975) explains the three characteristics essential to a gifted program. The first characteristic is content. The content should stress higher cognitive concepts and processes, "the greater complicity and higher levels of abstraction." Secondly, instructional strategies must be incorporated which go beyond basic absorption of knowledge to encourage development of higher level thinking processes. Lastly, the learning environment should be altered for at least a portion of the week allowing students of similar ability to work together.

The first two characteristics, content and process, are the elements of curriculum. In his book, Curriculum Planning For The Gifted, Dr. Fliegler emphasizes the importance of consideration of curriculum for the gifted.

> The future of the bright student is undeniably related to the decisions and opportunities provided for him by educators. His development can be guided to optimum levels, or it can be so misused that much of his potential will remain untapped. Upon the curriculum makers rests the prime responsibility of creating a productive climate to insure maximum growth"(Fliegler, 1961, p. 1).

In developing curriculum for the gifted, Kaplan feels that learning activities should be:

<u>Subject-Related</u>-related to something from which thinking and doing can be initiated.
<u>Process-Oriented</u>-empasize the development of thinking skills and processes rather than the mere acquisition of information.
<u>Doing-Centered</u>-focus on tasks which produce active involvement from the learner.
<u>Open-Ended Application</u>-allow for varied and personalized response.
<u>Student-Selected</u>-provide options for individual difference in need, preference, and capabilities.

Student-selected curriculum is a major focus in the Enrichment Triad Model, Joseph Renzulli's program design for gifted. Renzulli(1977) feels it essential to any gifted program that the students should have the "opportunity to pursue their own interest to whatever depth they so desire and to be allowed to pursue these interests in a manner that is consistent with their own preferred styles of learning." Additional support is given to this concept by Virgil Ward's (1961, p. 27) fundamental principles underlying differential education for the gifted, stating, "superior students should become acquainted with basic methods of inquiry within the various fields of knowledge."

In Providing Programs For The Gifted and Talented, Kaplan (1974,p.123) lists a variety of ways that content may be differentiated for the intellectually gifted child:

1. Accelerated or advanced content
2. Higher degree of complexity of content

3. Introduction of content beyond the prescribed curriculum
4. Student-selected content according to interest
5. Working with the abstract concepts in a content area
6. Level of resources used, use of resources beyond those reserved or designated for regular curriculum input
7. Type of resources available-use of information from multiple and varied resources; use of informational sources besides books.

The second characteristic, a change in instructional strategy or process, refers to the methods of thinking which receive emphasis. "Thinking skills can be classified according to the teacher/learning strategies of problem solving, creativity, inquiry and higher levels of cognitive operations" (Kaplan, 1974, p.94).

Renzulli (1977) sees process as the "path to learning", not a goal of learning and feels that process objectives are "things that 'just happen' in good learning situations."

Kaplan (1974,p.113) counters that the process of learning and thinking are "often of higher priority than the content of the subject" and that "it becomes vital that the teacher know and employ various teaching-learning models . . . in order to satisfy and stimulate the gifted and talented in a differentiated manner."

Guilford's Structure of the Intellect Model (1967) and Bloom's Taxonomy of Educational Objectives: Cognitive Domain (1956) are two of the models commonly associated with gifted programs.

> Guilford's Structure of the Intellect Model indicated three major dimensions of intellectual ability---operations, content, and products---that can be combined to form 120 defined intellectual ablities.
>
> Bloom's Taxonomy of Educational Objectives has six stages representing more complex kinds of thinking; knowledge, comprehension, application, analysis, synthesis and evaluation (Hill, 1969, pp. 25-27).

The models have been utilized in various ways in developing the curriculum of many programs for the gifted. Says Renzulli (1977, p. 8-9):

> I do not think that it would be presumptuous to state that gifted education has in many cases, taken the lead in promoting concern for the development of high level thought processes in general education. The rationale for redirecting our efforts away from an emphasis on content and toward the thinking and feeling processes is based on research studies which show that these processes are more widely applicable or transferable to new learning situations.

Morgan, Tennent and Gold (1980, p. 201) concur; "Certainly in designing a program for the gifted, considerations should be given to such cognitive models."

Included in the discussion of curriculum content and instructional strategies for the gifted should be needs of the highly creative child. Gallagher (1975) defines creativity as "the mental process where the expectation is that something new and original will be produced." This process is composed of four elements: "Sensing some kind of deficiency, formulating ideas or hypotheses, testing hypotheses and communicating results" (Torrance, 1962, p. 219).

Drews (1962) suggests conditions which facilitate creativity. The conditions include expectancy of original and superior work, encouragement, provisions of models in school and community, giving of awards, freedom to develop, and psychological acceptance of originality and superior performance.

Torrance(1962) puts emphasis on rewarding diverse contributions, developing minimum skills, utilizing opportunities, and developing values and purposes.

Williams (1972) suggests a model of thinking that includes the cognitive behaviors or strategies of:

Fluent Thinking----To think of the most; generation of a quantity; flow of thought; number of relevant responses
Flexible Thinking--To take different approaches; variety of kinds of ideas, detours in direction of thought.
Original Thinking--To think in novel or unique ways; unusual responses, clever ideas, production away from the obvious.
Elaboration--------To add on to; embellishing upon an idea, stretching or expanding upon things or ideas.

and the affective behaviors of:

Willingness to take risks---Courage to take a guess or function under conditions devoid of structure.
Preference for complexity---To seek thechallenge, to see gaps between how things are and how they could be; bring order out of chaos.
Curiosity------------------To want to be inquisitive; grope for new insights.
Imagination-----------------To have the power to visualize and build mental images; feel intuitively.

The third characteristic essential to a gifted program is a change of learning environment. Gallagher (1975) defines this as "either moving the youngster to a different setting or changing the nature of the setting in which he receives his instruction."

Grouping children for instruction has long been an accepted practice. Ability grouping makes possible many teaching and learning experiences which cannot be accomplished in the typical classroom. Ward (1962, p. 69) goes on to elaborate that:

> Grouping students according to selective ability patterns for all or part of their instruction is a practice which parallels the gifted student's ability to function in the classroom situation at intellectual levels not attainable by those of average ability. Only through ability grouping can the gifted student engage in stimulating discourse---discussion and debate---with his intellectual peers. This needed high level engagement of like minds cannot be carried on effectively or efficiently in the typically heterogeneous classroom.

Ability grouping tends to reduce the range of diversity of talent within a class and to increase the proportion of educational experiences and instruction in which a pupil may participate at approximately his own level of comprehension and problem-solving ability. Mirman (1971, p. 223) continues, "I am convinced that children learn about half from the teacher and more than half from each other."

However, many feel that the teacher is yet another important element of a gifted program. In studying elements of importance to programs for gifted students, Renzulli (1968) found that in addition to curriculum and student selection, that experts agreed that <u>the teacher</u> was a crucial element to the success of the program.

What characteristics typify a successful teacher of the gifted? In his research, Bishop (1968, p. 49) found that successful teachers were "mature, experienced, and superior intellectually. (They) were more interested in literature, the arts, and culture, had high personal achievement needs, and were seeking their own intellectual growth through teaching. They tended to be more student-centered, stimulating in the classroom and not surprisingly, were supportive of special educational programs for the gifted."

Bruch and Torrance (1972) felt that the teachers of the creatively gifted should:

1. Care about their pupils; be interested in assisting and guiding young people.
2. Be honest and willing to admit their mistakes.
3. Trust their pupils and have confidence in their ability to act responsibly.

Dr. Elizabeth Drews (1972, pp. 89-90) feels that teachers who have the greatest appeal to the gifted are those who combine two characteristics.

> First of all, they must know a great deal, and their knowledge should extend to the far reaches of their subjects.
>
> Secondly, they must have a superlative ability to relate to others . . . The creative teacher is one who has faith in young people and loves students, who is cooperative and kind, who is democratic and considerate.

What is this person's role in teaching the gifted? Renzulli (1977) feels that the primary role of a teacher in the program for gifted and talented students will be to provide each student with assistance in (1) identifying and structuring realistic solvable problems that are consistent with the student's interest (2) acquiring the necessary methodological resources and investigating skills that are necessary for solving these particular problems and (3) finding appropriate outlets for student products. Dressel and Grabow (1958) add that the teacher should expand and reinforce (1) Arousal of students' curiosity (2) Insist that work be redone until it is of high caliber and (3) Insist that students engage in activities requiring initiative and self-reliance.

With attention to content and process, learning environment and choice of teacher, what outcomes do we wish for the gifted child?

Gallagher (1975) would hope that such a student would have:

1. The ability to associate and interrelate concepts.
2. The ability to evaluate facts and arguments critically.
3. The ability to create new ideas and originate new lines of thought.
4. The ability to reason through complex problems.
5. The ability to understand other situations, other times, and other people, to be less bound by one's own peculiar environmental surroundings.

Renzulli (1977), Gallagher (1975), Goodlad (1964) and Beberman (1957) support the need for the learner to approach the subject matter as the specialist approaches it. Says Renzulli (1977, p.14), "Gifted kids can unquestionably function in the manner of true inquirers."

The intent of a gifted program is summarized by Morgan, Tennant and Gold (1980, p. 28):

> The focus of curriculum development, if it is to be a lasting program must go beyond providing additional content for gifted students to emphasize improving and challenging their thinking skills, encouraging their creative talents, allowing them opportunities to be producers of knowledge as well as consumers, and helping them evaluate and upgrade their products.

Math and the Gifted Child

Special Characteristics of the Pupil Gifted In Mathematics

The elementary child gifted in mathematics has a number of characteristics which relate to his curriculum needs and appropriate learning environment.

Gold (1965, p. 277) reports that the most commonly observed characteristic of gifted children is their "precocity in handling abstractions inherent in mathematics." One may expect, therefore, earlier readiness of many bright children for mathematics at all levels. This presumption is validated with Grossnickle's (1961, p. 59-60) list of identifiable qualities of the gifted pupil:

1. A high I.Q., usually above 120, as measured on standard mental tests.
2. Extraordinary memory to remember mathematical concepts and principles.
3. Ability to generalize about a quantitative situation from a few specific quantitative situations.
4. Resourcefulness in analyzing arithmetic data and process.
5. Keen quantitative insight into number relationships as expressed in verbal problems.
6. An unusually advanced knowledge of mathematics.

Keating (1972, p. 3-7) in his study of mathematically precocious youth identified several additional characteristics:

1. There is a slight favoring toward the second-born over only children and the other rankings of birth order. (McGurk and Lewis, 1972)
2. The higher the educational level of the parents the higher the mean score of their children.
3. Sex difference favors males.
4. Active interest in finding out things, discovering things, learning things.

Ernest Haggard (1957) investigated the emotional behavior of the gifted pupils in all areas of the curriculum. He found that pupils who excelled in arithmetic were most stable. He concluded (p. 37):

> The arithmetic achievers had by far the best-developed and the healthiest egos; both in relation to their own emotions and mental processes and in their greater maturity in dealing with the outside world of people and things.

The high arithmetic achievers could express their feelings freely and without guilt or anxiety. They were emotionally controlled and flexible and were capable of integrating their emotions, thoughts, and actions. Similarly, their intellectual processes tended to be spontaneous, flexible, assertive and creative. Of the subgroups studied, the arithmetic achiever showed the most independence of thought, were best at maintaining contact with reality and at avoiding being bound by its constraints, and could function most effectively in the realm of abstract symbols.

In their relation with authority figures and peers, they were more assertive, independent, and self-confident than were the children in the other subgroups.

Keating's (1972) findings agree in regard to emotional stability but he identified by 7th or 8th-grade an additional characteristic of this research group; a disillusionment with school in particular and academic pursuits in general. Says Keating (Keating and Stanley, 1972, p. 3-7):

> In the light of the tentatively confirmed expectation of lack of enthusiasm for school, it is indeed difficult to value highly some of the frequently heard arguments against significant restructuring of these students' educations because of potential harm to their social and emotional development. A number of studies suggest strongly that such a fear is unfounded (e.g. Pressey 1944; Oden, 1968). But the obverse of that concern is one voiced much less often: What is the potential harm to the social and emotional development of these students if they are required to remain in an unstimulating, hence, frustrating environment? It may be great.

Special Problems

The gifted mathematics program has suffered from several factors on the educational scene.

Experts surveyed for the purpose of this paper speak of the "unimaginative approach to arithmetic in the elementary school that (is) confined to routine drill on computation" (Gold, 1965, p. 272).

> Almost without exception, mathematics programs at the elementary school level are heavily computationally oriented. The many algorithms associated with computation generally are of little challenge to gifted pupils and provide little opportunity for interesting and challenging activities (Hersberger and Wheatley, 1980, p. 37).

This standard instruction has reduced mathematics to a subject in which only fairly low level cognitive skills are required. Even when instruction deals with problem-solving, it is in a way which requires the classification of a problem and the application of a standard method, usually learned by rote. Students who are successful with this method are very good at applying routine methods to routine problems. However, they are at a loss to deal with problems that do not easily fit one of the routines they have learned (Wavrick, 1980, p. 171).

> The preference for product over process as the central focus of gifted students is quite disturbing considering the abilities of the students. These persons are the individuals that society expects to take the lead in solving difficult problems of a highly divergent nature; problems which require excellence in thinking and learning skills (Hersberger and Wheatley, 1980, p. 37).

A math program whose major emphasis is drill and computation not only is lacking in imagination in content but presents a false impression of the field of mathematics and role of the mathematician to the learner. Says Gallagher (1975, p. 96):

> One of the first concerns of any serious program of mathematics introduced at the lower grades is to change the image in the minds of students and teachers of what a mathematician is and what he does. Too often the common portrait is of a person who gains great pleasure from manipulating numbers and who delights in the knowledge that all the answers have already been discovered. He has only to present the proper formulas and calculate the answer.

> Mathematics is too often viewed as an exact science where all the answers have been found and the student only has to learn the correct method to find them. Teachers who force their students to memorize the five-step method of an operation or who mark a problem incorrect because it does not follow the method used in the book, can kill the imagination needed to solve future problems (Jensen, 1976, p. 210).

It is important that children appreciate the creativity that can be a part of math, searching for alternate solutions, increasing their problem-solving abilities and recognizing the learning process as well as finding the "correct" answer.

The lock-step method of delivering math instruction is yet another concern. Gifted and talented children are typically two to five grade levels above their actual grade placement and if working on the same material as the rest of the class, are in essence, reviewing material learned two to three years previously. Stanley (1977, p. 85) has described the possible consequences of such a situation:

> He or she can daydream, be excessively meticulous in order to get perfect grades, harass the teacher, show off knowledge arrogantly in the class, or be truant. There is, however, no suitable way to while away the class hours when one already know much of the material and can learn the rest almost instantaneously as it is first presented. Boredom, frustration, and habits of gross inattention are almost sure to result.

Added to this is a lack of motivation and decline in attitude toward mathematics. The studies of Wheatly, et al (1979) have shown this to be true in Grade 4 to Grade 6. "If gifted students study mathematics from on-grade texts negative attitudes are likely to develop with little motivation for studying mathematics. (Wheatley, Hersberger, 1980, p. 37).

A final concern deals specifically with the teacher of mathematics. Some teachers, by virtue of their temperament or due to inadequate training or background, find it difficult to work with the gifted student.

This child represents a threat to their authority and as a result, the teacher is quick to squelch the student's ideas and resents his divergent thinking. Says Mirman (1971, pp. 218-219):

> This (inadequacy) is particularly crucial in the area of mathematics, where the average classroom teacher is totally unable to cope with, or provide for, the gifted child's conceptual strength and divergent approaches to the solution of problems. Such inadequacy can readily 'turn off' a youngster who has the potential to make a significant contribution or breakthrough at a later date.

The Arithmetic Program

An examination of problems in our present manner of dealing with the gifted elementary math student gives direction to planning a program to meet the child's needs.

Foremost a "mathematical program for gifted children should provide learning experiences that stimulate and sustain their interest, ability and imagination" (Education of the Gifted and Talented, 1972, p. 128).

Enrichment of the curriculum is the means most often suggested to meet the needs of the gifted student. But as Goldberg (1958, pp. 156-157) has written:

> Enrichment, like the weather, is something everybody talks about but few do anything about. We really don't know what enrichment is. Does it mean accelerated coverage of a standard course of study followed by advanced content in a given discipline, such as completing algebra in the eighth year and thus, in the twelfth year, having time for a course in calculus? Or does it mean dipping more deeply or extensively in selected areas . . . ? Or does it mean increased independent and creative work in some field of individual interest?

Perhaps the word enrichment is a misnomer; perhaps what is needed is not embellishment of existing course content but <u>different</u> content.

What are the specific content components that should be addressed in a curriculum for the mathematically gifted?

Students with high mathematics aptitudes should receive strong content-based programming in mathematics concepts and systems (Stanley et al, 1974) (Durden, 1979).

The research based on this approach as a way to begin conceptualizing a curriculum is compelling. "There is little richness to a curriculum that does not have a strong content base or focus" (VanTassel-Baska, 1981). "The structure of mathematics should be stressed at all levels. Topics and relationships of endurance should be given concentrated attention" (Scott, 1966, p. 97).

The curriculum development should be sequential in nature and accomplished in various stages. It should demonstrate a well-planned sequential development of increasingly difficult material, and include materials and activities which will provide for the development of skills in group and individual problem-solving and decision-making (Van Tassel-Baska, 1981).

Though the gifted child should be proficient in computational skills, this should not be the major emphasis of his curriculum. "The child should be able to perform basic operations, to understand their rationale, and to compute without making errors. Beyond this point, further drill brings diminishing returns"(Gold, 1965, p. 275).

These pupils need to be challenged with interesting problems requiring originality of thought (Hershberger, Wheatley, 1980, p. 37). "Giving four solutions to one problem is more challenging than solving four different problems of similar nature. The quantity of problems that the pupil solves is not as essential as the quality of thinking displayed by the pupil in problem solving" (Grossnickle, 1961, p. 80).

A growing recognition of the importance of creativity has led to the development of alternate forms of instruction, particularly for gifted students, which attempt to deal with higher order cognitive processes. "The intellectual processes of cognition and comprehension, knowledge or memory, divergent and convergent application, analysis and synthesis and evaluation as well as creativity must be considered indispensable to the mathematics program" (Clendening, 1980, p. 128).

Thus in designing a program suitable for the gifted child, it should be recognized that "growth of understanding is dependent upon concept exploration through challenging apparatus and concrete materials and cannot be restricted to mere symbolic manipulation" (Scott, 1966, p. 97).

It is necessary that opportunities are abundant for the student to explore thought-provocative material which leads to his discovery of certain mathematical principles and concepts (Bruner, 1966), (Gallagher, 1975). As Grossnickle (1961, p. 79) points out, the basic principles of arithmetic are not a body of subject-matter to be memorized. Learning the principles by rote and having examples identified will not guarantee understanding. "The way the pupil learns and discovers these principles is the important phase of this work."

Gold (1965, p. 274) summarizes the nature of the curriculum component in writing:

> The gifted child's superiority in abstract operations make the more recent approach in organizing the mathematics curriculum especially relevant. Emphasis is placed on the mental role of discovery by children of the meaning, nature and structure of the number system.

Yet another important objective in developing curriculum is "an appreciation for and interest in arithmetic as an indispensable language and functional resource in industry, commerce and the sciences." (Shane, 1958, pp. 229-230). There should be concentration on the interrelationships between and among bodies of knowledge (Van Tassel-Baska, 1981). Says Scott (in Gallagher, 1975, pp. 97-98), "Practical application of concepts, particularly those applications drawn from the natural sciences, are valuable to reinforcement and retention."

Says Gallagher (1975, p. 95), "The hiearchical nature of mathematical knowledge cries out for some means to individualize the instruction so that the bright student can continue to climb the abstract pyramid as fast as he or she is able . . . ."

Adds Gold (1965, p. 277),

> While readiness programs in reading and arithmetic are indeed important, the gifted child who enters school with experience in both these areas is likely to be discouraged and disappointed if his appetite to learn new things is ignored. By and large, the child who is ready for multiplication, division, fractions, ratios or decimals ought not to be held up simply because a given topic belongs to the next grade.

This does not mean, however, that the goal is to move the student through the <u>same</u> curriculum at a faster pace. Instead, the content needs to be rearranged and restructured around a conceptual framework rather than be taught in minute sections (Van Tassel-Baska, 1981), (Grossnickle, 1961), (Gallagher, 1975). "Good content acceleration allows for faster pacing of well-organized, compressed, and appropriate learning materials for the gifted" (Van Tassel-Baska, 1981, p. 2).

The grouping of students, classroom atmosphere and teacher qualities are final components of the program.

There is universal agreement in the research that regardless of the plan of segregation offered, some form of ability grouping is desirable. Children in a given class or grade show a wide range of ability in each phase of instruction. Furthermore, these variations generally increase from the lower to the upper elementary. One research study by Bruecker and Grossnickle (1959) based on results from the Stanford Achievement Test, administered in a small school system, reported ranges of 4.8 years for third-graders in both Arithmetic Reasoning and Arithmetic Computation. At the sixth-grade level, reported ranges were 6.0 years in Arithmetic Reasoning and 6.1 years in Arithmetic Computation (in Kramer, 1970, p. 331).

Although ability grouping will not create homogeneous groups, it will make the group less heterogeneous than groups formed at random (Grossnickle, 1961).

"Gifted children need the challenge and stimulation of being together for at least part of every school day. Their expectation levels need to be set high enough to stretch their potential ability to realize them"(Van Tassel-Baska, 1981).

The environment of the classroom is important as well.

As stated earlier, exploration is essential to the arithmetic program. Says Jensen (1976, p. 212), "If children are encouraged to explore a variety of methods, rather than forced to memorize one set method, not only will their knowledge of mathematical processes increase, but their enjoyment also will grow." In addition, "during the exploratory phase of a cycle a low-pressure environment is mandatory" (Wavrick, 1980, p. 170).

Secondly, a successful class for the gifted should be more individualized than most school mathematics programs.

This does not imply lack of guidance but rather the development of self-realization and increased student input as to tasks and investigations to be pursued.

> When students participate in a system where choices are available, a greater degree of responsibility is felt by the learner. Additionally, students become more proficient 'problem-sensers' as a result of determining exactly what constitutes a 'problem', and then working towards a solution (Hersberger, 1980, p. 38).

Of final importance to a successful arithmetic program is the teacher. Research reveals that the teacher, rather than the type of organization, was found to be significant in changes of interest on the part of pupils.

> If children gifted in mathematics are to enjoy opportunities in newer areas, they will need teachers who are qualified to lead them in the study of new material (Gold, 1965, p. 279).

It is necessary that the teacher must feel confident of his or her abilities and possess a healthy self-concept. It is highly probable that the teacher's limits of mathematical knowledge will be taxed or exceeded when working with the gifted. "It is essential that the teacher not feel threatened in this situation but can respond adequately, and in a manner which does not undermine the teacher-student relationship" (Hersberger and Wheatley, 1980, p. 39).

One of the main roles of the teacher is to structure the learning environment so that it is conducive to the development of the student's thinking ability (Wavrick,1980.p.169).

The teacher must act as a facilitator or resource manager, responding with appropriate materials when known, giving some direction to the child's inquiries, giving instruction in how the student can search for the necessary solutions amd materials (Hersberger, 1980, (Jensen, 1976).

Finally, "the teacher provides the support allowing a student to ultimately teach him or herself how to be creative in mathematics" (Wavrick, 1980, p. 171).

Summary and Conclusions

Historically, specific attention to the needs of the child gifted in mathematics has correlated closely with the history of gifted education in general in the United States.

Education of the gifted was given sporadic attention until the late 1950's when the Russian launching of Sputnik precipated a new interest in education and another rediscovery of the gifted and talented. Unfortunately, the interest was somewhat misdirected with America's educational system declared responsible for the failure to be first in space exploration. A widespread reform was called for. Suddenly, our gifted became a "commodity on the marketplace", a resource to be used to close the scientific "gap" between the U.S. and Russia (Houts, 1972).

Math and science education became areas on which reform movements focused. Federal funds were given to groups of educators to help formulate a new curriculum (Seattle Times, 1982).

But teachers and parents alike, did not like the "new math" and with the sixties came concern for civil rights and the disadvantaged. The gifted, we were sure, could fend for themselves.

After nearly a decade of waning attention, today's educators seem more prepared to revive the old enthusiasm for excellence that flashed for several years after Sputnik.

However, a great many changes have taken place in our society since the satellite launching. The kinds of excellence that we think we need today appear quite different from those thought necessary at that time. Our curriculum planning is more long-range in view. We are no longer grooming students to just service the critical requirements of our nation; "producing a breed of technocrats who possess only a pragmatic view of how their talents should be used" (Tannenbaum, 1972, p. 23).

We are now trying to meet the actual needs and interests of the gifted child, providing curriculum which encourages higher level thinking, which encourages problem-solving. We want students who are able to understand themselves, and are able to relate to others and their needs as well.

We have just stepped into the Computer Age. A decade ago, hand-held calculators were an expensive luxury. Today, parents vie with their children for computer time and programming is fast becoming a basic skill along with reading and writing. With computers playing an important part of so many academic fields and professions, the logical language of mathematics and those skilled in using it will be receiving ever-increasing attention in education once again.

Chapter 3

PROCEDURES

A teacher's manual of sample lessons for math enrichment has been developed for use by fourth, fifth, and sixth-grade students. These activities are planned for students of above-average ability to take place in a small-group setting.

A need for this type of material was determined from a variety of sources. The author has taught in Bellevue School District's program for gifted elementary students for ten years, has participated in Central Washington University's summer course in curriculum development for able learners since 1973, has taught several courses and conducted workshops for Seattle Pacific University in the area of curriculum development and during the summers of 1978, 1979 was a member of the State Writing Team, preparing sample curriculum units. In working with educators throughout the state, there is a continued request for materials and activities for the student who has completed activities within the basal text.

Currently, I am serving on a Bellevue District Task Force. The focus of concern? What to do with 4th, 5th, and 6th-graders who have mastered the material found in their grade-level text.

Though the math lessons in this manual treat a variety of subjects, all lessons share several features.

The same lesson-plan format is used for each activity. Learner's objectives are stated using Bloom's Cognitive Taxonomy. This model of instruction was chosen because of its familiarity to many educators. Bloom's Taxonomy has been incorporated by Madeline Hunter in her Instructional Theory Into Practice or I.T.I.P. I.T.I.P. teaching strategies have received emphasis in workshops and classes statewide.

Key materials are noted and the activities are sequentially outlined where possible.

Because enrichment activities can easily become an unrelated conglomerate, in assembling the lessons, "pattern" was chosen as a focus. Pattern is variously defined as an ideal person or thing worthy of imitation, as a model for making things; as an arrangement of form in design. Recent work in the psychology of perception indicates that recognition of patterns plays an important role in daily living and constitutes a large part of the way we organize and understand our world. A child's perception of patterns begins early in life, and patterns and the perceptions of patterns are basic to learning. Proceeding from the recognition of a pattern in a series of occurences to the prediction of an outcome or the formulation of a more general rule is a common method of learning (Thornton, 1977). Thus, activities that encourage the discovery of patterns in mathematics can be a fruitful area for teachers to explore in their classrooms. This concept is supported by the Educational Policies Commission. The" . . . abilities essential to perceiving patterns among abstract data are also essential to analyzing, deducting, or inferring. Such abilities may be developed through mathematics . . ." (Clendening and Davies, 1980, p. 38).

The teaching methods and content of the units place an emphasis in the use of higher level thinking skills, greater-than-normal depth of study, inductive learning and an early introduction of difficult course content. Wherever possible, manipulative activities are offered to encourage the learner to explore and recognize that math can go beyond paper and pencil computation.

These units are intended for use by teachers of able learners as well as by teachers who are interested in a format for writing curricula differentiated for the gifted.

Chapter 4

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The intent of this project is to assist the classroom teacher in meeting the needs of the intermediate student gifted in math. This student has mastered concepts covered in the basal text and requires materials which:

1. reinforce basic skills
2. provide opportunities for exploration
3. relate math to other academic areas and
4. provide stimulation.

Each activity in this manual is related to the focus of "pattern". A curriculum based on patterns involves providing learners experiences which require a <u>discovery</u> of the part embedded in a whole, <u>removing</u> a pattern-part from the whole and finally, a <u>restructuring</u> of the pattern within the whole. To perform these general tasks successfully, the learner must:

a. recognize the regularity of the pattern
b. be able to repeat the pattern
c. describe the regularity either verbally, quantitatively or visually
d. predict the next in sequence of the pattern, and lastly
e. alter the pattern to create a new pattern from the old.

The last two tasks, to <u>predict</u> and to <u>alter</u>, are areas to be emphasized in working with the gifted student. This emphasis on higher level thinking of analysis and synthesis differentiates gifted curriculum from that usually taught in a regular classroom.

Too often "enrichment" for the gifted is "busy work"; more of the same work at the same level is assigned. The concept of enrichment for these students should instead include the idea of increasing the depth of coverage and the degree of challenge of the work.

Due to decreased funding and an increase in mandates, today's elementary classroom teacher is working to meet an increasing number of needs of an increasing number of students with a reduced support staff. By ability-grouping students, as described in this project, and assigning them to a teacher with a positive and enthusiastic attitude toward the teaching of math, classroom teachers could devote more of their planning to the needs of one group, rather than to two or three ability groups.

The students would be stimulated and challenged by other students of like ability. Ability-grouped, their expectation levels could be set high enough to stretch their potential ability to realize them.

Even in districts such as Bellevue, which provide special programs for their elementary gifted students, there is a continuing need for extending and enriching math materials. Each of the four Enrichment Centers in Bellevue can serve only sixty students. Students must be advanced in both language arts <u>and</u> math to qualify for admittance. Students strong in just one of these areas will not qualify.

Because some students must transfer from their home school to attend the Enrichment Center, parents may decide against entering a qualified child in the program.

The Enrichment Center is a multicurricular program. Children attending the Center spend an average of one hour there, daily. Only a part of that time is devoted to math. Five hours remain in their school day. Most of those five hours are devoted to basic skills, including math.

Thus math materials are needed for the gifted child choosing not to attend a special program; for the child with above-average skills specific to the area of math; and for the child attending a Center who studies math in his regular classroom.

Finally, a project such as this can serve as an example of how curriculum is differentiated to meet the gifted child's needs. Activities should promote the higher level thinking skills of analysis, synthesis and evaluation. The student should be encouraged to explore and investigate, to look for interrelationships between and among bodies of knowledge. The content should offer appropriate challenge, earlier introduction to difficult coursework and greater than normal depth of study.

It must be kept in mind that this project is just a beginning. A focus and format for planning and teaching the curriculum is given. A teacher choosing to implement this program would need to inventory student abilities and interests and aided with the bibliography completing the manual, continue to plan and create challenging curriculum which allows for both pacing and depth.

BIBLIOGRAPHY

1. Abraham, Willard, Common Sense About Gifted Children, New York: Harper, 1958.

2. "After 'New-Math', A Shift to Problem Solving by Curriculum Concepts," The Seattle Times, Tuesday, September 28, 1982, p. B2.

3. Barbe, Walter, "Occupational Adjustments of the mentally gifted," Educating the Academically Able: A Book of Readings, eds. Lester D. and Alice Crow, New York: McKay, 1963.

4. Bishop, William E., "Successful Teachers Of The Gifted," Exceptional Children, 34, 1968.

5. Bloom, Benjamin S., ed., Taxonomy of Educational Objectives: The Classification of Educational Goals: Handbook I; Cognitive Domain, New York: David McKay Co., 1956.

6. Bruch, Catherine B. and E.P. Torrance, "Reaching The Creatively Gifted," The National Elementary Principal, Vol. LI, February, 1972, pp. 69-75.

7. Clendening, Corinne P. and Ruth Ann Davies, Creating Programs For The Gifted, New York: R.R. Bowker Company, 1980.

8. Curriculum For The Gifted and Talented, Council for Exceptional Children, Washington C.C.: U.S. Office of Gifted and Talented, 1978.

9. DeHaan, Robert and Robert Havighurst, Educating Gifted Children, Chicago: University of Chicago Press, 1961.

10. DeHaan, Robert F., "Identifying Gifted Children," School Review, 65, 1957.

11. Drews, Elizabeth M., "Leading Out and Letting Be," Today's Education, Vol. 65, No. 1, January/February, 1976.

12. Dressel, P.L. and J.M. Grabow, "The Gifted Evaluate Their High School Experience," Exceptional Children, 24, May, 1958, pp. 394-396.

13. Dunn, Rita S. and Gary E. Price, "The Learning Characteristics of Gifted Students," Gifted Child Quarterly, 24, Winter, 1980, pp. 33-36.

14. Durden, W., "The John Hopkins Program for Verbally Gifted Youth," Roeper Review, 2,(3), 1980, pp. 34-37.

15. Durr, William K., The Gifted Student, New York: Oxford Press, 1964.

16. Education of the Gifted and Talented, Report to Congress by the U.S. Commissioner of Education and background papers submitted to the U.S. Office of Education, Washington D.C.: U.S. Government Printing Office, 1972.

17. Fliegler, Louis A., Curriculum Planning For The Gifted, Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1961.

18. Gallagher, J.J., "Social status of children related to intelligence, propinquity, and social perception," Elementary School Journal, 1958, pp. 225-231.

19. ____________, Teaching The Gifted Child, Boston: Allyn and Bacon, Inc., 1975.

20. Getzels, J.W. and P.W. Jackson, Creativity and Intelligence, New York: Willey, 1962.

21. Gold, Milton J., Education Of The Intellectually Gifted, Columbus: Charles E. Merrill, 1965, pp. 254-282.

22. Goldberg, Miriam L., "Recent Research on the Talented," Teachers College Record, Vol. 60, December 1958, pp. 150-163.

23. ____________, "Research on the Gifted," The Teachers College Record, 1958.

24. Guilford, J.P., The Nature Of Human Intelligence, New York: McGraw-Hill Book Company, 1967.

25. Grace, H.A. and N.L. Booth, "Is the gifted child a social isolate?," Peabody Journal of Education, 35, 1958, pp. 195-196.

26. Grossnickle, Foster E., "The Arithmetic Program," in Curriculum Planning For The Gifted, L.A. Fliegler, ed., New Jersey: Prentice-Hall, Inc., 1961, pp. 58-86.

27. ____________, and Leo J. Brueckner, Discovering Meanings in Arithmetic, Philadelphia: John C. Winston Company, 1959.

28. Guilford, J.P., The Nature of Human Intelligence, New York: McGraw Hill Book Co., 1967.

29. Haggard, Ernest A., "Socialization, Personality, and Academic Achievement in Gifted Children," The School Review, Vol. 45, Winter, 1957, p. 379.

30. Hersberger, James and Grayson Wheatley, A Proposed Model for a Gifted Elementary School Mathematics Program," Gifted Child Quarterly, 24, Winter, 1980, pp. 37-40.

31. Hill, Mary Broderick, Enrichment: Programs for the Intellectually Gifted Pupils, California State Department of Education, 1969.

32. Hollingsworth, L., Children Above 180 I.Q., New York: World Book, 1942.

33. Houts, Paul L., ed., The National Elementary Principal, Vol LI, No. 5, February, 1972, p. 5.

34. Jensen, Linda R., "Using creativity in elementary school mathematics," The Arithmetic Teacher, Vol. 23, March 1976, pp. 210-214.

35. Kaplan, Sandra N., Providing Programs For The Gifted And Talented: A Handbook, Ventura, California: Office of the Ventura County Superintendent of Schools, June, 1974.

36. Keating, D.P. and J.C. Stanley, "Extreme measures for the exceptionally gifted in mathematics and science," Educational Researcher, 1 (9), 1972, pp. 3-7.

37. Leese, Joseph and L.A. Fliegler, "Problems and Practices," in L.A. Fliegler, ed., Curriculum Planning For The Gifted, Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1961, pp. 1-33.

38. Love, Harold D., Educating Exeptional Children In Regular Classrooms, Illinois: Charles C. Thomas, 1972, pp. 35-62.

39. McGurk, H. and M. Lewis, "Birth Order: A Phenomenon in Search of An Explanation," Research Bulletin in Stanley and Keating, Mathematical Talent, Baltimore: John Hopkins University Press, 1974, p. 31.

40. Miles, Catherine C., "Gifted Children," Manual of Child Psychology, New York: John Wiley, 1954, pp. 984-1063.

41. Miller, R.V., "Social Status and Socioempathetic Differences Among Mentally Superior, Mentally Typical and Mentally Retarded Children," Exceptional Children, 23, 1956, pp. 114-119.

42. Mirman, Norman, "Education of the Gifted in the 70's," Gifted Child Quarterly, 15, Autumn, 1971, pp. 217-224.

43. Morgan, Harry J., Carolyn G. Tennant and Milton J. Gold, Elementary And Secondary Level Programs For The Gifted And Talented, New York and London: Teachers College and Columbia University, 1980.

44. Nazarro, Jean N., Identification of the Gifted and Talented, Washington D.C.: U.S. Office of Gifted and Talented, 1979.

45. Ramos, Suzanne, "Don't Forget The Gifted," Teacher, Vol. 93, December, 1975, pp. 46-48.

46. Renzulli, Joseph S., The Enrichment Triad Model: A Guide For Developing Defensible Programs For The Gifted And Talented, Mansfield Center, Connecticut: Creative Learning Press, 1977.

47. ____________, "Identifying Key Features In Programs for the Gifted," Exceptional Children, 1968, pp. 217-221.

48. ____________, "What Makes Giftedness?," Phi Delta Kappan, November, 1978, pp. 180-184, 261.

49. Scott, L., "Trends In Elementary School Mathematics," in Gallagher, Teaching The Gifted, Boston: Allyn and Bacon, Inc., 1975, p. 97.

50. Smith, R.M., "The Relationship of Creativity to Social Class," Unitied States Office of Education Cooperative Research Project No. 2250, Pittsburgh: University of Pittsburgh, 1965.

51. Stanley, J.C., W.C. George and C.H. Solano, eds., _The Gifted and The Creative: A Fifty-Year Perspective_, Baltimore, Mds.: John Hopkins University Press, 1974.

52. Stanley, Julian C., Daniel P. Keating, and Lynn H. Fox, _Mathematical Talent_, Baltimore: John Hopkins University Press, 1974.

53. Strang, Ruth M., _Helping Your Gifted Child_, New York: E.P. Dutton and Company, Inc., 1960.

54. Tannenbaum, Abraham J., "A Backward and Forward Glance at the Gifted," _The National Elementary Principal_, February, Vol. LI, No. 5, 1972, pp. 14-23.

55. Terman, Louis M., "Genetic studies of genius," Vol. 4, _The Gifted Child Grows Up_, Stanford, Calif.: Stanford University Press, 1947.

56. ____________, _Mental and Physical Traits of a Thousand Gifted Children_, Stanford, Calif.: Stanford University Press, 1925.

57. ____________, and Melita Oden, "The Stanford Studies of the Gifted," ed. Paul Witty, _The Gifted Child_, Boston: Heath, 1951.

58. Torrance, E. Paul, "Creativity and Its Educational Implications For The Gifted," _The Gifted Child Quarterly_, Vol. 12, 1961, pp. 67-68.

59. Van Tassel-Baska, Joyce, "Appropriate Curriculum for the Gifted," mimeograph of speech given at Northwest Gifted Child Conference, Portland, Oregon, November, 1981.

60. Ward, Virgil, _Educating The Gifted---An Axiomatic Approach_, Columbus: Charles Merrill, 1961.

61. ____________, _The Gifted Student: A Manual For Program Improvement_, Atlanta: Southern Regional Education Board, 1962.

62. ____________, "Program Organization and Implementation," _Psychology and Education of the Gifted_, ed. Walter B. Barbe and Joseph S. Renzulli, New York: Irvington Publishers, Inc., 1975.

63. Wavrick, John J., "Mathematics Education For The Gifted Elementary School Students," _Gifted Child Quarterly_, 24, Fall, 1980, pp. 169-173.

64. Williams, Frank E., _A Total Creativity Program For Individualizing and Humanizing the Learning Process_: Vol. 2: Encouraging Creative Potential, New Jersey: Educational Technology Publications, 1972.

65. Witty, P.A., "Who Are The Gifted?," in N.B. Henry, ed., _Education of the Gifted_, Fifty-seventh Yearbook of the National Society for the Study of Education, Part II, Chicago: University of Chicago Press, 1958, p. 62.

66. ____________, ed., _The Gifted Child_, American Association for Gifted Children, Boston: D.C. Heath and Co., 1951.

APPENDIX

PATTERN AND MATHEMATICS
MATH ENRICHMENT ACTIVITIES FOR GIFTED FOURTH, FIFTH AND SIXTH GRADE CHILDREN

Prepared by

Janet K. Zuber

Bellevue School District

TABLE OF CONTENTS

INTRODUCTION

This manual provides a variety of math enrichment activities for fourth, fifth and sixth graders possessing advanced math skills. The activities require teacher direction and are suitable for group interaction.

With such a variety of needs vying for teacher attention, it is difficult to meet individual needs of the gifted math student in the heterogeneous classroom. Too often, such students view math as a discrete paper and pencil, computational pursuit, unrelated to other academic areas. Too much time is spent reviewing skills rather than extending activities and acquiring new skills.

Individualized programs frequently mean working alone. Without stimulation from peers and the teacher, student interest lags.

The gifted child requires lessons designed to reinforce and extend basic skills and to explore new concepts. They need the stimulation and challenge of working with other children of like abilities.

One way of meeting these needs is to group intermediate children with exceptional math abilities,from different classes and grade-levels, for a part of their school day. This would be a positive approach because lessons could be differentiated for greater depth and faster pacing; teacher and student stimulation would be possible; and limited to one or two math groups, teachers could reduce their number of preparations.

All lessons in this manual provide math experiences that go beyond paper and pencil computation. Many provide experiences with manipulative materials. Open-ended questions are asked and the learner is expected to work as a mathematician, making comparisons and discoveries, deriving rules and formulas.

Pattern is the theme or focus for all units. Mathematical patterns occur in every aspect of number relationships; the arithmetic operations exhibit many patterns or predictable associations between numbers; a number, itself, may exhibit a pattern, as in the instance of the repeating decimal; special sequences of numbers, such as the Fibonacci sequence, are mathematical patterns that display many interesting properties. In each area of mathematics, patterns can be found. Indeed, mathematics is the study of pattern.

Pattern is prevelant not only in mathematics. Nature's symmetries in flowers, the coloring of the various forms of sea life, a pebble dropped in a pond, are just a few patterns found in science. The rhythm of poetry, the beat of a melody, sentence structure; illustrate still other patterns.

Thus, pattern is easily interrelated to other subjects.

Each basic unit is paired with one or more extended lessons. The extended lessons provide increased depth to the initial activity.

Finally, the learner objectives of each lesson have been keyed to Bloom's Cognitive Taxonomy. Activities are designed to promote higher level thinking with an emphasis on analysis and synthesis. In keying lessons in this manner, it is hoped that the teacher would remain continually aware that concentrating on higher levels of behavior is an important and effective method of differentiating curriculum for the gifted child.

AN OVERVIEW

Following are five math units designed for intermediate gifted math students. The five units are unified by the concept of "pattern".

The first unit, WHAT'S MY PATTERNS and PATTERNS IN A SQUARE, deals with pattern in a visual manner. Much of the vocabulary associated with pattern is introduced in these lessons.

The second unit, ODD AND EVEN PYRAMIDS and MODULAR MATH, adds familiar math concepts to pattern and illustrates the concept in a visual manner.

The third unit, PATTERNS IN MULTIPLES OF NINE and THE SIEVE OF ERATOSTHENES, deals with number patterns found in a multiplication table.

The fourth unit, BASE TWO MINICOMPUTERS and BUILDING BASE TWO WITH COLORED CUBES, illustrates patterns found in place-value in a variety of number bases.

The final unit, SUMMATION SERIES, PATHS AND PATTERNS and SOAP BUBBLE PATHS, develops a pattern which is applied to science concepts and the scientific process of thinking.

BASIC UNIT	EXTENSION LESSON
WHAT'S MY PATTERN	PATTERNS IN A SQUARE
ODD AND EVEN PYRAMIDS	MODULAR MATH
PATTERNS IN MULTIPLES OF NINE	SIEVE OF ERATOSTHENES
BASE 2 MINICOMPUTERS	BUILDING BASE 2 WITH COLORED CUBES
SUMMATION SERIES	PATHS AND PATTERNS SOAP BUBBLE PATHS

BASIC UNIT

LESSON TITLE: WHAT'S MY PATTERN?

TIME: One Hour

KEY MATERIALS: E.S.S. Creature Cards, overhead projector 8 x 8 matrix for overhead, colored overhead markers, matrixes and colored cubes or markers for learners.

OBJECTIVES: The learner will demonstrate the ability to:

Knowledge/Comprehension	a.	Identify attributes
Application	b.	Relate attributes to a pattern
Synthesis	c.	Create a pattern

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

This first lesson is intended to introduce the learner to the "idea" of pattern. Pattern is created by attributes---color, shape, texture, etc.---and the learner needs to recognize these attributes, to choose certain attributes, to create his/her own patterns.

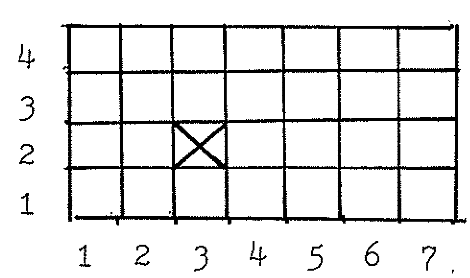
STAGES OF THE ACTIVITY:

1. Begin by showing the simplest Creature Cards and have the students distinguish which are in the set and which are not. As they go through the cards, ask the students to identify the attributes (both structure and number) of the set they retain. Examples: All the member of the set are spotted <u>or</u> all the member of the set have a tail and one spot <u>or</u> all the members of the set are closed shapes.

   Possible Extension

   Have the students create their own creature cards. They might add <u>color</u> as an additional attribute.

2. Show the students how to locate a particular square on a matrix. For example, X would be <u>3 over</u> and <u>2 up</u> or (3,2).



The teacher has a pattern (Figure A) and its attributes are color and place. Have the students begin guessing the pattern. Example: Is (3,2) blue? Is (5,4) red? As the pattern is revealed, they will see that it is symmetrical around both the horizontal and vertical axes. The words "symmetrical" and "axis" can be introduced at this time.

3. Give the students pattern sheets to create their own patterns with the requirement that they be symmetrical around at least one axis (horizontal, vertical, or diagonal). They may then pair off with other students and try guessing each others' patterns using their matrix and colored cubes to record data.

Extension

Have students begin looking for patterns in a series of numbers or words.

For Example:	Possible Pattern:
2, 4, 9, 16 . . .	All square numbers
1, 2, 4, 8, 16, 32 . . .	Each number twice the previous number
1, 3, 7, 15, 31 . . .	Doubled plus 1
Apple, Elephant, Igloo	All nouns beginning with vowels.
Apple, Igloo, Seattle	All nouns with double letters
Hydrogen, Carbon, Iron	All elements
California, Florida, Massachusetts	All states bordered by ocean

WHAT'S MY PATTERN?

These sheets may be run off for each student to create his or her own design.

DESIGNER: ______________________________

____

8	R	O	R	O	O	R	O	R
7	R	B	R	B	B	R	B	R
6	R	O	R	O	O	R	O	R
5	R	B	R	B	B	R	B	R
4	R	B	R	B	B	R	B	R
3	R	O	R	O	O	R	O	R
2	R	B	R	B	B	R	B	R
1	R	O	R	O	O	R	O	R
	1	2	3	4	5	6	7	8

TEACHER PATTERN (FIGURE A)

* AXIS OF SYMMETRY

EXTENSION LESSON

LESSON TITLE: PATTERNS IN SQUARES (Lesson Adapted from The Arithmetic Teacher, April, 1977, pp. 265-271).

TIME: Two or Three 1-Hour Sessions

KEY MATERIALS Felt tip markers or colored pencils, sheets of squares, rulers.

OBJECTIVES: The learner will demonstrate the ability to:

Knowledge/Comprehension	a.	Understand the meanings of congruence, symmetry and geometric transformations.
Knowledge	b.	Know fractional parts of a square.
Application/Synthesis	c.	Use patterns to create a design.

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

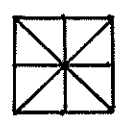
The learner will create and visualize pattern using concepts of fractions, symmetry and geometric transformation.

STAGES OF THE ACTIVITY:

Day 1

1. Have students divide square into eight equal sections. Have them share the variety of ways they have divided their respective squares and discuss the concept of congruence (coinciding exactly in all parts).

2. Choose one pattern which is divided into congruent sections and have the students color in exactly one half of the sections in as many different ways as possible.

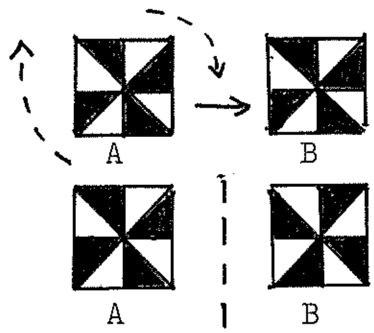
   Discussion Questions

   How many different designs are there? (13)

   How can you determine the maximum number of designs?

   How can you "prove" that you have discovered all the designs?

What is the students' definition of a "different" design? To clarify this concepts, review <u>congruence</u>, <u>rotation</u> and <u>flip</u>.



Rotation -- A is the same as B

Flip -- A is the same as B

Line of Symmetry

Possible Extension

Have the students divide the squares into other equal fractional parts (4ths, 16ths) and repeat the exercises.

Is it possible to predict how many different designs they can expect to find?

Day 2

3. Introduce rules and vocabulary governing (a) color of a design and (b) transformation of the pattern to discover patterns in design and create new designs.

What is my color rule?

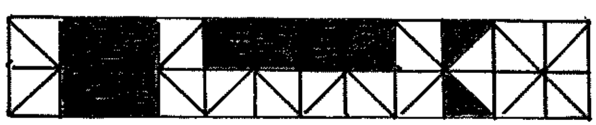
1/8 one color
1/4 second color
5/8 no color

Using this rule, how many different designs can you create?

Rule: 1/4 one color
1/2 second color
1/4 no color (or third color

Transformation

What is my transformation rule?



Rule: Flip-Rotate
Flip-Rotate

Rule: Flip-Slide
Flip-Slide

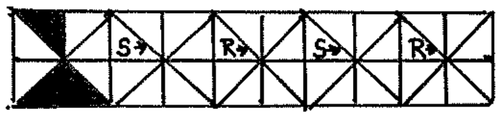
Vocabulary:

Slide-----to move from left to right

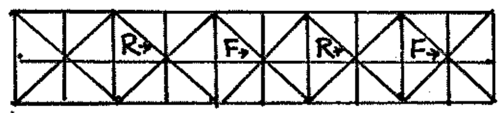
Flip------a reflection along the vertical line of symmetry

Rotation--a quarter turn of the design; clockwise

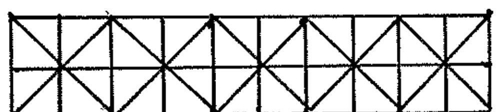
(a) Create a design using this transformation rule.



(b) Choose a pattern and create a design using this transformation rule.



(c) Create a design using your choice of pattern and rule.



Are there some patterns that look the same after they are flipped? rotated?

Day 3

Extensions to the Lesson

On a large piece of graph paper, have each student place his/her favorite design.

a. Have students decide on criteria for favorite designs, i.e., has symmetry, color pattern, etc.

b. Look for patterns in favorites. Is one design most popular? Are more "favorites" symmetrical? What is the average number of colors used in the favorite designs?

c. Using square gum erasers, have students carve a stamp of their favorite design and create their own pattern.

BASIC UNIT

LESSON TITLE: ODD AND EVEN PYRAMIDS

TIME: One Hour

KEY MATERIALS: Cubical counting blocks (two colors), overhead projector or chalk board.

OBJECTIVES: The learner will demonstrate the ability to:

Knowledge/Comprehension	a.	Identify a whole number as odd or even
Analysis	b.	Find patterns in the equations of odd and even sums.
Application	c.	Reproduce these patterns in a visual manner.

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

The learner is able to see how a simple, familiar math concept, odd and even, can be illustrated as a visual pattern and elaborated upon.

STAGES OF THE ACTIVITY:

1. Establish or review the concept of odd and even numbers.

   It is necessary that the learner understands that numbers which can be equally divided into two sets are even. Numbers which cannot be equally divided into two sets are odd. Given a random number of objects, the learner should be able to identify the number as odd or even by sorting by 2's, by matching groups in a one-to-one correspondence, or by discovering his/her own method for determining odd or even.

2. Using simple sums, discover pattern in addition.

   The simplest of addition equations may be divided into the following four patterns:

   a. Odd + Odd = Even
   b. Even + Even = Even
   c. Odd + Even = Odd
   d. Even + Odd = Odd

Give the learner addends (3+3=, 2+2=, 3+2=, 2+3=, etc.). The learner should complete the equation (3+3=6), label the parts odd or even (O+O=E), and list the information until a pattern is seen.

Possible Discussion Questions

Can the learner find exception to the pattern?

Does the size of the number affect the pattern?

Can a generalization or rule be made?

This pattern may be generalized to "sums of likes equal even, sums of unlikes equal odd" as illustrated in the four patterns given.

3. Visually demonstrate this rule.

Using counting blocks, colored chips or graph paper, "create" a number, designating one color of marker to be odd and a second color to be even.

Code:

O E E O E

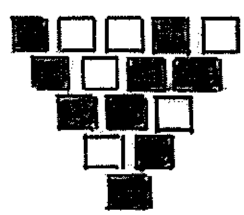
Using the rule derived in Step 2 (Odd + Odd = Even, etc.) "add" each two adjacent blocks and place their sum below them, at their intersection

Original Number

First Sum

Continue process until pyramid is formed.

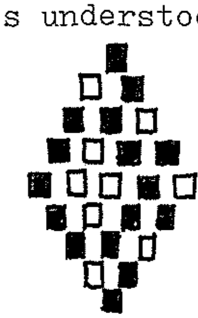
Original Number
First Sum
Second Sum
Third Sum
Fourth Sum



Possible Extensions

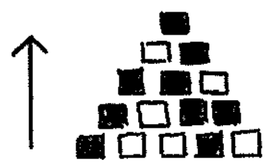
Once this process is understood the pattern can be:

a. Reflected



Line of Symmetry (original number)

b. Built in three-dimensions with blocks stacked upward in the same pattern.



c. Use this same concept to build multiplication pyramids.

CLOSURE:

Have the students look for color patterns and predict future patterns.

Have students discuss which patterns are most visually pleasing.

EXTENSION LESSON

LESSON TITLE: MODULAR MATH
(Lesson Adapted from Creative Teaching of Mathematics in the Elementary School, Alvin M. Westcott, James A. Smith, 1967).

TIME: Two or Three 1-Hour Sessions

KEY MATERIALS: Modular clocks, colored counting cubes

OBJECTIVES: The learner will demonstrate the ability to:

Comprehension	a.	understand the modular number system by citing examples of modular systems in our society.
Application	b.	use the modular clock to count, add and multiply.
Analysis	c.	analyze a modular structure built of (1) colored cubes (2) numbers, to derive patterns.
Analysis	d.	compare structures constructed using different modular number systems.
Synthesis	e.	create his/her own number system and build it using colored cubes.

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

The learner will look for patterns in a number system with a finite number of elements. The learner will look for both color patterns and number patterns.

BACKGROUND INFORMATION FOR THE TEACHER

Modular arithmetic can be said to run around in circles. In fact, modular arithmetic is often called clock arithmetic because it is cyclic in nature. Modular arithmetic involves number systems that employ a limited number of units and then repeat themselves. It can help to promote creative expression in students as well as foster other educational objectives. It can be used as a basic idea from which children may be able to "hitchhike" a variation of modular ideas.

The justification for inclusion of modular arithmetic in modern mathematics programs at the elementary school level are numerous. Most noteworthy are the following: (1) There are many examples of modular number systems that reside in our society---days of the week, months of the year, the clocks that tell us the time of day. (2) Modular arithmetic can be used as a system for checking addition, subtraction, multiplication, and division computation in our decimal system. (3) Because a modular number system operates within limits, it makes basic mathematics principles less complex to demonstrate to children. A modular number system is not as unwieldy as our open-ended decimal system.

By employing a modular number system, a teacher can help children to discover and examine the mechanics of the four basic processes within defined limits, analogous to placing the processes under a microscope.

A modular arithmetic system employs a limited number of symbols. For example, let us consider a modulus five (mod 5) system. Since we have chosen a mod 5 system, we can use only five symbols and five corresponding units to express all counting numbers. As the five symbols, let us use 0, 1, 2, 3, and 4.

A method of conceiving a mod system is to depict the units of such a system in a circular or clock-like manner. While the clock has 5 equal units, the largest numeral on the clock is 4. It is a mod 5 clock because it consists of 5 equal units depicted by 5 different numerals. The numeral 5 could be substituted for the zero on the clock, as the numeral 0 serves as the beginning and the end of the circular system.

STAGES OF THE ACTIVITY:

1. Establish the idea of modular or clock arithmetic.

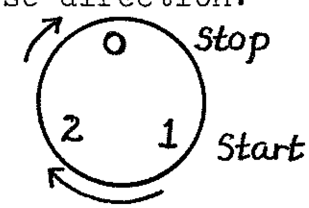
   a. "Modular arithmetic is often called clock arithmetic because it is cyclic in nature. What are some other cycles with which we are familiar besides the clock?"(Seasons, months of the year, weather and life cycles, etc.)

   b. "Like a clock, modular arithmetic contains a finite set of whole numbers. What is the set of numbers on a clock? How is a finite set of numbers different than our counting system?" (Our counting system is infinite; you can always name one larger number.)

c. (Demonstrate a modulo 3 clock).
"For example, in modulo 3 (also called mod 3), we use the three whole numbers 0, 1, and 2. These are the only three elements in the system, and these are the only three numbers that can result from any arithmetic operation. The hand on the clock starts at 0, 1, or 2 and moves around the face of the clock; the direction is determined by the operation, and the number of spaces is determined by the numbers used. The number on which the hand stops is the answer to the computation.

Example: 1 + 2 = 0 (mod 3)

"The hand starts at 1 and, since we are adding 2 to this number, moves 2 spaces in a clockwise direction."



"What would be the mod 3 number for our counting number 4? 6? 13? 24? Can you see a pattern? Can you develop a rule for changing a counting number to mod 3?"
(One possible rule: Counting Number divided by Mod = Quotient + Remainder. The Remainder is the modular number).

Example: What is 13 in mod 3?

13 - 3= 4 with a remainder of 1

13 = 1 (mod 3)

Possible Extension

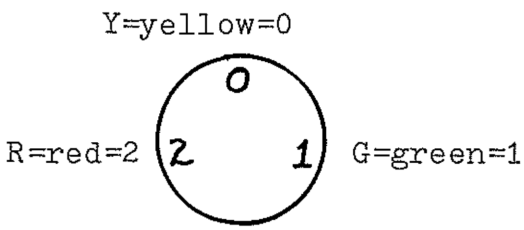
Have the students discover methods to do the remaining three operations using the modular clocks.

2. After discussing the mechanics of the mod systems and practicing with the clocks, explain to the students that they can "build" visual representations of the addition facts of any modular system. Colored counting cubes are used for this activity. A different color is assigned to each symbol of the mod system, then the cubes are used to build a triangular structure of the system.

To build the structure:

a. Have the student choose a different colored block to represent each number in the modular system.

b. Explain that the floor or table will represent the number zero.



Step 1. Begin the structure with the block representing 1

$\boxed{G}$

Step 2. Add 0 (table/floor) + Starting Block (G or 1)

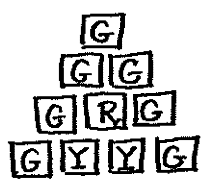
Starting Block

Table + $\boxed{G}$

$\boxed{G}$ + Table

Step 3. Table + $\boxed{G}$ = $\boxed{G}$, $\boxed{G}$ + $\boxed{G}$ = $\boxed{R}$, $\boxed{G}$ + Table = $\boxed{G}$

or/ 0+1=1, 1+1=2, 1+0=1



c. Have the students substitute numbers for colored cubes. Look for number patterns

```
      1
     1 1
    1 2 1
   1 0 0 1
  1 1 0 1 1
 1 2 1 1 2 1
1 0 0 2 0 0 1
```

Possible Extension Lessons

a. Invent one's own number system, build a visual structure of it and compute problems on paper with it.

b. Build a multiplication structure of the original mod system with which they are working and compare the pattern with that of the addition structure.

c. Build Pascal's Triangle using a variety of modular systems. Look for pattern within the structure. Compare structures built using different modular number systems.

d. Bezuszka, Stanley et. al., <u>Designs From Mathematical Patterns</u>, Palo Alto, California: Creative Publications, Inc., 1978.

<u>Designs From Mathematical Patterns</u> takes a variety of number structures including modular arithmetic operations, Latin Squares, Magic Squares and Fibonacci Numbers, defines the patterns and translates them into design. This book can serve as the textbook for an independent instructional unit or it can supplement and enrich the classroom textbook.

BASIC UNIT

LESSON TITLE: PATTERNS IN MULTIPLES OF NINE
(Lesson adapted from Starting Points. C.S. Banwell, et al., Oxford University Press, 1972, pp. 108-109).

TIME: Two or Three 1-Hour Sessions

KEY MATERIALS: Overhead or blackboard, paper and pencil.

OBJECTIVES: The learner will demonstrate the ability to:

Knowledge/Comprehension	Identify patterns in the multiples of nine to include:
Application	a. Pattern in place value
	b. Reversals
Analysis	c. Sums

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

By listing the multiples of 9, the learner will observe patterns in the one's and ten's places. In addition, he/she will find a pattern of reversals and a pattern of sums.

STAGES OF THE ACTIVITY:

1. Have the learner write the 9-times table, N x 9 where N=1, 2, 3, . . . 10. (See Figure A).

2. Have the learner look for patterns in this set of numbers.

   Possible Patterns

   A. Place Value
      1. In the one's place, the numbers begin at 9 and go down to zero (9, 8, 7 . . . 0).
      2. In the ten's place, the numbers begin at 1 and go up to 9 (1, 2, 3, . . . 9).

   B. Sums
      The sum of the digits of the product equals nine.
      "Using additions, can you find an easy way of remembering a product in the 9-times table?"

   C. Reversals of Products
      On the board write 18, 27, 72, 81.
      "I've written four of the products from our table.

What pattern do you see? Can you extend this pattern with the remaining products? (36, 63, and 45, 54)
How can we include 9 and 90?" (09 and 90)

3. Extend the times tables to N=11, 12, 13, . . . 33.

"Do you think the patterns we discovered will be found in our extended table?"

A. Place Value
   1. Patterns in the one's and ten's places continue.
   2. There is a pattern of 1's, 2's, etc., in the hundreds column.

B. Sums
"Do the sums of product's digits continue to add up to 9?"

Not always. Examples: 9 x 11, 9 x 21, 9 x 22
These products' digits add up to 18, but if you add again, the sum will be 9.

Example: $9 \times 11 = 99$
$9 + 9 = 18$
$1 + 8 = 9$

"How often do you find the product's sum of 18? Can you predict the next time it will occur?"

"Can you find other digit-sums besides 9 and 18? What would you predict they would be?"

Possible Extension

"Can you devise a formula, using place value and digit-sums data, to decide the product of any 9 x N?"

Ideas: If N is between 1 and 9, the ten's place (T) will be N-1 and the one's place will be 9-T.
Example: 9 x 6 where N = 6
Ten's place (T) N-1 or 6-1=5=T
One's place: 9-T or 9-5=4
Product = 54

Where $11 < N > 20$ the combined 100's and 10's place (HT) will be N-2.

Example: 9 x 12 where N=12
HT=N-2=12-2=10
Add the digits together; 1+0=1
Subtract from 9 to get the one's place; 9-1=8
Product: 9 x 12 = 108

The learner can extend this pattern. For example, where $21 < N > 30$, HT=N-3. The digit sum will always be 9 unless it falls into the 18 pattern.

"Does the pattern break down?"

C. Reversibility

Possible Extension

"We said earlier that 36 and 63 are reversals. 7 x 9 produces a product (63) that reverses the product of 4 x 9 (36). 7 is the reversal of 4 so R4=7. 9 x 9 produces a product (81) that reverses the product of 2 x 9 (18) so R2=9. Using these symbols, we could record the data of the reversals we have found so far."

"What patterns can we find in this data?"

One Possible Pattern

1 x 9 = 9	R1 = 10	1 + 0 = 1
10 x 9 = 90	R10 = 1	
2 x 9 = 18	R2 = 9	
9 x 9 = 81	R9 = 2	
90 x 9 = 810	R90 = 2	
20 x 9 = 180	R20 = 9	
	R8 = 3	
	R80 = 3	
	R200 = 9	
	R7 = 4	
	R70 = 4	
	R2000 = 9 etc.	

Possible Extensions

Investigate other tables for possible patterns.

3-Times Table

1	x	3	=	3	3	
2	x	3	=	6	6	
3	x	3	=	9	9	
4	x	3	=	12	1 + 2=3	
5	x	3	=	15	1 + 5=6	
6	x	3	=	18	1 + 8=9	
7	x	3	=	21	2 + 1=3	
8	x	3	=	24	2 + 4=6	
9	x	3	=	27	2 + 7=9	
10	x	3	=	30	3 + 0=3	

Digit-sums 3,6,9,3,6,9 . . .

"Does this pattern continue?

9 TIMES - TABLE

1 x 9 = 9
2 x 9 = 18 → 1 + 8 = 9
3 x 9 = 27 → 2 + 7 = 9 etc.
4 x 9 = 36
5 x 9 = 40 reversals
6 x 9 = 54
7 x 9 = 63
8 x 9 = 72
9 x 9 = 81
10 x 9 = 90
11 x 9 = 99 → 18 (digit-sum)
12 x 9 = 108
13 x 9 = 117
14 x 9 = 126
15 x 9 = 135
16 x 9 = 144
17 x 9 = 153
18 x 9 = 162
19 x 9 = 171
20 x 9 = 180
21 x 9 = 189 → 18 (digit-sum)
22 x 9 = 198 → 18
23 x 9 = 207
24 x 9 = 216
25 x 9 = 225
26 x 9 = 234
27 x 9 = 243
28 x 9 = 252
29 x 9 = 261
30 x 9 = 270
31 x 9 = 279 → 18 (digit-sum)
32 x 9 = 288 → 18
33 x 9 = 297 → 18

1

EXTENSION LESSON

LESSON TITLE: SIEVE OF ERATOSTHENES

TIME: Two 1-Hour Sessions

KEY MATERIALS: Overhead transparency of Sieve of Eratosthenes, copies of "Sieve", Prime Findin", Data Collection and worksheets for Factor Stacks, Factor Stacks, colored pens or pencils of four different colors.

OBJECTIVES: The learner will demonstrate the ability to:

Comprehension	a.	count by multiples
Comprehension/ Analysis	b.	recognize prime and composite numbers.
Analysis	c.	analyze number patterns.
Comprehension/ Analysis	d.	factor composite numbers.

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

By shading primes and multiples of primes, the student will create a graphic structure which can be analyzed for a variety of patterns.

BACKGROUND FOR THE TEACHER:

Eratosthenes was a Greek mathematician who lived in about 200 B.C. He developed an interesting way of locating prime numbers which we call the Sieve of Eratosthenes. In the following exercises, the students will use his method to locate all of the prime numbers between one and one hundred. They will also learn how many prime factors these composite numbers have, how to find them and what they are.

All of the counting or natural numbers, with the exception of 0 and 1, can be classified as either prime or composite. A prime number is a number which has only itself and 1 as factors. For example, the number 2 has only the factors 1 x 2, and the number 3 has only the factors 1 x 3. A composite number is a number that has two or more factors, not counting one times the number (1 x N). For example, 4 is a composite number because it has the factors 2 x 2, as well as the factors 1 x 4.

Mathematicians have agreed that the number 1 will not be considered prime. It is the identity number for multiplication. The Fundamental Theorem of Arithmetic states: Every natural number which is composite has only one prime factorization. As an example, the composite number 12 has the prime factorization 2 x 2 x 3. If the number 1 were to be considered prime, the factorization of 12 would not be unique since 2 x 2 x 3 x 1 x 1, etc., would have to be acceptable. The uniqueness of prime factors of a given number is very important in working with fractions.

STAGES OF THE ACTIVITY

Hour 1

1. Give directions and demonstrate, using the overhead projector and transparency. Instructions may be given orally or as a ditto for students to work through independently.

   "To complete the Sieve of Eratosthenes, you need only know how to count by multiples. If each set of multiples is marked with a different colored pen or pencil, a pattern will unfold and the prime factors become much more apparent. For this exercise, you will need a regular black lead pencil (or pen) plus pens or pencils of four other colors. Although the colors red, green, blue and yellow are listed in the following instructions, you may substitute any other four colors. It is very important, however, to be consistent and follow the pattern.

   a. Draw a red circle around the number 2 (the first prime number), and draw 3 slanting red lines through all multiples of 2.

   b. Draw a green circle around the number 3, and draw 3 slanting green lines through all multiples of 3. (Do you see a pattern?)

   c. Draw a blue circle around the number 5, and draw 3 slanting blue lines through every fifth number. (What pattern do the multiples of 5 form?)

   d. Draw a yellow circle around the number 7, and draw 3 slanting yellow lines through every seventh number. (These are the multiples of 7)

e. Draw a black circle around all remaining numbers that do not have slanting lines through them. These numbers (plus 2, 3, 5, and 7) are all the prime numbers between one and one hundred.

Stop and think about the following:

a. Why didn't we mark the multiples of 4 and 6?
b. Did we need to mark the multiples of 8, 9 or 10?
c. What is the next prime number after 7? What number do we get when we square that number (11 x 11)? Why do you think we could stop after we marked the multiples of 7?

2. Using their completed Sieve of Eratosthenes, have the students complete Prime Findin'.

3. Discuss patterns and discoveries the class found when analyzing their Sieve of Eratosthenes.

Hour 2

1. "In the space below each composite number on your completed Sieve of Eratosthenes, write the number of prime factors needed to make its product." (4 = 2 x 2 or two prime factors.) "The numbers will tell you the size of a stack needed to display a composite number in a Factor Stack.

2. Have the students complete "Data Collection and Worksheet for Factor Stacks".

TEACHER BACKGROUND-INFORMATION ON FACTOR STACKS

a. The top of the stack must be a composite number.
b. The first pair of factors may be any pair of factors for that number.
c. The bottom row of each stack must be entirely made up of primes. This row is called the "prime factorization" of the top composite number.

Examples of completed Factor Stacks.

d. The choice of size of stack needed to display a composite number depends on the number of prime factors in that number.

3. Using Factor Stack Sheets, have students factor their choice of numbers to fill the sheets. In completing the sheets, did they validate their predictions on the Data Sheet, questions 4-8?

Possible Extensions:

a. Explain how the information gained in factor stacks can be used to find Greatest Common Factors.

Example: What is the Greatest Common Factor of 36, 81?

What factors do they both share?

They both share two 3's.

Greatest Common Factor= 3 x 3 = 9

b. Teach the Euclidean Algorithm to find the Greatest Common Factor.

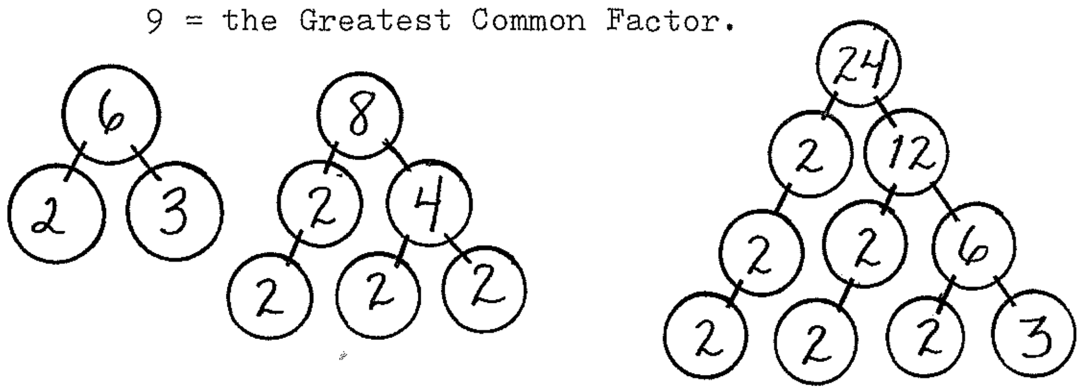
Example:

$$81 = 2 \times 36 + 9$$

$$36 = 4 \times 9 + 0$$

The number preceding 0 is the G.C.F thus

9 = the Greatest Common Factor.



Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Prime Findin'

Use your completed Sieve of Eratosthenes to answer these questions.

1. How many prime numbers less than 100? ________
2. How many even primes? ________ List________
3. How many primes end with 1? ________List________
4. How many primes begin with 7? ________ List________
5. Which columns on the Sieve have no primes? ________
6. Which column has the largest number of primes?________
7. Which columns have only one prime each? ________
8. Which columns have six primes? ________
9. Which columns have five primes? ________
10. Which row has the greatest number of primes.________
11. Twin Primes have only one natural number between them, such as 3 and 5 and 5 and 7. How many more Twin Primes are there on the Sieve?________

    List the pairs.________
12. How many primes end with 9? ________List________
13. How many primes begin with 9?________List________
14. How many primes end with 3? ________List________
15. How many primes begin with 3? ________List________
16. How many primes begin with 8?________List________
17. How many primes end with 8? ________List________

DATA COLLECTION AND WORKSHEET FOR FACTOR STACKS

Use your completed Sieve of Eratosthenes to answer these questions.

1. How many Prime numbers between 1 and 101? __________

2. How many of the Composite numbers have:

   (a) 2 prime factors? __________

   (b) 3 prime factors? __________

   (c) 4 prime factors? __________

   (d) 5 prime factors? __________

   (e) 6 prime factors? __________

3. What size Stack is required to display a composite number that has:

   (a) 2 prime factors? __________

   (b) 3 prime factors? __________

   (c) 4 prime factors? __________

4. List the composite numbers that will require a size 2 Stack.

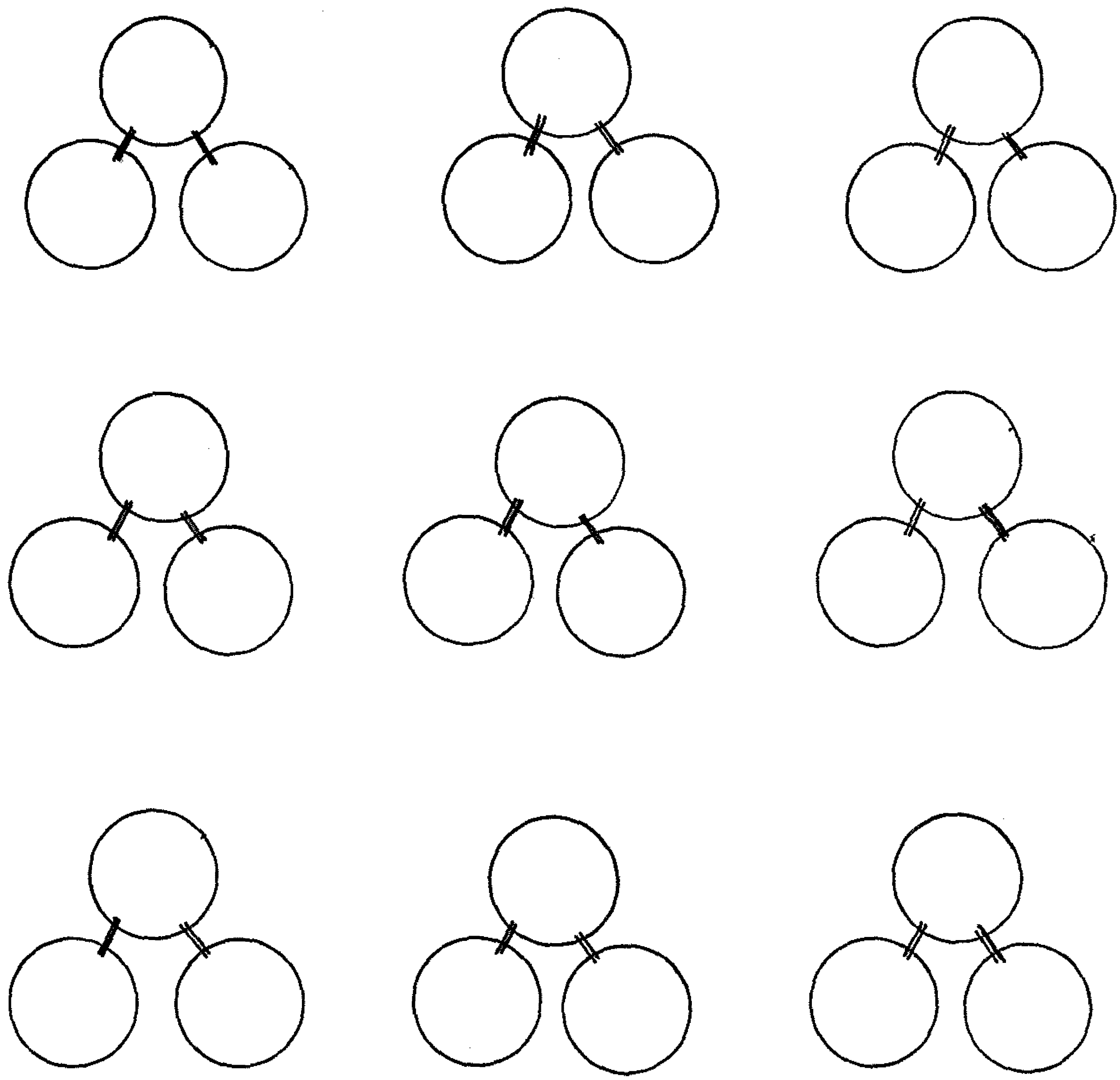
5. List those that will require a size 3 Stack.

6. A size 4 Stack.

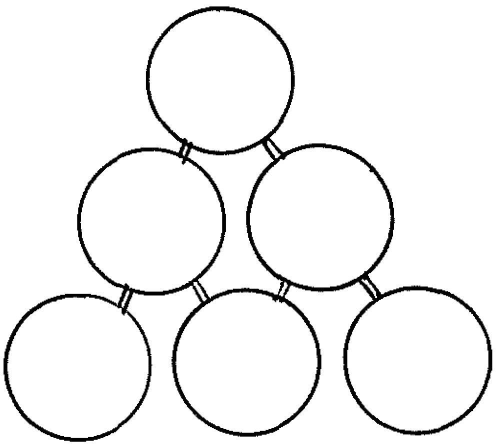
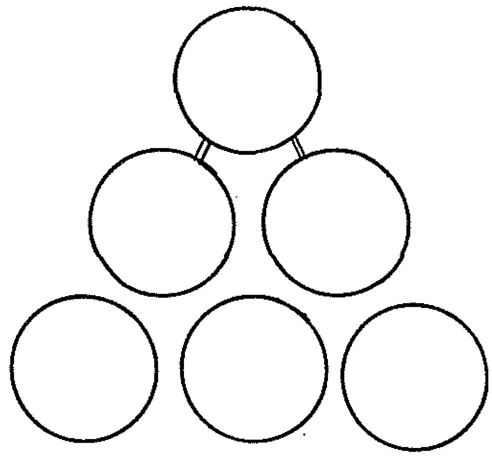
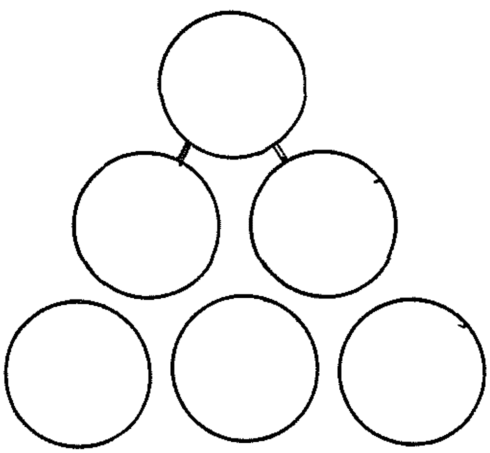
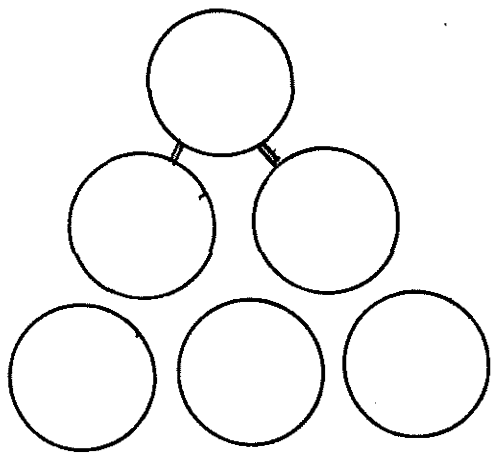
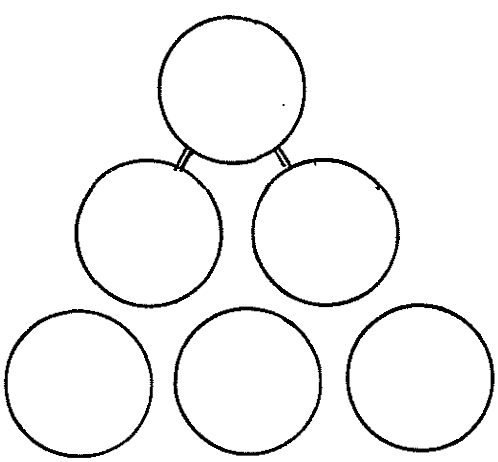
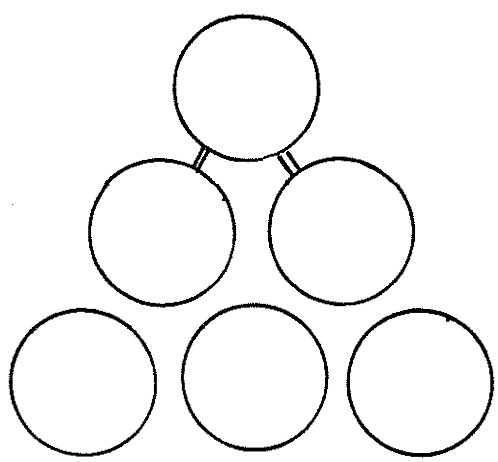
7. A size 5 Stack.

8. A size 6 Stack.

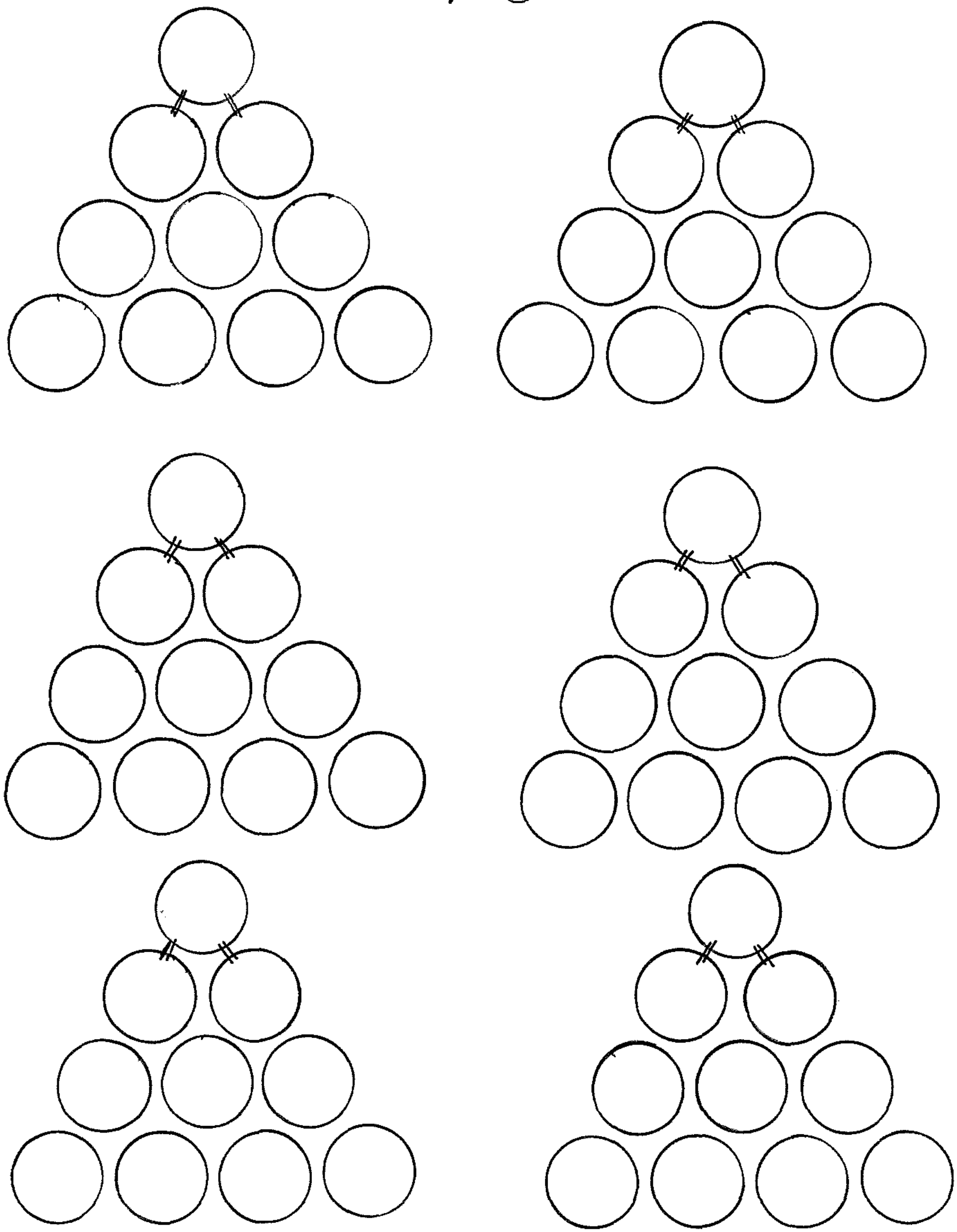
Factor Stacks



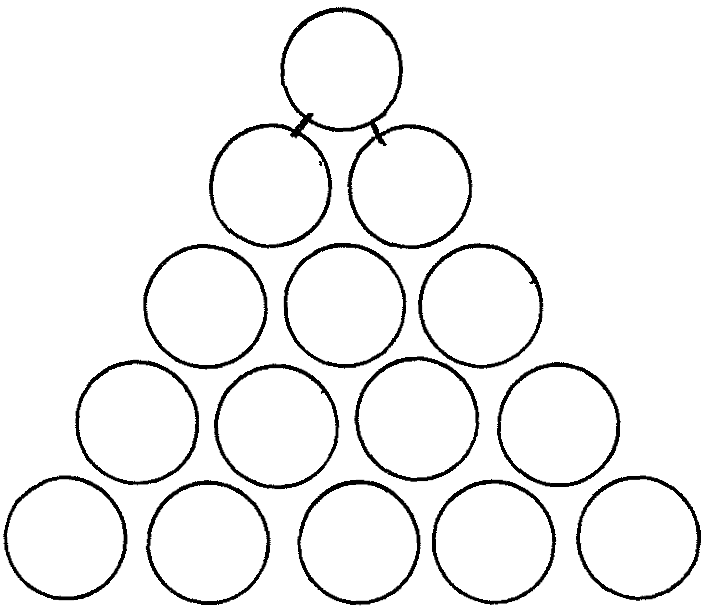
Factor Stacks

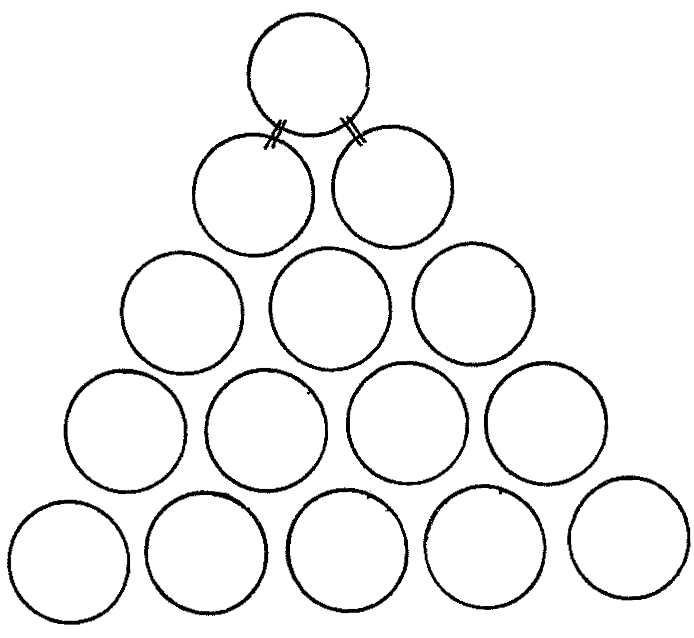







Factor Stacks

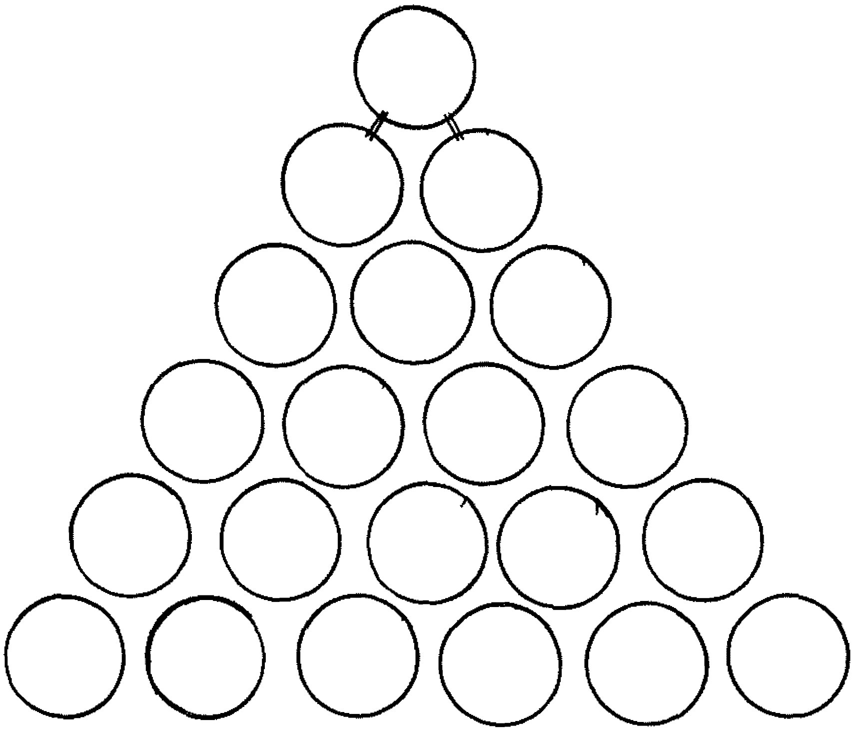


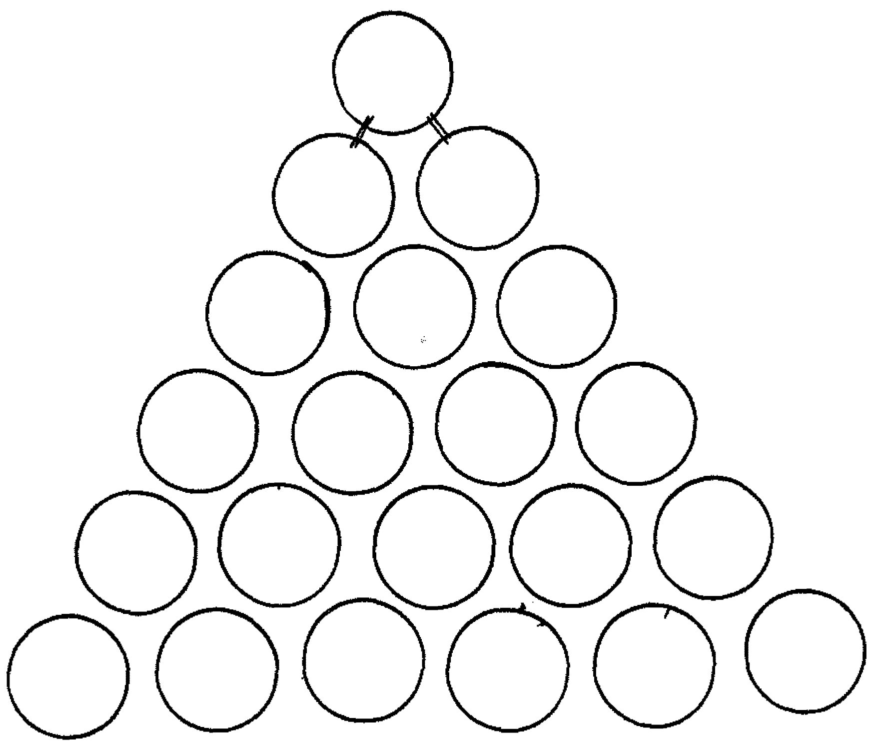
Factor Stacks





Factor Stacks





BASIC UNIT

LESSON TITLE: BASE 2 MINICOMPUTERS

TIME: One to Two 1-Hour Sessions

KEY MATERIALS: Overhead, Base 2 Minicomputers, markers (beans, chips), overhead transparency of a chess board, overhead pen.

OBJECTIVES: The learner will demonstrate the ability to:

Knowledge/Comprehension	a.	Read a number in Base 2
Knowledge/Application	b.	Convert a number from Base 2 to Base 10.

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

The learner will see common patterns in place-value in various number bases and will use these patterns to convert numbers from one number base to another.

STAGES OF THE ACTIVITY:

1. "Legend has it, that the King of Persia was so pleased with the inventor of chess that he wanted to reward the inventor with whatever he chose. After some thought, the inventor said he would like some wheat, a most valuable item in those days, and he would like it given to him in the following manner. (Demonstrate using markers and chessboard transparency.) He would like one piece of wheat placed in the first square, two pieces in the second square, four pieces in the third square, and to continue doubling square to square."

   "How many pieces would I need here (4th square) and here (5th square)?" (Replace markers with the numerals 1, 2, 4, 8, 16, etc.)

   "It turns out that by the time he got to the 64th square, in that square he would have 9 quintillion pieces of wheat!" (Write the numeral on the overhead--9,000,000,000,000,000,000.)

   "The inventor of chess would have had enough wheat to cover the country of Persia three inches deep! Since, in those days, Persia was four times as large as the state of California, with that much wheat, you could cover California more than a foot deep in wheat!"

"Now, today's lesson is not about chessboards or wheat, but about this special set of numbers formed when we take 1, double it, and continue this series." (List 1, 2, 4, 8, 16 on the overhead.) "It is possible to begin counting using each of these numbers only once."

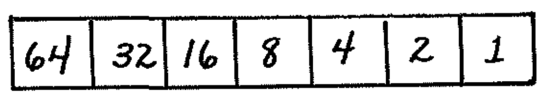
Demonstrate:

1 = 1
2 = 2
3 = 1 + 2
4 = 4
5 = 4 + 1
6 = 4 + 2
7 = 4 + 2 + 1
8 = 8

With paper and pencil, have students "count" to 25.

"Because of this property, this special set of numbers is used in computer work. With a 'light-on light-off' system, the computer can make any whole number."

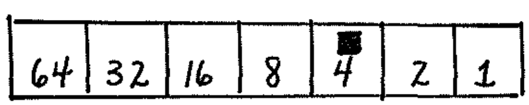
BASE 2 MINICOMPUTER

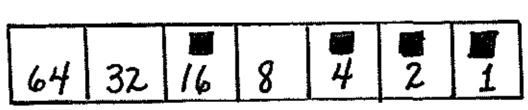


Use markers for LIGHT-ON
NO marker means LIGHT-OFF

Demonstrate:

LIGHT-ON LIGHT-OFF LIGHT-ON LIGHT-OFF equals 10





= 23

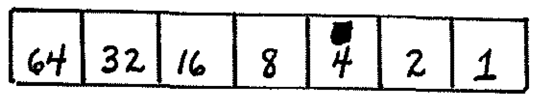
2. Give students numbers to set up on their computers in the same manner.

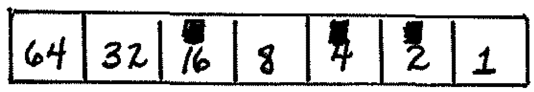
"Now, insted of writing LIGHT-ON LIGHT-OFF, we use numerals to give this information."

1 = LIGHT-ON

0 = LIGHT-OFF

Thus,

100 =  = 4

10110 =  = 22

"Since this special set of numberals are the place values for BASE 2, we indicate that by putting a small 2 in parentheses at the lower right of each number."

"Thus, $100_{(2)}$ = one 4, no 2's, no 1's = 4 Base 10 (Our common counting system)

3. List numbers in Base 2 and using their Base 2 Minicomputers, have the students practice converting from Base 2 to Base 10.

Possible Extension

Give the students riddles or messages in code using Base 2 and have them:

a. convert each number to Base 10, then

b. using the ordering of the alphabet as an additional pattern, change the Base 10 number to a letter.

Example:

Alphabet Pattern:

1	2	3	4	5	6	7	8	9	10	11	12	13
A	B	C	D	E	F	G	H	I	J	K	L	M
14	15	16	17	18	19	20	21	22	23	24	25	26
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Riddle: QUADRUPLETS

Code in Base 2: 110-1111-10101-10010 11-10010-11001-
1001-1110-111 1111-10101-10100
1100-1111-10101-100

Converted to Base 10: 6-15-21-18 3-18-25-9-14-7
15-21-20 12-15-21-4

Changed from numbers to letters: FOUR CRYING OUT LOUD

Have students then write and exchange their own messages!

EXTENSION LESSON

LESSON TITLE: BUILDING BASE 2 WITH COLORED CUBES

TIME: One Hour Session

KEY MATERIALS: Cubical counting blocks, Base 2 Computer, markers.

OBJECTIVES: The learner will demonstrate the ability to:

Knowledge/Application	a.	Count in Base 2
Comprehension/Analysis	b.	Explain patterns in Base 2
Analysis	c.	Predict future patterns

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

In this lesson, the Base 2 numerals will be visually symbolized using colored cubes. The blocks will form patterns in both color and number enabling the learner to see a pattern, to count in a number base and to predict future patterns.

STAGES OF THE ACTIVITY

1. "In our least session we became familiar with Base 2 and learned to convert a number from Base 2 to Base 10, our common counting system."

   "Today, we will work with Base 2 once again. Our Base 2 computers are going to help us count. We'll use colored blocks for 'counters'. I hope that their colors will help you to see patterns in the Base 2 counting system."

   Using the Base 2 computer and markers, review use of the computer and count sequentially from 1 to 8, each time having the learner state the number in Base 2.

   Example: (Teacher) "With your markers make a 1."

   (Student response)

64	32	16	8	4	2	1 ●

   T: "How is this number named in Base 2?" S: "One Base 2."

   T: "With your markers make a 2." S:

64	32	16	8	4	2 ●	1

T: "How is this number named?"

S: "One-Zero-Base Two."

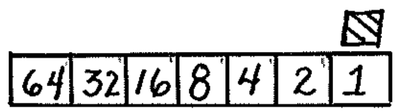
2. "Now, instead of using markers, we will use colored blocks or cubes to form our system."

"Since there are only two numerals in Base 2, one and zero, we will only use blocks of two colors. Let green (□) be our ZERO block and red (▧) be our ONE block.

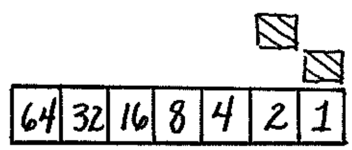
Code 1 = ▧

0 = □

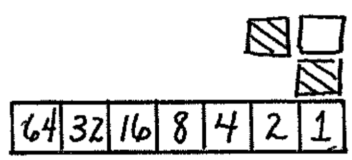
"Use your computer to help you keep track of place value. Begin by putting a "one-block" above the one on the computer."



"Now, make a 2 by putting a one-block above the 2." (Review the concept of "lights-on, lights-off" from the previous lesson.)

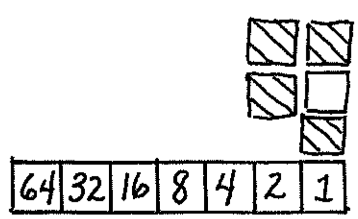


"Instead of keeping the space above the 1 blank, for light-off, we'll put down a zero-block (green) as a place holder."



"This can be read 'one-zero Base 2 (10_2)' meaning, 1 two and 0 ones.

"Put down blocks to make a 3."



"We now have a one-block in both places meaning 1 two and 1 one, and this is read as 'one one Base 2'." (Have the students continue to count in this manner using the Base 2 computer and colored blocks.

Refer to the Teacher's Key Base Two for the pattern. Students may also complete Chart 1, as they work, recording their data and comparing Base 2 with Base 10.)

3. When all the students have counted beyond 16, have them stop and look for patterns in their structure.

   Possible Patterns:

   a. Alternating colors in the one's place (one-zero-one-zero, etc.) or (red-green-red-green . . .).

   b. Alternating by groups of 2 in the 2's place (red, red, green, green, red, red . . .).

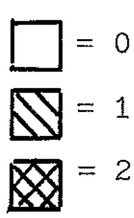
   c. Alternating by groups of 4 in the 4's place.

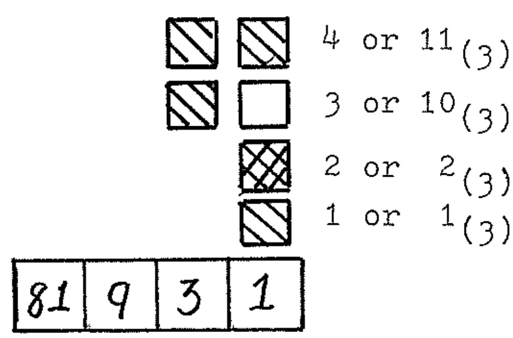
   Generalization: The number of blocks in one group names the place value.

4. Predict how many blocks will be "grouped" the next place to the left. Continue building your structure to determine if your prediction is correct.

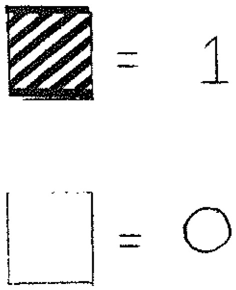
Possible Extensions

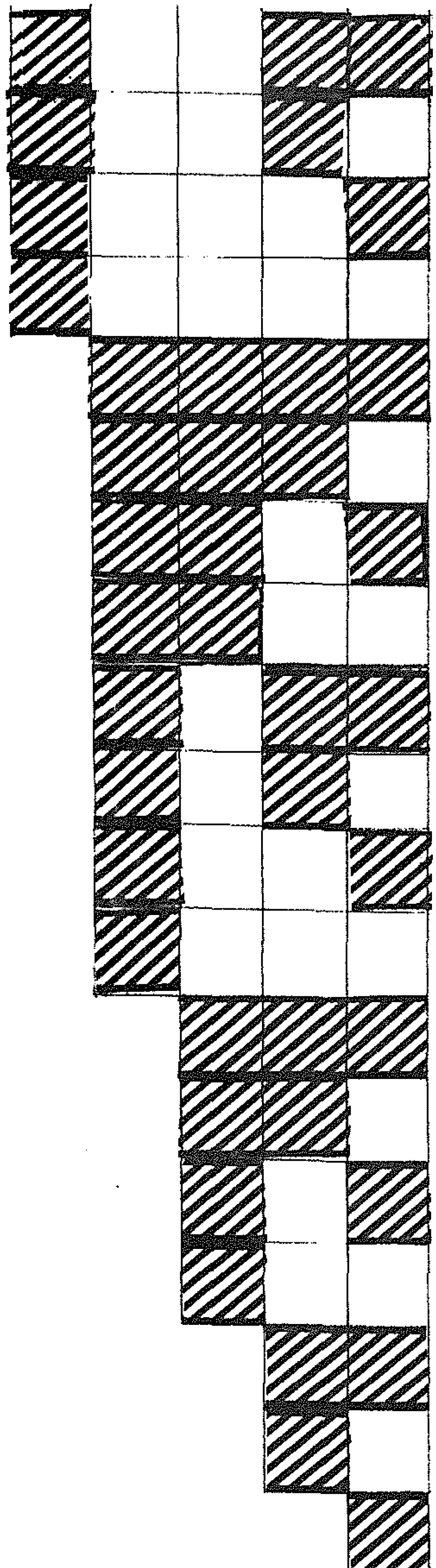
This same structure can be extended to other number bases. The numerals in the Base will determine the number of colors of cubes needed. Example: Base 3 contains the numerals 0, 1, and 2. Three colors of cubes are needed for constructing Base 3.





Have different groups of students build structures in various number bases, recording their data on charts similar to the one used for Base 2. Have them make pattern predictions for their own structure and then <u>compare</u> structures and look for additional patterns <u>between</u> structures.





19 = $10011_{(2)}$

18 = $10010_{(2)}$

17 = $10001_{(2)}$

16 = $10000_{(2)}$

15 = $1111_{(2)}$

14 = $1110_{(2)}$

13 = $1101_{(2)}$

12 = $1100_{(2)}$

11 = $1011_{(2)}$

10 = $1010_{(2)}$

9 = $1001_{(2)}$

8 = $1000_{(2)}$

7 = $111_{(2)}$

6 = $110_{(2)}$

5 = $101_{(2)}$

4 = $100_{(2)}$

3 = $11_{(2)}$

2 = $10_{(2)}$

1 = $1_{(2)}$

BASE 2
MINICOMPUTER

64	32	16	8	4	2	1

BASIC UNIT

LESSON TITLE: DISCOVERING A FORMULA FOR SUMMATION SERIES

TIME: One Hour

KEY MATERIALS: Colored blocks, overhead, chalk board

OBJECTIVES: The learner will demonstrate the ability to:

Application a. contruct a visual structure illustrating the formula for summation series.

Application b. use the summation formula to solve practical problems.

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

The learner will observe number patterns formed when listing series (a) numerically and (b) when building a visual structure from colored blocks.

STAGES OF THE ACTIVITY:

1. "If I were to hire you for a summer job in June, would you prefer me paying you \$10 per day or \$1 the first day, \$2 the next day, \$3 the third day and add one more dollar each day for thirty days?"

"How could you figure out which is the better deal?"

Adding a series of numbers is called a <u>summation</u>.

Two Methods For Deriving A Summation:

METHOD 1: List series to be summed:

$1 + 2 + 3 + 4$ (Can be written $\sum_{1}^{4}$

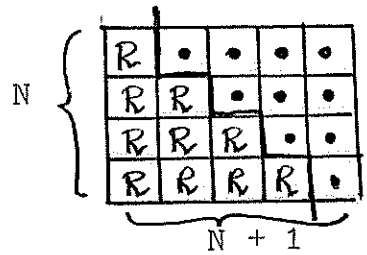
Reverse the series and add:

$$\begin{array}{r} 1 + 2 + 3 + 4 \\ + \; 4 + 3 + 2 + 1 \\ \hline 5 + 5 + 5 + 5 = 20 \end{array}$$

But we want only half of this sum. $20/2 = 10$

METHOD 2: Use one color block to illustrate the sum.

Let the number of $\boxed{R}$ = N



Complete the rectangle with another color block.

"What is the area of the rectangle?"

Area of the Rectangle= (N) x (N + 1)

"What is the area of the Triangle formed with the

R blocks?"

$$\text{Area of the Triangle} = \frac{(N) \times (N+1)}{2}$$

What if you want to find a summation not beginning with 0 or 1?

Example: 3 + 4 + . . . + 7

$$\sum_1^7 - \sum_1^2 = \frac{7 \times 8}{2} - \frac{2 \times 3}{2} = \frac{2 \times 3}{2} = 28-3= 25$$

Closure: Using either method, find a variety of summations. Which way would you like to have your salary paid?

Extension:

Bureloff, Morris, Number Triangles, Hayward, California: Activity Resources Co., Inc. (P.O. Box 4875) pp. 7-10.

EXTENSION LESSON

LESSON TITLE: PATHS AND PATTERNS

TIME: One Hour

KEY MATERIALS: Examples of "pattern", hexagon ditto, transparencies of a spiral and meander, geoboards and rubber bands.

OBJECTIVES: The learner will demonstrate the ability to:

Application a. use summation knowledge in a practical way.

Analysis b. relate data and see how this data relates to the real world.

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

The student will generate patterns using dots and lines as well as geoboards and rubber bands. Patterns will be related to their "functions" in nature.

STAGES OF THE ACTIVITY:

1. Introduce the idea of patterns using pictures, slides, examples of building materials (bricks, ceiling, tiles), children's clothing, etc.

   Review patterns seen in summation lesson (continually adding on more numbers to a series, adding one more line of blocks to a rectangle, patterns seen with the block rectangle).

2. "Let us generate some simple patterns using dots and lines.

   (a) Give students the "hexagon-dot ditto". "Connect all points in the array so that all points link up with the center point, either directly or indirectly, but so that any two two dots connect along only one point."

   (You may want to review the vocabulary: point, line, path, array, hexagon.)

   (b) Have students share their "paths" and give names to them (spiral, meander, explosion, etc.).

(c) Using the overhead transparency of the spiral and meander, have students predict which will have the longest path from the center to the outside. (Note: If we set the distance between adjacent dots equal to one unit, they both have a length of 90 units.)

(d) Using either an overhead geoboard or student's individual boards and string, test their predictions.

(e) Using geoboards and rubber bands, have students create a variety of patterns and record the length of their paths.

DATA SHEET

Pattern	Length of Paths	Average Length of Paths
Spiral	90	45.5
Meander	90	45.5
Explosion	233.1	3.37

(f) In addition, have students find the average length of paths. To do this, consider the path from the center point to the first point, the path from the center point through the first point to the second point, etc. Take all possible path lengths and divide by the number of possible paths to find average path lengths.

Point out usefulness of summation and symmetry in ascertaining these sums.

NOTES TO THE TEACHER:

What do these patterns tell us? They reveal, among other things, that spirals and explosions represent two extremes. The spiral is short, but it connects the points in an extremely circuitous manner. It might be a good path for a foraging worm or a visitor at a museum. Clearly, however, a spiral is not at all a suitable form for a tree which must transport nutrients between its central trunk and outermost leaves along a reasonably direct path.

The pattern of the explosion, on the other hand, minimizes travel distance between the center and each outlying point, but the total of all the travel distance is enormous. The pattern of the explosion might be suitable for a tree; a tree cannot sustain each of its leaves without separate branches.

It turns out that branching patterns are compromises between the single circuitous route of the spiral and the many direct routes of the explosion. Branching patterns obtain a short total length at the expense of only a little indirectness here and there. They effect a savings in the whole at the expense of only a few of the parts. Actually, they are incredibly good compromises. They may have no more total length than a spiral (in fact, they may even be shorter than a spiral, as we shall see) and only a slightly longer average path than the minimum found in the explosion. Branching, therefore, commands the best of both worlds; shortness as well as directness.

We have discovered several prototypical patterns: the spiral, the meander, the explosion and various forms of branching. We can describe those patterns explicitly in terms of four geometric attributes: 1. uniformity; 2. space fillings; 3. overall length; and 4. directness.

The spiral is beautifully uniform; it curves around on itself in a perfectly regular manner. It can fill all of two-dimensional space, being capable of infinite expansion and it is also quite short. But as we have seen, as measured by the mean of distances to its center, the spiral is extremely indirect.

The random meander turns out to be much like the spiral except that it is not uniform; it is quite turbulent and chaotic. Nevertheless, like the spiral, it can cover all of two-dimensional space, it is short and it is indirect.

The explosion is uniform in that it maintains constant angles between its rays. Note, however, that unlike the spiral or meander, it cannot fill all of space uniformly; it is much more dense closer to the source than far away. Furthermore, as we have seen, the sum of its constituent rays becomes very large. Nevertheless, it excels in directness---linking each point to the center as directly as possible.

Branching patterns are less uniform and display more variations in their details than either spirals or explosions, but they fill all of space, they are short and they are relatively direct. In addition, the branching pattern with regular triple junctions is shorter than any of the other patterns.

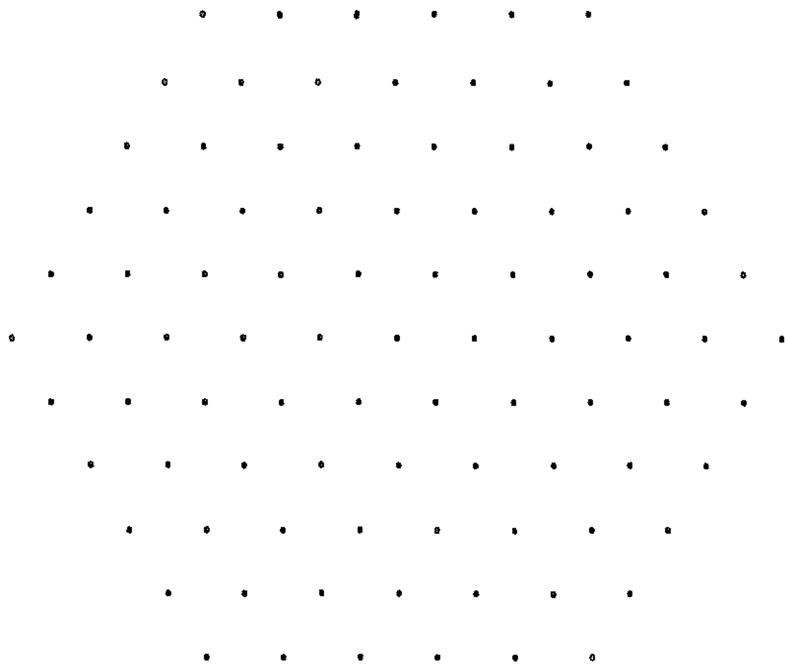
Possible Extension:

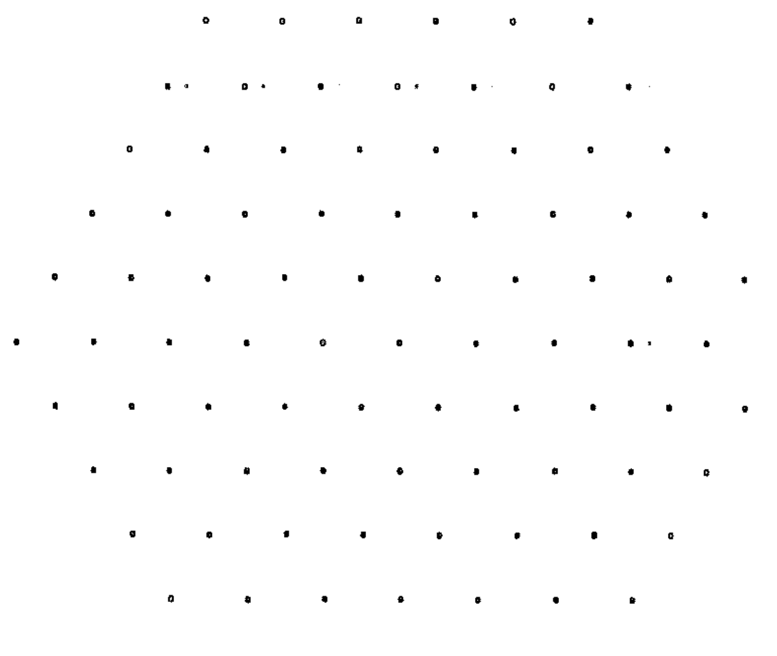
Use a similar appraoch using dots arranged in a rectangular array and a random array.

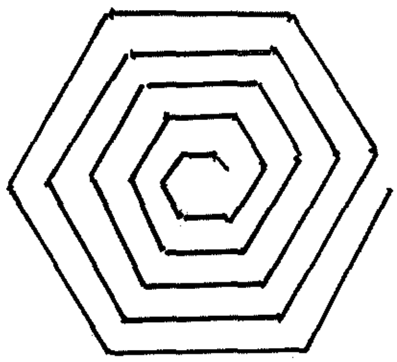
Source for more information:

Stevens, Peter S., Patterns In Nature, Boston, Toronto: An Atlantic Monthly Press Book, Little, Brown and Company, pp 37-48.

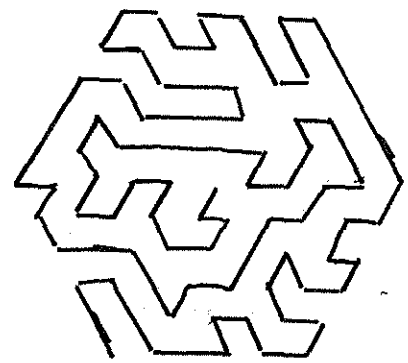
HEXAGON-DOT PAPER



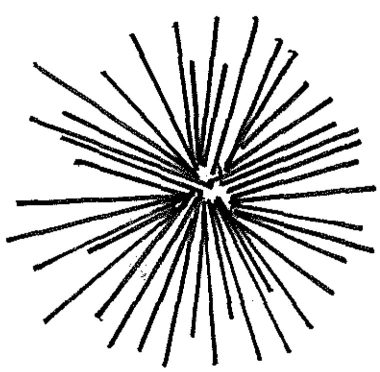




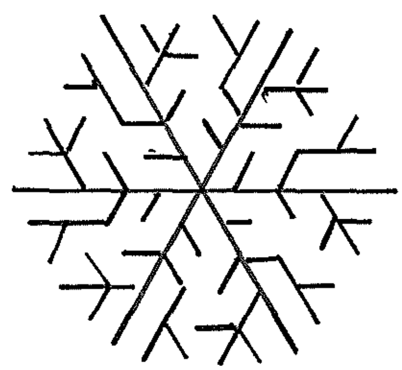
spiral



meander



explosion



branching

EXTENSION LESSON

LESSON TITLE: SOAPBUBBLE PATHS

TIME: 45 minutes

KEY MATERIALS: Overhead projector, plexiglass/glass sandwich, soap solution, protractor

OBJECTIVES: The learner will demonstrate the ability to:

Application Analysis Synthesis	Apply important aspects of problem solving by: a. using induction to go from specific to more general hypotheses. b. valuing making educated guesses to predict future cases. c. going from the general to the specific to form the hypotheses.

RELATION OF THIS LESSON TO THE TOPIC OF PATTERN:

The student will use the study of pattern to form and test hypotheses.

STAGES OF THE ACTIVITY:

1. Show the overhead transparency of the map.

   "You have just been dropped off at your mailbox following a heavy snowfall. You want to shovel a path to your house since you forgot your galoshes. Even worse, you've noticed that the headlights of your car are on! So you also have to shovel a path to your car! Your job---to shovel a path that connects all three points: the house, the car and the mailbox."

   Have the students come to the overhead and draw possible paths.

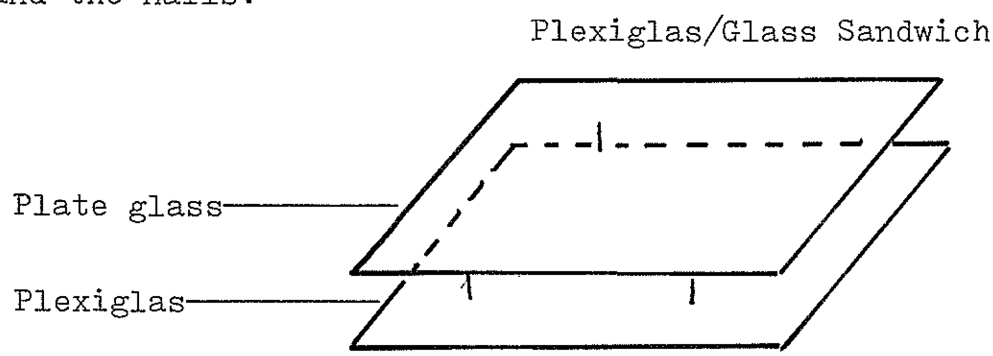
   "But since it's cold and you're hungry, you want to do as little work as possible. You don't want to lift one more shovelful of snow than is absolutely necessary. The problem then is to determine the <u>shortest</u> path that connects all three points.

2. Give the students dittos with spaced points similar to the overhead transparency.

"Consider various paths. Draw them and find their measure. Which is shortest?"

"Now, let's see how a soap bubble solves the problem."

Immerse the device in soapy solution. The film or bubble will form vertical planes between the plates and the nails.



"Has the bubble assumed a path similar to your solution?"

"I suggest that the bubble <u>has</u> solved the problem---that is <u>has</u> assumed the shortest path."

"Assuming that this bubble solution is the best one, try this problem:

A natural gas company with headquarters in Seattle, Tacoma and Spokane, wants to construct an underground pipeline connecting these three cities. Since the cost depends on the length of the pipeline, they are interested in finding the minimum path. See if you can determine the way to build it with the least amount of pipe."

(NOTE TO THE TEACHER: These problems are really adaptions of the "Steiner's Problem" treated by Jacob Steiner at the University of Berlin in the early nineteenth century. An analysis and discussion of this problem can be found in <u>What Is Mathematics?</u> by Courant and Robbins.)

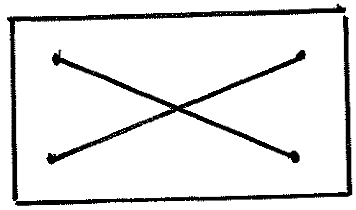
3. "Now give this problem a try:

Four cities, Endwell, Harmony, Roaring Branch and Moosap, are to be connected by a new freeway.

Of course, the people would like the shortest freeway possible, connecting the four cities so that it wouldn't cost too much. If you were the highway planner, how would you plan the highway for the shortest distance and least cost?"

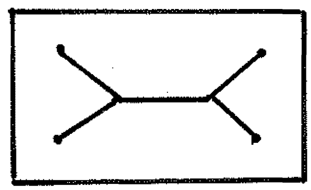
Common Guess

(A)



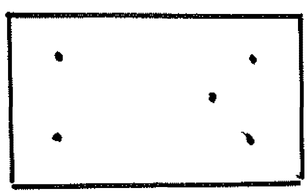
"Let's see how the bubble solves this problem:

(B)

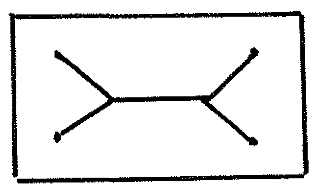


"Even though A was <u>incorrect</u>, it was a good <u>guess</u> or <u>hypothesis</u> based on patterns we observed with the earlier bubble and three nails."

"Here's another one:



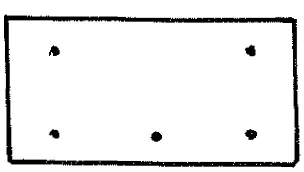
Solution:

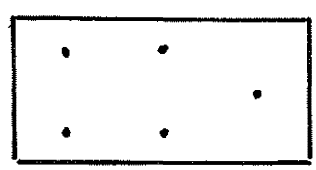


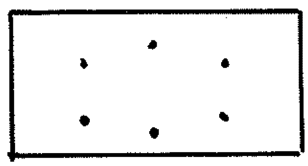
"Were you right?"

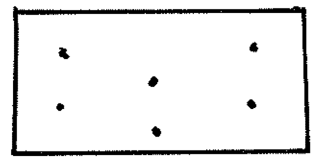
4. "List similarities or patterns you see in the three bubble solutions you've observed."

   "With these patterns in mind, sketch what you believe to be the shortest path connecting each of the following sets of points.









"Check each 'guess' or hypothesis with the soap bubble 'sandwich'."

5. If the students measure the angles found in the configurations, they will discover another pattern. In every configuration, the soap film "walls" join one another forming 120° angles.

6. Give the students the steps in the scientific method as follows and elicit from them what parts of the above lesson illustrate each step.

    1. Defining the problem

    2. Gathering controlled observations.

    3. Classifying and generalizing the facts.

    4. Forming and testing the hypotheses.

    5. Forming theories from tested hypotheses.

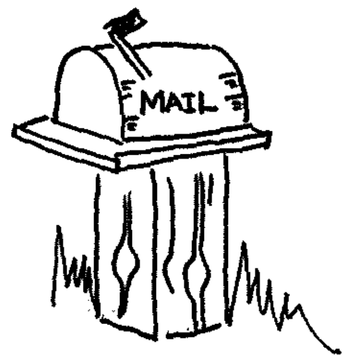
TO THE TEACHER:

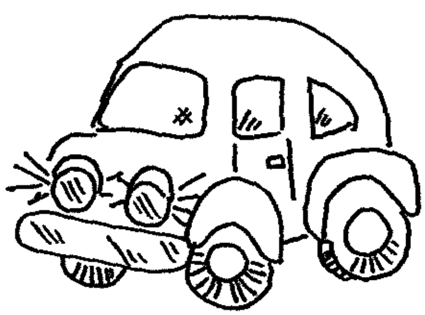
The problem was defined and observations gathered with the first few illustrations. As "facts" were gathered from observations, induction was used to go from specific to more general hypotheses.

With the four-nail problem, a generalization was made with insufficient evidence, resulting in an invalid conclusion.

Value was seen in making educated guesses to predict future cases. "Can you use the result or method for some other problem?"

If the students were successful on the latter soap bubble problems, they applied the generalization concerning angles of 120°. By going from the general to the specific, they used deductive reasoning to form their hypotheses.


MAIL



SELECTED BIBLIOGRAPHY

The following references are excellent math resources for use with intermediate-grade elementary students. They provide lessons, activities and materials which encourage the gifted child to operate as a mathematician; to explore; and to investigate and to discover.

Having begun a math enrichment curricula focus around PATTERN, these resources will provide ideas for many more lessons to continue the PATTERN theme.

1. Albarn, Keith, et al., The Language of Pattern, New York: Harper and Row, 1974.

2. Banwell C.S., K.D. Saunders and D.C. Tahta, Starting Points, London: Oxford University Press, 1972.

3. Bezuszka, Stanley, Margaret Kenney and Linda Silvey, Designs From Mathematical Patterns, Palo Alto, California: Creative Publications, 1970.

4. Carmen, Robert and Marilyn, "Number Patterns," Arithmetic Teacher, Vol. 17, No. 8, December, 1970, pp. 637-639.

5. Critchlow, Keith, Islamic Patterns. An Analytical and Cosmological Approach, New York: Schocken Books, 1976.

6. Garrison, Eunice, Line, Points and Patterns, Arizona: Think Ink Publications, 1981.

7. Hervey, Margaret and Bonnie H. Litwiller, "Polygonal Numbers: A Study Of Patterns," Arithmetic Teacher, Vol. 17, No. 1, January, 1970, pp. 33-38.

8. Hewitt, Frances, "Pattern for Discovery: Prime and Composite Numbers," Arithmetic Teacher, Vol. 13, No. 2, February, 1966, pp. 136-138.

9. Horne, Sylvia, Patterns And Puzzles In Mathematics, Chicago, Illinois: A Franklin Publication, 1970.

10. Jacobs, Harold R., Mathematics---A Human Endeavor, San Francisco, California: W.H. Freeman and Company, 1971.

11. Johnson, Donovan A., Activities in Mathematics: Patterns, Illinois: Scott, Foresman and Company, 1971.

12. Kennedy, Joe and Diane Thomas, Kaleidoscope Math, Palo Alto, California: Creative Publications, Inc., 1978.

13. Pedersen, Jean J. and Franz O. Armbruster, A New Twist, Menlo Park, California: Addison Wesley Publishing Company, 1979.

14. Proctor, Richard M., The Principles of Pattern, New York: Van Nostrand Reinhold Company, 1969.

15. Ranucci, E.R. and J.L. Teeters, Creating Escher-Type Drawings, Palo Alto, California: Creative Publications, 1977.

16. Roper, Ann and Linda Harvey, The Pattern Factory, Palo Alto, California: Creative Publications, 1980.

17. Stevens, Peter S., Patterns in Nature, Boston: Little, Brown and Company, 1974.